EXERCISE 3.1

Choose the correct answer from the given four options in the following questions:

Q1. Graphically, the pair of equations

$$6x - 3y + 10 = 0$$
$$2x - y + 9 = 0$$

represents two lines which are

(*a*) intersecting at exactly one point
(*b*) intersecting at exactly two points
(*c*) coincident
(*d*) parallel

Sol. (*d*): Here, $\frac{a_1}{a_2} = \frac{6}{2} = 3$, $\frac{b_1}{b_2} = \frac{-3}{-1} = 3$, $\frac{c_1}{c_2} = \frac{10}{9}$

$$\therefore \qquad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the system of linear equations is inconsistent (no solution) and graph will be a pair of parallel lines.

Q2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

(*a*) a unique solution (*b*) exactly two solutions
(*c*) infinitely many solutions (*d*) no solution

Sol. (*d*): Here, $\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}$, $\frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3}$, $\frac{c_1}{c_2} = \frac{5}{1}$

$$\therefore \qquad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the system of linear equations has no solution.

Q3. If a pair of linear equations is consistent, then the lines will be

(*a*) parallel (*b*) always coincident
(*c*) intersecting or coincident (*d*) always intersecting

Sol. (*c*): Condition for consistency

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ have unique solution (consistent) *i.e.*, intersecting at one point

or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (consistent lines, coincident or depend)

Q4. The pair of equations $y = 0$ and $y = -7$ has

(*a*) one solution (*b*) two solutions
(*c*) infinitely many solutions (*d*) no solution

Sol. (*d*): We know that equation of the form $y = a$ is a line parallel to x-axis at a distance 'a' from it. $y = 0$ is the equation of the x-axis and $y = -7$ is is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines. Therefore, there is no solution. Hence, (*d*) is the correct answer.

Q5. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(*a*) parallel (*b*) intersecting at (b, a)
(*c*) coincident (*d*) intersecting at (a, b)

Sol. (*d*): $x = a$ is the equation of a straight line parallel to the y-axis at a distance 'a' from it. Again, $y = b$ is the equation of a straight line parallel to the x-axis at a distance 'b' from it.

So, the pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

Hence, (*d*) is the correct answer.

Q6. For what value of k do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?

(*a*) $\frac{1}{2}$ (*b*) $\frac{-1}{2}$ (*c*) 2 (*d*) -2

Sol. (*c*):

$$3x - y = -8 \quad \text{...(i)}$$

$$6x - ky = -16 \quad \text{...(ii)}$$

For coincident lines,

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{-8}{-16}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$$

So, $k = 2$.

Q7. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(*a*) $\frac{-5}{4}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{15}{4}$ (*d*) $\frac{3}{2}$

Sol. (*c*): For parallel lines (or no solution)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{2} = \frac{2k}{5} \neq \frac{2}{-1}$$

$\Rightarrow \qquad 4k = 15$

$\Rightarrow \qquad k = \dfrac{15}{4}$

Q8. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is

(*a*) 3 (*b*) –3 (*c*) –12 (*d*) No value

Sol. (*d*): For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$$

Ratio I II III

From ratios I and II, $2c = 6 \Rightarrow c = 3$

From ratios I and III, $3c = 12 \Rightarrow c = 4$

As from the ratios, values of c are not common. So, there is no value of c for which lines have many solutions.

Q9. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be

(*a*) $10x + 14y + 4 = 0$ (*b*) $-10x - 14y + 4 = 0$

(*c*) $-10x + 14y + 4 = 0$ (*d*) $10x - 14y = -4$

Sol. (*d*): $-5x + 7y - 2 = 0$ ...(*i*)

$a_2x + b_2y + c_2 = 0$ ...(*ii*)

$\therefore$ For dependent system of linear equations

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{-5}{a_2} = \frac{7}{b_2} = \frac{-2}{c_2} = \frac{1}{k}$$

So, $a_2 = -5k,\quad b_2 = 7k,\quad c_2 = -2k$

$k = 0$, and 1, does not satisfy the required condition.

For $k = -2$, $a_2 = +10$, $b_2 = -14$ and $c_2 = +4$ satisfies the condition.

i.e., $\dfrac{-5}{+10} = \dfrac{7}{-14} = \dfrac{-2}{+4} = \dfrac{-1}{2}$ satisfies the condition.

Q10. A pair of linear equations which has a unique solution $x = 2$, $y = -3$ is

(*a*) $x + y = -1$ and $2x - 3y = -5$

(*b*) $2x + 5y = -11$ and $4x + 10y = -22$

(*c*) $2x - y = 1$ and $3x + 2y = 0$

(*d*) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

Sol. (*b* and *d*): As $x = 2$, $y = -3$ is unique solution of system of equations so these values must satisfy both equations.

(*a*) $x + y = -1$ and $2x - 3y = -5$
Put $x = 2$ and $y = -3$ in both the equations.
LHS $= x + y \Rightarrow 2 - 3 = -1$ (RHS)
LHS $= 2x - 3y \Rightarrow 2(2) - 3(-3) \Rightarrow 4 + 9 = 13 \neq$ RHS
(*b*) $2x + 5y = -11$ and $4x + 10y = -22$
Put $x = 2$ and $y = -3$ in both the equations.
LHS $= 2x + 5y \Rightarrow 2 \times 2 + 5(-3) \Rightarrow 4 - 15 = -11 =$ RHS
LHS $= 4x + 10y \Rightarrow 4(2) + 10(-3) \Rightarrow 8 - 30 = -22 =$ RHS
(*c*) $2x - y = 1$ and $3x + 2y = 0$
Put $x = 2$ and $y = -3$ in both the equations.
LHS $= 2x - y \Rightarrow 2(2) + 3 \Rightarrow 7 \neq$ RHS
LHS $= 3x + 2y \Rightarrow 3(2) + 2(-3) \Rightarrow 6 - 6 = 0 =$ RHS
(*d*) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$
$x - 4y = 14$ and $5x - y = 13$
Put $x = 2$ and $y = -3$ in both the equations.
LHS $= x - 4y \Rightarrow 2 - 4(-3) \Rightarrow 2 + 12 = 14 =$ RHS
LHS $= 5x - y \Rightarrow 5(2) - (-3) \Rightarrow 10 + 3 = 13 =$ RHS

Hence, the pair of equations is (*b*) and (*d*).

Q11. If $x = a$, $y = b$, is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(*a*) 3 and 5 (*b*) 5 and 3 (*c*) 3 and 1 (*d*) -1 and -3

Sol. (*c*): If (a, b) is the solution of the given equations, then it must satisfy the given equations so,

$$a - b = 2 \quad ...(i)$$
$$a + b = 4 \quad ...(ii)$$
$$\Rightarrow \quad 2a = 6 \quad \text{[Adding } (i) \text{ and } (ii)]$$
$$\Rightarrow \quad a = 3$$
$$\text{Now,} \quad 3 + b = 4 \quad \text{[From } (ii)]$$
$$\Rightarrow \quad b = 1$$

So, $(a, b) = (3, 1)$.

Q12. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(*a*) 35 and 15 (*b*) 35 and 20 (*c*) 15 and 35 (*d*) 25 and 25

Sol. (*d*): Let the number of ₹ 1 coins $= x$
and the number of ₹ 2 coins $= y$

So, according to the question

$$x + y = 50 \quad (i)$$
$$1x + 2y = 75 \quad ...(ii)$$
$$2x + 2y = 100 \quad [(i) \times 2]$$
$$\underline{\overset{-}{}1x \overset{-}{+} 2y = \overset{-}{}75} \quad \text{[From } (ii)]$$
$$x \quad = 25$$

Now, $25 + y = 50 \Rightarrow y = 25$ [From (*i*)]

So, $y = 25$ and $x = 25$.

Q13. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, (in years) of the son and the father are, respectively

(*a*) 4 and 24 (*b*) 5 and 30 (*c*) 6 and 36 (*d*) 3 and 24

Sol. (*c*): Let the present age of father be x years
and the present age of son be y years.

$\therefore$ According to the question, $x = 6y$ ...(*i*)

Age of father after four years = $(x + 4)$ years
and the age of son after four years = $(y + 4)$ years

Now, according to the question,

$$x + 4 = 4(y + 4) \quad ...(ii)$$

$$\Rightarrow \quad x + 4 = 4y + 16$$

$$\Rightarrow \quad 6y - 4y = 16 - 4 \quad [\because x = 6y]$$

$$\Rightarrow \quad 2y = 12$$

$$\Rightarrow \quad y = 6$$

$$\therefore \quad x = 6 \times 6 = 36 \text{ years} \quad [\text{From } (i)]$$

and $y = 6$ years

So, the present ages of the son and the father are 6 years and 36 years respectively.

EXERCISE 3.2

Q1. Do the following pair of linear equations have no solution? Justify your answer.

(*i*) $2x + 4y = 3$ and $12y + 6x = 6$

(*ii*) $x = 2y$ and $y = 2x$

(*iii*) $3x + y - 3 = 0$ and $2x + \frac{2}{3}y = 2$

Sol. The system of linear equations has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

(*i*) $2x + 4y = 3$ and $6x + 12y = 6$

Here, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$

$\therefore \frac{2}{6} = \frac{4}{12} \neq \frac{3}{6}$

So, the given system of linear equations has no solution.

(*ii*) $x - 2y = 0$ and $2x - y = 0$

Here, $\frac{a_1}{a_2} = \frac{1}{2}$

and $\frac{b_1}{b_2} = \frac{-2}{-1} = 2$

So, the given system of linear equations does not satisfy $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

(*iii*) $3x + y - 3 = 0$ and $2x + \frac{2}{3}y = 2$

Here, $\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{1}{2/3} = \frac{3}{2}$, $\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

So, the given system of linear equations does not satisfy $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Q2. Do the following equations represent a pair of coincident lines? Justifies your answer.

(*i*) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$

(*ii*) $-2x - 3y = 1$ and $6y + 4x = -2$

(*iii*) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Sol. Condition for coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{...}(i)$$

(*i*) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$

Here, $\frac{a_1}{a_2} = \frac{3}{7}$, $\frac{b_1}{b_2} = \frac{\frac{1}{7}}{3} = \frac{1}{21}$ and $\frac{c_1}{c_2} = \frac{3}{7}$

So, the given system of linear equations does not satisfy condition (*i*).

(*ii*) $-2x - 3y = 1$ and $6y + 4x = -2$

Here, $\frac{a_1}{a_2} = \frac{-2}{4} = \frac{-1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{1}{-2}$

So, the given system of linear equations does not satisfy given condition (*i*).

(*iii*) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Here, $\frac{a_1}{a_2} = \frac{\frac{1}{2}}{4} = \frac{1}{8}$, $\frac{b_1}{b_2} = \frac{1}{8}$ and $\frac{c_1}{c_2} = \frac{\frac{2}{5}}{\frac{5}{16}} = \frac{32}{25}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of linear equations does not satisfy condition (*i*).

Q3. Are the following pair of linear equations consistent? Justify your answer.

(*i*) $-3x - 4y = 12$ and $4y + 3x = 12$

(*ii*) $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$

(*iii*) $2ax + by = a$ and $4ax + 2by - 2a = 0,\ a, b \neq 0$

(*iv*) $x + 3y = 11$ and $2(2x + 6y) = 22$

Sol. For consistent system of linear equations $a, b \neq 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{(infinitely many solutions)}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{(unique solution)}$$

(*i*) $-3x - 4y = 12$ and $4y + 3x = 12$

Here, $\frac{a_1}{a_2} = \frac{-3}{3} = -1,\quad \frac{b_1}{b_2} = \frac{-4}{4} = -1$ and $\frac{c_1}{c_2} = \frac{12}{12} = 1$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations is inconsistent and has no solution.

(*ii*) $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$

Here, $\frac{a_1}{a_2} = \frac{3/5}{1/5} = 3,\quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{1/2}{1/6} = \frac{6}{2} = 3$

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given pair of linear equations is consistent and has unique solution.

(*iii*) $2ax + by = a$ and $4ax + 2by - 2a = 0$

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2},\quad \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{a}{2a} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given pair of linear equations is consistent and has infinitely many solutions.

(*iv*) $x + 3y = 11$ and $2(2x + 6y) = 22$

or $x + 3y = 11$ and $4x + 12y = 22$

Here, $\frac{a_1}{a_2} = \frac{1}{4}$, $\frac{b_1}{b_2} = \frac{3}{12} = \frac{1}{4}$ and $\frac{c_1}{c_2} = \frac{11}{22} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations is inconsistent and has no solution.

Q4. For the pair of equations, $\lambda x + 3y = -7$ and $2x + 6y = 14$, to have infinitely many solutions the value of λ should be 1. Is the statement true? Give reasons.

Sol. $\lambda x + 3y + 7 = 0$ ...(*i*)

$2x + 6y - 14 = 0$ ...(*ii*)

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{\lambda}{2},\ \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},\ \frac{c_1}{c_2} = \frac{7}{-14} = \frac{1}{-2}$$

So, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, for any value of λ.

Hence, the given statement is not true.

Q5. For all real values of c, the pair of equations $x - 2y = 8$ and $5x - 10y = c$ have a unique solution. Justify whether it is true or false.

Sol. (False) System of linear equations are

$$x - 2y = 8 \quad ...(i)$$

$$5x - 10y = c \quad ...(ii)$$

$$\therefore \frac{a_1}{a_2} = \frac{1}{5},\ \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ so system of linear equations can never have unique solution.

Hence, the given statement is false.

Q6. The line represented by $x = 7$ is parallel to x-axis. Justify whether the statement is true or not.

Sol. The line represented by $x = 7$ is of the form $x = a$. The graph of the equation is a line parallel to the y-axis.

Hence, the given statement is not true.

EXERCISE 3.3

Q1. For which value(s) of λ do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(*i*) no solution? (*ii*) infinitely many solutions?

(*iii*) a unique solution?

Sol. $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

(*i*) For no solution, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

$$\Rightarrow \quad \underset{\text{I}}{\frac{\lambda}{1}} = \underset{\text{II}}{\frac{1}{\lambda}} \neq \underset{\text{III}}{\frac{\lambda^2}{1}}$$

From ratio I and II, we get

$$\lambda^2 = 1$$

$$\Rightarrow \quad \lambda = \pm 1$$

(*ii*) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\Rightarrow \quad \lambda^2 = 1 \quad \Rightarrow \quad \lambda = \pm 1$$

Also, $\dfrac{\lambda}{1} = \dfrac{\lambda^2}{1}$

$$\Rightarrow \quad \lambda^2 = \lambda$$

$$\Rightarrow \quad \lambda^2 - \lambda = 0$$

$$\Rightarrow \quad \lambda(\lambda - 1) = 0$$

$$\Rightarrow \quad \lambda = 0, \quad \lambda = 1$$

$\therefore$ Common solution for which the pair of linear equations has infinitely many solutions is $\lambda = 1$ only.

(*iii*) For unique solution, $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

$$\therefore \quad \frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda^2 \neq 1 \quad \text{or} \quad \lambda \neq 1, -1$$

So, for unique solution all real values except $\lambda = 1, -1$.

Q2. For which value(s) of k will the pair of equations $kx + 3y = k - 3$ and $12x + ky = k$ has no solution?

Sol.
$$kx + 3y = k - 3$$
$$12x + ky = k$$

System of eqns. will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} \neq \frac{k-3}{k}$$

$$\Rightarrow \quad k^2 = 36$$

$$\Rightarrow \quad k = \pm 6$$

Also, $$\frac{3}{k} \neq \frac{(k-3)}{k}$$

$$\Rightarrow \quad k^2 - 3k \neq 3k$$

$$\Rightarrow \quad k^2 - 3k - 3k \neq 0$$

$$\Rightarrow \quad k^2 - 6k \neq 0$$

$$\Rightarrow \quad k(k-6) \neq 0$$

$$\Rightarrow \quad k \neq 0 \quad \text{and } k \neq 6$$

So, the value of k for which the system of linear equations has no solution is $k = -6$.

Q3. For which values of a and b, will the following pair of linear equations has infinitely many solutions?

$$x + 2y = 1$$

$$(a-b)x + (a+b)y = a + b - 2$$

Sol. For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \underset{\text{I}}{\frac{1}{(a-b)}} = \underset{\text{II}}{\frac{2}{(a+b)}} = \underset{\text{III}}{\frac{1}{(a+b-2)}}$$

From ratios I and II,

$$2a - 2b = a + b$$

$$\Rightarrow \quad a - 3b = 0 \qquad ...(i)$$

From ratios II and III,

$$2a + 2b - 4 = a + b$$

$$\Rightarrow \quad a + b = 4 \qquad ...(ii)$$

Now, solving (i) and (ii), we have

$$\begin{array}{rl} a - 3b = 0 & ...(i) \\ \underset{-}{} a \underset{-}{+} b = \underset{-}{} 4 & ...(ii) \\ \hline -4b = -4 & \end{array} \qquad \text{[Subtracting } (ii) \text{ from } (i)]$$

$$\Rightarrow \quad b = 1$$

and $$a = 4 - b \qquad \text{[From } (ii)]$$

$$\Rightarrow \quad a = 4 - 1$$

$$\Rightarrow \quad a = 3$$

Q4. Find the values of p in (*i*) to (*iv*) and p and q in (*v*) for the following pair of equations:

(*i*) $3x - y - 5 = 0$ and $6x - 2y - p = 0$, if the lines represented by these equations are parallel.

(*ii*) $-x + py = 1$ and $px - y = 1$, if the pair of equations has no solution.

(*iii*) $-3x + 5y = 7$ and $2px - 3y = 1$, if the lines represented by these equations are intersecting at a unique point.

(*iv*) $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$, if the pair of equations has a unique solution.

(*v*) $2x + 3y = 7$ and $2px + py = 28 - qy$, if the pair of equations has infinitely many solutions.

Sol. (*i*) Given equations are

$$3x - y - 5 = 0 \qquad ...(i)$$
$$6x - 2y - p = 0 \qquad ...(ii)$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-5}{-p} = \frac{5}{p}$$

The lines will be parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2} \neq \frac{5}{p}$$

$$\Rightarrow \quad p \neq 10$$

So, the given lines are parallel for all real values of p except 10.

(*ii*) Given pair of equations is

$$-x + py = 1 \qquad ...(i)$$
$$px - y = 1 \qquad ...(ii)$$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \quad \underset{\text{I}}{-\frac{1}{p}} = \underset{\text{II}}{\frac{p}{-1}} \neq \underset{\text{III}}{\frac{1}{1}}$$

From ratios I and II, $p^2 = 1$ or $p = \pm 1$

Using ratios II and III, $p \neq -1$

$\therefore$ For $p = 1$, the given equations have not any solution.

(*iii*) Pair of equations is

$$-3x + 5y = 7 \qquad ...(i)$$
$$2px - 3y = 1 \qquad ...(ii)$$

For unique solution, we have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \qquad \frac{-3}{2p} \neq \frac{5}{-3}$$

$$\Rightarrow \qquad 10p \neq +9 \quad \Rightarrow \quad p \neq \frac{9}{10}$$

Hence, the given equations have unique solution for all real values of p except $\frac{9}{10}$.

(*iv*) Pair of equations is

$$2x + 3y - 5 = 0 \qquad ...(i)$$

$$px - 6y - 8 = 0 \qquad ...(ii)$$

Pair of equations have unique solution if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \qquad \frac{2}{p} \neq \frac{3}{-6}$$

$$\Rightarrow \qquad 3p \neq -2 \times 6$$

$$\Rightarrow \qquad p \neq -\frac{12}{3}$$

$$\Rightarrow \qquad p \neq -4$$

Hence, the system of linear equations has unique solution for all real values of p except -4.

(*v*) Given system of linear equations is

$$2px + py = 28 - qy$$

i.e., $$2px + (p + q)y = 28 \qquad ...(i)$$

$$2x + 3y = 7 \qquad ...(ii)$$

The system of equations will have infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \qquad \underset{\text{I}}{\frac{2p}{2}} = \underset{\text{II}}{\frac{p+q}{3}} = \underset{\text{III}}{\frac{28}{7}}$$

Using ratios I and II we get,

$$\frac{2p}{2} = \frac{p+q}{3}$$

$$\Rightarrow \qquad 3p = p + q$$

$$\Rightarrow \qquad 2p - q = 0$$

$$\Rightarrow \qquad q = 2p \qquad ...(iii)$$

Using ratios I and III, we get

$$\frac{2p}{2} = \frac{28}{7} \quad \Rightarrow \quad p = 4$$

$\therefore$ $q = 2p = 2 \times 4 = 8$ [From (*iii*)]

$\therefore$ $q = 8 \quad \text{and} \quad p = 4$

Now, $$\frac{2p}{2} = \frac{p+q}{3} = \frac{28}{7}$$

$\Rightarrow$ $$\frac{p}{1} = \frac{p+q}{3} = \frac{4}{1}$$

By substituting the values of p and q, we have

$$4 = \frac{4+8}{3} = 4$$

$\Rightarrow$ $$4 = \frac{12}{3} = 4$$

$\Rightarrow$ $$4 = 4 = 4$$

Hence, the given system of equations has infinitely many solutions when $p = 4$ and $q = 8$.

Q5. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths will cross each other or not.

Sol. Two straight paths are represented by the equations

$x - 3y = 2$ and $-2x + 6y = 5$.

For the paths to cross each other i.e., to intersect each other, we must have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Now, $\frac{a_1}{a_2} = \frac{1}{-2} = \frac{-1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

Hence, the two straight paths do not cross each other.

Q6. Write a pair of linear equations which has the unique solution $x = -1$, and $y = 3$. How many such pairs can you write ?

Sol. For system of linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

the lines has unique solution $x = -1$ and $y = 3$ so it must satisfy the above equations.

$\therefore$ $$a_1(-1) + b_1(3) + c_1 = 0$$

and $$a_2(-1) + b_2(3) + c_2 = 0$$

$\Rightarrow$ $$-a_1 + 3b_1 + c_1 = 0 \qquad ...(i)$$

and $$-a_2 + 3b_2 + c_2 = 0 \qquad ...(ii)$$

The restricted values of a_1, a_2 and b_1, b_2 are only

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \qquad ...(iii)$$

So, all the real values of a_1, a_2, b_1, b_2 except condition (*iii*) can form so many linear equations which will satisfy equations (*i*) and (*ii*) and have solution $x = -1$ and $y = 3$.

We can have infinite number of lines passing through $(-1, 3)$, which is the solution of intersecting lines at this $(-1, 3)$ point.

So, infinite number of pairs of system of equations are possible which has unique solution $x = -1$ and $y = 3$.

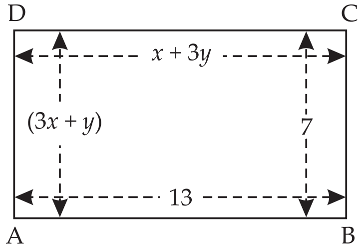
Q7. If $2x + y = 23$ and $4x - y = 19$, then find the values of $5y - 2x$ and $\frac{y}{x} - 2$.

Sol. Given equations are
$$2x + y = 23 \quad \text{...(}i\text{)}$$
$$4x - y = 19 \quad \text{...(}ii\text{)}$$
Adding equations (*i*) and (*ii*), we get
$$6x = 42$$
$$\Rightarrow \quad x = 7$$
Now,
$$2(7) + y = 23 \quad \text{[From (}i\text{)]}$$
$$\Rightarrow \quad y = 23 - 14$$
$$\Rightarrow \quad y = 9$$
So, $5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9 - 14}{7} = \frac{-5}{7}$

Hence, the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are 31 and $\frac{-5}{7}$ respectively.

Q8. Find the values of x and y in the following rectangle:



Sol. As the opposite sides of a rectangle are equal so by figure, we conclude that
$$3x + y = 7 \quad \text{...(}i\text{)}$$
$$x + 3y = 13 \quad \text{...(}ii\text{)}$$
$$9x + 3y = 21 \quad \text{[From (}i\text{)]}$$
$$x + 3y = 13$$
$$\underline{- \quad - \quad\quad -}$$
$$8x = 8 \quad \text{[Subtracting (}ii\text{) from (}i\text{)]}$$
$$x = 1$$

Now, $3(1) + y = 7$ [From (*i*)]

$\Rightarrow$ $y = 7 - 3$

$\Rightarrow$ $y = 4$

and $x = 1$

Hence, the required values of x and y are 1 and 4 respectively.

Q9. Solve the following pairs of linear equations:

(*i*) $x + y = 3.3, \quad \dfrac{0.6}{3x - 2y} = -1, \quad 3x - 2y \neq 0$

(*ii*) $\dfrac{x}{3} + \dfrac{y}{4} = 4, \quad \dfrac{5x}{6} - \dfrac{y}{8} = 4$

(*iii*) $4x + \dfrac{6}{y} = 15, \quad 6x - \dfrac{8}{y} = 14, \quad y \neq 0$

(*iv*) $\dfrac{1}{2x} - \dfrac{1}{y} = -1, \quad \dfrac{1}{x} + \dfrac{1}{2y} = 8, \quad x, y \neq 0$

(*v*) $43x + 67y = -24, \quad 67x + 43y = 24$

(*vi*) $\dfrac{x}{a} + \dfrac{y}{b} = a + b, \quad \dfrac{x}{a^2} + \dfrac{y}{b^2} = 2, \quad a, b \neq 0$

(*vii*) $\dfrac{2xy}{x + y} = \dfrac{3}{2}, \quad \dfrac{xy}{2x - y} = \dfrac{-3}{10}, \quad x + y \neq 0, \quad 2x - y \neq 0$

Sol. Some important rules for easy solution.

- Fraction in which constants are in denominators, convert the equation in the form of $ax + by = c$ by multiplying the equation both sides by LCM of denominators of equation.
- We can not multiply an equation by variable unless variable is not zero. If variables are not zero, then we can multiply by variables also.
- If in system of equations, x or y or both are in denominator and are symmetric no need to remove denominator.
- Remove decimals and again remove denominators by multiplying LCM of denominator to both sides.

(*i*) We have $x + y = 3.3$ ...(*i*)

$$\frac{0.6}{3x - 2y} = -1 \quad \text{...}(ii)$$

On multiplying eqn. (*i*) by 20, we get

$$20x + 20y = 66 \quad \text{...}(iii)$$

From eqn. (*ii*), we have

$$-3x + 2y = 0.6$$

On multiplying it by (–10), we get

$$30x - 20y = -6 \quad \text{...}(iv)$$

Now, adding (*iii*) and (*iv*), we get

$$30x - 20y = -6 \quad ...(iv)$$
$$20x + 20y = 66 \quad ...(iii)$$
$$\overline{50x \qquad = 60}$$

$\Rightarrow$ $x = \dfrac{60}{50} \Rightarrow x = \dfrac{6}{5} \Rightarrow x = 1.2$

Now, $10(1.2) + 10y = 33$ [From (*i*)]

$\Rightarrow$ $12 + 10y = 33$

$\Rightarrow$ $10y = 33 - 12$

$\Rightarrow$ $y = \dfrac{21}{10} \Rightarrow y = 2.1$

$\therefore$ The solution of the given system of equations is $x = 1.2$, and $y = 2.1$.

(*ii*) $\dfrac{x}{3} + \dfrac{y}{4} = 4$ ...(*i*) × LCM 12

$\dfrac{5x}{6} - \dfrac{y}{8} = 4$ ...(*ii*) × LCM 24

$$4x + 3y = 48 \quad ...(iii)$$
$$20x - 3y = 96 \quad ...(iv)$$
$$\overline{24x \qquad = 144}$$ [By adding above two equations]

$\Rightarrow$ $x = \dfrac{144}{24} \Rightarrow x = 6$

Now, $4x + 3y = 48$ [From (*iii*)]

On putting the value of $x = 6$, we have

$4(6) + 3y = 48$

$\Rightarrow$ $3y = 48 - 24$

$\Rightarrow$ $3y = 24$

$\Rightarrow$ $y = \dfrac{24}{3} \Rightarrow y = 8$

So, the solution of the given equations is $x = 6$ and $y = 8$

(*iii*) $4x + \dfrac{6}{y} = 15$ ...(*i*) × 6 or 3

$6x - \dfrac{8}{y} = 14$ ...(*ii*) × 4 or 2

(As y is in denominators and symmetric so no need to remove denominator and 6 and 4 are divisible by 2 so we can multiply (*i*), (*ii*) by 3 and 2 respectively)

$$12x + \frac{18}{y} = 45 \quad ...(iii)$$

$$12x - \frac{16}{y} = 28 \quad ...(iv)$$

$$- \qquad + \qquad -$$

$$\frac{18}{y} + \frac{16}{y} = 17 \text{ [By subtracting } (iv) \text{ from } (iii)]$$

$$\frac{18+16}{y} = \frac{17}{1}$$

$\Rightarrow$ $$17y = 34$$

$\Rightarrow$ $$y = \frac{34}{+17} \Rightarrow y = 2$$

Now, $$12x + \frac{18}{y} = 45 \quad \text{[From } (iii)]$$

$\Rightarrow$ $$12x + \frac{18}{2} = 45 \quad [\because y = 2]$$

$\Rightarrow$ $$12x = 45 - 9$$

$\Rightarrow$ $$12x = 36 \Rightarrow x = 3 \text{ and } y = 2$$

(*iv*) Given equations are

$$\frac{1}{2x} - \frac{1}{y} = -1 \quad ...(i) \times \frac{1}{2}$$

$$\frac{1}{x} + \frac{1}{2y} = 8 \quad ...(ii) \times 1,\ x, y \neq 0$$

[x, y both are in denominator and symmetric so no need to convert into linear equation hence, can be eliminated directly]

Multiplying eqn. (*i*) by the coefficient of $\frac{1}{y}$ in (*ii*) and vice versa, we have

$$\frac{1}{4x} - \frac{1}{2y} = -\frac{1}{2} \quad ...(iii)$$

$$\frac{1}{x} + \frac{1}{2y} = 8 \quad ...(iv)$$

$$\frac{1}{4x} + \frac{1}{x} = 8 - \frac{1}{2} \quad \text{[Adding eqns } (iii) \text{ and } (iv)]$$

$\Rightarrow$ $$\frac{1+4}{4x} = \frac{16-1}{2}$$

$\Rightarrow$ $$\frac{5}{4x} = \frac{15}{2}$$

$$\Rightarrow \quad 15 \times 4x = 5 \times 2$$

$$\Rightarrow \quad x = \frac{5 \times 2}{15 \times 4} \quad \Rightarrow \quad x = \frac{1}{6}$$

Now, $$\frac{1}{x} + \frac{1}{2y} = 8 \qquad \text{[From } (iv)\text{]}$$

$$\Rightarrow \quad \frac{1}{\left(\frac{1}{6}\right)} + \frac{1}{2y} = 8 \qquad \left[\because x = \frac{1}{6}\right]$$

$$\Rightarrow \quad 6 + \frac{1}{2y} = 8 \quad \Rightarrow \quad \frac{1}{2y} = 8 - 6$$

$$\Rightarrow \quad \frac{1}{2y} = \frac{2}{1} \quad \Rightarrow \quad 4y = 1 \quad \Rightarrow \quad y = \frac{1}{4}$$

So, $x = \dfrac{1}{6}$ and $y = \dfrac{1}{4}$.

(*v*) Given pair of equations are

$$43x + 67y = -24 \qquad \ldots(i)$$

$$67x + 43y = 24 \qquad \ldots(ii)$$

$$\Rightarrow \quad 110x + 110y = 0 \qquad \text{[Adding } (i) \text{ and } (ii)\text{]}$$

$$\Rightarrow \quad x + y = 0 \qquad \ldots(iii)$$

Subtracting (*ii*) from (*i*), we have

$$-24x + 24y = -48$$

$$\Rightarrow \quad -x + y = -2 \qquad \ldots(iv)$$

$$x + y = 0 \qquad \text{[From } (iii)\text{]}$$

$$\Rightarrow \quad 2y = -2 \qquad \text{[Adding } (iii) \text{ and } (iv)\text{]}$$

$$y = -1$$

From (*iii*), $$x + y = 0$$

$$\Rightarrow \quad x + (-1) = 0 \qquad [\because y = -1]$$

$$\Rightarrow \quad x = 1$$

and $$y = -1$$

Hence, the solution of the given equations is $x = 1$, $y = -1$.

(*vi*) Pair of linear equations is,

$$\frac{x}{a} + \frac{y}{b} = a + b \qquad \ldots(i) \times ab$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2 \qquad \ldots(ii) \times a^2b^2,\ a, b \neq 0$$

Multiplying (*i*) by LCM ab, and (*ii*) by LCM a^2b^2, we have

$$bx + ay = a^2b + ab^2 \qquad \ldots(iii) \times a^2 \text{ or } a$$

$$b^2x + a^2y = 2a^2b^2 \qquad \ldots(iv) \times a \text{ or } 1$$

$$\begin{array}{rl} abx + a^2y = a^3b + a^2b^2 & ...(v) \\ b^2x + a^2y = 2a^2b^2 & ...(iv) \\ - \quad - \quad\quad - & \text{[Subtracting } (vi) \text{ from } (v)] \\ \hline abx - b^2x = a^3b - a^2b^2 & \end{array}$$

$\Rightarrow \quad bx\,(a-b) = a^2b\,(a-b)$

$\Rightarrow \quad x = \dfrac{a^2b(a-b)}{b(a-b)}$

$\Rightarrow \quad x = a^2$

Now, $\quad bx + ay = a^2b + ab^2$ [From (iii)]

$\Rightarrow \quad b(a^2) + ay = a^2b + ab^2$ $[\because x = a^2]$

$\Rightarrow \quad ay = a^2b + ab^2 - a^2b$

$\Rightarrow \quad ay = ab^2$

$\Rightarrow \quad y = \dfrac{ab^2}{a} \quad \Rightarrow \quad y = b^2$

So, the solution of the given equations is $x = a^2$ and $y = b^2$.

(vii) We have: $\quad \dfrac{2xy}{x+y} = \dfrac{3}{2}$ ...(i)

$$\frac{xy}{2x-y} = \frac{-3}{10} \quad ...(ii)$$

$[(x + y) \neq 0 \text{ and } (2x - y) \neq 0)]$

Inversing the eqn. (i), we get

$$\frac{x+y}{2xy} = \frac{2}{3}$$

$\Rightarrow \quad \dfrac{x}{2xy} + \dfrac{y}{2xy} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{1}{2y} + \dfrac{1}{2x} = \dfrac{2}{3}$ ...(iii)

Inversing the eqn. (ii), we get

$$\frac{2x-y}{xy} = \frac{10}{-3}$$

$\Rightarrow \quad \dfrac{2x}{xy} - \dfrac{y}{xy} = \dfrac{-10}{3}$

$\Rightarrow \quad \dfrac{2}{y} - \dfrac{1}{x} = \dfrac{-10}{3}$

$\Rightarrow \quad \dfrac{2}{2y} - \dfrac{1}{2x} = \dfrac{-5}{3}$ ...(iv)

Now, $\quad \dfrac{2}{2y} - \dfrac{1}{2x} = \dfrac{-5}{3}$ [From (iv)]

and $$\frac{1}{2y} + \frac{1}{2x} = \frac{2}{3}$$ [From (*iii*)]

$$\frac{2}{2y} + \frac{1}{2y} = -\frac{5}{3} + \frac{2}{3}$$ [Adding (*iii*) and (*iv*)]

$$\Rightarrow \quad \frac{2+1}{2y} = \frac{-5+2}{3} \quad \Rightarrow \quad \frac{3}{2y} = \frac{-3}{3}$$

$$\Rightarrow \quad \frac{3}{2y} = -1 \quad \Rightarrow \quad y = -\frac{3}{2}$$

Now, $$\frac{2}{2y} - \frac{1}{2x} = \frac{-5}{3}$$ [From (*iv*)]

$$\Rightarrow \quad \frac{2}{y} - \frac{1}{x} = \frac{-10}{3}$$

$$\Rightarrow \quad \frac{2}{\left(\frac{-3}{2}\right)} - \frac{1}{x} = \frac{-10}{3} \qquad \left[\because y = \frac{-3}{2}\right]$$

$$\Rightarrow \quad \frac{4}{-3} - \frac{1}{x} = \frac{-10}{3}$$

$$\Rightarrow \quad -\frac{1}{x} = \frac{-10}{3} + \frac{4}{3}$$

$$\Rightarrow \quad \frac{-1}{x} = \frac{-6}{3}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{+2}{1} \quad \Rightarrow \quad 2x = 1 \quad \Rightarrow \quad x = \frac{1}{2}$$

So, $x = \frac{1}{2}$ and $y = \frac{-3}{2}$.

Q10. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$. Hence, find λ, if $y = \lambda x + 5$.

Sol. Given equations are

$$\frac{x}{10} + \frac{y}{5} - 1 = 0 \qquad \ldots(i) \times 20$$

and $$\frac{x}{8} + \frac{y}{6} = 15 \qquad \ldots(ii) \times 24$$

i.e., $$2x + 4y = 20 \qquad \ldots(iii)$$

$$3x + 4y = 360 \qquad \ldots(iv)$$

$$\begin{array}{rcrcl} 2x & + & 4y & = & 20 \quad \ldots(iii) \\ 3x & + & 4y & = & 360 \quad \ldots(iv) \\ - & & - & & - \\ \hline -x & & & = & -340 \end{array}$$

[Subtracting (*iv*) from (*iii*)]

$$\Rightarrow \quad 4x = +340$$

Now, $\quad 2x + 4y = 20 \quad$ [From (*iii*)]

$\Rightarrow \quad x + 2y = 10$

$\Rightarrow \quad 340 + 2y = 10 \quad [\because x = 340]$

$\Rightarrow \quad 2y = 10 - 340$

$\Rightarrow \quad 2y = -330$

$\Rightarrow \quad y = \frac{-330}{2} = -165$

and $\quad x = 340$

Now, $\quad y = \lambda x + 5 \quad$ [Given]

$\Rightarrow \quad -165 = \lambda(340) + 5 \quad [\because y = -165 \text{ and } x = 340]$

$\Rightarrow \quad -\lambda(340) = 5 + 165$

$\Rightarrow \quad -\lambda(340) = 170$

$\Rightarrow \quad \lambda = \frac{170}{-340} \Rightarrow \lambda = -\frac{1}{2}$

Hence, the solution of the given pair of equations is $x = 340$, $y = -165$ and $\lambda = -\frac{1}{2}$.

Q11. By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(*i*) $3x + y + 4 = 0$ and $6x - 2y + 4 = 0$

(*ii*) $x - 2y = 6$ and $3x - 6y = 0$

(*iii*) $x + y = 3$ and $3x + 3y = 9$

Sol. (*i*) Given equations are

$$3x + y + 4 = 0 \quad ...(i)$$

$$6x - 2y + 4 = 0 \quad ...(ii)$$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{-2} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{4}{4} = 1$$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given pair of equations is consistent and has unique solution.

$3x + y + 4 = 0 \quad$ [From (*i*)]

$\Rightarrow \quad \boxed{y = -3x - 4}$

If $x = 0$, $\quad y = -3(0) - 4 = 0 - 4 = -4$

$x = 1$, $\quad y = -3(1) - 4 = -3 - 4 = -7$

$x = 2$, $\quad y = -3(2) - 4 = -6 - 4 = -10$

x	0	1	2
y	-4	-7	-10

$6x - 2y + 4 = 0 \quad$ [From (*ii*)]

$\Rightarrow \quad 3x - y + 2 = 0$

$\Rightarrow \quad -y = -3x - 2$

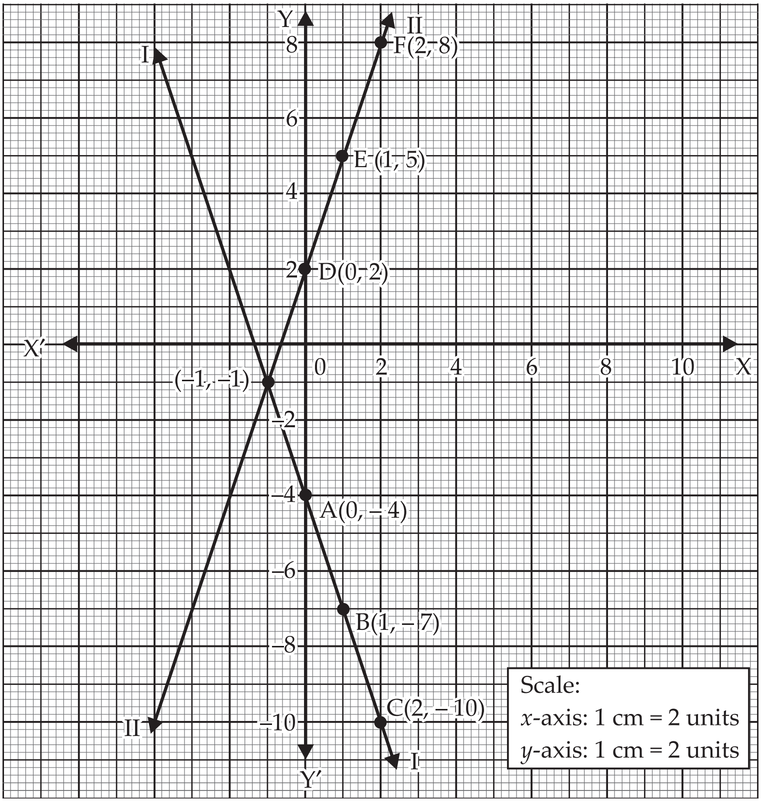
$\Rightarrow$ $\boxed{y = 3x + 2}$

If $x = 0$, $y = 3(0) + 2 = 0 + 2 = 2$

$x = 1$, $y = 3(1) + 2 = 3 + 2 = 5$

$x = 2$, $y = 3(2) + 2 = 6 + 2 = 8$

x	0	1	2
y	2	5	8



Intersecting point is (–1, –1) i.e., $x = -1$ and $y = -1$

(*ii*) Given equations are,

$$x - 2y = 6 \quad \text{...}(i)$$

$$3x - 6y = 0 \quad \text{...}(ii)$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-2}{-6} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{6}{0}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ System of equations is inconsistent. Hence, the lines represented by the given equations are parallel. So, the given equations have no solution.

(*iii*) Pair of equations is

$$x + y = 3 \quad \text{...}(i)$$
$$3x + 3y = 9 \quad \text{...}(ii)$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{3}{9} = \frac{1}{3}$$

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

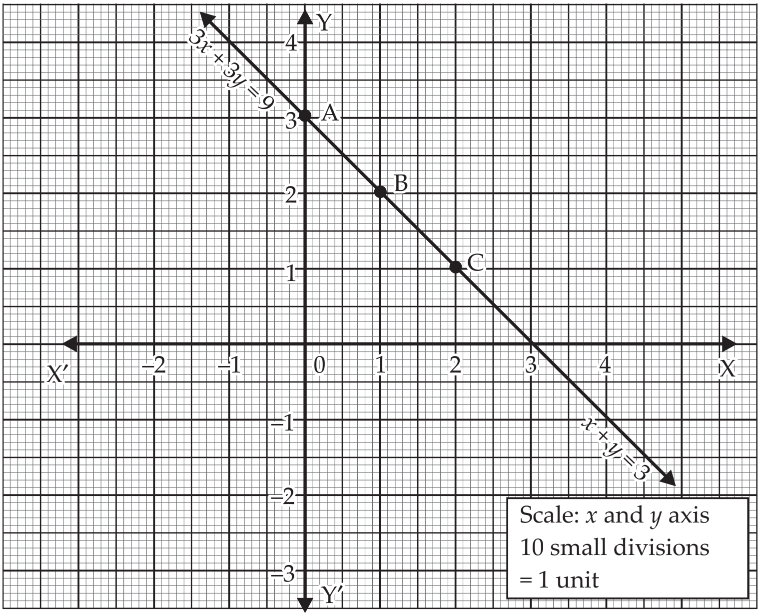
So, the system of given equations have infinitely many solutions. Graph will be overlapping so pair of equations is consistent.
As the lines are dependent so points on graph for both equations will be same. To draw, we can take any one equation.

$$x + y = 3$$
$$\Rightarrow \quad \boxed{y = 3 - x}$$

If $x = 0$, $y = 3 - 0 = 3$
$x = 1$, $y = 3 - 1 = 2$
$x = 2$, $y = 3 - 2 = 1$

Points for graph of equation (*i*) and (*ii*) are

x	0	1	2	3	4	5
y	3	2	1	0	–1	–2



So, the lines represented by the given equations are coinciding. Given equations are consistent.
Some solutions of system of equations are (0, 3), (1, 2), (2, 1), (3, 0), (4, –1) and (5, –2).

Q12. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

Sol. $2x + y = 4$ ...(*i*)

$\Rightarrow$ $\boxed{y = 4 - 2x}$

If $x = 0$, $y = 4 - 2(0) = 4 - 0 = 4$

$x = 1$, $y = 4 - 2(1) = 4 - 2 = 2$

$x = 2$, $y = 4 - 2(2) = 4 - 4 = 0$

x	0	1	2	3
y	4	2	0	-2
(i)	A	B	C	D

$2x - y = 4$ ...(*ii*)

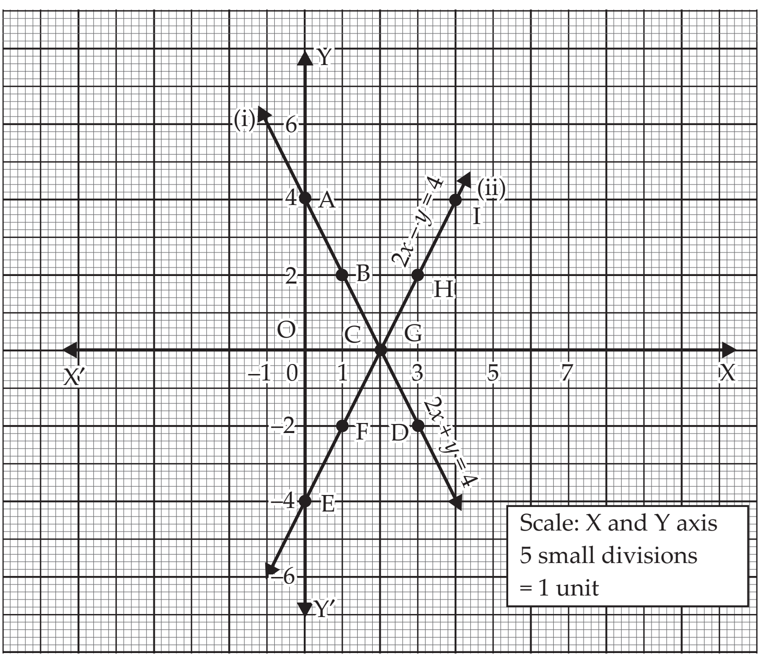
$\Rightarrow$ $\boxed{y = 2x - 4}$

If $x = 0$, $y = 2(0) - 4 = 0 - 4 = -4$

$x = 1$, $y = 2(1) - 4 = 2 - 4 = -2$

$x = 2$, $y = 2(2) - 4 = 4 - 4 = 0$

x	0	1	2	3	4
y	-4	-2	0	2	4
(ii)	E	F	G	H	I



Triangle formed by the lines with y-axis is ΔAEC. Coordinates of vertices are A(0, 4), E(0, $-$4) and C(2, 0).

$$\text{Area of } \Delta AEC = \frac{1}{2} \text{Base} \times \text{Altitude}$$
$$= \frac{1}{2} AE \times CO$$
$$= \frac{1}{2} \times [4-(-4)] \times (2-0)$$
$$= \frac{1}{2} \times 8 \times 2 = 8$$

$\therefore$ Area of ΔAEC = 8 square units

Q13. Write an equation of a line passing through the point representing the solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?

Ans. Given pair of linear equations is

$$x + y = 2 \quad \ldots(i)$$
$$2x - y = 1 \quad \ldots(ii)$$

Here, $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{1}{-1} = -1$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given pair of equations has unique solution.

For solution of equations, add (*i*) and (*ii*), we get

$$3x = 3$$

$\Rightarrow x = \frac{3}{3} \Rightarrow x = 1$

Now, $x + y = 2$ [From (*i*)]

$\Rightarrow 1 + y = 2 \Rightarrow y = 1$ [$\because x = 1$]

Hence, the solution of the given equations is $y = 1$ and $x = 1$.

Now, we have to find a line passing through (1, 1). We can make infinite linear equations passing through (1, 1). Some of the linear equations are given below:

Step I: Take any linear polynomial in x and y, let it be $8x - 5y$.

Step II: Put $x = 1$ and $y = 1$ in the above polynomial, i.e.

$$8(1) - 5(1) = 8 - 5 = 3$$

Step III: So, the required equation of line passing through (1, 1) is

$$8x - 5y = 3$$

Some more required equations are $2x - 3y = -1$, $3x - 2y = 1$ and $5x - 2y = 3$ and $x - y = 0$ etc.

Q14. If $(x + 1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a - 3b = 4$.

Sol. Let $f(x) = 2x^3 + ax^2 + 2bx + 1$

If $(x + 1)$ is a factor of $f(x)$, then by factor theorem $f(-1) = 0$.

$\therefore \quad f(-1) = 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$$\Rightarrow \quad -2 + a - 2b + 1 = 0$$
$$\Rightarrow \quad a - 2b = 1 \qquad ...(i)$$
$$2a - 3b = 4 \qquad ...(ii) \quad \text{[Given]}$$
$$2a - 4b = 2 \qquad [(i) \times 2]$$
$$\underline{\begin{array}{r} 2a - 3b = 4 \\ - \quad + \quad - \end{array}} \qquad \text{[From } (ii)\text{]}$$
$$-b = -2$$
$$\Rightarrow \quad b = 2$$
Now, $a - 2b = 1$ [From (i)]
$$\Rightarrow \quad a - 2(2) = 1 \qquad [\because b = 2]$$
$$\Rightarrow \quad a = 1 + 4$$
$$\Rightarrow \quad a = 5, \quad b = 2$$

Q15. The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y.

Sol. x, y and 40 are the measures of interior angles of a triangle.
$$\therefore \quad x + y + 40° = 180°$$
$$\Rightarrow \quad x + y = 140° \qquad ...(i)$$
The difference between x and y is 30° so
$$x - y = 30° \qquad ...(ii)$$
$$\underline{x + y = 140°} \qquad \text{[From } (i)\text{]}$$
$$2x = 170° \qquad \text{[Adding } (i) \text{ and } (ii)\text{]}$$
$$\Rightarrow \quad x = \frac{170}{2} = 85°$$
Now, $x + y = 140°$ [From (i)]
$$\Rightarrow \quad 85° + y = 140° \qquad [\because x = 85°]$$
$$\Rightarrow \quad y = 140° - 85°$$
$$\Rightarrow \quad y = 55°$$
and $x = 85°$

Q16. Two years ago, Salim was thrice as old as his daughter and six years later he will be four years older than twice her age. How old are they now?

Sol. Let the present age of Salim be x years.
Also, let the present age of his daughter be y years.

Age of Salim 2 years ago = $(x - 2)$ years
Age of Salim's daughter 2 years ago = $(y - 2)$ years

According to the question, we have

Age of Salim was = thrice × daughter [Given]
$$\Rightarrow \quad x - 2 = 3 \times (y - 2)$$
$$\Rightarrow \quad x - 2 = 3y - 6$$

$\Rightarrow \quad x - 3y = -4 \quad ...(i)$

Age of Salim 6 years later = $(x + 6)$ years

Age of Salim's daughter 6 years later = $(y + 6)$ years

According to the question, we have

$$x + 6 = 2(y + 6) + 4$$

$$\Rightarrow \quad x + 6 = 2y + 12 + 4$$

$$\Rightarrow \quad x - 2y = 16 - 6$$

$$\Rightarrow \quad x - 2y = 10 \quad (ii)$$

$$x - 3y = -4 \quad \text{[From } (i)\text{]}$$

$$- \quad + \quad\quad +$$

$$y = 14 \quad \text{[Subtracting } (i) \text{ from } (ii)\text{]}$$

Now, $x - 2y = 10$ [From (ii)]

$$\Rightarrow \quad x - 2(14) = 10 \quad [\because \; y = 14]$$

$$\Rightarrow \quad x = 10 + 28$$

$$\Rightarrow \quad x = 38$$

$\therefore$ Age of Salim at present = 38 years

and age of Salim's daughter at present = 14 years

Q17. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Sol. Let the present age of father be x years.

Also, let the sum of present ages of two children be y years.

The age of father is (=) twice ($\times 2$) the sum of the ages of two children

$$\Rightarrow \quad x = 2 \times (y) \quad \Rightarrow \quad x = 2y$$

$$\Rightarrow \quad x - 2y = 0 \quad ...(i)$$

Age of father 20 years later = $(x + 20)$ years

Increase in age of first children in 20 years = 20 years

Increase in age of second children in 20 years = 20 years

$\therefore$ Increase in the age of both children in 20 years = 20 + 20 = 40 years

$\therefore$ Sum of ages of both children 20 years later = $(y + 40)$

Now, according to the question, we have

Father will be (=) sum of ages of two children [Given]

$$\Rightarrow \quad x + 20 = y + 40$$

$$\Rightarrow \quad x - y = 20 \quad ...(ii)$$

$$\Rightarrow \quad 2y - y = 20 \quad [(\because x = 2y) \text{ from } (i)]$$

$$\Rightarrow \quad y = 20 \text{ years}$$

Now, $x = 2y$ [From (i)]

$$\Rightarrow \quad x = 2 \times 20$$

$$\Rightarrow \quad x = 40$$

$\therefore$ Age of father is 40 years.

Q18. Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

Sol. Let the numbers be $5x$ and $6x$ respectively. So, new numbers after subtracting 8 from each will be $(5x - 8)$ and $(6x - 8)$ respectively.

According to the question, ratio of new numbers is 4 : 5.

$$\therefore \quad \frac{5x-8}{6x-8} = \frac{4}{5}$$

$$\Rightarrow \quad 25x - 40 = 24x - 32$$

$$\Rightarrow \quad 25x - 24x = 40 - 32$$

$$\Rightarrow \quad x = 8$$

$\therefore$ Required numbers = $5x$ and $6x$ become 5×8, 6×8

i.e., Required numbers = 40 and 48

Q19. There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

Sol. Let the number of students initially in hall A be x.
and the number of students initially in hall B be y.

Case I: 10 students of hall A shifted to B

Now, number of students in hall A = $(x - 10)$

Now, number of students in hall B = $(y + 10)$

According to the question, number of students in both halls are equal.

$$\therefore \quad x - 10 = y + 10$$

$$\Rightarrow \quad x - y = 20 \qquad \text{...}(i)$$

Case II: 20 students are shifted from hall B to A, then

Number of students in hall A becomes = $x + 20$

Number of students in hall B becomes = $y - 20$

According to the question, students in hall A becomes twice of students in hall B.

$$\therefore \quad x + 20 = 2(y - 20)$$

$$\Rightarrow \quad x + 20 = 2y - 40$$

$$\Rightarrow \quad x - 2y = -60 \qquad (ii)$$

$$\begin{array}{rcl} x - y &=& 20 \qquad \text{[From } (i)\text{]} \\ - \quad + & & - \qquad \text{[Subtracting eqn. } (i) \text{ from } (ii)\text{]} \\ \hline -y &=& -80 \end{array}$$

$$\Rightarrow \quad y = 80$$

Now, $\quad x - y = 20 \qquad$ [From (i)]

$$\Rightarrow \quad x - 80 = 20$$

$\Rightarrow \quad x = 20 + 80$

$\Rightarrow \quad x = 100$

$\therefore$ Number of students initially in hall A = 100

and number of students initially in hall B = 80

Q20. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

Sol. Let the fixed charges for first two days = ₹x

Let the additional charges per day after 2 days = ₹ y

Latika paid ₹ 22 for six days. [Given]

2 days fixed charges + (6 – 2) days charges = 22

$\Rightarrow \quad x + 4y = 22$ ...(*i*)

Anand paid ₹ 16 for books kept for four days.

2 day's fixed charges + (4 – 2) day's additional charges = 16

$\Rightarrow \quad x + 2y = 16$ ...(*ii*)

$$\begin{array}{rcrl} x + 2y & = & 16 & \text{[From (ii)]} \\ x + 4y & = & 22 & \text{[From (i)]} \\ - \quad - & & - & \text{[Subtracting eqn. (i) from (ii)]} \\ \hline -2y & = & -6 & \end{array}$$

$\Rightarrow \quad y =$ ₹ 3 per day

Now, $\quad x + 2y = 16$ [From (*ii*)]

$\Rightarrow \quad x + 2(3) = 16$

$\Rightarrow \quad x = 16 - 6 \quad \Rightarrow \quad x = 10$

So, the fixed charges for first 2 days = ₹ 10

The additional charges per day after 2 days = ₹ 3 per day

Q21. In a competitive examination, 1 mark is awarded for each correct answer, while 1/2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Sol. Let the number of questions attempted correctly = x

Number of questions answered = 120

So, wrong answer attempted = $(120 - x)$

Marks awarded for right answer = $1 \times x = x$ marks

Marks deducted for $(120 - x)$ wrong answer = $\frac{1}{2}(120 - x)$

Totals marks awarded = 90

$\therefore \quad x - \frac{1}{2}(120 - x) = 90 \quad \Rightarrow \quad -60 + - = 90$

$\Rightarrow \quad x + \frac{x}{2} = 90 + 60 \quad \Rightarrow \quad \frac{3x}{2} = 150$

$$\Rightarrow \quad x = \frac{150 \times 2}{3} \quad \Rightarrow \quad x = 100$$

Hence, Jayanti answered 100 questions correctly.

Q22. The angles of a cyclic quadrilateral ABCD are $\angle A = (6x + 10)^\circ$, $\angle B = (5x)^\circ$, $\angle C = (x + y)^\circ$, $\angle D = (3y - 10)^\circ$. Find x and y, and hence the values of four angles.

Sol. The sum of opposite angles of a cyclic $\square$ABCD is 180° so

$$\angle A + \angle C = 180^\circ \quad \text{[Opposite } \angle\text{s of cyclic } \square]$$
$$\Rightarrow \quad (6x + 10) + (x + y) = 180^\circ$$
$$\Rightarrow \quad 6x + 10 + x + y = 180^\circ$$
$$\Rightarrow \quad 7x + y = 170^\circ \quad ...(i)$$

Also,
$$\angle B + \angle D = 180^\circ$$
$$\Rightarrow \quad 5x + (3y - 10) = 180^\circ$$
$$\Rightarrow \quad 5x + 3y = 180^\circ + 10^\circ$$
$$\Rightarrow \quad 5x + 3y = 190 \quad (ii)$$
$$21x + 3y = 510 \quad ...(iii) \text{ [From } (i)]$$
$$\underline{- \quad - \quad -}$$
$$-16x = -320 \quad \text{[Subtracting eqn. } (i) \text{ from } (ii)]$$
$$\Rightarrow \quad x = \frac{320}{16} \quad \Rightarrow \quad x = 20$$

Now,
$$7x + y = 170 \quad \text{[From } (i)]$$
$$\Rightarrow \quad 7(20) + y = 170$$
$$\Rightarrow \quad y = 170 - 140$$
$$\Rightarrow \quad y = 30$$

and
$$x = 20$$

$$\therefore \quad \angle A = (6x + 10)^\circ = (6 \times 20 + 10)^\circ = (120 + 10)^\circ = 130^\circ$$
$$\angle B = (5x)^\circ = (5 \times 20)^\circ = 100^\circ$$
$$\angle C = (x + y)^\circ = (20 + 30)^\circ = 50^\circ$$
$$\angle D = (3y - 10)^\circ = (3 \times 30 - 10)^\circ = (90 - 10)^\circ = 80^\circ$$

Hence, the values of x and y are 20 and 30 respectively. $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are 130°, 100°, 50°, 80° respectively.

EXERCISE 3.4

Q1. Graphically, solve the following pair of equations $2x + y = 6$ and $2x - y + 2 = 0$. Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

Sol. Given equation is $2x + y = 6$

$$\Rightarrow \quad \boxed{y = 6 - 2x} \quad \text{...}(i)$$

If $x = 0, y = 6 - 2(0) = 6$

$x = 1, y = 6 - 2(1) = 6 - 2 = 4$

$x = 2, y = 6 - 2(2) = 6 - 4 = 2$

x	0	1	2
y	6	4	2
I	A	B	C

Given equation is $2x - y + 2 = 0$

$\Rightarrow \quad \boxed{y = 2x + 2} \quad \ldots(ii)$

If $x = 0, \quad y = 2(0) + 2 = 0 + 2 = 2$

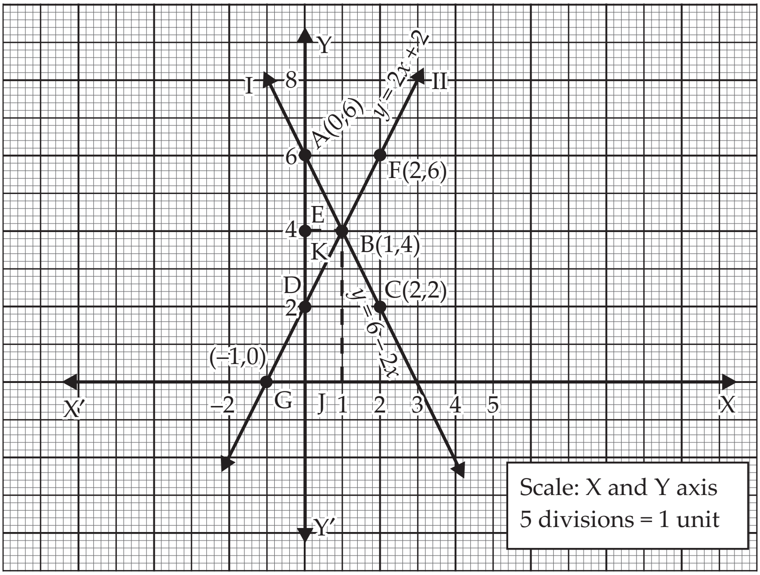
$x = 1, \quad y = 2(1) + 2 = 2 + 2 = 4$

$x = 2, \quad y = 2(2) + 2 = 4 + 2 = 6$

x	0	1	2
y	2	4	6
II	D	E	F

The area of Δ BGH formed by lines and X axis $= \frac{1}{2}$ GH $\times$ BJ

$$= \frac{1}{2}[3 - (-1)] \times (4 - 0) = \frac{1}{2} \times 4 \times 4 = 8 \text{ sq. units}$$



The area of Δ BAD formed by lines and Y axis $= \frac{1}{2}$ AD $\times$ KB

$$= \frac{1}{2}(6 - 2) \times (1 - 0) = \frac{1}{2} \times 4 \times 1 = 2 \text{ sq. units}$$

$\therefore \quad$ Ratio of areas of two Δs $= \dfrac{\text{Area } \Delta \text{ BGH}}{\text{Area } \Delta \text{ BAD}} = \dfrac{8}{2} = \dfrac{4}{1} = 4:1$

Q2. Determine graphically the vertices of the triangle formed by the lines $y = x$, $3y = x$, and $x + y = 8$.

Sol. Given equations are

$$y = x \quad ...(i)$$
$$x = 3y \quad ...(ii)$$
$$x + y = 8 \quad ...(iii)$$
$$\Rightarrow \quad y = 8 - x \quad \text{[From (iii)]}$$

If $x = 0$, $y = 8 - 0 = 8$
$x = 1$, $y = 8 - 1 = 7$
$x = 2$, $y = 8 - 2 = 6$

$y = 8 - x$

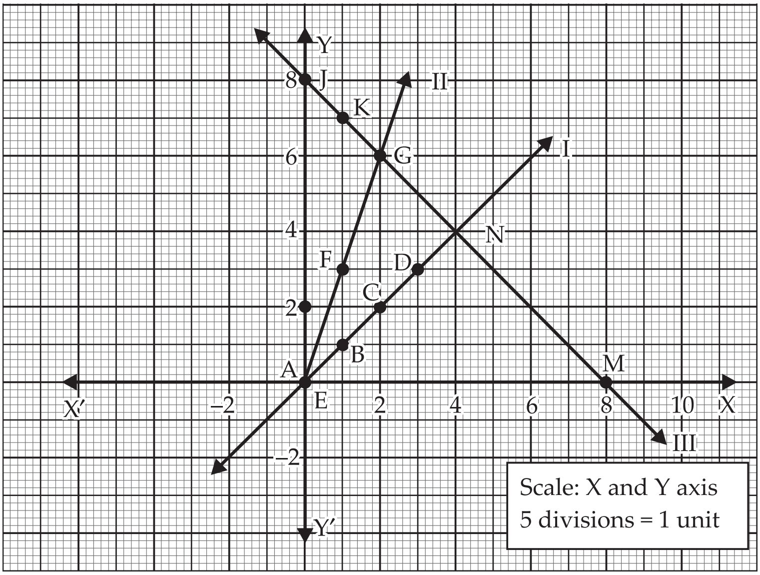
III	J	K	L	M
x	0	1	2	8
y	8	7	6	0

$y = x$

I	A	B	C	D
x	0	1	2	3
y	0	1	2	3

$x = 3y$

II	E	F	G	H
x	0	1	2	3
y	0	3	6	9



Hence, the vertices of Δ GNA formed by 3 lines are G(2, 6), N(4, 4) and A(0, 0).

Q3. Draw the graphs of the equations $x = 3$, $x = 5$, and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x-axis.

Sol. The given equations are

$$x = 3 \qquad \ldots(i)$$

$$x = 5 \qquad \ldots(ii)$$

$$2x - y - 4 = 0 \qquad \ldots(iii)$$

$$\Rightarrow \quad y = 2x - 4$$

If $x = 0$, then $y = 2(0) - 4 = 0 - 4 = -4$
$x = 1$, then $y = 2(1) - 4 = 2 - 4 = -2$
$x = 2$, then $y = 2(2) - 4 = 4 - 4 = 0$
$x = 3$, then $y = 2(3) - 4 = 6 - 4 = 2$
$x = 4$, then $y = 2(4) - 4 = 8 - 4 = 4$

$\Rightarrow$

$y = 2x - 4$

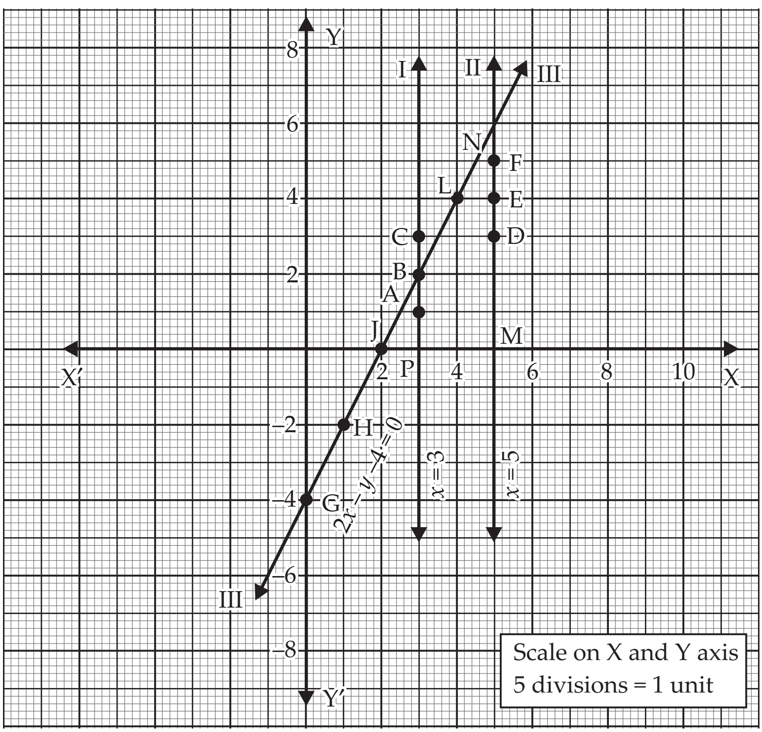
x	0	1	2	3	4
y	–4	–2	0	2	4
III	G	H	J	K	L

$x = 3$

x	3	3	3
y	1	2	3
I	A	B	C

$x = 5$

x	5	5	5
y	3	4	5
II	D	E	F



The coordinates of the vertices of the required □ PMNB are P(3,0), M(5,0), N(5,6) and B(3,2)

The quadrilateral formed by these given three lines and x-axis is $\square$ PMNB. It is trapezium. So, area of the required trapezium

$$= \frac{1}{2}\ (\text{BP} + \text{MN}) \times \text{PM}$$

$$= \frac{1}{2}[(2-0)+(6-0)]\,(5-3)$$

$$= \frac{1}{2} \times 8 \times 2\ = 8 \text{ square units}$$

Hence, the area of required $\square$ PMNB = 8 square units.

Q4. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Sol. Let the cost of a pen = ₹ x

Let the cost of a pencil box = ₹ y

$\therefore$ The cost of 4 pens and 4 pencil boxes = ₹ 100 [Given]

$$4x + 4y = 100 \qquad ...(i)$$

$$x + y = 25 \qquad ...(ii) \qquad \text{[By dividing } (i) \text{ by 4]}$$

Accoding to the second condition, we have

$$3x = y + 15$$

$$3x - y = 15 \qquad (iii)$$

$$x + y = 25 \qquad (ii)$$

$$4x = 40 \qquad \text{[Adding } (ii) \text{ and } (iii)\text{]}$$

$$\Rightarrow \qquad x = \frac{40}{4} = 10$$

Now, $x + y = 25$ [From (ii)]

$\Rightarrow$ $10 + y = 25$ [$\because$ $x = 10$)]

$\Rightarrow$ $y = 25 - 10 =$ ₹ 15

So, $x =$ ₹ 10 and $y =$ ₹ 15

Hence, the cost of a pen and a pencil box are ₹ 10 and ₹ 15 respectively.

Q5. Determine, algebraically, the vertices of the triangle formed by the lines, $3x - y = 3$, $2x - 3y = 2$ and $x + 2y = 8$.

Sol. Given linear equations are

$$3x - y = 3 \qquad ...(i)$$

$$2x - 3y = 2 \qquad ...(ii)$$

$$x + 2y = 8 \qquad ...(iii)$$

Let the intersecting points of lines (i) and (ii) is A, and of lines (ii) and (iii) is B and that of lines (iii) and (i) is C.

The intersecting point of (*ii*) and (*i*) can be find out by solving (*i*) and (*ii*) for (*x*, *y*).

$$3x - y = 3 \quad \text{[From } (i)\text{]}$$
$$2x - 3y = 2 \quad \text{[From } (ii)\text{]}$$

$$\begin{array}{rcrl} 9x - 3y &=& 9 & \text{...}(iv) \text{ [Multiplying eqn. } (i) \text{ by 3]} \\ 2x - 3y &=& 2 & \text{[From } (ii)\text{]} \\ - \quad + & & - & \\ \hline 7x &=& 7 & \text{[By subtracting } (ii) \text{ from } (iv)\text{]} \end{array}$$

$\Rightarrow \quad x = \dfrac{7}{7} \quad \Rightarrow \quad x = 1$

Now, $3x - y = 3$ [From (*i*)]

$\Rightarrow \quad 3(1) - y = 3$ [$\because \quad x = 1$]

$\Rightarrow \quad -y = 3 - 3 \quad \Rightarrow \quad -y = 0 \quad \Rightarrow \quad y = 0$

So, intersecting point of eqns. (*i*) and (*ii*) is A(1, 0).

Similarly, intersecting point B of eqns. (*ii*) and (*iii*) can be find out as follows:

$$2x - 3y = 2 \quad \text{[From } (ii)\text{]}$$
$$x + 2y = 8 \quad \text{[From } (iii)\text{]}$$

$$\begin{array}{rcrl} 2x - 3y &=& 2 & \text{[From } (ii)\text{]} \\ 2x + 4y &=& 16 & \text{...}(v) \text{ [By multiplying } (iii) \text{ by 2]} \\ - \quad - & & - & \\ \hline -7y &=& -14 & \text{[Subtracting } (v) \text{ from } (ii)\text{]} \end{array}$$

$\Rightarrow \quad y = \dfrac{14}{7} \quad \Rightarrow \quad y = 2$

Now, $x + 2y = 8$ [From (*iii*)]

$\Rightarrow \quad x + 2(2) = 8$

$\Rightarrow \quad x = 8 - 4$

$\Rightarrow \quad x = 4$

So, the coordinates of B are (4, 2).

Similarly, for intersecting point C of eqns. (*i*) and (*iii*), we have

$$3x - y = 3 \quad \text{[From } (i)\text{]}$$
$$x + 2y = 8 \quad \text{[From } (iii)\text{]}$$

Multiplying (*i*) by 2, we get

$$\begin{array}{rcrl} 6x - 2y &=& 6 & \text{...}(vi) \\ x + 2y &=& 8 & \text{[From } (iii)\text{]} \\ \hline 7x &=& 14 & \text{[Adding } (vi) \text{ and } (iii)\text{]} \end{array}$$

$\Rightarrow \quad x = \dfrac{14}{7} \quad \Rightarrow \quad x = 2$

Now, $3x - y = 3$ [From (*i*)]

$\Rightarrow \quad 3(2) - y = 3$

$\Rightarrow \quad -y = 3 - 6$

$\Rightarrow \quad -y = -3 \quad \Rightarrow \quad y = 3$

So, point C is (2, 3).

Hence, the vertices of ΔABC formed by given three linear equations are A(1, 0), B(4, 2) and C(2, 3).

Q6. Ankita travels 14 km to her home partly by rikshaw and partly by bus. She takes half an hour if she travels 2 km by rikshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rikshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rikshaw and of the bus.

Sol. Let the speed of rikshaw = x km/hr
and let the speed of bus = y km/hr

Case I: Time taken by rikshaw to travel 2 km = $\frac{\text{Distance}}{\text{Speed}} = \frac{2}{x}$ hr

Time taken by bus to travel (14 – 2) km (remaining) = $\frac{12}{y}$ hr

Total time taken by rikshaw (2 km) and bus (12 km) = $\frac{1}{2}$ hr

$$\therefore \quad \frac{2}{x} + \frac{12}{y} = \frac{1}{2} \quad \text{...}(i)$$

Case II: Time taken by rikshaw to travel 4 km = $\frac{4}{x}$ hr

Time taken by bus to travel remaining (14 – 4) km = $\frac{10}{y}$ hr

$\therefore$ Total time in case II = $\frac{1}{2}$ hr + 9 min

$$\therefore \quad \frac{4}{x} + \frac{10}{y} = \frac{1}{2}\text{hr} + \frac{9}{60}\text{hr} \quad \Rightarrow \quad \frac{4}{x} + \frac{10}{y} = \frac{30+9}{60}$$

$$\Rightarrow \quad \frac{4}{x} + \frac{10}{y} = \frac{39}{60} \quad \Rightarrow \quad \frac{4}{x} + \frac{10}{y} = \frac{13}{20} \quad \text{...}(ii)$$

Multiplying equation (*i*) by 2, we get

$$\frac{4}{x} + \frac{24}{y} = \frac{2}{2} \quad \text{...}(iii)$$

Now, subtracting (*iii*) from (*ii*), we get

$$\frac{4}{x} + \frac{10}{y} = \frac{13}{20}$$

$$\frac{4}{x} + \frac{24}{y} = \frac{2}{2}$$
$$- \quad - \qquad -$$
$$\frac{10}{y} - \frac{24}{y} = \frac{13}{20} - \frac{2}{2}$$

$\Rightarrow \quad \dfrac{10-24}{y} = \dfrac{13-20}{20} \quad \Rightarrow \quad -\dfrac{14}{y} = \dfrac{-7}{20}$

$\Rightarrow \quad 7y = 14 \times 20 \quad \Rightarrow \quad y = \dfrac{14 \times 20}{7}$

$\Rightarrow \quad y = 40$ km/hr

Now, $\quad \dfrac{2}{x} + \dfrac{12}{y} = \dfrac{1}{2}$ [From (*i*)]

$\Rightarrow \quad \dfrac{2}{x} + \dfrac{12}{(40)} = \dfrac{1}{2} \quad \Rightarrow \quad \dfrac{2}{x} = \dfrac{1}{2} - \dfrac{3}{10}$

$\Rightarrow \quad \dfrac{2}{x} = \dfrac{5-3}{10} \quad \Rightarrow \quad \dfrac{2}{x} = \dfrac{2}{10}$

$\Rightarrow \quad x = 10$ km/hr

Hence, the speeds of rikshaw and bus are 10 km/hr and 40 km/hr respectively.

Q7. A person, rowing at the rate of 5 km/hr in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

Sol. Let the speed of the stream be x km/hr.
And, the speed of the boat in still water = 5 km/hr
Speed of the boat upstream = $(5 - x)$ km/hr
Speed of the boat downstream = $(5 + x)$ km/hr

Time taken in rowing 40 km upstream = $\dfrac{40}{5-x}$ hrs

Time taken in rowing 40 km downstream = $\dfrac{40}{5+x}$ hrs

According to the question, we have
Time taken in 40 km upstream = 3 × Time taken in 40 km downstream

$\therefore \quad \dfrac{40}{5-x} = \dfrac{3 \times 40}{5+x}$

$\Rightarrow \quad \dfrac{1}{5-x} = \dfrac{3}{5+x}$

$\Rightarrow \quad -3x + 15 = x + 5$

$$\Rightarrow \quad -3x - x = 5 - 15$$
$$\Rightarrow \quad -4x = -10$$
$$\Rightarrow \quad x = \frac{10}{4}$$
$$\Rightarrow \quad x = 2.5 \text{ km/hr}$$

Hence, the speed of stream is 2.5 km/hr.

Q8. A motorboat can travel 30 km upstream and 28 km downstream in 7 hrs. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Sol. Let speed of boat in still water = x km/hr
and the speed of the stream = y km/hr
Speed of motor boat upstream = $(x - y)$ km/hr
Speed of motor boat downstream = $(x + y)$ km/hr

Case I: Time taken by motor boat in 30 km upstream = $\frac{30}{x-y}$ hr

Time taken by motor boat in 28 km downstream = $\frac{28}{x+y}$ hr

$$\therefore \quad \frac{30}{(x-y)} + \frac{28}{(x+y)} = 7$$
$$\Rightarrow \quad 2\left[\frac{15}{(x-y)} + \frac{14}{(x+y)}\right] = 7$$
$$\Rightarrow \quad \frac{15}{x-y} + \frac{14}{x+y} = \frac{7}{2} \quad \text{...}(i)$$

Case II: Time taken by motor boat in 21 km upstream = $\frac{21}{x-y}$ hr

Time taken by motor boat to return 21 km downstream = $\frac{21}{x+y}$ hr

$$\therefore \quad \frac{21}{x-y} + \frac{21}{x+y} = 5$$
$$\Rightarrow \quad 21\left[\frac{1}{x-y} + \frac{1}{x+y}\right] = 5$$
$$\Rightarrow \quad \frac{1}{x-y} + \frac{1}{x+y} = \frac{5}{21} \quad \text{...}(ii)$$
$$\frac{15}{x-y} + \frac{14}{x+y} = \frac{7}{2} \quad \text{[From } (i)\text{]}$$

As equations (both) are symmetric to $(x - y)$ and $(x + y)$ so we can eliminate either $(x - y)$ or $(x + y)$.

Multiplying (*ii*) by 14, we get

$$\frac{14}{(x-y)} + \frac{14}{(x+y)} = \frac{70}{21} \quad \text{...}(iii)$$

$$\frac{15}{(x-y)} + \frac{14}{x+y} = \frac{7}{2} \quad \text{[From } (i)\text{]}$$

$$- \quad - \quad - \quad \text{[Subtracting } (i) \text{ from } (iii)\text{]}$$

$$\frac{14}{(x-y)} - \frac{15}{(x-y)} = \frac{10}{3} - \frac{7}{2}$$

$$\Rightarrow \quad \frac{14-15}{(x-y)} = \frac{20-7\times 3}{3\times 2}$$

$$\Rightarrow \quad \frac{-1}{(x-y)} = \frac{-1}{6}$$

$$\Rightarrow \quad (x-y) = 6 \quad \text{...}(iv)$$

Now, substituting $x - y = 6$ in (*ii*), we have

$$\frac{1}{(x-y)} + \frac{1}{(x+y)} = \frac{5}{21}$$

$$\Rightarrow \quad \frac{1}{6} + \frac{1}{(x+y)} = \frac{5}{21} \quad \Rightarrow \quad \frac{1}{(x+y)} = \frac{5}{21} - \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{(x+y)} = \frac{2\times 5 - 7\times 1}{3\times 7\times 2} \quad \Rightarrow \quad \frac{1}{(x+y)} = \frac{3}{42}$$

$$\Rightarrow \quad \frac{1}{(x+y)} = \frac{1}{14}$$

$$\Rightarrow \quad x + y = 14 \quad \text{...}(v)$$

$$x - y = 6 \quad \text{[From } (iv)\text{]}$$

$$2x = 20 \quad \text{[Subtracting } (iv) \text{ from } (v)\text{]}$$

$$\Rightarrow \quad x = 10 \text{ km/hr}$$

Now, $x + y = 14$ [From (*v*)]

$$\Rightarrow \quad 10 + y = 14$$

$$\Rightarrow \quad y = 4 \text{ km/hr}$$

Hence, the speed of motorboat and stream are 10 km/hr and 4 km/hr respectively.

Q9. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Sol. Let the two digit number $= xy$

$$= 10\,x + y$$

According to the question:

$$\text{Number} = 8(x+y) - 5$$
$$\Rightarrow \quad 10x + y = 8x + 8y - 5$$
$$\Rightarrow \quad 10x - 8x + y - 8y = -5$$
$$\Rightarrow \quad 2x - 7y = -5 \quad \text{...}(i)$$

Also,
$$\text{Number} = 16(x-y) + 3 = 10x + y$$
$$\Rightarrow \quad 10x + y = 16x - 16y + 3$$
$$\Rightarrow \quad -6x + 17y = 3 \quad \text{...}(ii)$$

Multiplying (i) by 3, we get

$$6x - 21y = -15 \quad \text{...}(iii)$$

Adding (iii) and (ii), we have

$$\begin{array}{r} -6x + 17y = 3 \\ 6x - 21y = -15 \\ \hline -4y = -12 \end{array}$$

$$\Rightarrow \quad y = 3$$

Now,
$$2x - 7y = -5 \quad \text{[From } (i)\text{]}$$
$$\Rightarrow \quad 2x - 7(3) = -5$$
$$\Rightarrow \quad 2x = -5 + 21$$
$$\Rightarrow \quad 2x = 16$$
$$\Rightarrow \quad x = 8$$

So, the number is $xy = 83$.

We can also find another number if possible.

$$16(y-x) + 3 = 10x + y$$
$$\Rightarrow \quad 16y - 16x + 3 = 10x + y$$
$$\Rightarrow \quad -16x - 10x + 16y - y = -3$$
$$\Rightarrow \quad -26x + 15y = -3 \quad \text{...}(iv)$$

$$\begin{array}{rl} -26x + 15y = -3 & \text{...}(iv) \\ 26x - 91y = -65 & (i) \times 13 \\ \hline -76y = -68 & \text{[Adding above 2 eqns.]} \end{array}$$

$$\Rightarrow \quad y = \frac{68}{76} = \frac{17}{19}$$

But x, y can never be in fraction or negative.

Hence, the required number = 83

Q10. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs ₹ 2530. Also, one reserved first class ticket and one reserved first class half ticket from station A to B costs ₹ 3810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.

Sol. Let the cost of full fare from station A to B = ₹ x
and the reservation charges per ticket = ₹ y
Cost of one full ticket from A to B = ₹ 2530
i.e., (1 fare + 1 reservation) charges = ₹ 2530

i.e., $$x + y = 2530 \quad ...(i)$$

Cost of 1 full and one, half ticket from station A to B = ₹ 3810
i.e., (1 full ticket) + (1/2 ticket) charges = ₹ 3810

i.e., $(x + y) + (1/2 \text{ fare} + \text{reservation}) = 3810$

i.e., $$(x + y) + \frac{1}{2}x + y = 3810$$

$$\Rightarrow \quad \frac{3}{2}x + 2y = 3810$$

$$\Rightarrow \quad 3x + 4y = 7620 \quad ...(ii)$$

Multiplying (*i*) by 3, we get

$$3x + 3y = 7590 \quad ...(iii)$$

Subtracting (*iii*) from (*ii*), we get

$$\begin{array}{rrrl} 3x + & 4y = & 7620 & ...(ii) \\ 3x + & 3y = & 7590 & (iii) \\ - \quad - & - & & \\ \hline & y = & ₹\ 30 & \end{array}$$

Now, $$x + y = 2530 \quad \text{[From } (i)]$$

$$\Rightarrow \quad x + 30 = 2530 \quad (\because \quad y = 30)$$

$$\Rightarrow \quad x = 2530 - 30$$

$$\Rightarrow \quad x = ₹\ 2500$$

Hence, full fare and reservation charges of a ticket from station A to B are ₹ 2500 and ₹ 30 respectively.

Q11. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount; thereby getting a sum of ₹ 1008. If she had sold saree at 10% profit and the sweater at 8% discount. She would have got ₹ 1028, then find out the cost price of the saree and the list price (price before discount) of the sweater.

Sol. Let the cost price of a saree = ₹ x
and the list price of sweater = ₹ y

Case I:

(S.P. of saree at 8% profit) + (S.P. of a sweater at 10% discount)= ₹ 1008

$$\Rightarrow \quad \frac{(100+8)}{100}x + \frac{(100-10)}{100}y = 1008$$

$$\Rightarrow \quad 108x + 90y = 100800$$

$$\Rightarrow \quad 6x + 5y = 5600 \qquad ...(i)$$

Case II:

(S.P. of saree at 10% profit) + (S.P. of a sweater at 8% discount)= ₹ 1028

$$\Rightarrow \quad \frac{(100+10)}{100}x + \frac{(100-8)}{100}y = 1028$$

$$\Rightarrow \quad 110x + 92y = 102800 \qquad ...(ii)$$

Dividing (*ii*) by 2, we have

$$55x + 46y = 51400 \qquad ...(iii)$$

Again, multiplying (*iii*) by 5, we get

$$275x + 230y = 257000 \qquad ...(iv)$$

Multiplying (*i*) by 46, we get

$$276x + 230y = 257600 \qquad ...(v)$$

Subtracting (*v*) from (*iv*), we get

$$\begin{array}{rcl} 275x + 230y &=& 257000 \qquad ...(iv) \\ 276x + 230y &=& 257600 \qquad ...(v) \\ - \quad - & & - \\ \hline -x &=& 257000 - 257600 \end{array}$$

$$\Rightarrow \quad -x = -600$$

$$\Rightarrow \quad x = ₹\ 600$$

Now, $\quad 6x + 5y = 5600$ [From (*i*)]

$$\Rightarrow \quad 6 \times 600 + 5y = 5600 \qquad [\because x = 600]$$

$$\Rightarrow \quad 5y = 5600 - 3600$$

$$\Rightarrow \quad y = \frac{2000}{5}$$

$$\Rightarrow \quad y = 400$$

Hence, the C.P. of a saree and L.P. of sweater are ₹ 600, ₹ 400 respectively.

Q12. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investments in two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

Sol. Let the money invested in scheme A = ₹ x

and the money invested in scheme B = ₹ y

Case I: Susan invested ₹ x at 8% p.a. + Susan invested ₹ y at 9% p.a. = 1860

$$\Rightarrow \quad \frac{x \times 8 \times 1}{100} + \frac{y \times 9 \times 1}{100} = 1860$$

$$\Rightarrow \quad 8x + 9y = 186000 \quad ...(i)$$

Case II: Interchanging the amount in schemes A and B, we have

$$\frac{9 \times x}{100} + \frac{8 \times y}{100} = (1860 + 20)$$

$$\Rightarrow \quad 9x + 8y = 188000 \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$\begin{array}{rl} 9x + \ \ 8y = 188000 & ...(ii) \\ 8x + \ \ 9y = 186000 & ...(i) \\ \hline 17x + 17y = 374000 & \end{array}$$

$$\Rightarrow \quad x + y = 22000 \quad ...(iii)$$

On subtracting (*i*) and (*ii*), we get $x - y = 2000$ ...(*iv*)

Now,
$$\begin{array}{rl} x - y = \ \ 2000 & ...(iv) \\ x + y = 22000 & ...(iii) \\ \hline 2x \quad = 24000 & \text{[Adding (iv) and (iii)]} \end{array}$$

$$\Rightarrow \quad x = ₹\ 12000$$

Now, $x + y = 22000$ [From (*iii*)]

$$\Rightarrow \quad y = 22000 - 12000$$

$$\Rightarrow \quad y = ₹\ 10,000$$

Hence, the amount invested in schemes A and B are ₹ 12000 and ₹ 10,000 respectively.

Q13. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

Sol. Let the number of bananas in lot A = x
and the number of bananas in lot B = y

Case I: S.P. of 3 bananas of lot A = ₹ 2

$$\Rightarrow \quad \text{S.P. of 1 banana of lot A} = ₹\frac{2}{3}$$

$$\Rightarrow \quad \text{S.P. of } x \text{ bananas of lot A} = \frac{2}{3}x$$

Now, S.P. of 1 banana of lot B = ₹ 1

$$\Rightarrow \quad \text{S.P. of } y \text{ bananas of lot B} = ₹\ y$$

$\therefore \qquad \dfrac{2x}{3} + y = 400$

$\Rightarrow \qquad 2x + 3y = 1200 \qquad ...(i)$

Case II: $\quad x + \dfrac{4}{5}y = 460$

$\Rightarrow \qquad 5x + 4y = 2300 \qquad ...(ii)$

Multiplying (i) by 4, we get

$$8x + 12y = 4800 \qquad ...(iii)$$

Also, multiplying (ii) by 3, we get

$$15x + 12y = 6900 \qquad ...(iv)$$

Now,
$$\begin{array}{rcl} 15x + 12y & = & 6900 \qquad ...(iv) \\ 8x + 12y & = & 4800 \qquad ...(iii) \\ - \quad - & & - \\ \hline 7x & = & 2100 \end{array}$$
[On subtracting (iii) from (iv)]

$\Rightarrow \qquad x = \dfrac{2100}{7}$

$\Rightarrow \qquad x = 300$

Now, $\quad 2x + 3y = 1200 \qquad$ [From (i)]

$\Rightarrow \qquad 2(300) + 3y = 1200$

$\Rightarrow \qquad 3y = 1200 - 600$

$\Rightarrow \qquad y = \dfrac{600}{3}$

$\Rightarrow \qquad y = 200$

Hence, the total number of bananas = $(x + y) = (300 + 200) = 500$.