

CHAPTER 2

ELECTRIC POTENTIAL & CAPACITANCE

ELECTRIC POTENTIAL

Electric potential at a point in electric field is defined to be equal to the minimum work done by an external agent in moving a unit positive charge from infinity or a reference point to that point against the electrical force of the field.

If W is the work done by external agent in bringing a positive test charge q_0 from infinity to a point then the potential V at that point.

$$V = \frac{W_{\text{ext}}}{q_0} \quad \text{Unit of potential is joule/coulomb or volt. (S.I. unit)}$$

Electric potential difference

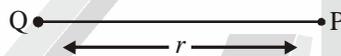
Potential difference between two points is equal to the minimum work done in moving a unit positive test charge from one point to the other.

$$V_B - V_A = \frac{W_{AB}}{q_0} \Rightarrow W_{AB} = q_0 (V_B - V_A)$$

Relation between electric field and electric potential

$$\left[E = -\frac{dV}{dr} \right] \quad * \text{ Electric Field is always directed from higher potential to lower potential.}$$

Electric Potential Due to point charge



$$V_P = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

Superposition Theorem

The potential at any point due to a group of point charges is the algebraic sum of the potentials contributed at the same point by all the individual point charges. $V = V_1 + V_2 + V_3 + \dots$

The electric potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal

charge elements in which the distribution may be divided $V = \int dV$; $V = \int \frac{dq}{4\pi\epsilon_0 r}$

ELECTRIC POTENTIAL DUE TO VARIOUS CHARGE DISTRIBUTION

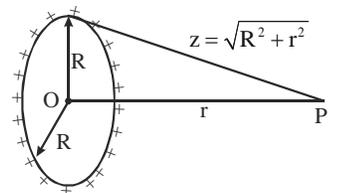
Electric Potential due to a Charged Ring

A charge Q is uniformly distributed over the circumference of a ring. Let us calculate the electric potential at an axial point at a distance r from the centre of the ring.

The electric potential at P due to the charge element dq of the ring is given by

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{Z} = \frac{1}{4\pi\epsilon_0} \frac{dq}{(R^2 + r^2)^{1/2}}$$

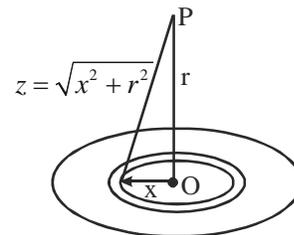
Hence, the electric potential at P due to the uniformly charged ring is given by



$$V = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{(R^2 + r^2)^{1/2}} = \frac{1}{4\pi\epsilon_0} \frac{1}{(R^2 + r^2)^{1/2}} \int dq = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + r^2}}.$$

Electric Potential Due to a Charged Disc at a Point on the Axis

A non-conducting disc of radius 'R' has a uniform surface charge density $\sigma \text{ C/m}^2$. Let us calculate the potential at a point on the axis of the disc at a distance 'r' from its centre. The symmetry of the disc tells us that the appropriate choice of element is a ring of radius x and thickness dx. All points on this ring are at the same distance $Z = \sqrt{x^2 + r^2}$, from the point P. The charge on the ring is $dq = \sigma dA = \sigma(2\pi x \, dx)$ and so the potential due to the ring is



$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{Z} = \frac{1}{4\pi\epsilon_0} \frac{\sigma(2\pi x \, dx)}{\sqrt{x^2 + r^2}}$$

Since potential is scalar. The potential due to the whole disc is given by

$$V = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{x \, dx}{\sqrt{x^2 + r^2}} = \frac{\sigma}{2\epsilon_0} \left[(x^2 + r^2)^{1/2} \right]_0^R = \frac{\sigma}{2\epsilon_0} \left[(R^2 + r^2)^{1/2} - r \right]$$

Let us see this expression at large distance when $r \gg R$.

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}, \text{ where } Q = \pi r^2 \sigma \text{ is the total charge on the disc.}$$

Thus, we conclude that at large distance, potential due to the disc is the same as that of a point charge Q.

Electric Potential Due to a Shell

A shell of radius R has a charge Q uniformly distributed over its surface. Let us calculate the potential at a point

(a) outside the shell; ($r > R$):

At points outside a uniform spherical distribution, the electric field is $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$

since \vec{E} is radial, $\vec{E} \cdot d\vec{r} = E dr$; since $V(\infty) = 0$, we have

$$V(\infty) - V(r) = -\int_{\infty}^r \vec{E} \cdot d\vec{r} \quad 0 - V = -\int_r^{\infty} \frac{Q}{4\pi\epsilon_0 r^2} dr \Rightarrow V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

We see that the potential due to a uniformly charged shell is the same as that due to a point charge Q at the centre of the shell.

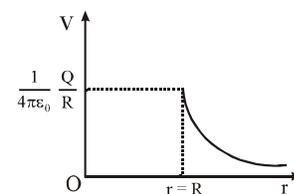
(b) At an internal Point: inside the shell ($r < R$).

At points inside the shell, $E = 0$. So, the work done in bringing a unit positive charge from a point on the surface to any point inside the shell is zero. Thus, the potential has a fixed value at all points within the spherical shell and is equal to the potential at the surface.

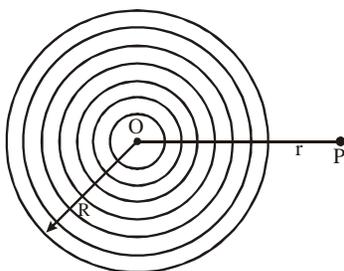
$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

Variation of electric potential with the distance from the centre (r)

All the above results hold for a "conducting sphere also whose charge lies entirely on the outer surface.



Electric Potential due to a Non-conducting Charged Sphere



A charge Q is uniformly distributed throughout a non-conducting spherical volume of radius R . Let us find expressions for the potential at an (a) external point ($r > R$); (b) internal point ($r < R$) where r is the distance of the point from the centre of the sphere.

(a) At an external point

Let O be the centre of a non-conducting sphere of radius R , have a charge Q distributed uniformly over its entire volume.

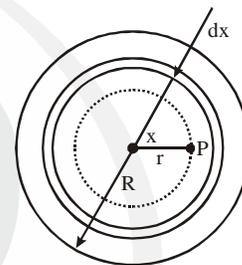
Let us divide the sphere into a large number of thin concentric shells carrying charges q_1, q_2, q_3, \dots etc.

The potential at the point P due to the shell of charge q_1 is $\frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$.

Now, potential is a scalar quantity. Therefore the potentials V due to the whole sphere is equal to the sum of the potentials due to all the shells.

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r} + \dots = \frac{1}{4\pi\epsilon_0 r} [q_1 + q_2 + q_3 + \dots]$$

But $q_1 + q_2 + q_3 + \dots = Q$, the charge on the sphere $\therefore V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$



(b) Potential at an internal point

Suppose the point P lies inside the sphere at a distance r from the centre O , if we draw a concentric sphere through the point P , the point P will be external for the solid sphere of radius r , and internal for the outer spherical shell of internal radius r and external radius R .

The charge on the inner solid spheres $\frac{4}{3}\pi r^3 \rho$. Therefore the potential V_1 at P due to this sphere is given

$$\text{by } V_1 = \frac{1}{4\pi\epsilon_0} \frac{4/3\pi r^3 \rho}{r} = \frac{r^2 \rho}{3\epsilon_0}$$

Let us now find the potential at P due to the outer spherical shell. Let us divide this shell into a number of thin concentric shells and consider one such shell of radius x and infinitesimally small thickness dx . The volume of this shell = surface area \times thickness = $4\pi x^2 dx$. The charge on this shell, $dq = 4\pi x^2 dx \rho$. The potential at P due to this shell

$$dV_2 = \frac{1}{4\pi\epsilon_0} \frac{dq}{x} = \frac{1}{4\pi\epsilon_0} \frac{4\pi x^2 (dx)\rho}{x} = \frac{\rho x dx}{\epsilon_0}$$

The potential V_2 at P due to the whole shell of internal radius r and external radius R is given by

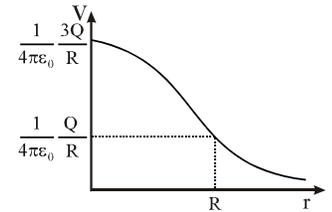
$$V_2 = \int_r^R \frac{\rho}{\epsilon_0} x dx = \frac{\rho}{\epsilon_0} \left| \frac{x^2}{2} \right|_r^R = \frac{\rho(R^2 - r^2)}{2\epsilon_0}$$

Since the potential is a scalar quantity, the total potential V at P is given by

$$V = V_1 + V_2 = \frac{r^2 \rho}{3\epsilon_0} + \frac{\rho(R^2 - r^2)}{2\epsilon_0} = \frac{\rho(3R^2 - r^2)}{6\epsilon_0}$$

$$\text{But } \rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad \therefore V = \frac{1}{4\pi\epsilon_0} \frac{Q}{2R^3} [3R^2 - r^2]$$

Variation of potential with distance from centre.



Equipotential surface

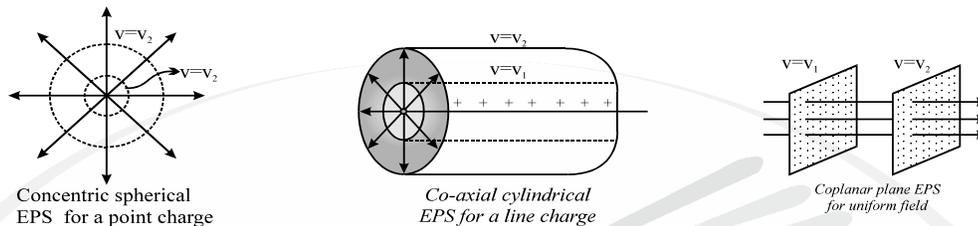
If all the points of the surface are at the same potential, the surface is called an equipotential surface.

Work done in the moving a charge between any two points on an equipotential surface is zero.

Equipotential surfaces can never cross each other because there will be two normals at the point of intersection giving two different directions of electric field which is absurd.

Equipotential surface are always perpendicular to lines of force.

For a point charge the equipotential surface is spherical. For a line charge equipotential surface is cylindrical and for uniform field the equipotential surface is planar.



Equipotential surfaces are closely spaced where electric field intensity is large and widely spaced where electric field intensity is small.

Properties of Equipotential surface

- (i) Potential difference between two points in an equipotential surface is zero.
- (ii) If a test charge q_0 is moved from one point to the other on such a surface, the electric potential energy $q_0 V$ remains constant.
- (iii) No work is done by the electric force when the test charge is moved along this surface.
- (iv) Two equipotential surfaces can never intersect each other because otherwise the point of intersection will have two potentials which is of course not acceptable.
- (v) Field lines and equipotential surfaces are always mutually perpendicular.

ELECTROSTATICS POTENTIAL ENERGY

In the figure, if a charge $+q$ is moved from B to C in the electric field of charge $+Q$, the work will have to be done by some outside agent in pushing the charge $+q$ against the force of field of $+Q$.

This situation is very similar to that of a mass moved in gravitational field of earth away from it. Work done against the gravitational pull of earth is stored in Gravitational potential energy and can be recovered back. Similarly in electric field, work done against an electric field is stored in the form of electric potential energy & can be recovered back. If the charge $+q$ is taken back from C to B, the electric force will try to accelerate the charge and hence to recover the potential energy stored in the form of kinetic energy.

As the work done against an electric field can be recovered back, electrostatic forces and fields fall under the category of conservative forces and fields. Another property of these fields is that the work done is independent of path taken from the one point to the another.

Potential Energy of a System of Two Point Charges

The potential energy possessed by a system of two-point charges q_1 and q_2 separated by a distance r is the work required to bring them to this arrangement from infinity. This electrostatic potential energy is

$$\text{given by } U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

Note : While writing potential or potential energy charges must be multiplied with their signs.

Electric Potential Energy of a System of Point Charges

The electric potential energy of such a system is the work done in assembling this system starting from infinite separation between any two-point charges.

For a system of point charges q_1, q_2, \dots, q_n , the potential energy is
$$U = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} \quad (i \neq j)$$

It simply means that we have to consider all the pairs that are possible.

Important points regarding Electrostatic potential energy

(i) Work done required by an external agency to move a charge q from A to B in an electric field with constant speed $W_{A \rightarrow B} = q[V_B - V_A]$

(ii) When a charge q is let free in an electric field, it loses potential energy and gains kinetic energy, if it goes from A to B, then loss in potential energy = gain in kinetic energy

$$\text{or } q(V_B - V_A) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

ILLUSTRATIONS

1. The electric field in a region is given by $\vec{E} = (A/x)^3 \hat{i}$. Write a suitable SI unit for A. Write an expression for the potential in the region assuming the potential at infinity to be zero.

Sol. The SI unit of electric field is N/C or V/m. Thus, The unit of A is $\frac{N \cdot m^3}{C}$ or $V \cdot m^2$.

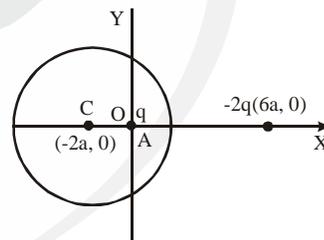
$$V(x, y, z) = - \int_{\infty}^{(x, y, z)} \vec{E} \cdot d\vec{r} = - \int_{\infty}^{(x, y, z)} \frac{A dx}{x^3} = \frac{A}{2x^2}.$$

2. Two points charge q and $-2q$ are placed at a distance $6a$ apart. Find the locus of the point in the plane of charges where the field potential is zero.

Sol. Let us take the charge on X-axis;
 q at A (0, 0) and $-2q$ at B(6a, 0)

$$\text{Potential at a point P(x, y) is } V = \frac{q}{4\pi\epsilon_0 \sqrt{x^2 + y^2}} + \frac{-2q}{4\pi\epsilon_0 \sqrt{(x - 6a)^2 + y^2}}$$

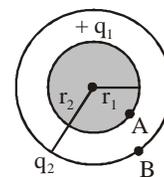
$$V = 0 \Rightarrow \frac{q^2}{x^2 + y^2} = \frac{4q^2}{(x - 6a)^2 + y^2}$$



PRACTICE PROBLEM

1. What is electrostatic potential due to a point charge?
2. Why is electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface?
3. Name the physical quantity which has its unit joule coulomb⁻¹. Is it a scalar or vector?
4. Do electrons tend to go to regions of high potential or low potential?
5. What is the potential due to a point charge?
6. Is electrostatic potential necessarily zero at a point where electric field strength is zero. Illustrate your answer.
7. Can two equipotential surfaces intersect each other? Justify your answer.

8. What is the amount of work done in moving a $200 \mu\text{C}$ charge between two points 5 cm apart on an equipotential surface?
9. A small sphere of radius r_1 and charge q_1 enclosed by a spherical shell of radius r_2 and charge q_2 . Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.
10. Equipotential surfaces are perpendicular to field lines. Why?
11. Two point charges $3 \mu\text{C}$ and $-3 \mu\text{C}$ are placed at points A and B, 5 cm apart.
- Draw the equipotential surface of the system
 - Why do equipotential surfaces get close to each other near the point charge?
12. What is relation between electric field and potential?
13. Let V be the electric potential at a given point. Then the electric field E_x along x-direction at that point is given by :
- $\int_0^\infty V dx$
 - $\frac{dV}{dx}$
 - $-\frac{dV}{dx}$
 - $-V \frac{dV}{dx}$
14. Potential at any point inside a charged hollow sphere :
- increases with distance
 - is a constant
 - decreases with distance from centre
 - is zero
15. A solid spherical conductor is given a charge. The electrostatic potential of the conductor is
- constant throughout the conductor
 - largest at the centre
 - largest on the surface
 - largest somewhere b/w the centre & the surface



CAPACITANCE

CAPACITORS

Capacitor is an arrangement of two conductors carrying charges of equal magnitudes and opposite sign and separated by an insulating medium.

- The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q , we mean that the positively charged conductor has charge $+Q$ and negatively charged conductor has a charge $-Q$.
- The positively charged conductor is at a higher potential than the negatively charged conductor. The potential difference V between the conductor is proportional to the charge magnitude Q and the ratio Q/V is known as capacitance C of the capacitor.

$$C = \frac{Q}{V}$$

Units of capacitance is farad (F). The capacitance is usually measured in microfarad (μF).

$$1 \mu\text{F} = 10^{-6} \text{ F}$$

- In a circuit, a capacitor is represented by the symbol : 
- Capacitors work as a charge – storing or energy – storing devices. A capacitor stores energy in the form of electric field.

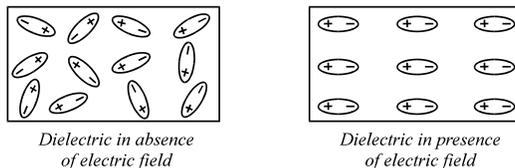
DIELECTRICS

In dielectric materials, effectively there are no free electrons.

In monatomic materials the centre of the negative charge coincides with the centre of the positive charge whereas in polyatomic materials, on the other hand, the center of the negative charge may or may not coincide with the centre of the positive charge distribution. If it does not coincide, each molecule behaves as a dipole with dipole moment \vec{p} . Such materials are known as polar materials.

If such a material is placed in an electric field, the individual dipoles experience torque due to the field and they try to align along the field.

The charge appearing on the surface of a dielectric when placed in an electric field is called **induced charge**. As the induced charge appears due to a shift in the electrons bound to the nuclei, this charge is also called bound charge.



Because of the induced charges, an extra electric field is produced inside the material. If \vec{E}_0 be the applied field due to external sources and \vec{E}_p be the field due to polarization. The resultant field is $\vec{E} = \vec{E}_0 + \vec{E}_p$. For homogeneous and isotropic dielectrics, the direction of \vec{E}_p is opposite to the direction of \vec{E}_0 . The resultant field \vec{E} is in the same direction as the applied field \vec{E}_0 but its magnitude is reduced. We

can write
$$\vec{E} = \frac{\vec{E}_0}{K}$$

where K is a constant for given dielectric which has a value greater than one. This constant K is called the dielectric constant or relative permittivity of the dielectric.

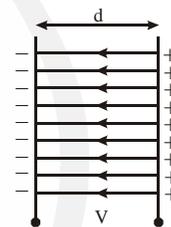
Types of Capacitor

- (a) Parallel plate capacitor (b) Spherical capacitor (c) Cylindrical capacitor

Parallel Plate Capacitor

The parallel plate capacitor consist of two metal plates placed parallel to each other and separated by a distance that is very small as compared to the dimension of the plates.

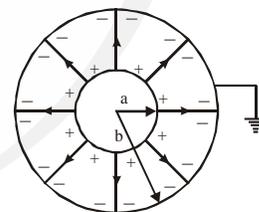
The capacitance is given by
$$C = \frac{K \epsilon_0 A}{d}$$



Where K the dielectric constant of the medium between the plates, d is separation between the plates and A is the area of each plate.

Spherical Capacitor

A spherical capacitor consist of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential. The inner surface of the outer sphere has an equal negative charge.

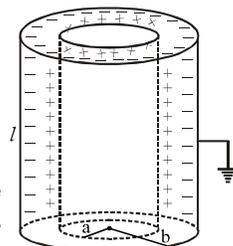


The capacitance is given by
$$C = \frac{4\pi K \epsilon_0 ab}{b - a}$$

Where K is the dielectric constant and a and b are inner and outer radius of the sphere.

Cylindrical Capacitor

Cylindrical capacitor consist of two co-axial cylinders of radii a and b and length l. The electric fields exists in the region between the cylinders. Let K be the dielectric constant of the material between the cylinders. The capacitance is given by:



The capacitance is given by:
$$C = \frac{2\pi K \epsilon_0 \ell}{\ln \frac{b}{a}}$$

Where K is the dielectric constant, a and b are inner and outer radius of the sphere and ℓ is the length of the cylinder.

CAPACITORS IN SERIES AND PARALLEL COMBINATION

Series Combinations

When two or more than two capacitors are connected in such a way that plates of capacitors are connected with each other the combination is known as series. [Only first plate of first capacitor and second plate of last capacitor is connected to source.]



When capacitors are connected in series, the magnitude of charge Q on each capacitor is same. The potential difference across C_1 and C_2 is different i.e., V_1 and V_2 . $Q = C_1 V_1 = C_2 V_2$

The total potential difference across combination is : $V = V_1 + V_2$; $V = \frac{Q}{C_1} + \frac{Q}{C_2}$; $\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$

The ratio Q/V is called as the equivalent capacitance C between point a and b.

The equivalent capacitance C is given by : $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

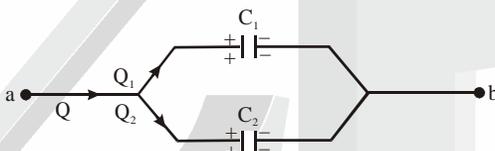
The potential difference across C_1 and C_2 is V_1 and V_2 respectively, given as follows:

$$V_1 = \frac{C_2 V}{C_1 + C_2} \text{ \& } V_2 = \frac{C_1 V}{C_1 + C_2}$$

In case of more than two capacitors, the relation is : $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \dots \Rightarrow \frac{1}{C_{eq}} = \sum_{i=1}^n \frac{1}{C_i}$

Parallel Combinations

When two or more than two capacitors are connected in such a way that one plate of all capacitors are connected to one point and other plate of all capacitors are connected to other single point such a combination arrangement of capacitors is known as parallel combination.



When capacitors are connected in parallel, the potential difference V across each is same and the charge on C_1 , C_2 is different i.e., Q_1 and Q_2 .

The total charge is Q given as : $Q = Q_1 + Q_2$; $Q = C_1 V + C_2 V$; $\frac{Q}{V} = C_1 + C_2$

Equivalent capacitance between a and b is : $C = C_1 + C_2$

The charges on capacitors is given as : $Q_1 = \frac{C_1}{C_1 + C_2} Q$; $Q_2 = \frac{C_2}{C_1 + C_2} Q$

In case of more than two capacitors, $C = C_1 + C_2 + C_3 + C_4 + C_5 + \dots$

Energy Stored by Capacitor

Consider a capacitor of capacity C connected to a source of which maintains a constant potential difference V across it. Let at any instant charge on capacitor is q which increased to $q + dq$ in next instant.

Potential difference between the plates at the instant charge on capacitor is q , is $\Rightarrow V = \frac{q}{C}$

When extra charge dq is transferred, the increment of work required, $dW = Vdq$

$$\therefore \int dW = \int_0^Q V dq \Rightarrow \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} \quad ; \quad \therefore \text{energy stored} = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

Work done by battery

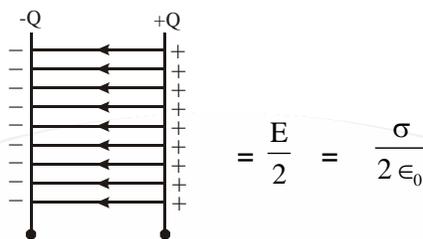
$W = \text{Charge that flow through battery} \times \text{battery EMF}$

Whenever there is a charging of capacitor. Work done by battery, in part is stored as electrostatic energy in between capacitor plate and remaining is dissipated as heat due to charge flow through connecting wires.

In general, Heat produced is given by $\Rightarrow H = (\text{Work done by battery} - \text{Energy stored in capacitor})$

Force on any plate of parallel plate capacitor due to other

Intensity of the field at surface of any plate due to other is half of the field between plates



Force for area dS on any plate $dF = \sigma ds \frac{E}{2} = \frac{\sigma^2 ds}{2 \epsilon_0}$; Net force on any plate $F = \int dF = \frac{\sigma^2 A}{2 \epsilon_0}$

Force per unit area $= \frac{\sigma^2}{2 \epsilon_0}$

ILLUSTRATIONS

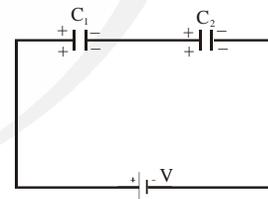
1. Two capacitors of capacitance $C_1 = 6 \mu F$ and $C_2 = 3 \mu F$ are connected in series across a cell of emf 18 V. Calculate:
- (a) the equivalent capacitance (b) the potential difference across each capacitor
- (c) the charge on each capacitor.

Sol. (a) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 3}{6 + 3} = 2 \mu F.$

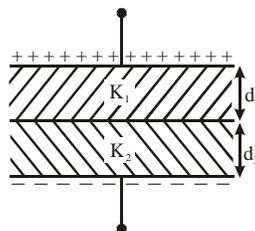
(b) $V_1 = \frac{C_2}{C_1 + C_2} V = \frac{3}{6 + 3} \times 18 = 6 \text{ volts}$; $V_2 = \frac{C_1}{C_1 + C_2} V = \frac{6}{6 + 3} \times 18 = 12 \text{ volts}$

Note that the smaller capacitor C_2 has a larger potential difference across it.

(c) $Q_1 = Q_2 = C_1 V_1 = C_2 V_2$ charge on each capacitor $= C_{eq} V = 2 \mu F \times 18 \text{ volts} = 36 \mu C$

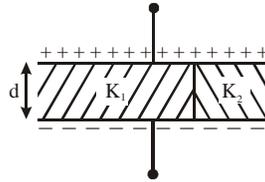


2. Find the capacitance of the system in which dielectric is filled as shown in the figure. Each plates are of area A .



Sol. $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$; $\frac{1}{C} = \frac{d_1}{k_1 \epsilon_0 A} + \frac{d_2}{k_2 \epsilon_0 A}$; $C = \frac{\epsilon_0 A}{\frac{d_1}{k_1} + \frac{d_2}{k_2}}$

3. Find the capacitance of the system in which dielectric is filled as shown in the figure.



Sol. $C = C_1 + C_2$; $C = \frac{k_1 \epsilon_0 A_1}{d} + \frac{k_2 \epsilon_0 A_2}{d}$; $C = \frac{\epsilon_0}{d} [k_1 A_1 + k_2 A_2]$.

4. An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.

Sol. Suppose the capacitance of the capacitor is C and the emf of the battery is V . The charge given to the capacitor is $Q = CV$. The work done by the battery is $W = QV$

The battery supplies this energy. The energy stored in the capacitor is $U = \frac{1}{2} CV^2 = \frac{1}{2} QV$

The remaining energy $QV - \frac{1}{2} QV = \frac{1}{2} QV$ is lost as heat.

Thus, half the energy supplied by the battery is lost as heat.

5. A capacitor of capacitance C is charged by connecting it to a battery of emf ϵ . The capacitor is now disconnected and reconnected to the battery with the polarity reversed. Calculate the heat developed in the connecting wires.

Sol. When the capacitor is connected to the battery, a charge $Q = C\epsilon$ appears on one plate and $-Q$ on the other. When the polarity is reversed, a charge $-Q$ appears on the first plate and $+Q$ on the second. A charge $2Q$, therefore, passes through the battery from the negative to the positive terminal. The battery does a work. $W = (2Q)\epsilon = 2C\epsilon^2$ in the process. The energy stored in the capacitor is the same in the two cases. Thus, the work done by the battery appears as heat in the connecting wires. The heat produced is, therefore, $2C\epsilon^2$.

6. A parallel plate air capacitor is made using two plates 0.2 m square, spaced 1 cm apart. It is connected to a 50 V battery.

- (a) what is the capacitance? (b) what is the charge on each plate?
 (c) what is the energy stored in the capacitor? (d) what is the electric field between the plates?
 (e) if the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts?

Sol. (a) $C_0 = \frac{\epsilon_0 A}{d_0} = \frac{8.85 \times 10^{-12} \times 0.2 \times 0.2}{0.01}$; $C_0 = 3.54 \times 10^{-5} \mu\text{F}$

(b) $Q_0 = C_0 V_0 = (3.54 \times 10^{-5} \times 50) \mu\text{C} = 1.77 \times 10^{-3} \mu\text{C}$

(c) $U_0 = 1/2 C_0 V_0^2 = 1/2 (3.54 \times 10^{-11}) (50)^2$; $U_0 = 4.42 \times 10^{-8} \text{J}$.

(d) $E_0 = \frac{V_0}{d_0} = \frac{50}{0.01} = 5000 \text{V/m}$.

(e) If the battery is disconnected, the charge on the capacitor plates remains constant while the potential difference between plates can change.

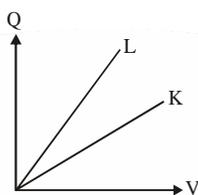
$C = \frac{A \epsilon_0}{2d} = 1.77 \times 10^{-5} \mu\text{F}$; $Q = Q_0 = 1.77 \times 10^{-3} \mu\text{C}$; $V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0 = 100 \text{volts}$.

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q_0^2}{(C_0/2)} = 2U_0 = 8.84 \times 10^{-8} \text{ J} ; E = \frac{V}{c} \frac{2V_0}{2d_0} = E_0 = 5000 \text{ V/m.}$$

work has to be done against the attraction of plates when they are separated. This gets stored in the energy of the capacitor.

PRACTICE PROBLEM

1. What is a net charge on a charged capacitor?
2. How will you obtain maximum capacitance from three given condensers?
3. How much work must be done to charge a $24 \mu\text{F}$ capacitor, when the potential difference between the plates is 500 V ?
4. The following graph shows the variation of charge Q with voltage V , for two capacitors K and L . In which capacitor is more electrostatic energy stored?



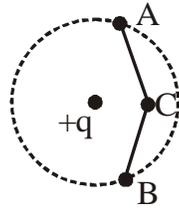
5. How does an electrically polarised object differ from an electrically charged object?
6. What is the capacitance of a parallel plate capacitor when a dielectric is placed between the plates.
7. Energy stored in a capacitor is:
 - a. $1/2 QV$
 - b. QV
 - c. $1/QV$
 - d. $2/QV$
8. A parallel plate condenser with oil between the plates (dielectric constant of oil $K = 2$) has a capacitance C . If the oil is removed, then capacitance of the capacitor becomes
 - a. $\sqrt{2} C$
 - b. $2C$
 - c. $C/\sqrt{2}$
 - d. $C/2$
9. A point charge q is held at a distance $2a$ from the centre of an isolated, uncharged conductance sphere of radius a . The potential of the sphere is
 - a. $\frac{q}{8\pi\epsilon_0 a}$
 - b. $\frac{q}{4\pi\epsilon_0 a}$
 - c. $\frac{q}{2\pi\epsilon_0 a}$
 - d. zero
10. When you double the charge on a capacitor, capacitance becomes :
 - a. double
 - b. half
 - c. one-fourth
 - d. remains same

EXERCISE

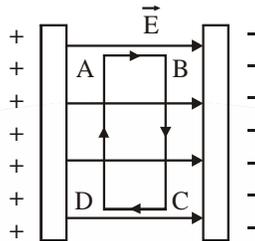
ELECTRIC POTENTIAL AND POTENTIAL ENERGY

1. Why are equipotential surface are perpendicular to field lines?
2. Derive an expression for potential at a point
 - a. due to point charge
 - b. due to group of charges
 - c. due to an electric dipole.
 - d. the charge inside the cube.
3. Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively.
 - a. What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?
 - b. Obtain the dependence of potential on the distance r of a point from the origin when $\frac{r}{a} \gg 1$.
 - c. How much work is done in moving a small test charge from point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x -axis? Does the answer change if the path of the test charge between the same points is not along the x -axis?

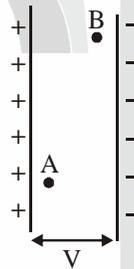
- Write down the relation between the electric field and electric potential at a point.
- A and B are two conducting spheres of the same radius, A being solid and B hollow. Both are charged to the same potential. What will be the relation between the charges on the two spheres?
- If a point charge $+q$ is taken first from A to C and then from C to B of a circle (figure) drawn with another point charge $+q$ at centre, then along which path more work will be done?



- A uniform electric field E exists between two charged plates as shown in figure. What would be the work done in moving a charge q along the closed rectangular path ABCDA.

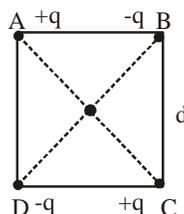


- Draw an equipotential surface for a point charge $Q > 0$.
- Two protons A and B are placed between two parallel plates having a potential difference V as shown in figure.



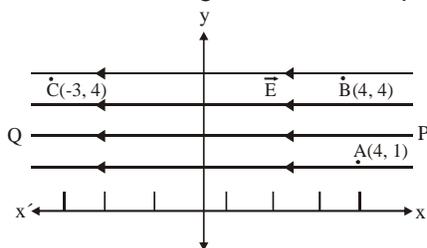
Will these protons experience equal or unequal force?

- An uncharged insulated conductor A is brought near a charged insulated conductor B. What happens to charge and potential of B?
- What is the work done in moving a 2×10^{-6} C point charge from corner A to corner B of a square ABCD when a $10 \mu\text{C}$ charge exists at the centre of the square?
- The electric field and electric potential at any point due to a point charge kept in air is 20 NC^{-1} and 400V respectively. Compute the magnitude of this charge.
- What is the potential at the centre of a square of each side 1.0 m, when four charges $+1 \times 10^{-8}$ C, -2×10^{-8} C, $+3 \times 10^{-8}$ C and $+2 \times 10^{-8}$ C are placed at the four corners of the square?
- Two point charges A and B of value $+5 \mu\text{C}$ and $+6 \mu\text{C}$ are kept 12 cm apart in air. Calculate the work done when charge B is moved by 2 cm towards charge A.
- Four charges are arranged at the corners of a square ABCD of side d as shown in figure. Find the work required to put together this arrangement.



16. What is an equipotential surface?

A uniform electric field \vec{E} of 300 NC^{-1} is directed along PQ. A, B and C are three points in the field having x and y coordinates (in metre) as shown in the figure. Calculate potential difference between the points :



- a. A and B b. B and C c. A and C

17. Deduce dimensional formula of potential difference.

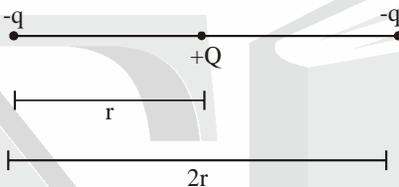
18. Two charges $5 \times 10^{-8} \text{ C}$ and $-3 \times 10^{-8} \text{ C}$ are located 16 cm apart. At what points on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

19. Two charges $+2\mu\text{C}$ and $-2\mu\text{C}$ are placed at points A and B, 6 cm apart :

- a. Identify and draw equipotential surface of the system.
b. What is the direction of the electric field at every point on this surface?

20. Name the physical quantity whose SI unit is J/C. Is it a scalar or a vector quantity?

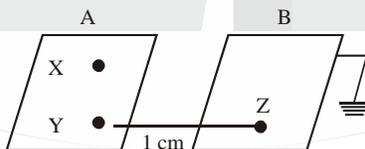
21. The total potential energy of these arrangement is zero. Find the ratio of q/Q .



22. Calculate the potential at the centre of a square of side $\sqrt{2} \text{ m}$, which carries at its four corners charges of 2 nC, + 1 nC -2 nC and -3 nC respectively.

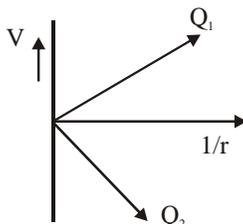
23. Three insulated concentric metal spheres A, B and C have radii R_1 , R_2 and R_3 respectively and have charges Q_1 , Q_2 and Q_3 . What is the potential and intensity of electric field at a point P between the spheres B and C at a distance r from the centre O of the spheres.

24. Two identical plane metallic surface 'A' and 'B' are kept parallel to each other in air separated by a distance of 1 cm as in figure and the surface 'A' is given a positive potential of 10V and outer surface of 'B' is earth.



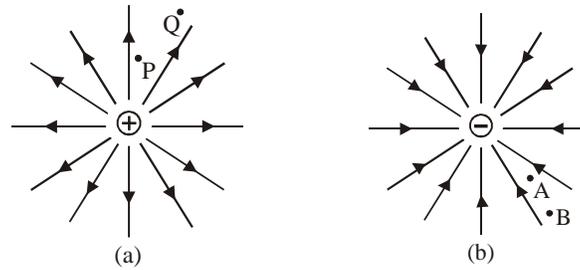
- a. What is the magnitude and the direction of uniform electric field between points 'Y' and 'Z'?
b. What is the work done in moving a charge of $20\mu\text{C}$ from point 'X' to point 'Y'?

25. Two graph drawn below, shows the variation of electrostatic potential 'V' with $1/r$ (r being distance of the field point from the point charge) for two point charges Q_1 and Q_2 .

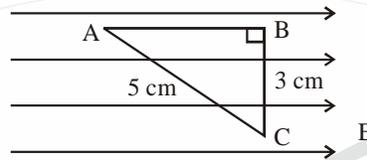


- a. What are the sign of two charges? b. Which of the two charges has larger magnitude?

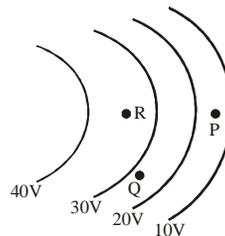
26. Figure (a) and (b) show the field lines of a single positive and negative charges respectively.



- Give the signs of the potential difference : $V_P - V_Q$; $V_B - V_A$.
 - Give the sign of the potential energy difference of a small $-ve$ charge between the points Q & P; A & B.
 - Give the sign of the work done by the field in moving a small positive charge from Q to P.
 - Give the sign of the work done by an external agency in moving a small negative charge from B to A
 - Does the kinetic energy of a small negative charge increase or decrease in going from B to A?
27. A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure.

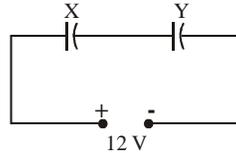


- Calculate the potential difference between A and C.
 - At which point (of the two) is the electric potential more and why?
- What will be the effect on potential at a point if the medium of dielectric constant K is introduced?
 - Define the term potential energy charge q at a distance r in an external field.
 - A charge of 8mC is located at the origin. Calculate the work done in taking a small charge of $-2 \times 10^{-9}\text{C}$ from a point $P(0, 0, 3\text{ cm})$ to a point $Q(0, 4\text{ cm}, 0)$ via a point $R(0, 6\text{ cm}, 9\text{ cm})$.
 - A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q .
 - A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surface of the shell?
 - Is the electric field intensity inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.
 - Two point charges $3\mu\text{C}$ and $-3\mu\text{C}$ are placed at point A and B, 5 cm apart.
 - Draw the equipotential surface of the system.
 - Why do equipotential surface get close to each other near the point charge?
 - Where is electric field intensity maximum and where is it minimum out of the three given points P, Q and R in the given figure of lines of constant potential in a electric field.

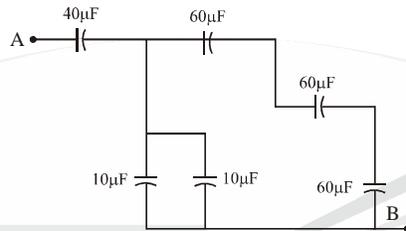


- Draw three equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along z -direction. How are these surfaces different from that of a constant electric field along z -direction?
- Derive an expression for potential at a point.
 - due to point charge
 - due to a group of charges
 - due to an electric dipole

49. An isolated air capacitor of capacitance C is charged to a potential V . Now the charging battery is disconnected and a dielectric slab of dielectric constant 3 is inserted between its plates, completely filling the space between the plates, then how do the following change:
a. Capacitance **b.** Potential difference **c.** Field between the plates **d.** Energy stored by the capacitor
50. X and Y are two parallel plate capacitors in figure, having the same area of plates and same separation between the plates. X has air between the plates and Y contains a dielectric medium of $\epsilon_r = 5$.

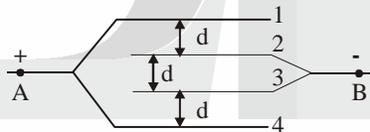


- a.** Calculate the potential difference between the plates of X and Y. or potential drop across X and Y.
b. What is the ratio of electrostatic energy stored in X and Y?
51. Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in the figure.

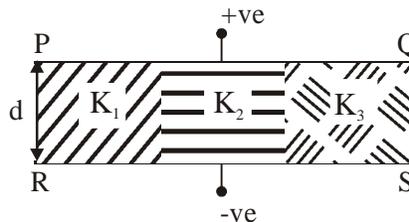


Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points A and B.

52. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacity of the capacitor becomes $\frac{2}{3}$ rd of its original value. What is the new distance between the plates?
53. Show mathematically that sharing of charges between two capacitors (or conductors), is always accompanied with some loss of electrostatic energy.
54. Find the total capacitance between the points A and B in the arrangement of 4 metal plates, each of area A and separated by a distance d , shown in under given figure.

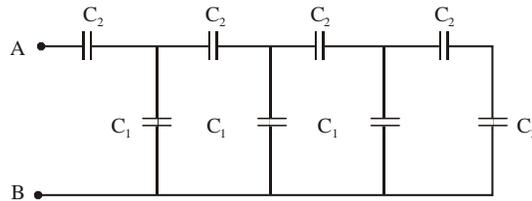


55. Two parallel plates PQ and RS are kept distance 'd' apart. Area of each plate is 'A'. The space between them is filled with three dielectric slabs of identical size having dielectric constants K_1 , K_2 and K_3 , respectively as shown in the figure.

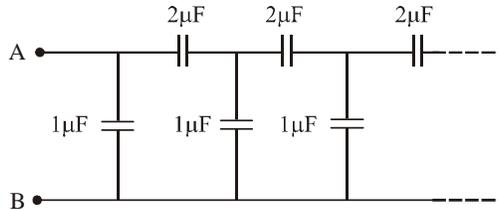


Find the capacitance of the capacitor.

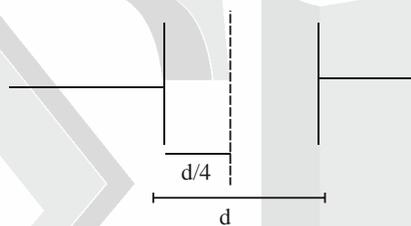
56. Find the equivalent capacitance of the combination shown in figure between points A and B.
 It is given that $C_1 = 1 \mu\text{F}$ and $C_2 = 2 \mu\text{F}$



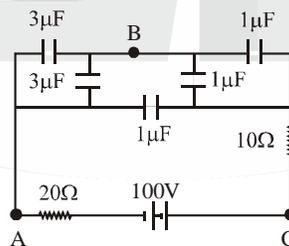
57. Find the capacitance of the infinite ladder of capacitors shown in figure, between points A and B.



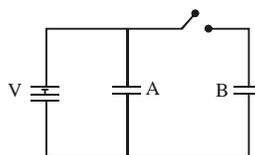
58. Derive an expression for the energy stored in a parallel plate capacitor. On charging a parallel plate capacitor to a potential V , the spacing between the plates is halved and a dielectric medium of $\epsilon_r = 10$ is introduced between the plates, without disconnecting the d.c. source. Explain using suitable expressions, how the: **a.** capacitance **b.** electric field, and **c.** energy density of the capacitor change
59. An earth plate is placed in between two plates with charge density $+\sigma$ and $-\sigma$. Find the charge density in the earth plate.
60. A capacitor of capacitance C has distance 'd' between the plates of a parallel plate capacitor. A very thin wire is placed at a distance $d/4$ in it. Find the new capacitance



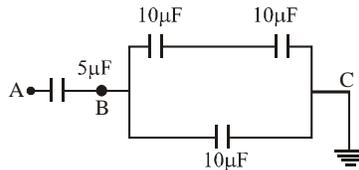
61. Three capacitors of equal capacitance, when connected in series, have a net capacitance of C_1 and when connected in parallel have capacitance of C_2 . What will be the value of C_1/C_2 ?
62. In the figure below, what is the potential difference between the point A and B and between B and C respectively in steady state



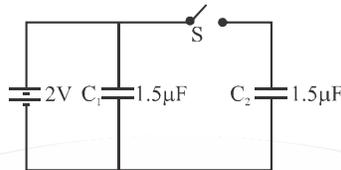
63. Figure given below shows two identical parallel plate capacitors connected to a battery with switch S closed. The switch is now opened and the free space between the plate of capacitors is filled with a dielectric of dielectric constant 3. What will be the ratio of total electrostatic energy stored in both capacitors before and after the introduction of the dielectric



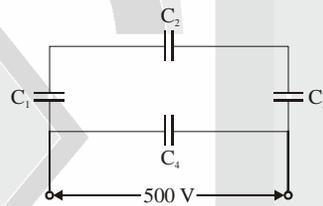
64. In the given circuit if point C is connected to the earth and a potential of $+2000V$ is given to the point A, the potential at B is



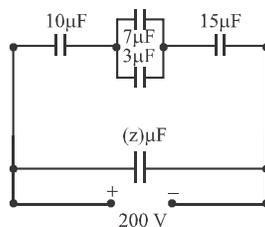
65. What is the net charge on a charged capacitor and why?
66. Capacitance of a capacitor depends on which factor.?
67. Calculate the total battery voltage if two capacitors are connected in series with the battery. The value of capacitors are $6\mu\text{F}$ and $12\mu\text{F}$ and the voltage across the $6\mu\text{F}$ capacitor is 2 V.
68. Two identical capacitors C_1 and C_2 each of $1.5\mu\text{F}$ capacitance, connected to a battery of 2 V. Initially switch S is closed. After some time S is left open and dielectric slabs of dielectric constant $K = 2$ are inserted to fill completely the space between the plates of the two capacitors. How will the



- a. charge & b. potential difference between the plates of the capacitors be affected after the slabs are inserted?
69. Define the dielectric constant of a medium. Briefly explain the capacitance of a parallel plate capacitor increases, on introducing dielectric medium between the plates.
70. A network of four capacitors each of $12\mu\text{F}$ capacitance if connected to a 500 V supply as shown in the figure. Determine



- a. equivalent capacitance of the network b. charge on each capacitor.
71. Deduce an expression showing relation between electric field and potential gradient.
72. Define electron-volt. Express it in joule.
73. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the distance between plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?
74. A parallel plate capacitor is charged by a battery. After sometime, the battery is disconnected and a dielectric slab of dielectric constant K is inserted between the plates. How would
- a. the capacitances b. the energy stored in the capacitor, be affected? Justify your answer.
75. Explain what would happen if in capacitor in above question, a 3 mm thick mica sheet of (dielectric constant = 6) were inserted between the plates :
- a. while the voltage supply remained connected b. after the supply was disconnected.
76. A system of capacitors, connected as shown, has a total energy of 160 mJ stored in it. Obtain the value of the equivalent capacitance of this system and value of Z.



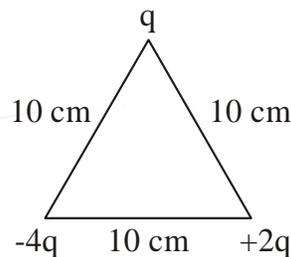
77. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?
78. Explain the underlying principle of working of a parallel plate capacitor. If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expressions for
 - a. the electric field at point between the two plates.
 - b. the potential difference between the plates.
 - c. the capacitances of the capacitor so formed.
79. a. A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and a dielectric slab is inserted to completely fill the space between the plates. How will
 - i. its capacitance,
 - ii. electric field between the plates and
 - iii. energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expression for each case.
 b. Sketch the pattern of electric field lines due to:
 - i. a conducting sphere having negative charge on it.
 - ii. an electric dipole.
80. Deduce the expression for equivalent capacitance, when three capacitances are connected in parallel.
81. Derive an expression for the energy stored in a parallel plate capacitor. On charging a parallel plate capacitor to a potential V , the spacing between the plates is halved and a dielectric medium of $\epsilon_r = 10$ is introduced between the plates, without disconnecting the DC source. Explain, using suitable expressions, how the
 - a. electric field and
 - b. energy density of the capacitor change?
82. Can a Van-de-Graff generator be used to accelerate neutrons? Why?
83. How is leakage of charge minimized in a Van-de-Graaff's generator?
84. Define electrostatic shielding.
85. Write the principle of van de Graaff generator.

PREVIOUS YEARS QUESTIONS

ELECTRIC POTENTIAL AND POTENTIAL ENERGY

1. Define electric potential energy. [Delhi 2008C]
2. Define electrostatic potential.
3. What is the geometrical shape of equipotential surface due to single isolated charge? [Delhi 2013]
4. Distinguish between electric potential and potential energy.
5. Why there is no work done in moving a charge from one point to another on an equipotential surface? [Foreign 2012]
6. Equipotential surfaces are perpendicular to field lines. Why?
7. Why is the potential inside a hollow spherical charged conductor is constant and has the same value as on its surface? [Foreign 2012]
8. Two spherical conductors A and B of radii r_A and r_B ($r_A > r_B$) are given equal amounts of charge. In which direction will the charge flow when these spheres are brought in contact? Give reason for your answer.
9. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero. [All India 2009C, 2011C, 2013]
10. Plot a graph comparing the variation of potential V and electric field E due to a point charge q as a function of distance r from the point charge. [Foreign 2010, Delhi 2012]
11. Draw an equipotential surface in a uniform electric field.
12. Two charges of $5\mu\text{C}$ and $-5\mu\text{C}$ are placed at points A and B 2 cm apart. Depict an equipotential surface of the system. [Delhi 2013 C]

13. A dipole, with its charge, $-q$ and $+q$, located at the points $(0, -b, 0)$ and $(0 + b, 0)$, is present in a uniform electric field E . The equipotential surfaces of this field are planes parallel to the $Y-Z$ plane.
- What is the direction of the electric field E ?
 - How much torque would the dipole experience in this field? [Delhi 2010C]
14. Derive the expression for the electric potential at any point along the axial line of an electric dipole? [Delhi 2008]
15. Derive a relation between potential gradient and electric field strength.
16. Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d . [Delhi 2010, 2014]
17. A 500 mC charge is at the centre of a square of side 10 cm . Find the work done in moving a charge of 10 mC between two diagonally opposite points on the square. [Delhi 2008]
18. Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown : Here $q = 1.6 \times 10^{-10} \text{ C}$. [Delhi 2008]

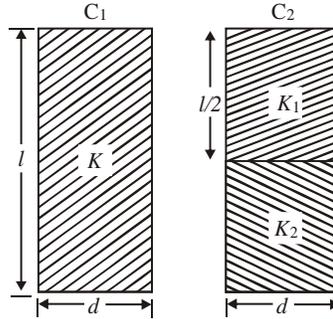


19. Calculate the potential at a point 9 cm away from a charge $4 \times 10^{-7} \text{ C}$. Hence obtain the work done in bringing a charge of $2 \times 10^{-9} \text{ C}$ from infinity to that point.
20. Two charges $+q$ and $-q$ are located at points $A(0, 0, -2)$ and $B(0, 0, 2)$ respectively. How much work will be done in moving a test charge from point $P(4, 0, 0)$ to $Q(-5, 0, 0)$? [Delhi 2009]
21. Calculate the amount of work done to dissociate a system of three charges $1 \mu\text{C}$, $1 \mu\text{C}$ and $-4 \mu\text{C}$ placed on the vertices of an equilateral triangle of side 10 cm . [All India 2013C]
22. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero? [Delhi 2016]

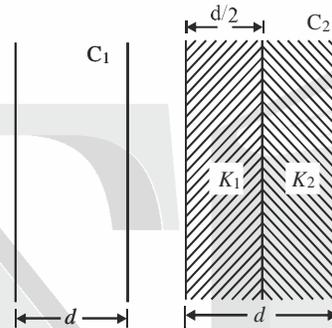


23. Define capacitance of a capacitor. Give its S.I. unit.
24. Differentiate between a dielectric and a conductor. [Delhi 2012]
25. A parallel plate capacitor is charged to a potential difference, V by a DC source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change?
(i) Electric field between the plates. (ii) Capacitance. (iii) Energy stored in the capacitor. [Delhi 2010]
26. The electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region. Why? [Foreign 2010]
27. What are the uses of capacitor? [Foreign 2012]
28. State the effect of filling a dielectric in a capacitor with battery connected.
29. State the effect of filling a dielectric in a capacitor after disconnecting the battery.
30. Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor. [All India 2008, Delhi 2014]
31. Find the ratio of the potential differences that must be applied across the parallel and the series combination of two identical capacitors so that the energy stored, in the two cases, becomes the same. [Foreign 2010]

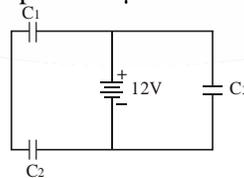
32. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. [Foreign 2010, All India 2013, Delhi 2013]
33. Two identical parallel plate (air) capacitors C_1 and C_2 have capacitance C each. The space between their plates is now filled with dielectrics as shown. If the two capacitors still have equal capacitance, obtain the relation between dielectric constants K , K_1 and K_2 .



34. You are given an air filled parallel plate capacitor C_1 . The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in C_2 . Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d . A is a filled parallel plate capacitor. [Foreign 2011]



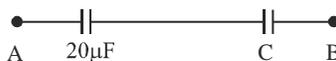
35. A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of $100 \mu\text{C}/\text{m}^2$. Calculate the : i. charge on the sphere ii. total electric flux passing through the sphere [Delhi 2008]
36. Two parallel plate capacitor, X and Y, have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\epsilon_r = 4$.
 i. Calculate capacitance of each capacitor if equivalent capacitance of the combination is $4 \mu\text{F}$.
 ii. Calculate the potential difference between the plates of X and Y.
 iii. What is the ratio of electrostatic energy stored in X and Y? [Delhi 2009]
37. Three identical capacitors C_1 , C_2 , & C_3 of capacitance μF each are connected a 12V battery as shown. [Delhi 2009]



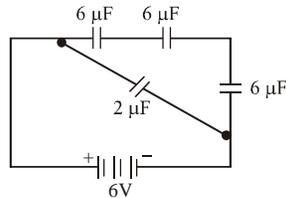
- Find:** i. Charge on each capacitor ii. Equivalent capacitance of the network iii. Energy stored in the network of capacitors
38. The equivalent capacitance of the combination between A and B in the given figure is $15 \mu\text{F}$. Calculate the capacitance of capacitor C.



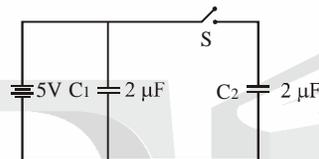
39. The equivalent capacitance of the combination of two capacitors between A and B in the given figure is $4 \mu\text{F}$.



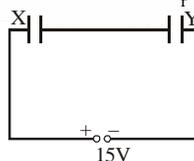
- i. Calculate capacitance of the capacitor C.
 ii. Calculate charge on each capacitor if a 12V battery is connected across terminals A and B.
 iii. What will be the potential drop across each capacitor? **[Delhi 2009C]**
40. A capacitor of 200 pF is charged by a 300V battery. The battery is then disconnected and the charge capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. **[All India 2010, Foreign 2012]**
41. Four capacitors of values $6\ \mu\text{F}$, $6\ \mu\text{F}$, $6\ \mu\text{F}$ and $2\ \mu\text{F}$ are are connected to a 6V battery as shown in the figure. Determine.



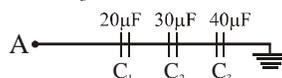
- i. Equivalent capacitance of the network. **ii.** The charge on each capacitor. **[Delhi 2010C]**
42. Figure shows two identical capacitors C_1 and C_2 , each of $2\ \mu\text{F}$ capacitance, connected to a battery of 5V. Initially switch 'S' is closed. After some time S is left open and dielectric slabs of dielectric constant $K = 5$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted? **[Delhi 2011]**



43. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is $360\ \mu\text{C}$. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes $120\ \mu\text{C}$.
 i. Calculate the potential V and the unknown capacitance C.
 ii. What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V? **[Delhi 2013]**
44. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is $300\ \mu\text{C}$. When potential across the capacitor is reduced by 100 V, the charge stored in it becomes $100\ \mu\text{C}$. Calculate the potential V and the unknown capacitance. What will be the charge stored in the capacitor if the voltage applied had increased by 100 V? **[Delhi 2013]**
45. a. Derive the expression for the energy stored in parallel plate capacitor. Hence obtain the expression for the energy density of the electric field. **[Delhi 2015]**
 b. A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than stored initially in the single capacitor.
46. Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\epsilon_r = 4$. **[Delhi 2016]**



- a. Calculate capacitance of each capacitor if equivalent capacitance of the combination is $4\ \mu\text{F}$.
 b. Calculate the potential difference between the plates of X and Y.
 c. Estimate the ratio of electrostatic energy stored in X and Y.
47. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V. $C_1 = 20\ \mu\text{F}$, $C_2 = 30\ \mu\text{F}$, $C_3 = 15\ \mu\text{F}$. **[Delhi 2016]**



EXERCISE ANSWERS

3. a. zero b. $\frac{Kp \cos \theta}{r^2}$ c. zero 4. $\left(\vec{E} = \frac{dV}{dr}\right)$ 5. $q_1 = q_2$ 6. $W_{AC} > W_{CA}$ 7. zero 9. equal 10. no change

11. zero 12. $0.9 \mu\text{C}$ 13. $360\sqrt{2}$ volt 14. 0.45J 15. $\frac{-3Kq^2}{d^2}$ 16. a. zero b. -2100 volt c. -2100 volt

17. $\text{ML}^2 \text{t}^{-3} \text{A}^{-1}$ 18. 10cm from $5 \times 10^{-8} \text{C}$ 19. a. at mid point b. Perpendicular 20. Electric Potential, Scalar

21. $q/Q = 4$ 22. -18 volt 24. a. 1000N/C b. zero 25. a. $Q_1 : +ve$; $Q_2 : -ve$ b. $Q_2 > Q_1$

26. a. +ve, +ve b. -ve, -ve c. (+ve) d. K.E. decrease 27. a. 0.04E b. Point A 28. $1/K$ times 30. 1.2J

31. b. Yes 33. E_R Minimum ; E_p Maximum 38. $x = 2\text{m}$ from $4Q$ charge, 1.44mJ 40. $2 \times 10^4 \text{NC}^{-1}$

41. Minimum number of capacitors = 18 42. Equivalent capacitance = $200/3 \text{PF}$: $Q_1 = 10^{-8} \text{C}$; $Q_2 = 10^{-8} \text{C}$;
 $Q_3 = 10^{-8} \text{C}$; $Q_4 = 2 \times 10^{-8} \text{C}$; $V_1 = 100 \text{V}$; $V_2 = 50 \text{V}$; $V_3 = 50 \text{V}$; $V_4 = 200 \text{V}$

43. $2.67 \times 10^{-2} \text{J}$ 44. and $q_3 = \left(\frac{C_3}{C_1 + C_2 + C_3}\right) Q$ 46. $C_1 = 4 \mu\text{F}$ or $12 \mu\text{F}$; $C_2 = 12 \mu\text{F}$ or $4 \mu\text{F}$

47. $C = \frac{2\epsilon_0 A}{d} \left[\frac{K_1 K_2}{K_1 + K_2}\right]$ 48. a. Charge will remain unchanged b. New P.D. $V' = \frac{Q}{C'} = 2V = 2\epsilon_0$

c. New E.F. $V' = \frac{V'}{d'} = \frac{2\epsilon_0}{2d} = \frac{\epsilon_0}{d} = E = \text{original}$ d. New energy stored = $2u = \text{twice the original}$

49. a. Capacitance = 3 times the original capacitance = $3C$ b. New P.D. = $\frac{V}{3}$ (becomes $\frac{1}{3}$ rd)

c. New field = $\frac{E}{3}$ (becomes $\frac{1}{3}$ rd) d. New energy stored = $\frac{1}{3} u$. (becomes $\frac{1}{3}$ rd)

50. $\frac{U_1}{U_2} = \frac{C_2}{C_1} = 5$ 51. $C = 20 \mu\text{F}$. Total charge $Q = 2 \times 10^{-3} \text{C}$ 52. $D = 8 \text{mm}$ 54. $C = \frac{3\epsilon_0 A}{d}$

55. $C = \frac{3\epsilon_0 A}{3d} (K_1 + K_2 + K_3)$ 56. $C = 1 \mu\text{F}$ 57. $C = 2 \mu\text{F}$ 60. $C = \frac{\epsilon_0 A}{3d}$ (No change) 61. $C_1 : C_2 = 1 : 9$

62. $V_{AB} = 25 \text{V}$; $V_{BC} = 75 \text{V}$ 63. $U_1 : U_2 = 3 : 5$ 64. $V_B = 500 \text{V}$ 67. $V = 3 \text{V}$ 68. $C_1 = 3 \mu\text{F}$ $C_2 = 3 \mu\text{F}$
 $V_1 = 2 \text{V}$ $V_2 = 2 \text{V}$

70. Equivalent Capacitance $C = 16 \mu\text{F}$; Charge on C_1, C_2 and $C_3 = 200 \mu\text{C}$. on $C_4 = 6000 \mu\text{C}$.

73. $Q = 1.77 \times 10^{-9} \text{C}$. 74. Equivalent capacitance = $8 \mu\text{F}$; $Z = \frac{17}{4} \mu\text{F}$ 77. $6 \times 10^{-6} \text{J}$.