Series **EF1GH/1**

SET~3

प्रश्न-पत्र कोड
Q.P. Code **65/1/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

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निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*
Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
Please check that this question paper contains **23** printed pages.

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
Please check that this question paper contains **38** questions.

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* **38** *questions.* ***All*** *questions are* ***compulsory.***

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

2. The direction ratios of a line parallel to z-axis are :

(a) $< 1, 1, 0 >$ (b) $< 1, 1, 1 >$

(c) $< 0, 0, 0 >$ (d) $< 0, 0, 1 >$

3. Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that she missed the target exactly once is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{9}$ (d) $\frac{1}{9}$

4. The function $f(x) = |x| - x$ is :

(a) continuous but not differentiable at $x = 0$.

(b) continuous and differentiable at $x = 0$.

(c) neither continuous nor differentiable at $x = 0$.

(d) differentiable but not continuous at $x = 0$.

5. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are :

(a) $\pm \sqrt{7}$ (b) 0

(c) ± 5 (d) 25

6. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$

7. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) $\{0\}$ (b) $(0, \infty)$

(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

8. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

9. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2 (b) 3

(c) 4 (d) 5

10. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2 (b) 2

(c) 3 (d) -3

11. If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is :

(a) 6 (b) -6

(c) 0 (d) -7

12. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) -8 only

(c) 64 (d) 8 or -8

13. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

14. The primitive of $\dfrac{2}{1 + \cos 2x}$ is

(a) $\sec^2 x$ (b) $2 \sec^2 x \tan x$

(c) $\tan x$ (d) $-\cot x$

15. $\displaystyle\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to :

(a) $\dfrac{1}{\sqrt{3}}$ (b) $-\dfrac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

16. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5
(b) 2
(c) 3
(d) 4

17. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3
(b) -3
(c) $-\frac{17}{3}$
(d) $\frac{17}{3}$

18. For what value of λ, the projection of vector $\hat{i} + \lambda\hat{j}$ on vector $\hat{i} - \hat{j}$ is $\sqrt{2}$?

(a) -1
(b) 1
(c) 0
(d) 3

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

20. *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If the product of two positive numbers is 9, find the numbers so that the sum of their squares is minimum.

22. (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

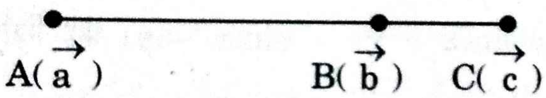
OR

(b) Evaluate :

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

23. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.

24. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

25. If $x = \sqrt{a^{\tan^{-1} t}}$, $y = \sqrt{a^{\cot^{-1} t}}$, then show that $x\frac{dy}{dx} + y = 0$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Evaluate :

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$

OR

(b) Find :

$$\int \frac{1}{\cos(x-a)\ \cos(x-b)}\, dx$$

27. Evaluate :

$$\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2\cos x)]\, dx$$

28. Find :

$$\int \frac{x^2}{(x^2+4)(x^2+9)}\, dx$$

29. (a) Find the general solution of the differential equation : $\frac{dy}{dx} - \frac{2y}{x} = \sin\frac{1}{x}$.

OR

(b) Find the particular solution of the differential equation : $\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$, given that when $x = \frac{\pi}{4}$, $y = 0$.

30. Solve the following linear programming problem graphically :

Maximize $z = 600x + 400y$

subject to the constraints :

$$x + 2y \le 12,$$
$$2x + y \le 12,$$
$$x + 1{\cdot}25y \ge 5,$$
$$x, y \ge 0$$

31. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \le X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method :

$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

$$3x + 2y - 2z = 3$$

33. (a) Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines.

OR

(b) Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

34. Find the area of the region bounded by the lines $y = 4x + 5$, $x + y = 5$ and $4y = x + 5$, using integration.

35. A relation R is defined on a set of real numbers $\mathbb{R}$ as

$$R = \{(x, y) : x \,.\, y \text{ is an irrational number}\}.$$

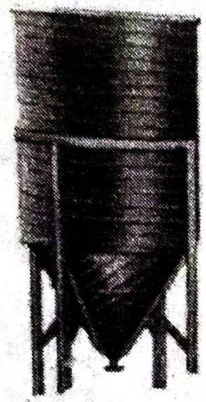
Check whether R is reflexive, symmetric and transitive or not.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r. *1*

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. *1*

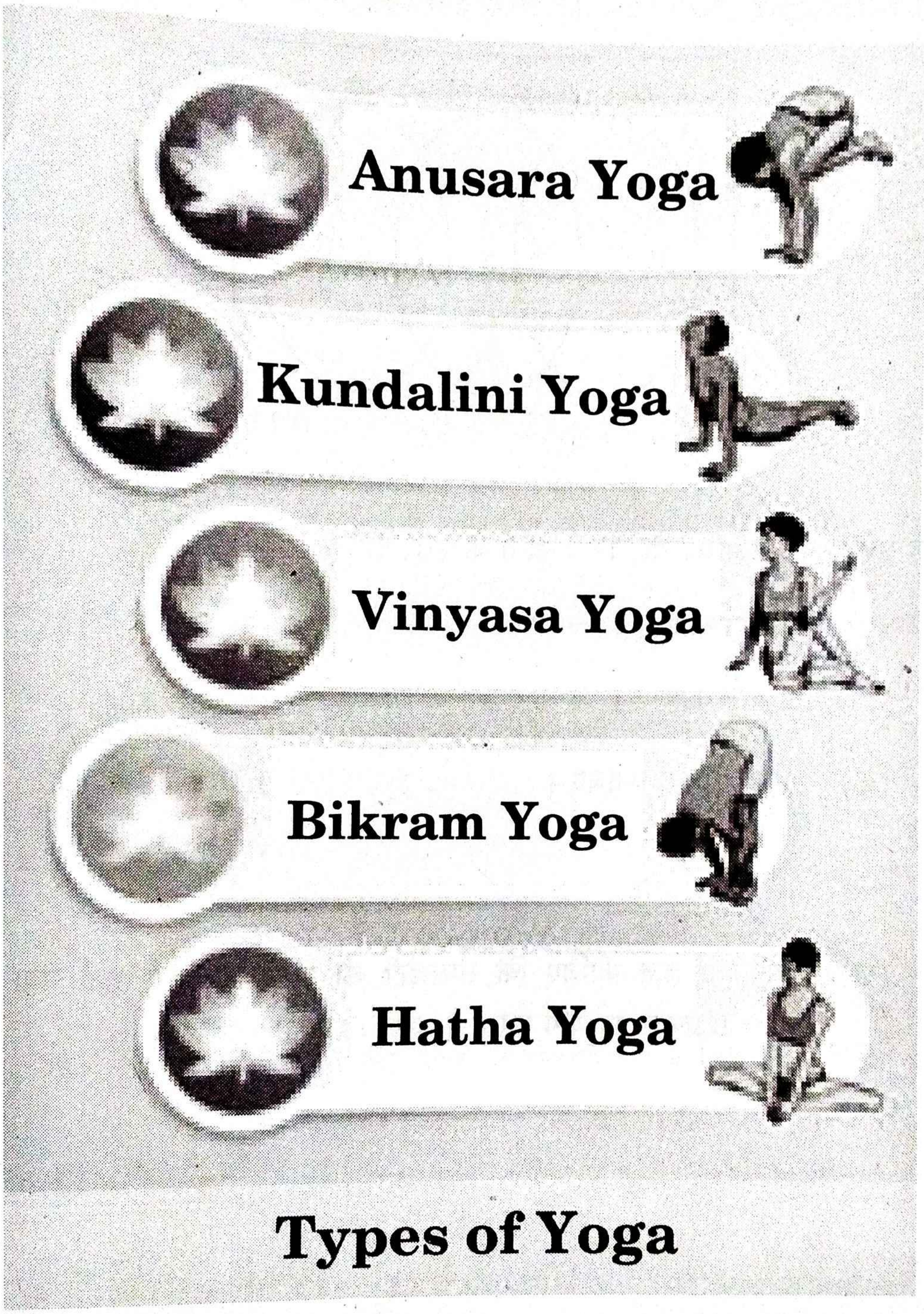
(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. *2*

OR

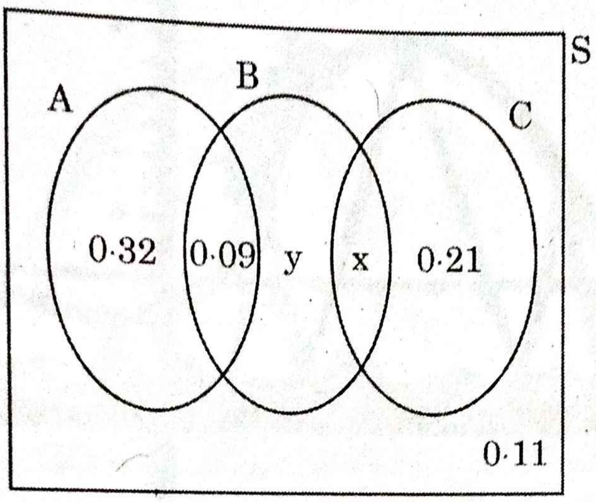
(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm. *2*

Case Study – 2

37. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below :



The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.



On the basis of the above information, answer the following questions :

(i) Find the value of x. *1*

(ii) Find the value of y. *1*

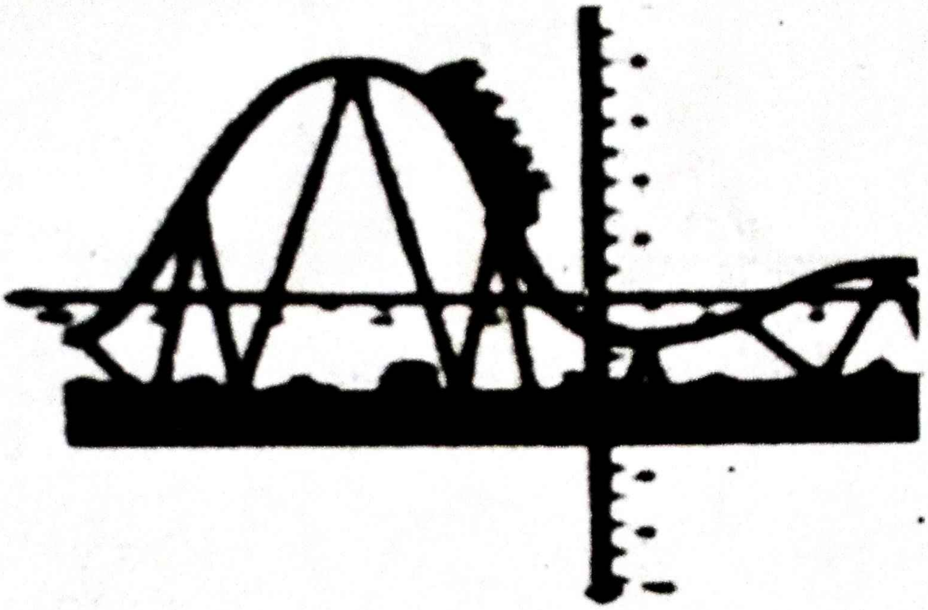
(iii) (a) Find $P\left(\frac{C}{B}\right)$. 2

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. 2

Case Study – 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(i) Find the value of 'a'. 2

(ii) Find $f''(x)$ at $x = 1$. 2