9.3 EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

Sol. The given differential equation is

$$\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x}$$

Separating the variables, we get

$$\frac{dy}{2^y} = \frac{dx}{2^x} \quad \Rightarrow 2^{-y}dy = 2^{-x}dx$$

Integrating both sides, we get

$$\int 2^{-y}\,dy = \int 2^{-x}\,dx$$

$$\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + c \quad \Rightarrow -2^{-y} = -2^{-x} + c\log 2$$

$$\Rightarrow \quad -2^{-y} + 2^{-x} = c\log 2$$

$$\Rightarrow \quad 2^{-x} - 2^{-y} = k \qquad [\text{where } c\log 2 = k]$$

Q2. Find the differential equation of all non vertical lines in a plane.

Sol. Equation of all non vertical lines are $y = mx + c$

Differentiating with respect to x, we get $\frac{dy}{dx} = m$

Again differentiating w.r.t. x we have $\frac{d^2y}{dx^2} = 0$

Hence, the required equation is $\frac{d^2y}{dx^2} = 0$.

Q3. Given that $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$. Find the value of x when $y = 3$.

Sol. Given equation is

$$\frac{dy}{dx} = e^{-2y}$$

$$\Rightarrow \quad \frac{dy}{e^{-2y}} = dx \quad \Rightarrow e^{2y}\,.\,dy = dx$$

Integrating both sides, we get

$$\int e^{2y}dy = \int dx \quad \Rightarrow \quad \frac{1}{2}e^{2y} = x + c$$

Put $y = 0$ and $x = 5$

$\Rightarrow \frac{1}{2}e^{0} = 5 + c \Rightarrow c = \frac{1}{2} - 5 = -\frac{9}{2}$

$\therefore$ The equation becomes $\frac{1}{2}e^{2y} = x - \frac{9}{2}$

Now putting $y = 3$, we get

$$\frac{1}{2}e^{6} = x - \frac{9}{2} \quad \Rightarrow \quad x = \frac{1}{2}e^{6} + \frac{9}{2}$$

Hence the required value of $x = \frac{1}{2}(e^{6} + 9)$.

Q4. Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$.

Sol. Given differential equation is

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

Dividing by $(x^2 - 1)$, we get

$$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{}{(x^2 - 1)^2}$$

It is a linear differential equation of first order and first degree.

$\therefore \quad P = \frac{2x}{x^2 - 1}$ and $Q = \frac{1}{(x^2 - 1)^2}$

Integrating factor I.F. = $e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1}dx} = e^{\log(x^2 - 1)} = (x^2 - 1)$.

$\therefore$ Solution of the equation is

$y \times \text{I.F.} = \int Q \,.\, \text{I.F.}\, dx + \text{C}$

$\Rightarrow \quad y \times (x^2 - 1) = \int \frac{1}{(x^2 - 1)^2} \times (x^2 - 1)\, dx + \text{C}$

$\Rightarrow \quad y\,(x^2 - 1) = \int \frac{1}{x^2 - 1}dx + \text{C} \quad \Rightarrow \quad y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| + \text{C}$

Hence the required solution is $y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| + \text{C}$.

Q5. Solve the differential equation $\frac{dy}{dx} + 2xy = y$.

Sol. Given equation is $\frac{dy}{dx} + 2xy = y$.

$$\Rightarrow \quad \frac{dy}{dx} = y - 2xy \quad \Rightarrow \frac{dy}{dx} = y\,(1-2x) \quad \Rightarrow \frac{dy}{y} = (1-2x)\,dx$$

Integrating both sides, we have

$$\int \frac{dy}{dy} = \int (1-2x)dx \Rightarrow \log y = x - 2.\frac{x^2}{2} + \log c$$

$$\Rightarrow \quad \log y = x - x^2 + \log c \Rightarrow \log y - \log c = x - x^2$$

$$\Rightarrow \quad \log\frac{y}{c} = x - x^2 \Rightarrow \frac{y}{c} = e^{x-x^2}$$

$$\therefore \quad y = y = c\,.\,e^{x-x^2}$$

Hence, the required solution is $y = c\,.\,e^{x-x^2}$.

Q6. Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$.

Sol. Given equation is $\frac{dy}{dx} + ay = e^{mx}$.

Here, $P = a$ and $Q = e^{mx}$

$\therefore$ I.F $= e^{\int P dx} = e^{\int a\,.\,dx} = e^{ax}$.

Solution of equation is $y \times$ I.F $= \int Q$ I.F $dx + c$

$$\Rightarrow \quad y.e^{ax} = \int e^{mx}\,.\,e^{ax}dx + c \Rightarrow y\,.\,e^{ax} = \int e^{(m+a)x}dx + c$$

$$\Rightarrow \quad y.e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + c \Rightarrow y = \frac{e^{(m+a)x}}{(m+a)}.\,e^{-ax} + c.e^{-ax}$$

$$\therefore \quad y = \frac{e^{mx}}{(m+a)} + c\,.\,e^{-ax}$$

Hence the required solution is $y = \frac{e^{mx}}{(m+a)} + c\,.\,e^{-ax}$.

Q7. Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$.

Sol. Given that: $\frac{dy}{dx} + 1 = e^{x+y}$

Put $x + y =$ t

$$\therefore \quad 1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\therefore \quad \frac{dt}{dx} = e^t \quad \Rightarrow \frac{dt}{e^t} = dx \quad \Rightarrow e^{-t}dt = dx$$

Integrating both sides, we have

$$\int e^{-t}dt = \int dx \quad \Rightarrow -e^{-t} = x + c$$

$$\Rightarrow \quad -e^{-(x+y)} = x + c \Rightarrow \frac{-1}{e^{x+y}} = x + c \Rightarrow (x + c)\, e^{x+y} = -1$$

Hence, the required solution is $(x + c)\,.\,e^{x \;\; y} + 1 = 0$.

Q8. Solve: $ydx - xdy = x^2y\, dx$.

Sol. Given equation is $ydx - xdy = x^2y\, dx$.

$$\Rightarrow \quad y\, dx - x^2 y\, dx = xdy$$

$$\Rightarrow \quad y\,(1 - x^2)\, dx = xdy$$

$$\Rightarrow \quad \left(\frac{1 - x^2}{x}\right) dx = \frac{dy}{y} \quad \Rightarrow \left(\frac{1}{x} - x\right) dx = \frac{dy}{y}$$

Integrating both sides we get

$$\int \left(\frac{1}{x} - x\right) dx = \int \frac{dy}{y}$$

$$\Rightarrow \log x - \frac{x^2}{2} = \log y + \log c$$

$$\Rightarrow \log x - \frac{x^2}{2} = \log yc \Rightarrow \log x - \log c = \frac{x^2}{2} \Rightarrow \log \frac{x}{yc} = \frac{x^2}{2}$$

$$\Rightarrow \quad \frac{x}{yc} = e^{x^2/2} \quad \Rightarrow \frac{yc}{x} = e^{-x^2/2} \Rightarrow yc = xe^{-x^2/2}$$

$$\therefore \quad y = \frac{1}{c}\,.\, xe^{-x^2/2} \quad \Rightarrow y = kxe^{-x^2/2} \qquad \left[\because\ k = \frac{1}{c}\right]$$

Hence, the required solution is $y = kxe^{-x^2/2}$.

Q9. Solve the differential equation $\dfrac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0, x = 0$.

Sol. Given equation is

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\Rightarrow \quad \frac{dy}{dx} = 1\,(1 + x) + y^2(1 + x)$$

$$\Rightarrow \quad \frac{dy}{dx} = (1 + x)\,(1 + y^2) \Rightarrow \frac{dy}{1 + y^2} = (1 + x)\, dx$$

Integrating both sides, we get

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx \;\Rightarrow\; \tan^{-1} y = x + \frac{x^2}{2} + c$$

Put $x = 0$ and $y = 0$, we get $\tan^{-1}(0) = 0 + 0 + c \Rightarrow c = 0$

$$\therefore \quad \tan^{-1} y = x + \frac{x^2}{2} \;\Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

Hence, the required solution is $y = \tan\left(x + \frac{x^2}{2}\right)$.

Q10. Find the general solution of $(x + 2y^3)\frac{dy}{dx} = y$.

Sol. Given equation is $(x + 2y^3)\frac{dy}{dx} = y$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x+2y^3} \;\Rightarrow \frac{dx}{dy} = \frac{x+2y^3}{y}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x}{y} + \frac{2y^3}{y} \;\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2$$

Here $P = -\frac{1}{y}$ and $Q = 2y^2$.

$\therefore$ Integrating factor I.F. = $e^{\int P dy} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = e^{\log \frac{1}{y}} = \frac{1}{y}$.

So the solution of the equation is

$$x.\text{I.F.} = \int \text{Q.I.F.}dy + c$$

$$x.\frac{1}{y} = \int 2y^2.\frac{1}{y}dy + c$$

$$\Rightarrow \quad \frac{x}{y} = 2\int y\,dy + c \Rightarrow \frac{x}{y} = 2.\frac{y^2}{2} + c \Rightarrow \frac{x}{y} = y^2 + c$$

So $\quad x = y^3 + cy = y(y^2 + c)$

Hence, the required solution is $x = y\,(y^2 + c)$.

Q11. If $y(x)$ is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$ and $y\,(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

Sol. Given equation is

$$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$$

$$\Rightarrow \quad \left(\frac{2+\sin x}{\cos x}\right)\frac{dy}{dx} = -(1+y) \quad \Rightarrow \quad \frac{dy}{(1+y)} = -\left(\frac{\cos x}{2+\sin x}\right)dx$$

Integrating both sides, we get

$$\int \frac{dy}{1+y} = -\int \frac{\cos x}{2+\sin x}\,dx$$

$$\Rightarrow \quad \log|1+y| = -\log|2+\sin x| + \log c$$

$$\Rightarrow \log|1+y| + \log|2+\sin x| = \log c$$

$$\Rightarrow \quad \log(1+y)(2+\sin x) = \log c \Rightarrow (1+y)(2+\sin x) = c$$

Put $x = 0$ and $y = 1$, we get

$(1 + 1)(2 + \sin 0) = c \quad \Rightarrow \; 4 = c$

$\therefore$ equation is $(1 + y)(2 + \sin x) = 4$

Now put $x = \frac{\pi}{2}$

$$\therefore \quad (1+y)\left(2+\sin\frac{\pi}{2}\right) = 4$$

$$\Rightarrow \quad (1+y)(2+1) = 4 \quad \Rightarrow \; 1+y = \frac{4}{3} \quad \Rightarrow \; y = \frac{4}{3} - 1 \Rightarrow \frac{1}{3}$$

So, $y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

Hence, the required solution is $y\left(\frac{\pi}{2}\right) = \frac{1}{3}$.

Q12. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then show that $y(1) = -\frac{1}{2}$.

Sol. Given equation is

$$(1+t)\frac{dy}{dt} - ty = 1 \quad \Rightarrow \; \frac{dy}{dt} - \left(\frac{t}{1+t}\right)y = \frac{1}{1+t}$$

Here, $P = \frac{-t}{1+t}$ and $Q = \frac{1}{1+t}$

$$\therefore \text{ Integrating factor I.F. } = e^{\int P\,dt} = e^{\int \frac{-t}{1+t}dt} = e^{-\int \frac{1+t-1}{1+t}dt}$$

$$= e^{-\int\left(1-\frac{1}{1+t}\right)dt} = e^{-\left[t-\log(1+t)\right]}$$

$$= e^{-t+\log(1+t)} = e^{-t} \cdot e^{\log(1+t)}$$

$\therefore$ I.F. $= e^{-t} \cdot (1 + t)$

Required solution of the given differential equation is

$$y\,.\,\text{I.F.} = \int Q\,.\,\text{I.F.}\; dt + c$$

$$\Rightarrow \quad y\,.\,e^{-t}(1+t) = \int \frac{1}{(1+t)}\,.\,e^{-t}\,.\,(1+t)\,dt + c$$

$$\Rightarrow \quad y\,.\,e^{-t}(1+t) = \int e^{-t}\,dt + c$$

$$\Rightarrow \quad y\,.\,e^{-t}(1+t) = -e^{-t} + c$$

Put $t = 0$ and $y = -1$ $[\because y(0) = -1]$

$$\Rightarrow \quad -1\,.\,e^{0}\,.\,1 = -e^{0} + c$$

$$\Rightarrow \quad -1 = -1 + c \quad \Rightarrow c = 0$$

So the equation becomes

$$ye^{-t}(1+t) = -e^{-t}$$

Now put $t = 1$

$$\therefore \quad y\,.\,e^{-1}(1+1) = -e^{-1}$$

$$\Rightarrow \quad 2y = -1 \Rightarrow y = -\frac{1}{2}$$

Hence $y(1) = -\frac{1}{2}$ is verified.

Q13. Form the differential equation having $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$ where A and B are arbitrary constants, as its general solution.

Sol. Given equation is $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$

$$\frac{dy}{dx} = 2\sin^{-1}x\,.\,\frac{1}{\sqrt{1-x^2}} + A\,.\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

Multiplying both sides by $\sqrt{1-x^2}$, we get

$$\sqrt{1-x^2}\,\frac{dy}{dx} = 2\sin^{-1}x - A$$

Again differentiating w.r.t x, we get

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\,.\,\frac{1\times(-2x)}{2\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2}\,\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\,\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

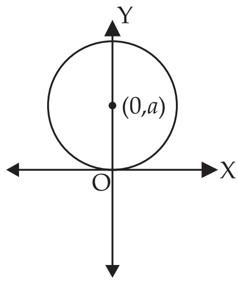
Multiplying both sides by $\sqrt{1-x^2}$, we get

$$\Rightarrow \quad (1-x^2)\,\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} - 2 = 0$$

Which is the required differential equation.

Q14. Form the differential equation of all circles which pass through origin and whose centres lie on y-axis.

Sol. Equation of circle which passes through the origin and whose centre lies on y-axis is



$$(x-0)^2+(y-a)^2=a^2$$

$$\Rightarrow \quad x^2+y^2+a^2-2ay=a^2$$

$$\Rightarrow \quad x^2+y^2-2ay=0 \qquad ...(i)$$

Differentiating both sides w.r.t. x we get

$$\Rightarrow \quad 2x+2y.\frac{dy}{dx}-2a.\frac{dy}{dx}=0$$

$$\Rightarrow \quad x+y\frac{dy}{dx}-a.\frac{dy}{dx}=0 \quad \Rightarrow \quad x+(y-a).\frac{dy}{dx}=0$$

$$y-a=\frac{x}{dy/dx}$$

$$\therefore \quad a=y+\frac{-x}{dy/dx}$$

Putting the value of a in eq. (i), we get

$$x^2+y^2-2\left(y+\frac{x}{dy/dx}\right)y=0$$

$$\Rightarrow \quad x^2+y^2-2y^2-\frac{2xy}{\frac{dy}{dx}}=0 \quad \Rightarrow \quad x^2-y^2=\frac{2xy}{\frac{dy}{dx}}$$

$$\therefore \quad (x^2-y^2)\frac{dy}{dx}-2xy=0$$

Hence, the required differential equation is

$$(x^2-y^2)\frac{dy}{dx}-2xy=0$$

Q15. Find the equation of a curve passing through origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$.

Sol. Given equation is

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2$$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$$

Here, P = $\dfrac{2x}{1+x^2}$ and Q = $\dfrac{4x^2}{1+x^2}$

Integrating factor I.F. = $e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$

$\therefore$ Solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$$

$$\Rightarrow \quad y(1+x^2) = \int \frac{4x^2}{1+x^2} \times (1+x^2)\, dx + c$$

$$\Rightarrow \quad y(1+x^2) = \int 4x^2 dx + c$$

$$\Rightarrow \quad y(1+x^2) = \frac{4}{3}x^3 + c \qquad \text{...}(i)$$

Since the curve is passing through origin *i.e.*, (0, 0)

$\therefore$ Put $y = 0$ and $x = 0$ in eq. (i)

$$0\,(1+0) = \frac{4}{3}(0)^3 + c \;\Rightarrow\; c = 0$$

$\therefore$ Equation is $y(1+x^2) = \dfrac{4}{3}x^3 \;\Rightarrow\; y = \dfrac{4x^3}{3(1+x^2)}$

Hence, the required solution is $y = \dfrac{4x^3}{3(1+x^2)}$.

Q16. Solve : $x^2 . \dfrac{dy}{dx} = x^2 + xy + y^2$

Sol. Given equation is $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Put $y = vx$ [$\because$ it is a homogeneous differential equation]

$$\therefore \quad \frac{dy}{dx} = v + x . \frac{dv}{dx}$$

$$\therefore \quad v + x . \frac{dv}{dx} = \frac{x^2 + vx^2 + v^2x^2}{x^2}$$

$$\Rightarrow \quad v + x . \frac{dv}{dx} = \frac{x^2(1 + v + v^2)}{x^2}$$

$$\Rightarrow \quad v + x . \frac{dv}{dx} = 1 + v + v^2 \;\Rightarrow\; x . \frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = 1 + v^2 \quad \Rightarrow \quad \frac{dv}{1+v^2} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \tan^{-1} v = \log x + c \quad \Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log x + c$$

Hence, the required solution is $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$.

Q17. Find the general solution of the differential equation

$$(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

Sol. Given equation is

$$(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (x - e^{\tan^{-1} y})\frac{dy}{dx} = -(1+y^2) \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-(1+y^2)}{x - e^{\tan^{-1} y}}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x - e^{\tan^{-1} y}}{-(1+y^2)} \quad \Rightarrow \quad \frac{dx}{dy} = -\frac{x}{(1+y^2)} + \frac{e^{\tan^{-1} y}}{1+y^2}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1} y}}{1+y^2}$$

Here, $\mathrm{P} = \frac{1}{1+y^2}$ and $\mathrm{Q} = \frac{e^{\tan^{-1} y}}{1+y^2}$

$\therefore$ Integrating factor I.F. $= e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

$\therefore$ Solution is

$$x \text{ . I.F.} = \int Q \text{ .I.F. } dy + c$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \cdot e^{\tan^{-1} y} dy + c$$

Put $e^{\tan^{-1} y} = \mathrm{t}$

$$\therefore \quad e^{\tan^{-1} y} \cdot \frac{1}{1+y^2} dy = dt$$

$$\therefore \quad x \cdot e^{\tan^{-1} y} = \int t \cdot dt + c$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \frac{1}{2} t^2 + c$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \frac{1}{2}(e^{\tan^{-1} y})^2 + c \quad \Rightarrow x = \frac{1}{2}(e^{\tan^{-1} y}) + \frac{c}{e^{\tan^{-1} y}}$$

$$\Rightarrow \quad 2x = e^{\tan^{-1} y} + \frac{2c}{e^{\tan^{-1} y}}$$

$$\Rightarrow 2x \cdot e^{\tan^{-1} y} = (e^{\tan^{-1} y})^2 + 2c$$

Hence, this is the required general solution.

Q18. Find the general solution of $y^2 dx + (x^2 - xy + y^2)\, dy = 0$.

Sol. The given equation is $y^2 dx + (x^2 - xy + y^2)\, dy = 0$.

$$\Rightarrow \quad y^2 dx = -(x^2 - xy + y^2)\, dy$$

$$\Rightarrow \quad \frac{dx}{dy} = -\frac{x^2 - xy + y^2}{y^2}$$

Since it is a homogeneous differential equation

$\therefore$ Put $x = vy \Rightarrow \dfrac{dx}{dy} = v + y \cdot \dfrac{dv}{dy}$

So, $$v + y \cdot \frac{dv}{dy} = -\left(\frac{v^2y^2 - vy^2 + y^2}{y^2}\right)$$

$$\Rightarrow \quad v + y \cdot \frac{dv}{dy} = -\frac{y^2(v^2 - v + 1)}{y^2}$$

$$\Rightarrow \quad v + y \cdot \frac{dv}{dy} = (-v^2 + v - 1) \quad \Rightarrow y \cdot \frac{dv}{dy} = -v^2 + v - 1 - v$$

$$\Rightarrow \quad y \cdot \frac{dv}{dy} = -v^2 - 1 \quad \Rightarrow \frac{dv}{(v^2+1)} = -\frac{dy}{y}$$

Integrating both sides, we get

$$\Rightarrow \quad \int \frac{dv}{(v^2+1)} = -\int \frac{dy}{y} \quad \Rightarrow \tan^{-1} v = -\log y + c$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x}{y}\right) + \log y + c$$

Hence the required solution is $\tan^{-1}\left(\dfrac{x}{y}\right) + \log y = c$.

Q19. Solve: $(x + y)(dx - dy) = dx + dy$. [Hint: Substitute $x + y = z$ after separating dx and dy]

Sol. Given differential equation is

$$(x + y)(dx - dy) = dx + dy$$

$$\Rightarrow \quad (x + y)\, dx - (x + y)\, dy = dx + dy$$

$$\Rightarrow \quad -(x + y)\, dy - dy = dx - (x + y)\, dx$$

$$\Rightarrow \quad -(x+y+1)\,dy = -(x+y-1)\,dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

Put $x + y = z$

$$\therefore \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} - 1$$

So, $$\frac{dz}{dx} - 1 = \frac{z-1}{z+1}$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{z-1}{z+1} + 1 \quad \Rightarrow \quad \frac{dz}{dx} = \frac{z-1+z+1}{z+1}$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{2z}{z+1} \quad \Rightarrow \quad \frac{z+1}{z}\,dz = 2\,.\,dx$$

Integrating both sides, we get

$$\int \frac{z+1}{z}\,dz = 2\int dx$$

$$\Rightarrow \quad \int\left(1+\frac{1}{z}\right)dz = 2\int dx$$

$$\Rightarrow \quad z + \log|z| = 2x + \log|c|$$

$$\Rightarrow \quad x + y + \log|x+y| = 2x + \log|c|$$

$$\Rightarrow \quad y + \log|x+y| = x + \log|c|$$

$$\Rightarrow \quad \log|x+y| = x - y + \log|c|$$

$$\Rightarrow \quad \log|x+y| - \log|c| = (x-y)$$

$$\Rightarrow \quad \log\left|\frac{x+y}{c}\right| = (x-y) \quad \Rightarrow \quad \frac{x+y}{c} = e^{x-y}$$

$$\therefore \quad x + y = c\,.\,e^{x-y}$$

Hence, the required solution is $x + y = c\,.\,e^{x-y}$.

Q20. Solve: $2(y+3) - xy\,.\,\frac{dy}{dx} = 0$, given that $y(1) = -2$.

Sol. Given differential equation is

$$2(y+3) - xy\,.\,\frac{dy}{dx} = 0$$

$$\Rightarrow \quad xy\,.\,\frac{dy}{dx} = 2y + 6$$

$$\Rightarrow \quad \left(\frac{y}{2y+6}\right)dy = \frac{dx}{x} \quad \Rightarrow \quad \frac{1}{2}\left(\frac{y}{y+3}\right)dy = \frac{dx}{x}$$

Integrating both sides, we get

$$\Rightarrow \quad \frac{1}{2}\int \frac{y}{y+3}\,.\,dy = \int \frac{dx}{x} \Rightarrow \frac{1}{2}\int \frac{y+3-3}{y+3}\,dy = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int\left(1-\frac{3}{y+3}\right)dy = \int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\int 1\,.\,dy - \frac{3}{2}\int \frac{1}{y+3}\,dy = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}y - \frac{3}{2}\log|y+3| = \log x + c$$

Put $x = 1$, $y = -2$

$$\Rightarrow \quad \frac{1}{2}(-2) - \frac{3}{2}\log|-2+3| = \log(1) + c$$

$$\Rightarrow \quad -1 - \frac{3}{2}\log(1) = \log(1) + c$$

$$\Rightarrow \quad -1 - 0 = 0 + c \qquad [\because \log(1) = 0]$$

$$\therefore \quad c = -1$$

$\therefore$ equation is

$$\frac{1}{2}y - \frac{3}{2}\log|y+3| = \log x - 1$$

$$\Rightarrow \quad y - 3\log|y+3| = 2\log x - 2$$

$$\Rightarrow \quad y - \log|(y+3)^3| = \log x^2 - 2$$

$$\Rightarrow \quad \log|(y+3)^3| + \log x^2 = y + 2$$

$$\Rightarrow \quad \log|x^2(y+3)^3| = y + 2 \quad \Rightarrow x^2(y+3)^3 = e^{y+2}$$

Hence, the required solution is $x^2(y+3)^3 = e^{y+2}$.

Q21. Solve the differential equation $dy = \cos x\,(2 - y\,\text{cosec}\,x)\,dx$ given that $y = 2$ when $x = \frac{\pi}{2}$.

Sol. The given differential equation is

$$dy = \cos x\,(2 - y\,\text{cosec}\,x)\,dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos x\,(2 - y\,\text{cosec}\,x) \Rightarrow \frac{dy}{dx} = 2\cos x - y\cos x\,.\,\text{cosec}\,x$$

$$\Rightarrow \quad \frac{dy}{dx} = 2\cos x - y\cot x \quad \Rightarrow \frac{dy}{dx} + y\cot x = 2\cos x$$

Here, P = $\cot x$ and Q = $2\cos x$.

$\therefore$ Integrating factor I.F. = $e^{\int P dx} = e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

$\therefore$ Required solution is $y \times$ I.F = $\int Q \times \text{I.F.}\,dx + c$

$\Rightarrow \quad y \,.\, \sin x = \int 2 \cos x \,.\, \sin x \, dx + c$

$\Rightarrow \quad y \,.\, \sin x = \int \sin 2x \, dx + c \quad \Rightarrow y \,.\, \sin x = -\dfrac{1}{2} \cos 2x + c$

Put $x = \dfrac{\pi}{2}$ and $y = 2$, we get

$$2 \sin \frac{\pi}{2} = -\frac{1}{2} \cos \pi + c$$

$\Rightarrow \quad 2(1) = -\dfrac{1}{2}(-1) + c \quad \Rightarrow 2 = \dfrac{1}{2} + c \;\Rightarrow\; c = 2 - \dfrac{1}{2} = \dfrac{3}{2}$

$\therefore$ The equation is $y \sin x = -\dfrac{1}{2} \cos 2x + \dfrac{3}{2}$.

Q22. Form the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.

Sol. Given that $Ax^2 + By^2 = 1$

Differentiating w.r.t. x, we get

$$2A \,.\, x + 2By \frac{dy}{dx} = 0$$

$\Rightarrow \quad Ax + By \,.\, \dfrac{dy}{dx} = 0 \quad \Rightarrow \; By \,.\, \dfrac{dy}{dx} = -Ax$

$\therefore \quad \dfrac{y}{x} \cdot \dfrac{dy}{dx} = -\dfrac{A}{B}$

Differentiating both sides again w.r.t. x, we have

$$\frac{y}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x.\dfrac{dy}{dx} - y.1}{x^2} \right) = 0$$

$\Rightarrow \dfrac{yx^2}{x} \cdot \dfrac{d^2y}{dx^2} + x.\left(\dfrac{dy}{dx}\right)^2 - y.\dfrac{dy}{dx} = 0$

$\Rightarrow \quad xy.\dfrac{d^2y}{dx^2} + x.\left(\dfrac{dy}{dx}\right)^2 - y.\dfrac{dy}{dx} = 0 \;\Rightarrow\; xy.y'' + x.(y')^2 - y.y' = 0$

Hence, the required equation is

$$xy.y'' + x.(y')^2 - y.y' = 0$$

Q23. Solve the differential equation $(1 + y^2) \tan^{-1}x \, dx + 2y\,(1 + x^2)\, dy = 0$.

Sol. Given differential equation is

$(1 + y^2) \tan^{-1}x \, dx + 2y\,(1 + x^2)\, dy = 0$

$\Rightarrow \quad 2y\,(1 + x^2)\, dy = -(1 + y^2) \,.\, \tan^{-1}x \,.\, dx$

$\Rightarrow \quad \dfrac{2y}{1+y^2} dy = -\dfrac{\tan^{-1} x}{1+x^2} . \, dx$

Integrating both sides, we get

$$\int \frac{2y}{1+y^2}dy = -\int \frac{\tan^{-1} x}{1+x^2}.\,dx$$

$$\Rightarrow \quad \log\left|1+y^2\right| = -\frac{1}{2}\left(\tan^{-1} x\right)^2 + c$$

$$\Rightarrow \quad \frac{1}{2}\left(\tan^{-1} x\right)^2 + \log\left|1+y^2\right| = c$$

Which is the required solution.

Q24. Find the differential equation of system of concentric circles with centre (1, 2).

Sol. Family of concentric circles with centre (1, 2) and radius 'r' is

$$(x-1)^2 + (y-2)^2 = r^2$$

Differentiating both sides w.r.t., x we get

$$2(x-1)+2(y-2)\frac{dy}{dx} = 0 \Rightarrow (x-1)+(y-2)\frac{dy}{dx} = 0$$

Which is the required equation.

LONG ANSWER TYPE QUESTIONS

Q25. Solve: $y+\frac{d}{dx}(xy) = x\,(\sin x + \log x)$

Sol. The given differential equation is

$$y+\frac{d}{dx}(xy) = x\,(\sin x + \log x)$$

$$\Rightarrow \quad y + x\,.\frac{dy}{dx} + y = x\,(\sin x + \log x)$$

$$\Rightarrow \quad x\frac{dy}{dx} = x\,(\sin x + \log x) - 2y$$

$$\Rightarrow \quad \frac{dy}{dx} = (\sin x + \log x) - \frac{2y}{x} \Rightarrow \frac{dy}{dx} + \frac{2}{x}y = (\sin x + \log x)$$

Here, $P = \frac{2}{x}$ and $Q = (\sin x + \log x)$

Integrating factor I.F. $= e^{\int P dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

$\therefore$ Solution is

$$y \times \text{I.F.} = \int Q.\text{I.F.}\,dx + c$$

$$\Rightarrow \quad y\,.\,x^2 = \int (\sin x + \log x)\, x^2 dx + c \qquad ...(1)$$

Let I

$$= \int (\sin x + \log x)\, x^2 dx$$
$$= \int \underset{\text{I}}{x^2} \underset{\text{II}}{\sin x}\, dx + \int \underset{\text{II}}{x^2} \underset{\text{I}}{\log x}\, dx$$
$$= \left[x^2 . \int \sin x\, dx - \int \left(D(x^2) . \int \sin x\, dx\right) dx\right] +$$
$$\left[\log x . \int x^2\, dx - \int \left(D(\log x) . \int x^2 dx\right) dx\right]$$
$$= \left[x^2(-\cos x) - 2\int -x\cos x\, dx\right] + \left[\log x . \frac{x^3}{3} - \int \frac{1}{x} . \frac{x^3}{3} dx\right]$$
$$= \left[-x^2 \cos x + 2\left(x \sin x - \int 1 . \sin x\, dx\right)\right] + \left[\frac{x^3}{3}\log x - \frac{1}{3}\int x^2 dx\right]$$
$$= -x^2 \cos x + 2x \sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3$$

Now from eq (1) we get,

$$y . x^2 = -x^2 \cos x + 2x \sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3 + c$$
$$\therefore \quad y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x\log x}{3} - \frac{1}{9}x + c . x^{-2}$$

Hence, the required solution is

$$y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x\log x}{3} - \frac{1}{9}x + c . x^{-2}$$

Q26. Find the general solution of $(1 + \tan y)(dx - dy) + 2xdy = 0$.

Sol. Given that: $(1 + \tan y)(dx - dy) + 2xdy = 0$

$$\Rightarrow \quad (1 + \tan y)\, dx - (1 + \tan y)\, dy + 2xdy = 0$$
$$\Rightarrow \quad (1 + \tan y)\, dx - (1 + \tan y - 2x)\, dy = 0$$
$$\Rightarrow \quad (1 + \tan y)\frac{dx}{dy} = (1 + \tan y - 2x) \quad \Rightarrow \frac{dx}{dy} = \frac{1 + \tan y - 2x}{1 + \tan y}$$
$$\Rightarrow \quad \frac{dx}{dy} = 1 - \frac{2x}{1 + \tan y} \quad \Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

Here, $P = \dfrac{2}{1 + \tan y}$ and $Q = 1$

Integrating factor I.F.

$$= e^{\int \frac{2}{1+\tan y} dy} = e^{\int \frac{2\cos . y}{\sin y + \cos y} dy}$$
$$= e^{\int \frac{\sin y + \cos y - \sin y + \cos y}{(\sin y + \cos y)} dy} = e^{\int \left(1 + \frac{\cos y - \sin y}{\sin y + \cos y}\right) dy}$$
$$= e^{\int 1 . dy} . e^{\int \frac{\cos y - \sin y}{\sin y + \cos y} dy}$$
$$= e^y . e^{\log(\sin y + \cos y)} = e^y . (\sin y + \cos y)$$

So, the solution is $x \times \text{I.F.} = \int Q \times \text{I.F.}\, dy + c$

$\Rightarrow \quad x \cdot e^y (\sin y + \cos y) = \int 1 \cdot e^y (\sin y + \cos y) dy + c$

$\Rightarrow \quad x \cdot e^y (\sin y + \cos y) = e^y \cdot \sin y + c$

$$\left[\because \int e^x [f(x) + f'(x)]\, dx = e^x f(x) + c\right]$$

$\Rightarrow \quad x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$

Hence, the required solution is $x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$.

Q27. Solve : $\dfrac{dy}{dx} = \cos(x+y) + \sin(x+y)$. [**Hint:** Substitute $x + y = z$]

Sol. Given that : $\dfrac{dy}{dx} = \cos(x+y) + \sin(x+y)$

Put $x + y = v$, on differentiating w.r.t. x, we get,

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \quad \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\therefore \quad \frac{dv}{dx} - 1 = \cos v + \sin v$$

$$\Rightarrow \quad \frac{dv}{dx} = \cos v + \sin v + 1$$

$$\Rightarrow \quad \frac{dv}{\cos v + \sin v + 1} = dx$$

Integrating both sides, we have

$$\int \frac{dv}{\cos v + \sin v + 1} = \int 1 \cdot dx$$

$$\Rightarrow \quad \int \frac{dv}{\left(\dfrac{1 - \tan^2 \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + \dfrac{2 \tan \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + 1\right)} = \int 1 \cdot dx$$

$$\Rightarrow \int \frac{\left(1 + \tan^2 \frac{v}{2}\right)}{1 - \tan^2 \frac{v}{2} + 2 \tan \frac{v}{2} + 1 + \tan^2 \frac{v}{2}}\, dv = \int 1 \cdot dx$$

$$\Rightarrow \quad \int \frac{\sec^2 \frac{v}{2}}{2 + 2 \tan \frac{v}{2}}\, dv = \int 1 \cdot dx$$

Put $2 + 2 \tan \dfrac{v}{2} = t$

$$2 \cdot \frac{1}{2} \sec^2 \frac{v}{2} dv = dt \Rightarrow \sec^2 \frac{v}{2} dv = dt$$

$$\Rightarrow \quad \int \frac{dt}{t} = \int 1 \, . \, dx$$

$$\Rightarrow \quad \log |t| = x + c$$

$$\Rightarrow \quad \log \left| 2 + 2 \tan \frac{v}{2} \right| = x + c$$

$$\Rightarrow \log \left| 2 + 2 \tan \left(\frac{x+y}{2} \right) \right| = x + c \Rightarrow \log 2 \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c$$

$$\Rightarrow \quad \log 2 + \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c$$

$$\Rightarrow \quad \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c - \log 2$$

Hence, the required solution is

$$\log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + K \qquad [c - \log 2 = \text{K}]$$

Q28. Find the general solution of $\frac{dy}{dx} - 3y = \sin 2x$.

Sol. Given equation is $\frac{dy}{dx} - 3y = \sin 2x$.

Here, P = –3 and Q = sin 2 x

$\therefore$ Integrating factor I.F. = $e^{\int P dx} = e^{\int -3 dx} = e^{-3x}$

$\therefore$ Solution is

$$y \times \text{I.F.} = \int \text{Q} \, . \, \text{I.F.} \, dx + c$$

$$\Rightarrow \quad y \, . \, e^{-3x} = \int \sin 2x \, . \, e^{-3x} \, dx + c$$

$$\text{Let} \quad \text{I} = \int \underset{\text{I}}{\sin 2x} \, . \, \underset{\text{II}}{e^{-3x}} \, dx$$

$$\Rightarrow \quad \text{I} = \sin 2x \, . \int e^{-3x} dx - \int \left(D (\sin 2x) \, . \int e^{-3x} dx \right) dx$$

$$\Rightarrow \quad \text{I} = \sin 2x \, . \, \frac{e^{-3x}}{-3} - \int 2 \cos 2x \, . \, \frac{e^{-3x}}{-3} \, dx$$

$$\Rightarrow \quad \text{I} = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \int \underset{\text{I}}{\cos 2x} \, . \, \underset{\text{II}}{e^{-3x}} \, dx$$

$$\Rightarrow \quad \text{I} = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[\cos 2x \, . \int e^{-3x} dx - \int [D \cos 2x \, . \int e^{-3x} dx] \, dx \right]$$

$$\Rightarrow \quad I = \frac{e^{-3x}}{-3}\sin 2x + \frac{2}{3}\left[\cos 2x \cdot \frac{e^{-3x}}{-3} - 2\sin 2x \cdot \frac{e^{-3x}}{-3}\right]dx$$

$$\Rightarrow \quad I = \frac{e^{-3x}}{-3}\sin 2x - \frac{2}{9}\cos 2x \cdot e^{-3x} - \frac{4}{9}\int \sin 2x \cdot e^{-3x}\,dx$$

$$\Rightarrow \quad = \frac{e^{-3x}}{-3}\sin 2x - \frac{2}{9}e^{-3x}\cos 2x - \frac{4}{9}I$$

$$\Rightarrow \quad I + \frac{4}{9}I = \frac{e^{-3x}}{-3}\sin 2x - \frac{2}{9}e^{-3x}\cos 2x$$

$$\Rightarrow \quad \frac{13\,I}{9} = -\frac{1}{9}\left[3\,e^{-3x}\sin 2x + 2\,e^{-3x}\cos 2x\right]$$

$$\Rightarrow \quad I = -\frac{1}{13}e^{-3x}\left[3\sin 2x + 2\cos 2x\right]$$

$\therefore$ The equation becomes

$$y \cdot e^{-3x} = -\frac{1}{13}e^{-3x}\left[3\sin 2x + 2\cos 2x\right] + c$$

$$\therefore \quad y = -\frac{1}{13}\left[3\sin 2x + 2\cos 2x\right] + c \cdot e^{3x}$$

Hence, the required solution is

$$y = -\left[\frac{3\sin 2x + 2\cos 2x}{13}\right] + c \cdot e^{3x}$$

Q29. Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is $\frac{x^2 + y^2}{2xy}$.

Sol. Given that the slope of tangent to a curve at (x, y) is

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

It is a homogeneous differential equation

So, put $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x \cdot vx}$$

$$\Rightarrow \quad v + x \cdot \frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = \frac{1+v^2}{2v} - v \quad \Rightarrow x \cdot \frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = \frac{1-v^2}{2v} \quad \Rightarrow \frac{2v}{1-v^2}dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{2v}{1-v^2}dv = \int \frac{dx}{x} \Rightarrow -\log|1-v^2| = \log x + \log c$$

$$\Rightarrow \quad -\log\left|1-\frac{y^2}{x^2}\right| = \log x + \log c \Rightarrow -\log\left|\frac{x^2-y^2}{x^2}\right| = \log x + \log c$$

$$\Rightarrow \log\left|\frac{x^2}{x^2-y^2}\right| = \log|xc| \quad \Rightarrow \frac{x^2}{x^2-y^2} = xc$$

Since, the curve is passing through the point (2, 1)

$$\therefore \quad \frac{(2)^2}{(2)^2-(1)^2} = 2c \quad \Rightarrow \frac{4}{3} = 2c \quad \Rightarrow \quad c = \frac{2}{3}$$

Hence, the required equation is

$$\frac{x^2}{x^2-y^2} = \frac{2}{3}x \quad \Rightarrow 2(x^2-y^2) = 3x$$

Q30. Find the equation of the curve through the point (1, 0) if the slope of the tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$.

Sol. Given that the slope of the tangent to the curve at (x, y) is

$$\frac{dy}{dx} = \frac{y-1}{x^2+x} \quad \Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

Integrating both sides, we have

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \frac{dx}{x^2+x+\frac{1}{4}-\frac{1}{4}} \quad \text{[making perfect square]}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow \quad \log|y-1| = \frac{1}{2\times\frac{1}{2}}\log\left|\frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}}\right| + \log c$$

$$\Rightarrow \quad \log|y-1| = \log\left|\frac{x}{x+1}\right| + \log c$$

$$\Rightarrow \quad \log|y-1| = \log\left|c\left(\frac{x}{x+1}\right)\right|$$

$$\therefore \quad y-1 = \frac{cx}{x+1} \Rightarrow (y-1)(x+1) = cx$$

Since, the line is passing through the point (1, 0), then

$(0-1)(1+1) = c(1) \Rightarrow c = 2.$

Hence, the required solution is $(y-1)(x+1) = 2x$.

Q31. Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

Sol. Here, slope of the tangent of the curve = $\frac{dy}{dx}$

and the difference between the abscissa and ordinate = $x - y$.

$\therefore$ As per the condition, $\frac{dy}{dx} = (x-y)^2$

Put $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \quad \frac{dy}{dx} = 1 - \frac{dv}{dx}$$

$\therefore$ the equation becomes

$$1 - \frac{dv}{dx} = v^2 \Rightarrow \frac{dv}{dx} = 1 - v^2 \Rightarrow \frac{dv}{1-v^2} = dx$$

Integrating both sides, we get

$$\int \frac{dv}{1-v^2} = \int dx$$

$$\Rightarrow \quad \frac{1}{2}\log\left|\frac{1+v}{1-v}\right| = x + c \Rightarrow \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x + c \quad ...(1)$$

Since, the curve is passing through (0, 0)

then $\frac{1}{2}\log\left|\frac{1+0-0}{1-0+0}\right| = 0 + c \Rightarrow c = 0$

∴ On putting $c = 0$ in eq. (1) we get

$$\frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x \quad \Rightarrow \quad \log\left|\frac{1+x-y}{1-x+y}\right| = 2x$$

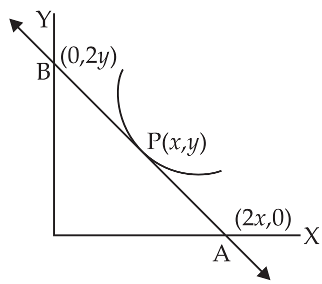
$$\therefore \qquad \frac{1+x-y}{1-x+y} = e^{2x}$$

$$\Rightarrow \qquad (1+x-y) = e^{2x}(1-x+y)$$

Hence, the required equation is $(1 + x - y) = e^{2x}(1 - x + y)$.

Q32. Find the equation of a curve passing through the point (1, 1), if the tangent drawn at any point P (x, y) on the curve meets the coordinate axes at A and B such that P is the mid point of AB.

Sol. Let P (x, y) be any point on the curve and AB be the tangent to the given curve at P.



P is the mid point of AB (given)

∴ Coordinates of A and B are $(2x, 0)$ and $(0, 2y)$ respectively.

∴ Slope of the tangent AB = $\frac{2y-0}{0-2x} = -\frac{y}{x}$

$$\therefore \qquad \frac{dy}{dx} = -\frac{y}{x} \quad \Rightarrow \quad \frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int\frac{dy}{y} = -\int\frac{dx}{x} \Rightarrow \log y = -\log x + \log c$$

$$\Rightarrow \qquad \log y + \log x = \log c \quad \Rightarrow \log yx = \log c$$

$$\therefore \qquad yx = c$$

Since, the curve passes through (1, 1)

$$\therefore \qquad 1 \times 1 = c \quad \therefore \quad c = 1$$

$$\Rightarrow \qquad yx = 1$$

Hence, the required equation is $xy = 1$.

Q33. Solve: $x\frac{dy}{dx} = y(\log y - \log x + 1)$

Sol. Given that: $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$$\Rightarrow \quad x\frac{dy}{dx} = y\left[\log\left(\frac{y}{x}\right)+1\right] \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right)+1\right]$$

Since, it is a homogeneous differential equation.

$\therefore$ Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x \cdot \dfrac{dv}{dx}$

$$\therefore \quad v + x \cdot \frac{dv}{dx} = \frac{vx}{x}\left[\log\left(\frac{vx}{x}\right) + 1\right]$$

$$\Rightarrow \quad v + x \cdot \frac{dv}{dx} = v\,[\log v + 1]$$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = v\,[\log v + 1] - v \quad \Rightarrow x \cdot \frac{dv}{dx} = v\,[\log v + 1 - 1]$$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = v \cdot \log v \quad \Rightarrow \quad \frac{dv}{v \log v} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x}$$

Put $\log v = t$ on L.H.S.

$$\frac{1}{v}\,dv = dt$$

$$\therefore \quad \int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\log|t| = \log|x| + \log c$$

$$\Rightarrow \quad \log|\log v| = \log xc \quad \Rightarrow \log v = xc$$

$$\Rightarrow \quad \log\left(\frac{y}{x}\right) = xc$$

Hence, the required solution is $\log\left(\dfrac{y}{x}\right) = xc$.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer from the given four options in each of the Exercises from 34 to 75 (M.C.Q.)

Q34. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right) \text{ is}$$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) not defined

Sol. The degree of the given differential equation is not defined because the value of sin $\left(\dfrac{dy}{dx}\right)$ on expansion will be in increasing power of $\left(\dfrac{dy}{dx}\right)$.

Hence, the correct option is (*d*).

Q35. The degree of the differential equation $\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2} = \dfrac{d^2y}{dx^2}$ is (*a*) 4 (*b*) $\frac{3}{2}$ (*c*) not defined (*d*) 2

Sol. The given differential equation is

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \left(\frac{d^2y}{dx^2}\right)$$

Squaring both sides, we have

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of the given differential equation is 2.

Hence, the correct option is (*d*).

Q36. The order and degree of the differential equation

$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{4}}+x^{\frac{1}{5}}=0$ respectively are

(*a*) 2 and not defined (*b*) 2 and 2

(*c*) 2 and 3 (*d*) 3 and 3

Sol. Given differential equation is

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{4}}+x^{\frac{1}{5}} = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{4}} = -x^{\frac{1}{5}}$$

Since the degree of $\frac{dy}{dx}$ is in fraction.

So, the degree of the differential equation is not defined as the order is 2.

Hence, the correct option is (*a*).

Q37. If $y = e^{-x}$ (A cos x + B sin x), then y is a solution of

(*a*) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$ (*b*) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(*c*) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$ (*d*) $\frac{d^2y}{dx^2}+2y=0$

Sol. Given equation is $y = e^{-x}$ (A cos x + B sin x)

Differentiating both sides, w.r.t. x, we get

$$\frac{dy}{dx} = e^{-x}(-A\sin x + B\cos x) - e^{-x}(A\cos x + B\sin x)$$

$$\frac{dy}{dx} = e^{-x}(-A\sin x + B\cos x) - y$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = e^{-x}(-\text{A}\cos x - \text{B}\sin x) - e^{-x}(-\text{A}\sin x + \text{B}\cos x) - \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -e^{-x}(\text{A}\cos x + \text{B}\sin x) - \left[\frac{dy}{dx} + y\right] - \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y - \frac{dy}{dx} - y - \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y \quad \Rightarrow \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

Hence, the correct option is (*c*)

Q38. The differential equation for $y = \text{A}\cos \alpha x + \text{B}\sin \alpha x$, where A and B are arbitrary constants is:

(*a*) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (*b*) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

(*c*) $\frac{d^2y}{dx^2} + \alpha y = 0$ (*d*) $\frac{d^2y}{dx^2} - \alpha y = 0$

Sol. Given equation is : $y = \text{A}\cos \alpha x + \text{B}\sin \alpha x$

Differentiating both sides w.r.t. x, we have

$$\frac{dy}{dx} = -\text{A}\sin \alpha x \,.\, \alpha + \text{B}\cos \alpha x \,.\, \alpha$$
$$= -\text{A}\,\alpha \sin \alpha x + \text{B}\,\alpha \cos \alpha x$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\text{A}\,\alpha^2 \cos \alpha x - \text{B}\,\alpha^2 \sin \alpha x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\alpha^2 (\text{A}\cos \alpha x + \text{B}\sin \alpha x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\alpha^2 y \Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = 0$$

Hence, the correct option is (*b*).

Q39. Solution of differential equation $x\,dy - y\,dx = 0$ represents:

(*a*) a rectangular hyperbola

(*b*) parabola whose vertex is at origin

(*c*) straight line passing through origin

(*d*) a circle whose centre is at origin.

Sol. The given differential equation is

$$xdy - ydx = 0$$

Sol. The given differential equation is

$$xdy - ydx = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log y = \log x + \log c \Rightarrow \log y = \log xc$$

$\Rightarrow$ $y = xc$ which is a straight line passing through the origin.

Hence, the correct answer is (c).

Q40. Integrating factor of the differential equation

$\cos x \,.\, \frac{dy}{dx} + y \sin x = 1$ is

(a) cos x (b) tan x (c) sec x (d) sin x

Sol. The given differential equation is

$$\cos x \,.\, \frac{dy}{dx} + y \sin x = 1$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{\sin x}{\cos x} y = \frac{1}{\cos x} \quad \Rightarrow \quad \frac{dy}{dx} + \tan x \, y = \sec x$$

Here, P = tan x and Q = sec x

$\therefore$ Integrating factor $= e^{\int P dx} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

Hence, the correct option is (c).

Q41. Solution of differential equation

$\tan y \sec^2 x \, dx + \tan x \sec^2 y \, dy = 0$ is

(a) $\tan x + \tan y = k$ (b) $\tan x - \tan y = k$

(c) $\frac{\tan x}{\tan y} = k$ (d) $\tan x \,.\, \tan y = k$

Sol. The given differential equation is

$$\tan y \sec^2 x \, dx + \tan x \sec^2 y \, dy = 0$$

$$\Rightarrow \tan x \sec^2 y \, dy = - \tan y \sec^2 x \, dx$$

$$\Rightarrow \quad \frac{\sec^2 y}{\tan y} \,.\, dy = \frac{-\sec^2 x}{\tan x} \,.\, dx$$

Integrating both sides, we get

$$\Rightarrow \quad \int \frac{\sec^2 y}{\tan y} dy = \int \frac{-\sec^2 x}{\tan x} dx$$

$$\Rightarrow \quad \log |\tan y| = - \log |\tan x| + \log c$$

$$\Rightarrow \quad \log |\tan y| + \log |\tan x| = \log c$$

Q42. Family $y = Ax + A^3$ of curves is represented by the differential equation of degree

(a) 1 (b) 2 (c) 3 (d) 4

Sol. Given equation is $y = Ax + A^3$

Differentiating both sides, we get

$$\frac{dy}{dx} = \text{A which has degree 1.}$$

Hence, the correct answer is (a).

Q43. Integrating factor of $x\frac{dy}{dx} - y = x \quad - \quad x$ is

(a) x (b) $\log x$

(c) $\frac{1}{x}$ (d) $-x$

Sol. The given differential equation is

$$x\frac{dy}{dx} - y = x^4 - 3x \quad \Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$$

Here, $\text{P} = -\frac{1}{x}$ and $\text{Q} = x^3 - 3$

So, integrating factor $= e^{\int P dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x}$

Hence, the correct option is (c).

Q44. Solution of $\frac{dy}{dx} - y = 1$, $y(0) = 1$ is given by

(a) $xy = -e^x$ (b) $xy = -e^{-x}$

(c) $xy = -1$ (d) $y = 2e^x - 1$

Sol. The given differential equation is

$$\frac{dy}{dx} - y = 1$$

Here, $\text{P} = -1$, $\text{Q} = 1$

$\therefore$ Integrating factor, I.F. $= e^{\int P dx} = e^{\int -1\, dx} = e^{-x}$

So, the solution is

$$y \times \text{I.F.} = \int \text{Q. I.F.}\, dx + c$$

$$\Rightarrow \quad y \times e^{-x} = \int 1.\, e^{-x}\, dx + c$$

$$\Rightarrow \quad y\,.\, e^{-x} = -e^{-x} + c$$

Put $x = 0$, $y = 1$

$$\Rightarrow \quad 1\,.\, e^0 = -e^0 + \text{c}$$

$$\Rightarrow \quad 1 = -1 + c \qquad \therefore c = 2$$

So the equation is $y \,.\, e^{-x} = -e^{-x} + 2$

$\Rightarrow \qquad y = -1 + 2e^{x} = 2e^{x} - 1$

Hence, the correct option is (*d*).

Q45. The number of solutions of $\dfrac{dy}{dx} = \dfrac{y+1}{x-1}$ when $y(1) = 2$ is

(*a*) none (*b*) one

(*c*) two (*d*) infinite

Sol. The given differential equation is $\dfrac{dy}{dx} = \dfrac{y+1}{x-1}$

$$\Rightarrow \qquad \frac{dy}{y+1} = \frac{dx}{x-1}$$

Integrating both sides, we get

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}$$

$$\Rightarrow \qquad \log(y+1) = \log(x-1) + \log c$$

$$\Rightarrow \quad \log(y+1) - \log(x-1) = \log c$$

$$\Rightarrow \qquad \log\left|\frac{y+1}{x-1}\right| = \log c \quad \Rightarrow \quad \frac{y+1}{x-1} = c$$

Put $x = 1$ and $y = 2$

$$\Rightarrow \qquad \frac{2+1}{1-1} = c \qquad \therefore\ c = \infty$$

$$\therefore \qquad \frac{y+1}{x-1} = \frac{1}{0} \quad \Rightarrow \quad x - 1 = 0 \quad \Rightarrow \quad x = 1$$

Hence, the correct option is (*b*).

Q46. Which of the following is a second order differential equation?

(*a*) $(y')^2 + x = y^2$ (*b*) $y'y'' + y = \sin x$

(*c*) $y''' + (y'')^2 + y = 0$ (*d*) $y' = y^2$

Sol. Second order differential equation is $y'y'' + y = \sin x$

Hence, the correct option is (*b*).

Q47. Integrating factor of the differential equation

$$(1 - x^2)\frac{dy}{dx} - xy = 1 \text{ is}$$

(*a*) $-x$ (*b*) $\dfrac{x}{1+x^2}$

(*c*) $\sqrt{1-x^2}$ (*d*) $\dfrac{1}{2}\log(1-x^2)$

Sol. The given differential equation is

$$(1 - x^2)\frac{dy}{dx} - xy = 1$$

$$\Rightarrow \qquad \frac{dy}{dx} - \frac{x}{1-x^2} \cdot y = \frac{1}{1-x^2}$$

Here, $P = -\dfrac{x}{1-x^2}$ and $Q = \dfrac{1}{1-x^2}$

$\therefore$ Integrating factor

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{-x}{1-x^2} dx} = e^{\frac{1}{2}\log(1-x^2)} = \sqrt{1-x^2}$$

Hence, the correct option is (*c*).

Q48. $\tan^{-1}x + \tan^{-1}y = c$ is the general solution of the differential equation:

(*a*) $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$ (*b*) $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$

(*c*) $(1 + x^2)\, dy + (1 + y^2)\, dx = 0$

(*d*) $(1 + x^2)\, dx + (1 + y^2)\, dy = 0$

Sol. Given equation is $\tan^{-1}x + \tan^{-1}y = c$

Differentiating w.r.t. x, we have

$$\frac{1}{1+x^2} + \frac{1}{1+y^2} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{1}{1+y^2}\right)\frac{dy}{dx} = -\left(\frac{1}{1+x^2}\right) \Rightarrow \frac{dy}{dx} = -\left(\frac{1+y^2}{1+x^2}\right)$$

$$\Rightarrow (1 + x^2)\, dy = -(1 + y^2)\, dx$$

$$\Rightarrow (1 + x^2)\, dy + (1 + y^2)\, dx = 0$$

Hence the correct option is (*c*).

Q49. The differential equation $y\dfrac{dy}{dx} + x = c$ represents:

(*a*) Family of hyperbolas (*b*) Family of parabolas

(*c*) Family of ellipses (*d*) Family of circles

Sol. Given differential equation is

$$y\frac{dy}{dx} + x = c$$

$$\Rightarrow y\frac{dy}{dx} = c - x \Rightarrow y\, dy = (c - x)\, dx$$

$\therefore$ Integrating both sides, we get

$$\int y\, dy = \int (c - x)\, dx$$

$$\Rightarrow \frac{y^2}{2} = cx - \frac{x^2}{2} + k \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} - cx = k$$

$\Rightarrow$ $x^2 + y^2 - 2cx = 2k$ which is a family of circles.

Hence, the correct option is (*d*).

Q50. The general solution of $e^x \cos y\, dx - e^x \sin y\, dy = 0$ is:

(*a*) $e^x \cos y = k$ (*b*) $e^x \sin y = k$

(*c*) $e^x = k \cos y$ (*d*) $e^x = k \sin y$

Sol. The given differential equation is

$$e^x \cos y\, dx - e^x \sin y\, dy = 0$$

$$\Rightarrow e^x (\cos y\, dx - \sin y\, dy) = 0$$

$$\Rightarrow \cos y\, dx - \sin y\, dy = 0 \qquad [\because e^x \neq 0]$$

$$\Rightarrow \sin y\, dy = \cos y\, dx \Rightarrow \frac{\sin y}{\cos y} dy = dx$$

Integrating both sides, we have

$$\int \frac{\sin y}{\cos y} dy = \int dx$$

$$\Rightarrow -\log|\cos y| = x + \log k \Rightarrow \log \frac{1}{\cos y} - \log k = x$$

$$\Rightarrow \log\left(\frac{1}{k\cos y}\right) = x \Rightarrow \frac{1}{k\cos y} = e^x$$

$$\Rightarrow \frac{1}{k} = e^x \cos y \Rightarrow e^x \cos y = c \qquad \left[c = \frac{1}{k}\right]$$

Hence, the correct option is (*a*).

Q51. The degree of the differential equation:

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0 \text{ is}$$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

Sol. The degree of the given differential equation is 1 as the power of the highest order is 1.

Hence, the correct option is (*a*).

Q52. The solution of the differential equation

$$\frac{dy}{dx} + y = e^{-x},\ y(0) = 0 \text{ is}$$

(*a*) $y = e^x (x - 1)$ (*b*) $y = xe^{-x}$

(*c*) $y = x\, e^{-x} + 1$ (*d*) $y = (x + 1)\, e^{-x}$

Sol. The given differential equation is

$$\frac{dy}{dx} + y = e^{-x}$$

Since, it is a linear differential equation

$\therefore$ P = 1 and Q = e^{-x}

$\therefore$ I.F = $e^{\int 1 . dx} = e^x$

So, the solution is

$$y \times \text{I.F.} = \int \text{Q . I.F.}\, dx + c \Rightarrow y \,.\, e^x = \int e^{-x} \,.\, e^x dx + c$$

$$\Rightarrow y \,.\, e^x = \int 1 \,.\, dx + c \Rightarrow y \,.\, e^x = x + c$$

Put $x = 0$, $y = 0$, we have $0 = 0 + c \quad \therefore c = 0$

So, the solution is $y\, e^x = x \quad \Rightarrow \quad y = x \, . \, e^{-x}$

Hence, the correct option is (*b*).

Q53. Integrating factor of the differential equation

$$\frac{dy}{dx} + y \tan x - \sec x = 0 \text{ is}$$

(*a*) $\cos x$ (*b*) $\sec x$

(*c*) $e^{\cos x}$ (*d*) $e^{\sec x}$

Sol. Given differential equation is

$$\frac{dy}{dx} + y \tan x - \sec x = 0 \Rightarrow \frac{dy}{dx} + y \tan x = \sec x$$

Here, P = $\tan x$ and Q = $\sec x$

$\therefore$ I.F. = $e^{\int P dx} = e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$

Hence, the correct option is (*b*).

Q54. The solution of the differential equation

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \text{ is}$$

(*a*) $y = \tan^{-1} x$ (*b*) $y - x = k(1 + xy)$

(*c*) $x = \tan^{-1} y$ (*d*) $\tan(xy) = k$

Sol. The given differential equation is

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \quad \Rightarrow \quad \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating both sides, we get

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\Rightarrow \quad \tan^{-1} y = \tan^{-1} x + c \Rightarrow \tan^{-1} y - \tan^{-1} x = c$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y-x}{1+xy}\right) = c$$

$$\Rightarrow \quad \frac{y-x}{1+xy} = \tan c \quad \Rightarrow \quad \frac{y-x}{1+xy} = k \quad [k = \tan c]$$

$$\Rightarrow \quad y - x = k(1 + xy)$$

Hence, the correct option is (*b*).

Q55. The integrating factor of the differential equation

$$\frac{dy}{dx} + y = \frac{1+y}{x} \text{ is:}$$

(*a*) $\frac{x}{e^x}$ (*b*) $\frac{e^x}{x}$ (*c*) xe^x (*d*) e^x

Sol. The given differential equation is

$$\frac{dy}{dx} + y = \frac{1+y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1+y}{x} - y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x} + y\frac{(1-x)}{x} \Rightarrow \frac{dy}{dx} - \left(\frac{1-x}{x}\right)y = \frac{1}{x}$$

Here, $P = -\left(\frac{1-x}{x}\right)$ and $Q = \frac{1}{x}$

$\therefore$ Integrating factor I.F. $= e^{\int P dx} = e^{\int \frac{x-1}{x} dx} = e^{\int\left(1-\frac{1}{x}\right)dx}$

$= e^{(x - \log x)} = e^x . e^{-\log x}$

$= e^x . e^{\log \frac{1}{x}} = e^x . \frac{1}{x}$

Hence, the correct option is (*b*).

Q56. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equations?

(*a*) $\frac{dy}{dx} + my = 0$ (*b*) $\frac{dy}{dx} - my = 0$

(*c*) $\frac{d^2y}{dx^2} - m^2y = 0$ (*d*) $\frac{d^2y}{dx^2} + m^2y = 0$

Sol. The given equation is $y = ae^{mx} + be^{-mx}$

On differentiation, we get $\frac{dy}{dx} = a . me^{mx} - b . me^{-mx}$

Again differentiating w.r.t., we have

$$\frac{d^2y}{dx^2} = am^2 e^{mx} + bm^2 e^{-mx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = m^2 (ae^{mx} + be^{-mx}) \Rightarrow \frac{d^2y}{dx^2} = m^2 y \Rightarrow \frac{d^2y}{dx^2} - m^2 y = 0$$

Hence, the correct option is (*c*).

Q57. The solution of the differential equation $\cos x \sin y \, dx + \sin x \cos y \, dy = 0$ is

(*a*) $\frac{\sin x}{\sin y} = c$ (*b*) $\sin x \sin y = c$

(*c*) $\sin x + \sin y = c$ (*d*) $\cos x \cos y = c$

Sol. The given differential equation is

$\cos x \sin y \, dx + \sin x \cos y \, dy = 0$

$$\Rightarrow \quad \sin x \cos y \, dy = -\cos x \sin y \, dx$$

$$\Rightarrow \quad \frac{\cos y}{\sin y} dy = -\frac{\cos x}{\sin x} dx \Rightarrow \cot y \, dy = -\cot x \, dx$$

Integrating both sides, we have

$$\Rightarrow \quad \int \cot y \, dy = -\int \cot x \, dx$$

$\Rightarrow \quad \log|\sin y| = -\log|\sin x| + \log c$

$\Rightarrow \quad \log|\sin y| + \log|\sin x| = \log c$

$\Rightarrow \quad \log|\sin y \,.\, \sin x| = \log c \Rightarrow \sin x \sin y = c$

Hence, the correct option is (*b*).

Q58. The solution of $x\dfrac{dy}{dx} + y = e^x$ is:

(*a*) $y = \dfrac{e^x}{x} + \dfrac{k}{x}$ (*b*) $y = xe^x + cx$

(*c*) $y = x \,.\, e^x + k$ (*d*) $x = \dfrac{e^y}{y} + \dfrac{k}{y}$

Sol. The given differential equation is $x\dfrac{dy}{dx} + y = e^x$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$$

Here P = $\dfrac{1}{x}$ and Q = $\dfrac{e^x}{x}$

$\therefore$ Integrating factor I.F. = $e^{\int \frac{1}{x} dx} = e^{\log|x|} = x$

So, the solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + k \Rightarrow y \times x = \int \frac{e^x}{x} \times x\, dx + k$$

$$\Rightarrow \quad y \times x = \int e^x\, dx + k \quad \Rightarrow y \times x = e^x + k$$

$$\therefore \quad y = \frac{e^x}{x} + \frac{k}{x}$$

Hence, the correct option is (*a*).

Q59. The differential equation of the family of curves $x^2 + y^2 - 2ay = 0$, where a is arbitrary constant, is:

(*a*) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$ (*b*) $2(x^2 + y^2)\dfrac{dy}{dx} = xy$

(*c*) $2(x^2 - y^2)\dfrac{dy}{dx} = xy$ (*d*) $(x^2 + y^2)\dfrac{dy}{dx} = 2xy$

Sol. The given equation is

$$x^2 + y^2 - 2ay = 0 \quad ..(1)$$

Differentiating w.r.t. x, we have

$$2x + 2y \,.\, \frac{dy}{dx} - 2a\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x + y\frac{dy}{dx} - a\frac{dy}{dx} = 0 \quad \Rightarrow \quad x + (y - a)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (y - a)\frac{dy}{dx} = -x \quad \Rightarrow y - a = \frac{-x}{dy/dx}$$

$$\Rightarrow \quad a = y + \frac{x}{dy/dx} \Rightarrow a = \frac{y \cdot \frac{dy}{dx} + x}{\frac{dy}{dx}}$$

Putting the value of a in eq. (1) we get

$$x^2 + y^2 - 2y\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right] = 0$$

$$\Rightarrow \quad (x^2 + y^2)\frac{dy}{dx} - 2y\left(y\frac{dy}{dx} + x\right) = 0$$

$$\Rightarrow \quad (x^2 + y^2)\frac{dy}{dx} - 2y^2\frac{dy}{dx} - 2xy = 0$$

$$\Rightarrow \quad (x^2 + y^2 - 2y^2)\frac{dy}{dx} = 2xy \Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy$$

$\therefore$ Hence the correct option is (a).

Q60. Family $y = Ax + A^3$ of curves will correspond to a differential equation of order

(a) 3 (b) 2 (c) 1 (d) not defined

Sol. The given equation is

$$y = Ax + A^3$$

Differentiating both sides, we get $\frac{dy}{dx} = \text{A}$

Again differentiating both sides, we have $\frac{d^2y}{dx^2} = 0$

So the order of the differential equation is 2.

Hence, the correct option is (b).

Q61. The general solution of $\frac{dy}{dx} = 2xe^{x^2 - y}$ is :

(a) $e^{x^2 - y} = c$ (b) $e^{-y} + e^{x^2} = c$

(c) $e^y = e^{x^2} + c$ (d) $e^{x^2 + y} = c$

Sol. The given differential equation is

$$\frac{dy}{dx} = 2x \cdot e^{x^2 - y}$$

$$\Rightarrow \quad \frac{dy}{dx} = 2x \cdot e^{x^2} \cdot e^{-y} \Rightarrow \frac{dy}{e^{-y}} = 2x \cdot e^{x^2}\,dx$$

Integrating both sides, we have

$$\int \frac{dy}{e^{-y}} = \int 2x \cdot e^{x^2}\,dx \Rightarrow \int e^y\,dy = \int 2x \cdot e^{x^2}\,dx$$

Put in RHS $x^2 = t$ $\therefore$ $2x\,dx = dt$

$\therefore \quad \int e^y dy = \int e^t dt$

$\Rightarrow \quad e^y = e^t + c \quad \Rightarrow e^y = e^{x^2} + c$

Hence, the correct option is (*c*).

Q62. The curve for which the slope of the tangent at any point is equal to the ratio of the abscissa to the ordinate of the point is:

(*a*) an ellipse (*b*) parabola
(*c*) circle (*d*) rectangular hyperbola

Sol. Since, the slope of the tangent to the curve = $x : y$

$$\therefore \quad \frac{dy}{dx} = \frac{x}{y} \Rightarrow ydy = xdx$$

Integrating both sides, we get $\int y\,dy = \int x\,dx$

$$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + c \quad \Rightarrow y^2 = x^2 + 2c$$

$\Rightarrow \quad y^2 - x^2 = 2c = k$ which is rectangular hyperbola.

Hence, the correct option is (*d*).

Q63. The general solution of the differential equation

$\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$ is:

(*a*) $y = c\,.\,e^{\frac{-x^2}{2}}$ (*b*) $y = c\,.\,e^{\frac{x^2}{2}}$
(*c*) $y = (x + c)\,.\,e^{\frac{x^2}{2}}$ (*d*) $y = (c - x)\,e^{\frac{x^2}{2}}$

Sol. The given differential equation is

$$\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy \quad \Rightarrow \frac{dy}{dx} - xy = e^{\frac{x^2}{2}}$$

Since it is linear differential equation where P = $-x$ and Q = $e^{\frac{x^2}{2}}$

$\therefore$ Integrating factor I.F. = $e^{\int Pdx} = e^{\int -x\,dx} = e^{-\frac{x^2}{2}}$

So, the solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx + c$$

$$\Rightarrow \quad y \times e^{-\frac{x^2}{2}} = \int e^{\frac{x^2}{2}}\,e^{-\frac{x^2}{2}}\,dx + c$$

$$\Rightarrow \quad y \times e^{-\frac{x^2}{2}} = \int e^0\,dx + c$$

$$\Rightarrow \quad y \times e^{-\frac{x^2}{2}} = \int 1\,.\,dx + c \quad \Rightarrow y \times e^{-\frac{x^2}{2}} = x + c$$

$\therefore \qquad y = (x + c)\, e^{\frac{x^2}{2}}$

Hence the correct option is (*c*).

Q64. The solution of the equation $(2y - 1)\, dx - (2x + 3)\, dy = 0$ is

(*a*) $\dfrac{2x-1}{2y+3} = k$ (*b*) $\dfrac{2y+1}{2x-3} = k$

(*c*) $\dfrac{2x+3}{2y-1} = k$ (*d*) $\dfrac{2x-1}{2y-1} = k$

Sol. The given differential equation is

$$(2y - 1)\, dx - (2x + 3)\, dy = 0 \Rightarrow (2x + 3)\, dy = (2y - 1)\, dx$$

$$\Rightarrow \quad \frac{dy}{2y-1} = \frac{dx}{2x+3}$$

Integrating both sides, we get

$$\int \frac{dy}{2y-1} = \int \frac{dx}{2x+3}$$

$$\Rightarrow \quad \frac{1}{2}\log|2y-1| = \frac{1}{2}\log|2x+3| + \log c$$

$$\Rightarrow \quad \log|2y-1| = \log|2x+3| + 2\log c$$

$$\Rightarrow \quad \log|2y-1| - \log|2x+3| = \log c^2$$

$$\Rightarrow \quad \log\left|\frac{2y-1}{2x+3}\right| = \log c^2$$

$$\Rightarrow \quad \frac{2y-1}{2x+3} = c^2 \Rightarrow \frac{2x+3}{2y-1} = \frac{1}{c^2}$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = k, \text{ where } k = \frac{1}{c^2}$$

Hence, the correct option is (*c*).

Q65. The differential equation for which $y = a\cos x + b\sin x$ is a solution, is:

(*a*) $\dfrac{d^2y}{dx^2} + y = 0$ (*b*) $\dfrac{d^2y}{dx^2} - y = 0$

(*c*) $\dfrac{d^2y}{dx^2} + (a + b)\, y = 0$ (*d*) $\dfrac{d^2y}{dx^2} + (a - b)\, y = 0$

Sol. The given equation is

$$y = a\cos x + b\sin x$$

$$\frac{dy}{dx} = -a\sin x + b\cos x$$

$$\frac{d^2y}{dx^2} = -a\cos x - b\sin x$$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -(a\cos x + b\sin x) \Rightarrow \dfrac{d^2y}{dx^2} = -y \quad \Rightarrow \dfrac{d\ y}{dx} + y = 0$

Hence, the correct option is (*a*).

Q66. The solution of $\dfrac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is:

(*a*) $y = e^{-x}(x-1)$ (*b*) $y = x \,.\, e^{x}$

(*c*) $y = x\,e^{-x} + 1$ (*d*) $y = x \,.\, e^{-x}$

Sol. The given differential equation is $\dfrac{dy}{dx} + y = e^{-x}$

Since, it is a linear differential equation then P = 1 and Q = e^{-x}

Integrating factor I.F. = $e^{\int P dx} = e^{\int 1 . dx} = e^{x}$

$\therefore$ Solution is

$y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$

$\Rightarrow \quad y \times e^x = \int e^{-x} \times e^x dx + c \quad \Rightarrow y \times e^x = \int e^0 dx + c$

$\Rightarrow \quad y \times e^x = \int 1 . dx + c \quad \Rightarrow y \times e^x = x + c$

Put $y = 0$ and $x = 0$

$\therefore \quad 0 = 0 + c \quad \therefore \quad c = 0$

$\therefore$ equation is $y \times e^x = x$

So $y = x \,.\, e^{-x}$

Hence, the correct option is (*d*).

Q67. The order and degree of the differential equation

$$\left[\frac{d^3y}{dx^3}\right]^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4 \text{ are}$$

(*a*) 1, 4 (*b*) 3, 4

(*c*) 2, 4 (*d*) 3, 2

Sol. The given differential equation is

$$\left[\frac{d^3y}{dx^3}\right]^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$$

Here the highest derivative is $\dfrac{d^3y}{dx^3}$.

$\therefore$ the order of the differential equation is 3 and since, the power of highest order is 2

$\therefore$ its degree is 2

Hence, the correct option is (d).

Q68. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$ are:

(*a*) $2, \frac{3}{2}$ (*b*) 2, 3

(*c*) 2, 1 (*d*) 3, 4

Sol. The given differential equation is

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$$

Here, the highest derivative is 2,

$\therefore$ order = 2

and the power of the highest derivative is 1

$\therefore$ degree = 1

Hence, the correct option is (*c*).

Q69. The differential equation of the family of curves $y^2 = 4a(x + a)$ is:

(*a*) $y^2 = 4\frac{dy}{dx}\left(x+\frac{dy}{dx}\right)$ (*b*) $2y \cdot \frac{dy}{dx} = 4a$

(*c*) $y \cdot \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 0$ (*d*) $2x \cdot \frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2 - y$

Sol. The given equation of family of curves is

$$y^2 = 4a(x + a)$$

$$\Rightarrow \quad y^2 = 4ax + 4a^2 \quad ...(1)$$

Differentiating both sides, w.r.t. x, we get

$$2y \cdot \frac{dy}{dx} = 4a$$

$$\Rightarrow \quad y \cdot \frac{dy}{dx} = 2a \Rightarrow \frac{y}{2}\frac{dy}{dx} = a$$

Now, putting the value of a in eq. (1) we get

$$y^2 = 4x\left(\frac{y}{2}\frac{dy}{dx}\right)+4\left(\frac{y}{2}\cdot\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad y^2 = 2xy\frac{dy}{dx}+y^2\left(\frac{dy}{dx}\right)^2 \quad \Rightarrow y = 2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2$$

$\Rightarrow \quad 2x \cdot \frac{dy}{dx} + y \cdot \left(\frac{dy}{dx}\right)^2 - y = 0$

Hence, the correct option is (*d*).

Q70. Which of the following is the general solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0?$$

(*a*) $y = (Ax + B) \cdot e^x$ (*b*) $y = (Ax + B)\, e^{-x}$

(*c*) $y = Ae^x + Be^{-x}$ (*d*) $y = A \cos x + B \sin x$

Sol. The given differential equation is

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

Since the above equation is of second order and first degree

$\therefore \quad D^2y - 2Dy + y = 0$, where $D = \frac{d}{dx}$

$\Rightarrow \quad (D^2 - 2D + 1)\, y = 0$

$\therefore$ auxiliary equation is

$$m^2 - 2m + 1 = 0 \quad \Rightarrow \quad (m-1)^2 = 0 \quad \Rightarrow \quad m = 1, 1$$

If the roots of Auxiliary equation are real and equal say (m) then $CF = (c_1 x + c_2) \cdot e^{mx}$

$\therefore \quad CF = (Ax + B)\, e^x$

So $\quad y = (Ax + B) \cdot e^x$

Hence, the correct option is (*a*).

Q71. General solution of $\frac{dy}{dx} + y \tan x = \sec x$ is:

(*a*) $y \sec x = \tan x + c$ (*b*) $y \tan x = \sec x + c$

(*c*) $\tan x = y \tan x + c$ (*d*) $x \sec x = \tan y + c$

Sol. The given differential equation is $\frac{dy}{dx} + y \tan x = \sec x$

Since, it is a linear differential equation

$\therefore \quad$ P = $\tan x$ and Q = $\sec x$

Integrating factor I.F. = $e^{\int P dx} = e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$

$\therefore$ Solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$$

$\Rightarrow \quad y \times \sec x = \int \sec x \cdot \sec x\, dx + c$

$\Rightarrow \quad y \sec x = \int \sec^2 x\, dx + c \quad \Rightarrow y \sec x = \tan x + c$

Hence, the correct option is (*a*).

Q72. Solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is:

(*a*) $x\,(y + \cos x) = \sin x + c$ (*b*) $x\,(y - \cos x) = \sin x + c$

(*c*) $xy \cos x = \sin x + c$ (*d*) $x\,(y + \cos x) = \cos x + c$

Sol. The given differential equation is $\frac{dy}{dx} + \frac{y}{x} = \sin x$

Since, it is a linear differential equation

$\therefore \quad P = \frac{1}{x}$ and $Q = \sin x$

Integrating factor I.F. $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore$ Solution is $\quad y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$

$$y \times x = \int \sin x \,.\, x\, dx + c \Rightarrow y \times x = \int x \sin x\, dx + c$$

$$\Rightarrow \quad yx = x \,.\int \sin x\, dx - \int \left(D(x) \int \sin x\, dx\right) dx + c$$

$$\Rightarrow \quad yx = x(-\cos x) - \int -\cos x\, dx$$

$$\Rightarrow \quad yx = -x \cos x + \int \cos x\, dx \Rightarrow yx = -x\cos x + \sin x + c$$

$$\Rightarrow \quad yx + x\cos x = \sin x + c$$

$$\Rightarrow \quad x\,(y + \cos x) = \sin x + c$$

Hence, the correct option is (*a*).

Q73. The general solution of the differential equation $(e^x + 1)\, y\, dy = (y + 1)\, e^x\, dx$ is:

(*a*) $(y + 1) = k\,(e^x + 1)$

(*b*) $y + 1 = e^x + 1 + k$

(*c*) $y = \log\,[k\,(y + 1)\,(e^x + 1)]$

(*d*) $y = \log \left\{\frac{e^x + 1}{y + 1}\right\} + k$

Sol. The given differential equation is

$$(e^x + 1)\, y\, dy = (y + 1)\, e^x\, dx$$

$$\Rightarrow \quad \frac{y}{y+1} dy = \frac{e^x}{e^x + 1} dx$$

Integrating both sides, we get

$$\int \frac{y}{y+1} dy = \int \frac{e^x}{e^x + 1} dx$$

$$\Rightarrow \quad \int \frac{y + 1 - 1}{y + 1} dy = \int \frac{e^x}{e^x + 1} dx$$

$$\Rightarrow \quad \int 1 \,.\, dy - \int \frac{1}{y+1} dy = \int \frac{e^x}{e^x + 1} dx$$

$\Rightarrow \quad y - \log|y+1| = \log|e^x+1| + \log k$

$\Rightarrow \quad y = \log|y+1| + \log|e^x+1| + \log k$

$\Rightarrow \quad y = \log|k(y+1)(e^x+1)|$

Hence, the correct option is (*c*).

Q74. The solution of differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

(*a*) $y = e^{x-y} - x^2e^{-y} + c$ (*b*) $e^y - e^x = \frac{x^3}{3} + c$

(*c*) $e^x + e^y = \frac{x^3}{3} + c$ (*d*) $e^x - e^y = \frac{x^3}{3} + c$

Sol. The given differential equation is

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^x.e^{-y} + x^2.e^{-y} \Rightarrow \frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow \quad \frac{dy}{e^{-y}} = (e^x + x^2)\,dx \Rightarrow e^y.dy = (e^x + x^2)\,dx$$

Integrating both sides, we have

$$\int e^y dy = \int (e^x + x^2)\,dx$$

$$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + c \Rightarrow e^y - e^x = \frac{x^3}{3} + c$$

Hence, the correct option is (*b*).

Q75. The solution of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2} \text{ is:}$$

(*a*) $y(1+x^2) = c + \tan^{-1}x$ (*b*) $\frac{y}{1+x^2} = c + \tan^{-1}x$

(*c*) $y\log(1+x^2) = c + \tan^{-1}x$ (*d*) $y(1+x^2) = c + \sin^{-1}x$

Sol. The given differential equation is

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$$

Since, it is a linear differential equation

$P = \frac{2x}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

Integrating factor I.F. $= e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$

$\therefore$ Solution is $y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx + c$

$$\Rightarrow \quad y\,(1+x^2) = \int \frac{1}{(1+x^2)^2} \times (1+x^2)\,dx + c$$

$$\Rightarrow \quad y\,(1+x^2) = \int \frac{1}{(1+x^2)}\,dx + c \Rightarrow y\,(1+x^2) = \tan^{-1} x + c$$

Hence, the correct option is (*a*).

Q76. Fill in the blanks of the following (*i* to *xi*):

(*i*) The degree of the differential equation $\frac{d^2y}{dx^2} + e^{dy/dx} = 0$ is ____________ .

(*ii*) The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = x$ is ____________ .

(*iii*) The number of arbitrary constants in the general solution of a differential equation of order three is ____________ .

(*iv*) $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{1}{x}$ is an equation of the type ____________ .

(*v*) General solution of the differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is given by ____________ .

(*vi*) The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is ____________ .

(*vii*) The solution of $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ is ____________ .

(*viii*) The solution of the differential equation $y\,dx + (x + xy)\,dy = 0$ is ____________ .

(*ix*) General solution of $\frac{dy}{dx} + y = \sin x$ is ____________ .

(*x*) The solution of differential equation $\cot y\;dx = x\;dy$ is ____________ .

(*xi*) The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is ____________ .

Sol. (*i*) The degree of the differential equation $\frac{d^2y}{dx^2} + e^{dy/dx} = 0$ is **not defined**.

(*ii*) The given differential equation is $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = x$

Squaring both sides, we gett

$$1+\left(\frac{dy}{dx}\right)^2 = x^2$$

So, the degree of the equation is **2**.

(*iii*) The number of arbitrary constants in the solution is **3**.

(*iv*) The given differential equation $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{1}{x}$ is of the type $\frac{dy}{dx} + Py = Q$.

(*v*) General solution of the differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is given by $x \times \text{I.F.} = \int Q \times \text{I.F.}\, dy + c$

$\Rightarrow \quad x \,.\, e^{\int P_1\, dy} = \int Q_1 \,.\, e^{\int P_1\, dy} dy + c.$

(*vi*) The given differential equation is $x\frac{dy}{dx} + 2y = x^2$

$\Rightarrow \qquad \frac{dy}{dx} + \frac{2}{x} y = x.$

Since, it is linear differential equation

$\therefore \quad \text{P} = \frac{2}{x}$ and $\text{Q} = x$

Integrating factor I.F. = $e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$

$\therefore$ Solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$$

$\Rightarrow \quad y \,.\, x^2 = \int x \,.\, x^2\, dx + c \quad \Rightarrow y \,.\, x^2 = \int x^3\, dx + c$

$\Rightarrow \quad y \,.\, x^2 = \frac{1}{4} x^4 + c \quad \Rightarrow y = \frac{1}{4} x^2 + c \,.\, x^{-2}$

Hence, the solution is $y = \frac{1}{4} x^2 + c \,.\, x^{-2}$.

(*vii*) The given differential equation is

$$(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$\Rightarrow \quad \frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{4x^2}{1 + x^2}$

Since it is a linear differential equation

$\therefore \quad P = \frac{2x}{1 + x^2}$ and $Q = \frac{4x^2}{1 + x^2}$

Integrating factor I.F. = $e^{\int P dx} = e^{\int \frac{2x}{1 + x^2} dx} = e^{\log(1 + x^2)} = (1 + x^2)$

$\therefore$ Solution is $\quad y \times \text{I.F} = \int Q \times \text{I.F.}\, dx + c$

$$\Rightarrow \quad y \times (1 + x^2) = \int \frac{4x}{1+x^2} \times (1+x^2)\, dx + c$$

$$\Rightarrow \quad y \times (1 + x^2) = \int 4x^2\, dx + c \quad \Rightarrow y \times (1 + x^2) = \frac{4}{3}x^3 + c$$

$$\Rightarrow \quad y = \frac{4}{3}\frac{x^3}{(1+x^2)} + c\,(1+x^2)^{-1}$$

Hence, the required solution is $y = \mathbf{\frac{4}{3}\frac{x^3}{(1+x^2)} + c\,(1+x^2)^{-1}}$.

(*viii*) The given differential equation is

$$ydx + (x + xy)\, dy = 0$$

$$\Rightarrow \quad (x + xy)\, dy = -y\, dx \quad \Rightarrow x\,(1 + y)\, dy = -y\, dx$$

$$\Rightarrow \quad \frac{1+y}{y}\, dy = -\frac{1}{x}\, dx$$

Integrating both sides, we get

$$\int \frac{1+y}{y}\, dy = -\int \frac{1}{x}\, dx$$

$$\Rightarrow \quad \int \left(\frac{1}{y} + 1\right) dy = -\int \frac{1}{x}\, dx$$

$$\Rightarrow \quad \log y + y = -\log x + \log c$$

$$\Rightarrow \quad \log x + \log y + \log e^y = \log c$$

$$\Rightarrow \quad \log (xy \,.\, e^y) = \log c$$

$$\therefore \quad xy = c\, e^{-y}$$

Hence, the required solution is $\boldsymbol{xy = c\, e^{-y}}$.

(*ix*) The given differential equation is $\frac{dy}{dx} + y = \sin x$

Since, it it a linear differential equation

$\therefore$ P = 1 and Q = $\sin x$

Integrating factor I.F. = $e^{\int P dx} = e^{\int 1 . dx} = e^x$

$\therefore$ Solution is $y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$

$$\Rightarrow \quad y \,.\, e^x = \int \sin x \,.\, e^x dx + c \qquad ...(1)$$

Let $\mathrm{I} = \int \underset{\mathrm{I}}{\sin x} \,.\, \underset{\mathrm{II}}{e^x}\, dx$

$$\mathrm{I} = \sin x \,.\, \int e^x dx - \int \left(D(\sin x) \,.\, \int e^x dx\right) dx$$

$$\mathrm{I} = \sin x \,.\, e^x - \int \underset{\mathrm{I}}{\cos x} \,.\, \underset{\mathrm{II}}{e^x}\, dx$$

$$\mathrm{I} = \sin x \,.\, e^x - \left[\cos x \,.\, \int e^x dx - \int \left(D\,(\cos x) \int e^x dx\right) dx\right]$$

$$I = \sin x \,.\, e^x - \left[\cos x \,.\, e^x - \int - \sin x \,.\, e^x dx\right]$$

$$I = \sin x \,.\, e^x - \cos x \,.\, e^x - \int \sin x \,.\, e^x dx$$

$$I = \sin x \,.\, e^x - \cos x \,.\, e^x - I$$

$\Rightarrow I + I = e^x (\sin x - \cos x)$

$\Rightarrow \quad 2I = e^x (\sin x - \cos x)$

$\therefore \quad I = \dfrac{e^x}{2}(\sin x - \cos x)$

From eq. (1) we get

$$y \,.\, e^x = \frac{e^x}{2}(\sin x - \cos x) + c$$

$$y = \left(\frac{\sin x - \cos x}{2}\right) + c \,.\, e^{-x}$$

Hence, the required solution is

$$y = \left(\frac{\sin x - \cos x}{2}\right) + c \,.\, e^{-x}$$

(x) The given differential equation is $\cot y \, dx = x \, dy$

$\Rightarrow \dfrac{dy}{\cot y} = \dfrac{dx}{x} \Rightarrow \tan y \, dy = \dfrac{dx}{x}$

Integrating both sides, we get

$$\int \tan y \, dy = \int \frac{dx}{x} \Rightarrow \log \sec y = \log x + \log c$$

$\Rightarrow \quad \log \sec y - \log x = \log c$

$\Rightarrow \quad \log \left|\dfrac{\sec y}{x}\right| = \log C$

$\therefore \quad \dfrac{\sec y}{x} = C \Rightarrow \dfrac{x}{\sec y} = \dfrac{1}{C} \Rightarrow \dfrac{x}{\sec y} = C \quad \left[\dfrac{1}{c} = C\right]$

$\therefore \quad x = C \sec y$

Hence, the required solution is $x = C \sec y$.

(xi) The given differential equation is

$$\frac{dy}{dx} + y = \frac{1+y}{x}$$

$\Rightarrow \quad \dfrac{dy}{dx} + y = \dfrac{1}{x} + \dfrac{y}{x}$

$\Rightarrow \quad \dfrac{dy}{dx} + y - \dfrac{y}{x} = \dfrac{1}{x} \quad \Rightarrow \dfrac{dy}{dx} + \left(1 - \dfrac{1}{x}\right)y = \dfrac{1}{x}$

Here P = $\left(1 - \frac{1}{x}\right)$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \left(1 - \frac{1}{x}\right) dx} = e^{(x - \log x)}$

$= e^x . e^{-\log x} = e^x . e^{\log \frac{1}{x}} = e^x . \frac{1}{x}$

Hence, the required I.F. = $e^x . \frac{1}{x}$.

Q77. State True or False for the following:

(*i*) Integrating factor of the differential equation of the form $\frac{dx}{dy} + P_1 x = Q_1$ is given by $e^{\int P_1 dy}$.

(*ii*) Solution of the differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is given by x . I.F. $= \int (\text{I.F.})\, Q_1 dy$.

(*iii*) Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = f(x, y)$, where $f(x, y)$ is a homogeneous function of zero degree is $y = vx$.

(*iv*) Correct substitution for the solution of the differential equation of the type $\frac{dx}{dy} = g(x, y)$, where $g(x, y)$ is a homogeneous function of the degree zero is $x = vy$.

(*v*) Number of arbitrary constants in the particular solution of a differential equation of order two is two.

(*vi*) The differential equation representing the family of circles $x^2 + (y - a)^2 = a^2$ will be of order two.

(*vii*) The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ is $y^{2/3} - x^{2/3} = c$.

(*viii*) Differential equation representing the family of curves $y = e^x (A \cos x + B \sin x)$ is $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

(*ix*) The solution of differential equation $\frac{dy}{dx} = \frac{x + 2y}{x}$ is $x + y = kx^2$.

(*x*) Solution of $\frac{xdy}{dx} = y + x \tan \frac{y}{x}$ is $\sin\left(\frac{y}{x}\right) = cx$.

(*xi*) The differential equation of all non-horizontal lines in a plane is $\frac{d^2x}{dy^2} = 0$.

Sol. (*i*) True

I.F. of the given differential equation

$$\frac{dx}{dy} + P_1 x = Q \text{ is } e^{\int P_1 dy}$$

(*ii*) True (*iii*) True (*iv*) True (*v*) False

Since particular solution of a differential equation has no arbitrary constant.

(*vi*) False

We know that the order of the differential equation is equal to the number of arbitrary constants.

(*vii*) True

The given differential equation is

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}} \quad \Rightarrow \quad \frac{dy}{y^{1/3}} = \frac{dx}{x^{1/3}}$$

Integrating both sides, we get

$$\int \frac{dy}{y^{1/3}} = \int \frac{dx}{x^{1/3}} \quad \Rightarrow \int y^{-1/3}\, dy = \int x^{-1/3}\, dx$$

$$\Rightarrow \frac{1}{-\frac{1}{3}+1} y^{-1/3+1} = \frac{1}{-1/3+1} . x^{-1/3+1} + c$$

$$\Rightarrow \quad \frac{3}{2} y^{2/3} = \frac{3}{2} x^{2/3} + c$$

$$\Rightarrow \quad y^{2/3} = x^{2/3} + \frac{2}{3}c \quad \Rightarrow \quad y^{2/3} - x^{2/3} = k \left[k = \frac{2}{3}c\right]$$

(*viii*) True

Given equation is

$$y = e^x (A \cos x + B \sin x)$$

Differentiating both sides, we get

$$\frac{dy}{dx} = e^x (-A \sin x + B \cos x) + (A \cos x + B \sin x)\, e^x$$

$$\frac{dy}{dx} = e^x (-A \sin x + B \cos x) + y$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = e^x(-A\cos x - B\sin x) + (-A\sin x + B\cos x)\,.\,e^x + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -e^x(A\cos x + B\sin x) + \frac{dy}{dx} - y + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -y - y + 2\frac{dy}{dx} \qquad \therefore \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

(ix) True

The given differential equation is

$$\frac{dy}{dx} = \frac{x+2y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + 2\frac{y}{x} \quad \Rightarrow \quad \frac{dy}{dx} - \frac{2y}{x} = 1$$

Here, $P = \frac{-2}{x}$ and $Q = 1$

Integrating factor I.F. $= e^{\int \frac{-2}{x}dx} = e^{-2\log x} = e^{\log x^{-2}} = \frac{1}{x^2}$

$\therefore$ Solution is $y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx + c$

$$\Rightarrow \quad y \times \frac{1}{x^2} = \int 1 \times \frac{1}{x^2}\, dx + c$$

$$\Rightarrow \quad \frac{y}{x^2} = \int \frac{1}{x^2}\, dx + c \quad \Rightarrow \quad \frac{y}{x^2} = -\frac{1}{x} + c$$

$$\Rightarrow \quad y = -x + cx^2 \quad \Rightarrow \quad y + x = cx^2$$

(x) True

The given differential equation is

$$x\frac{dy}{dx} = y + x\tan\left(\frac{y}{x}\right)$$

$$x\frac{dy}{dx} = -x\tan\left(\frac{y}{x}\right) = y$$

$$\Rightarrow \quad \frac{dy}{dx} - \tan\left(\frac{y}{x}\right) = \frac{y}{x} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow \quad v + x\,.\,\frac{dv}{dx} = \frac{vx}{x} + \tan\left(\frac{vx}{x}\right)$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \tan v \quad \Rightarrow \quad x\frac{dv}{dx} = \tan v$$

$$\Rightarrow \quad \frac{dv}{\tan v} = \frac{dx}{x} \quad \Rightarrow \quad \cot v\, dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \cot v\, dv = \int \frac{dx}{x} \Rightarrow \log \sin v = \log x + \log c$$

$$\Rightarrow \quad \log \sin v - \log x = \log c \quad \Rightarrow \quad \log \sin \frac{y}{x} = \log xc$$

$$\therefore \quad \sin \frac{y}{x} = xc$$

(xi) True

Let $y = mx + c$ be the non-horizontal line in a plane

$$\therefore \quad \frac{dy}{dx} = m \quad \text{and} \quad \frac{d^2y}{dx^2} = 0.$$