EXERCISE 9.1

Choose the correct answer from the given four options:

Q1. If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is the tangent to the other circle is

(*a*) 3 cm (*b*) 6 cm (*c*) 9 cm (*d*) 1 cm

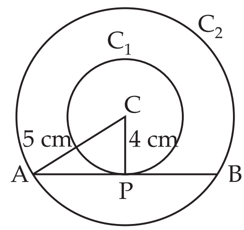
Sol. (*b*): C_1, C_2 are concentric circles with their centre C.

Chord AB of circle C_2 touches C_1 at P

AB is tangent at P and PC is radius at P.

So, CP $\perp$ AB.

$\Rightarrow \angle P = 90°$, CP = 4 cm and CA = 5 cm (Given)



$\therefore$ In right angle ΔPAC,

$$AP^2 = AC^2 - PC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$\Rightarrow$ AP = 3 cm

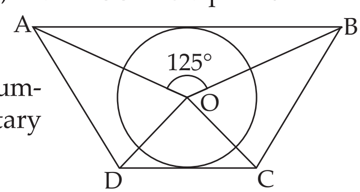
Perpendicular from centre to chord bisects the chord.

So, AB = 2AP = 2 × 3 = 6 cm. Hence, verifies option (*b*).

Q2. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to

(*a*) 62.5° (*b*) 45°

(*c*) 35° (*d*) 55°



Sol. (*d*): We know that a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

$$\therefore \quad \angle AOB + \angle COD = 180°$$

$$125° + \angle COD = 180°$$

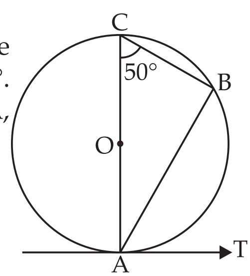
$$\angle COD = 180° - 125° = 55°.$$

Hence, verifies option (*d*).

Q3. In the given figure, AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to

(*a*) 65° (*b*) 60°

(*c*) 50° (*d*) 40°



Sol. (*c*): AC is diameter.

$\Rightarrow \quad \angle B = 90° \quad (\angle$ in a semi-circle)

$\therefore \quad \angle BAC = 180° - \angle C - \angle B$ [Angle sum property of a triangle]

$\Rightarrow \quad \angle BAC = 180° - 50° - 90° = 180° - 140° = 40°$

Tangent AT at A and radius OA at A arc at 90°.

So, $\angle OAT = 90°$

$\therefore \quad \angle OAB + \angle BAT = 90°$

$\Rightarrow \quad 40° + \angle BAT = 90°$

$\Rightarrow \quad \angle BAT = 90° - 40°$

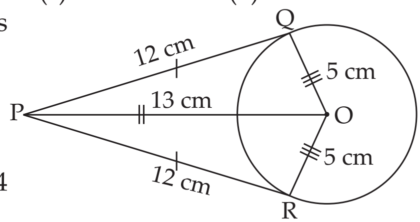
$\Rightarrow \quad \angle BAT = 50°.$

Hence, verifies option (*c*).

Q4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(*a*) 60 cm^2 (*b*) 65 cm^2 (*c*) 30 cm^2 (*d*) 32.5 cm^2

Sol. (*a*): PQ is tangent and QO is radius at contact point Q.



$\therefore \quad \angle PQO = 90°$

$\therefore$ By Pythagoras theorem,

$$PQ^2 = OP^2 - OQ^2$$
$$= 13^2 - 5^2 = 169 - 25 = 144$$

$\Rightarrow \quad PQ = 12$ cm

$\therefore \quad \Delta OPQ \cong \Delta OPR$ [By SSS criterion of congruence]

$\therefore \quad$ Area of ΔOPQ = ar ΔOPR

Area of quadrilateral QORP = 2 ar (ΔOPR)

$$= 2 \times \frac{1}{2} \text{ base} \times \text{altitude}$$

$$= RP \times OR = 12 \times 5 = 60 \text{ cm}^2$$

Hence, verifies the option (*a*).

Q5. At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

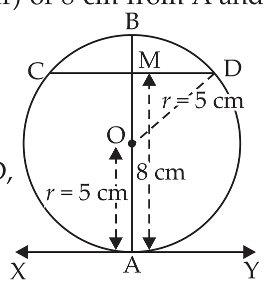
(*a*) 4 cm (*b*) 5 cm (*c*) 6 cm (*d*) 8 cm

Sol. (*d*): XAY is tangent and AO is radius at contact point A of circle.

AO = 5 cm

$\therefore \quad \angle OAY = 90°$

CD is another chord at distance (perpendicular) of 8 cm from A and CMD || XAY meets AB at M.



Join OD.

OD = 5 cm

OM = 8 – 5 = 3 cm

$\angle OMD = \angle OAY = 90°$ Now, in right angled ΔOMD,

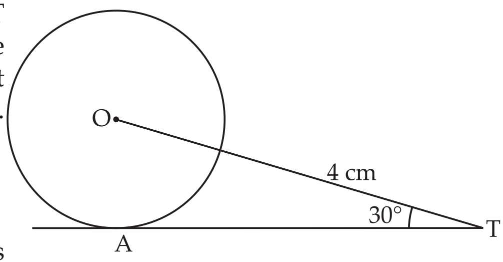
$$MD^2 = OD^2 - MO^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$\Rightarrow \quad MD = 4$ cm

Perpendicular from centre O of circle bisect the chord. So CD = 2MD = 2 × 4 = 8 cm.

Hence, length of chord CD = 8 cm, which verifies option (*d*).

Q6. In the given figure, AT is a tangent to the circle with centre 'O' such that OT = 4 cm and $\angle OTA = 30°$. Then AT is equal to

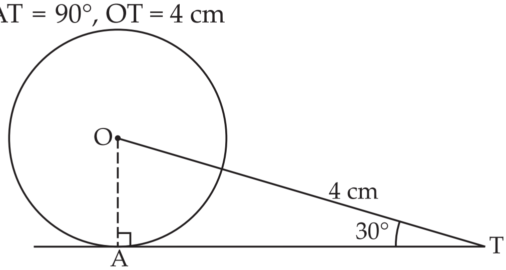


(*a*) 4 cm (*b*) 2 cm

(*c*) $2\sqrt{3}$ cm (*d*) $4\sqrt{3}$ cm

Sol. (*c*): Join OA. OA is radius and AT is tangent at contact point A.

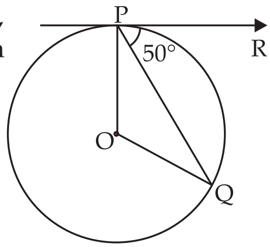
So, $\angle OAT = 90°$, OT = 4 cm [Given]



Now, $\dfrac{AT}{4} = \dfrac{\text{Base}}{\text{Hypotenuse}} = \cos 30° \Rightarrow AT = 4 \times \dfrac{\sqrt{3}}{2} = 2\sqrt{3}$ cm.

Hence, verifies the option (*c*).

Q7. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to



(*a*) 100° (*b*) 80°

(*c*) 90° (*d*) 75°

Sol. (*a*): OP is radius and PR is tangent at P.

$$
\begin{aligned}
\text{So,} \quad & \angle OPR = 90° \\
\Rightarrow \quad & \angle OPQ + 50° = 90° \\
\Rightarrow \quad & \angle OPQ = 90° - 50° \\
\Rightarrow \quad & \angle OPQ = 40°
\end{aligned}
$$

In ΔOPQ,

$OP = OQ$ [Radii of same circle]

$\therefore \quad \angle Q = \angle OPQ = 40°$

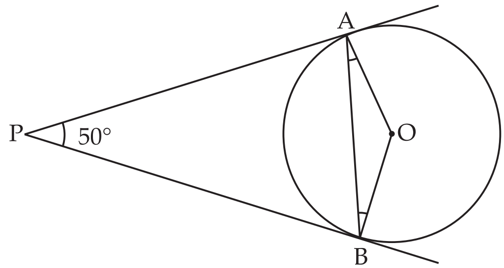
[Angles opposite to equal sides are equal]

$$
\begin{aligned}
\text{But,} \quad \angle POQ &= 180° - \angle P - \angle Q \\
&= 180° - 40° - 40° = 180° - 80° = 100°
\end{aligned}
$$

$\Rightarrow \quad \angle POQ = 100°.$

Hence, verifies the option (*a*).

Q8. In the given figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to



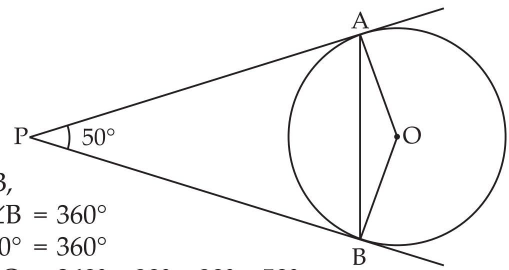
(*a*) 25° (*b*) 30°
(*c*) 40° (*d*) 50°

Sol. (*a*): In ΔOAB, we have

$OA = OB$ [Radii of same circle]

$\therefore \angle OAB = \angle OBA$ [Angles opposite to equal sides are equal]

As OA and PA are radius and tangent respectively at contact point A.



So, $\angle OAP = 90°$. Similarly, $\angle OBP = 90°$

Now, in quadrilateral PAOB,

$$\angle P + \angle A + \angle O + \angle B = 360°$$
$$\Rightarrow 50° + 90° + \angle O + 90° = 360°$$
$$\Rightarrow \angle O = 360° - 90° - 90° - 50°$$
$$\Rightarrow \angle O = 130°$$

Again, in ΔOAB,

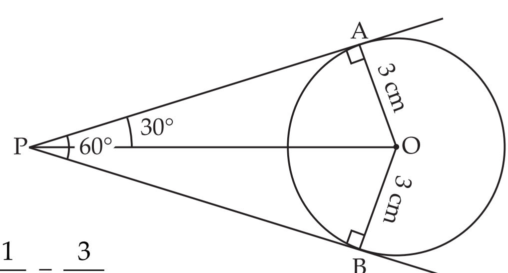
$$\angle O + \angle OAB + \angle OBA = 180°$$
$$\Rightarrow 130° + \angle OAB + \angle OAB = 180° \quad [\because \angle OBA = \angle OAB]$$
$$\Rightarrow 2\angle OAB = 180° - 130° = 50°$$
$$\Rightarrow \angle OAB = 25°$$

Hence, $\angle OAB = 25°$ which verifies option (*a*).

Q9. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(*a*) $\frac{3}{2}\sqrt{3}$ cm (*b*) 6 cm (*c*) 3 cm (*d*) $3\sqrt{3}$ cm

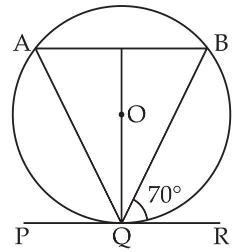
Sol. (*d*): $\because$ OA and PA are the radius and the tangent respectively at contact point A of a circle of radius OA = 3 cm. So, $\angle PAO = 90°$.



In right angled ΔPOA,

$$\tan 30° = \frac{OA}{PA} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{PA}$$

$\Rightarrow$ PA = $3\sqrt{3}$ which verifies the option (*d*).

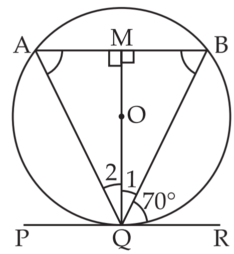


Q10. In the given figure, if PQR is the tangent to a circle at Q, whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to

(*a*) 20° (*b*) 40°
(*c*) 35° (*d*) 45°

Sol. (*b*): AB || PQR [Given]

$\angle B = \angle BQR = 70°$ [Alternate interior angles]



and $\angle OQR = \angle AMQ$ [Alternate interior angles]

As PQR and OQ are tangent and radius at contact point Q

$\therefore \quad \angle OQR = 90°$

$\Rightarrow \quad \angle 1 + \angle 70° = 90°$

$\Rightarrow \quad \angle 1 = 90° - 70° = 20°$

$\therefore$ $\angle AMO = 90°$ and perpendicular from centre to chord bisect the chord

So, $\quad MA = MB$

$\angle QMA = \angle QMB$ [Each 90°]

$MQ = MQ$ [Common]

$\therefore \quad \Delta QMA \cong \Delta QMB$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle A = \angle B$

$\Rightarrow \quad \angle A = 70°$ [$\because \angle B = 70°$]

$\therefore \quad \angle A + \angle AMQ + \angle 2 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 70° + 90° + \angle 2 = 180°$

$\Rightarrow \quad \angle 2 = 180° - 160°$

$\Rightarrow \quad \angle 2 = 20°$

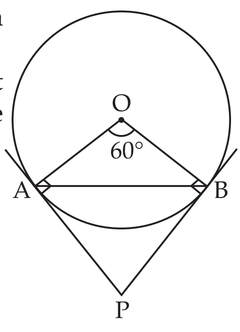
$\therefore \quad \angle AQB = \angle 1 + \angle 2 = 20° + 20° = 40°$

Hence, verifies option (*b*).

EXERCISE 9.2

Write True or False and justify your answer in each of the following:

Q1. If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is also 60°.



Sol. False: Chord AB subtends $\angle 60°$ at O.

$\therefore$ AP and OA are tangent and radius at A.

$\therefore$ $\angle OAP = 90°$

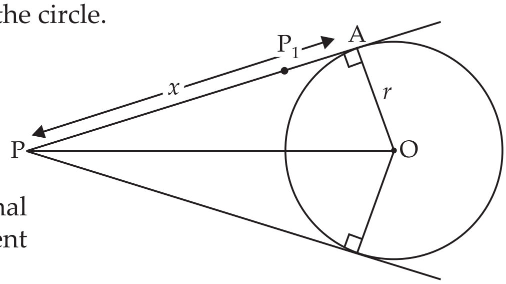
Similarly, $\angle OBP = 90°$

In quadrilateral OAPB,

$$\angle O + \angle P + \angle OAP + \angle OBP = 360°$$
$$\Rightarrow \quad 60° + \angle P + 90° + 90° = 360°$$
$$\Rightarrow \quad \angle P = 360° - 240°$$
$$\Rightarrow \quad \angle P = 120°$$

Hence, the given statement is false.

Q2. The length of tangent from an external point on a circle is always greater than the radius of the circle.



Sol. False: Consider any point P external to a circle away from O.

Now, draw tangent PA on the circle. Clearly,

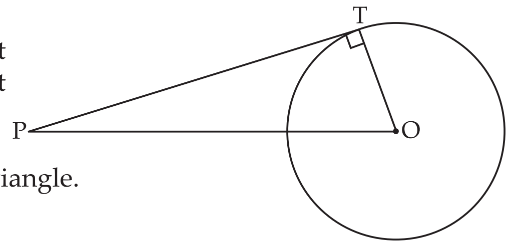
PA > r [$\because$ P is external to circle and P is at sufficient distance]

Now, again consider any point P_1 on the tangent AP very near to contact point A of tangent PA, $P_1A < AO$

So, it is clear that the length of the tangent PA and P_1A are greater and smaller respectively than radius OA.

Hence, the length of the tangent from an external point of a circle may or may not be greater than the radius of the circle. Hence, the given statement is false.

Q3. The length of the tangent from an external point P on a circle with centre O is always less than OP.



Sol. True:

PT and OT are the tangent and radius respectively at contact point T.

So, $\angle OTP = 90°$

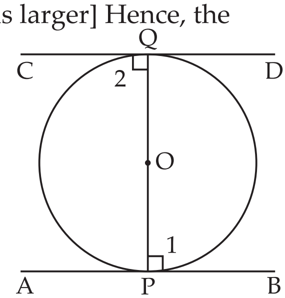
$\Rightarrow$ ΔOPT is right angled triangle.

Again, in ΔOPT

$\because \quad \angle T > \angle O$

$\therefore \quad OP > PT$ [Side opposite to greater angle is larger] Hence, the given statement is true.

Q4. The angle between two tangents to a circle may be 0°.



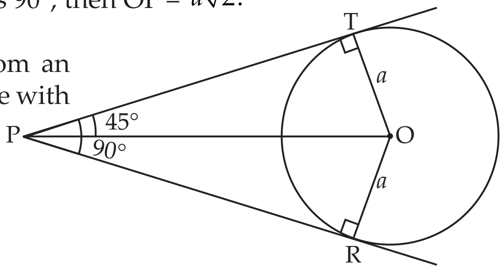
Sol. True:

Consider the diameter POQ of a circle with centre O. The tangent at P and Q are drawn, as we know the radius and tangent at contact point are perpendicular so $\angle 1 = \angle 2 = 90°$. These are alternate angles so the tangent APB || CQD *i.e.*, angle between two tangents to a circle may be zero.

Hence, the given statement is true.

Q5. If the angle between two tangents drawn from a point P to a circle of radius '*a*' and centre O is 90°, then OP = $a\sqrt{2}$.

Sol. True.



Consider a tangent PT from an external point P on a circle with radius '*a*'.

OT and PT are radius and tangent respectively at contact point T.

$\therefore \quad \angle T = 90°$

As $\quad \Delta OPT \cong \Delta OPR$ [By SSS criterion of congruence]

$\therefore \quad \angle OPT = \angle OPR = \dfrac{90°}{2} = 45°$

$\therefore$ In right angle ΔOPT,

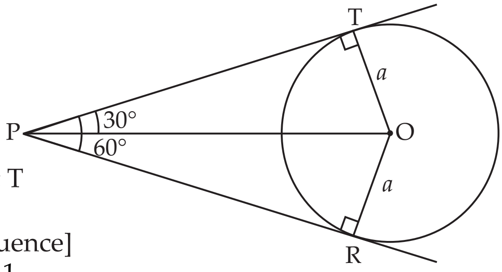
$$\sin 45° = \frac{OT}{OP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{a}{OP}$$

$$\Rightarrow \quad OP = \sqrt{2}a\,.$$

Hence, the given statement is true.

Q6. If the angle between two tangents drawn from a point P to a circle of radius '*a*' and centre O is 60°, then OP = $a\sqrt{3}$.



Sol. False: PT and OT are tangent and radius respectively at contact point T.

$\therefore \quad \angle OTP = 90°$

$\Rightarrow \Delta OTP$ is right angle Δ at T

As $\quad \Delta OPT \cong \Delta OPR$

[By SSS criterion of congruence]

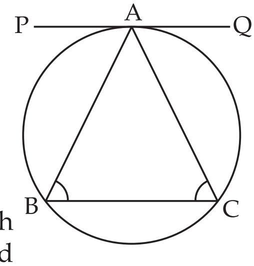
$\Rightarrow \quad \angle OPT = \angle OPR = \dfrac{1}{2} \times 60° = 30°$

$\therefore$ In right angle ΔOPT,

$$\sin 30° = \frac{OT}{OP} \Rightarrow \frac{1}{2} = \frac{a}{OP} \Rightarrow OP = 2a$$

Hence, the given statement is false.

Q7. The tangent to the circumcircle of an isosceles ΔABC at A, in which AB = AC, is parallel to BC.



Sol. True.

A ΔABC, inscribed in a circle in which $AB = AC$.

PAQ is tangent at A.

AB is chord.

$\therefore \quad \angle PAB = \angle C \quad ...(i)$

$\because$ Angle $\angle PAB$ formed by chord (AB) with tangent is equal to the angle $\angle C$ formed by chord AC in alternate segment.

In ΔABC,

$AB = AC$ [Given]

$\therefore \quad \angle B = \angle C$ [$\because$ Angles opposite to equal sides are equal] ...(*ii*)

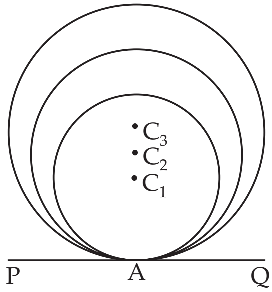
From (*i*) and (*ii*), $\angle B = \angle PAB$

These are alternate interior angles.

So, $PAQ \parallel BC$

Hence, the given statement is true.

Q8. If a number of circles touch a given line segment PQ at a point A, then their centres lies on the perpendicular bisector of PQ.



Sol. False:

C_1A and PAQ are radius and tangent at contact point A.

$\therefore \quad \angle C_1AP = 90° \Rightarrow C_1A \perp PQ$

Similarly, $\angle C_2AP = 90° \Rightarrow C_2A \perp PQ$

$\angle C_3AP = 90° \Rightarrow C_3A \perp PQ$

We know that perpendicular on any point of a segment PQ may be only one.

So, point segments C_1A, C_2A, C_3A, C_4A, ... will be on a line.

$\Rightarrow C_1A, C_2A, C_3A, C_4A$ will lie on a line, which is perpendicular on PQ at A.

As A is not mid point of PQ. So, the perpendicular AB will not be perpendicular bisector of PQ.

Hence, the given statement is false.

Q9. If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

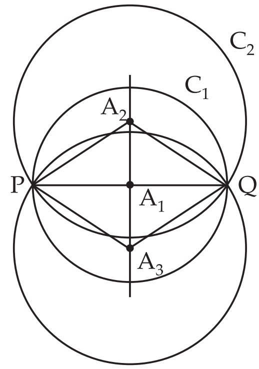
Sol. True: Centre of any circle passing through the end points P and Q of a line segment are equidistant from P and Q.

$\therefore \quad A_1P = A_1Q$

$A_2P = A_2Q$

$A_3P = A_3Q$

as we know that any point on perpendicular bisector of a segment is equidistant from the end points of the segment. Hence, A_1, A_2, A_3 points are the centres of circles passing through the end points P and Q of a segment PQ or the centres of circles lie on the perpendicular bisector of PQ.

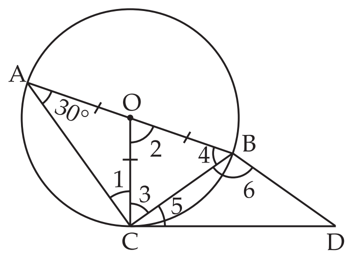


Q10. AB is a diameter of a circle and AC is its chord such that $\angle BAC = 30°$. If the tangent at C intersects AB extended at D, then BC = BD.

Sol. True:

CD is a tangent at contact point C. AOB is diameter which meets tangent produced at D.

Chord AC makes $\angle A = 30°$ with diameter AB.



To prove: BD = BC

Proof: In ΔOAC,

$OA = OC = r$ [Radii of same circle]

$\angle 1 = \angle A$ [$\angle$s opp. to equal sides are equal]

$\Rightarrow \quad \angle 1 = 30°$ [$\because \angle A = 30°$]

Exterior $\angle BOC = \angle 2 = \angle 1 + \angle A = (30° + 30°) = 60°$

Now, in ΔOCB,

$OC = OB$ [Radii of same circle]

$\therefore \quad \angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

$\angle 3 + \angle 4 + \angle COB = 180°$

$\Rightarrow \quad \angle 3 + \angle 3 + 60° = 180°$ [Angle sum property of triangle]

$\Rightarrow \quad 2\angle 3 = 180° - 60° = 120°$

$\Rightarrow \quad \angle 3 = 60° = \angle 4$

$\angle 6 + \angle 4 = 180°$ [Linear pair axiom]

$\Rightarrow \quad \angle 6 = 180° - \angle 4$

$= 180° - 60°$

$\Rightarrow \quad \angle 6 = 120°$

$\because$ Tangent CD and radius CO are at contact point C.

$\therefore \quad \angle OCD = 90°$

$\Rightarrow \quad \angle 3 + \angle 5 = 90°$

$\Rightarrow \quad 60° + \angle 5 = 90°$

$\Rightarrow \quad \angle 5 = 30°$

Now, in ΔBCD, we have

$$\angle D + \angle 5 + \angle 6 = 180° \quad \text{[Angle sum property of a triangle]}$$

$$\Rightarrow \quad \angle D = 180° - \angle 5 - \angle 6$$

$$= 180° - 30° - 120° = 180° - 150°$$

$$\Rightarrow \quad \angle D = 30°$$

$$\therefore \quad \angle D = \angle 5 = 30°$$

$$\Rightarrow \quad BC = BD$$

[Sides opposite to equal $\angle$s of a triangle are equal]

Hence, verifies the given statement true.

EXERCISE 9.3

Q1. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

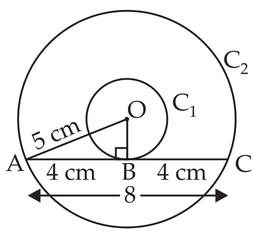
Sol. Given: Two concentric circles C_1 and C_2 with centre O.

Chord AC of circle C_2 is tangent of circle C_1 at B.

We know that tangent AC and radius BO at point B are perpendicular.

$\therefore$ Perpendicular from centre to chord bisects the chord.

$$\therefore \quad AB = CB = \frac{AC}{2} = \frac{8}{2} = 4 \text{ cm}$$



In right angle ΔABO,

$$OB^2 = OA^2 - AB^2$$

[By Pythagoras theorem]

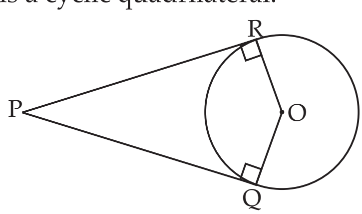
$$= 5^2 - 4^2 = 25 - 16 = 9$$

$$\Rightarrow \quad OB = 3 \text{ cm}$$

Hence, radius of circle C_1 is 3 cm.

Q2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Sol. Given: Tangents PR and PQ from an external point P to a circle with centre O.



To prove: Quadrilateral QORP is cyclic.

Proof: RO and RP are the radius and tangent respectively at contact point R.

$\therefore \quad \angle PRO = 90°$

Similarly, $\quad \angle PQO = 90°$

In quadrilateral QORP, we have

$$\angle P + \angle R + \angle O + \angle Q = 360°$$

$$\Rightarrow \quad \angle P + \angle 90° + \angle O + \angle 90° = 360°$$

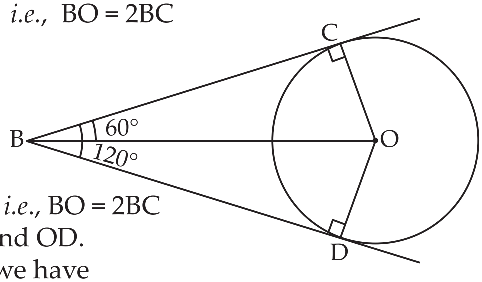
$$\Rightarrow \quad \angle P + \angle O = 360° - 180° = 180°$$

These are opposite angles of quadrilateral QORP and are supplementary.

$\therefore$ Quadrilateral QORP is cyclic. Hence, proved.

Q3. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that $\angle DBC = 120°$, prove that

$$BC + BD = BO, \textit{ i.e., } BO = 2BC$$



Sol. Given: A circle with centre O.

Tangents BC and BD are drawn from an external point B such that $\angle DBC = 120°$.

To prove: $BC + BD = BO$, *i.e.*, $BO = 2BC$

Construction: Join OB, OC and OD.

Proof: In ΔOBC and ΔOBD, we have

$$OB = OB \quad \text{[Common]}$$
$$OC = OD \quad \text{[Radii of same circle]}$$
$$BC = BD \quad \text{[Tangents from an external point are equal in length]} \quad ...(i)$$

$\therefore \quad \Delta OBC \cong \Delta OBD$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle OBC = \angle OBD$ (CPCT)

$\therefore \quad \angle OBC = \frac{1}{2}\angle DBC = \frac{1}{2} \times 120°$ $[\because \angle CBD = 120°$ given$]$

$\Rightarrow \quad \angle OBC = 60°$

OC and BC are radius and tangent respectively at contact point C.

So, $\quad \angle OCB = 90°$

Now, in right angle ΔOCB, $\quad \angle OBC = 60°$

$\therefore \quad \cos 60° = \frac{BC}{BO}$

$\Rightarrow \quad \frac{1}{2} = \frac{BC}{BO}$

$\Rightarrow \quad OB = 2BC$

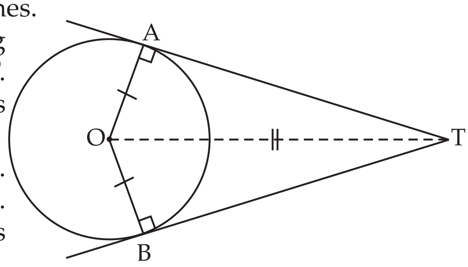
Hence, proved (*ii*) part.

$\Rightarrow \quad OB = BC + BC$

$\Rightarrow \quad OB = BC + BD$ $[\because BC = BD$ from $(i)]$

Hence, proved.

Q4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.



Sol. Given: Two intersecting lines AT and BT intersect at T. A circle with centre O touches the above lines at A and B.

To prove: OT bisects the $\angle ATB$.

Construction: Join OA and OB.

Proof: OA is radius and AT is tangent at A.

$\therefore \quad \angle OAT = 90^\circ$

Similarly, $\angle OBT = 90^\circ$

In ΔOTA and ΔOTB, we have

$\angle OAT = \angle OBT = 90^\circ$

$OT = OT$ [Common]

$OA = OB$ [Radii of same circle]

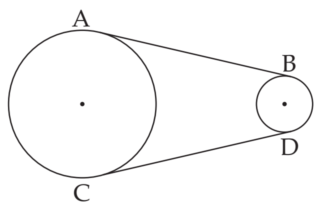
$\therefore \quad \Delta OTA \cong \Delta OTB$ [By RHS criterion of congruence]

$\Rightarrow \quad \angle OTA = \angle OTB$ [CPCT]

$\Rightarrow$ Centre of circle 'O' lies on the angle bisector of $\angle ATB$.

Hence, proved.

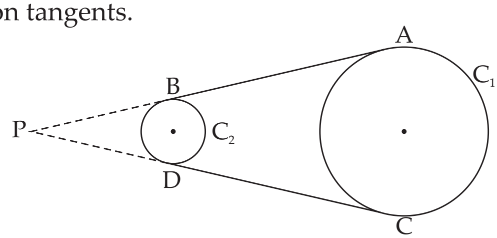
Q5. In the given figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.



Sol. Given: Circles C_1 and C_2 of radii r_1 and r_2 respectively and $r_1 < r_2$. AB and CD are two common tangents.

To prove: AB = CD

Construction: Produce AB and CD upto point P where both tangents meet.



Proof: Tangents from an external point to a circle are equal.

For circle C_1, $PB = PD$ ...(*i*)

and for circle C_2, $PA = PC$ ...(*ii*)

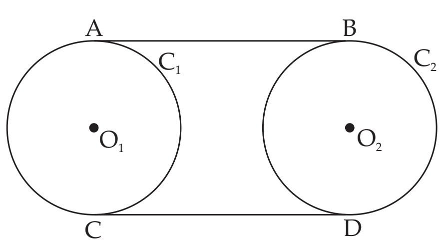
Subtracting (*i*) from (*ii*), we have

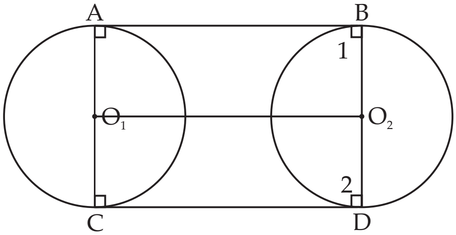
$$PA - PB = PC - PD$$

$$\Rightarrow \quad AB = CD.$$

Hence, proved.

Q6. In Question 5 above, if radii of the two circles are equal, prove that AB = CD.





Sol. Given: Two circles of equal radii, two common tangents, AB and CD on circles C_1 and C_2.

To prove: AB = CD

Construction: Join O_1A, O_1C and O_2B and O_2D. Also, join O_1O_2.

Proof: Since tangent at any point of a circle is perpendicular to the radius to the point of contact.

$\therefore$ $\angle O_1AB = \angle O_2BA = 90°$

As $O_1A = O_2B$, so O_1ABO_2 is a rectangle.

Since opposite sides of a rectangle are equal,

$\therefore$ $AB = O_1O_2$ ...(*i*)

Similarly, we can prove that O_1CDO_2 is a rectangle.

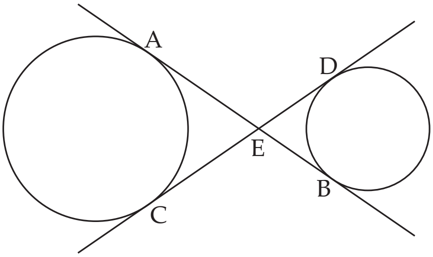
$\therefore$ $O_1O_2 = CD$ ...(*ii*)

From (*i*) and (*ii*), we get

$$AB = CD.$$

Hence, proved.

Q7. In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



Sol. Given: Two non-intersecting circles are shown in the figure. Two intersecting tangents AB and CD intersect at E. E point is between the circles and outside also.

To prove: AB = CD

Proof: We know that tangents drawn from an external point (E) to a circle are equal. Point E is outside of both the circles.

So, $EA = EC$ ...(*i*)

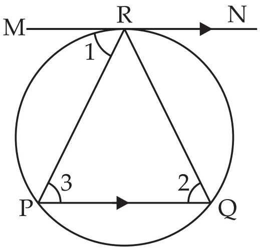
$EB = ED$ ...(*ii*)

$\Rightarrow$ $EA + EB = EC + ED$ [Adding (*i*) and (*ii*)]

$\Rightarrow$ $AB = CD$

Hence, proved.

Q8. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.



Sol. Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN

To prove: R bisects the arc PRQ.

Construction: Join RP and RQ.

Proof: Chord RP subtends $\angle 1$ with tangent MN and $\angle 2$ in alternate segment of circle so $\angle 1 = \angle 2$.

MRN || PQ

$\therefore$ $\angle 1 = \angle 3$ [Alternate interior angles]

$\Rightarrow$ $\angle 2 = \angle 3$

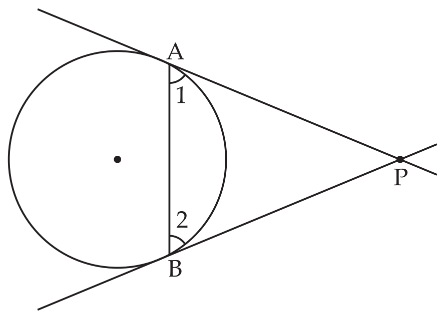
$\Rightarrow$ PR = RQ [Sides opp. to equal $\angle$s in ΔRPQ]

$\because$ Equal chords subtend equal arcs in a circle so

arc PR = arc RQ

or R bisect the arc PRQ. Hence, proved.

Q9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Sol. Given: A chord AB of a circle, tangents AP and BP at A and B respectively are drawn.

To prove: $\angle PAB = \angle PBA$

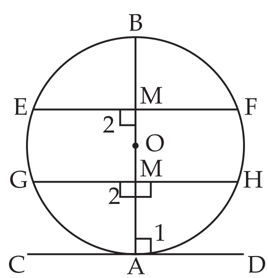
Proof: We know that tangents drawn from an external point P to a circle are equal so PA = PB.

$\angle \Rightarrow 2 = \angle 1$

[Angles opposite to equal sides of a triangle are equal]

Hence, tangents PA and PB make equal angles with chord AB. Hence, proved.

Q10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.



Sol. Given: A circle with centre O and AOB is diameter.

CAD is a tangent at A. Chord EF || tangent CAD

To prove: AB bisects any chord EF || CAD.

Proof: OA radius is perpendicular to tangent CAD.

$\therefore$ $\angle 1 = 90°$

CAD || EF [Given]

$\therefore$ $\angle 1 \angle = 2 = 90°$ [Alternate interior angles]

Point M is on diameter which passes through centre O.

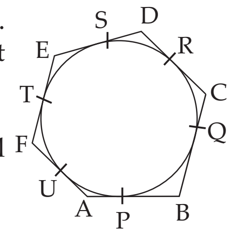
$\because$ Perpendicular drawn from centre to chord bisect the chord. Hence, AB bisects any chord EF || CAD.

EXERCISE 9.4

Q1. If a hexagon ABCDEF circumscribe a circle, then prove that

$$AB + CD + EF = BC + DE + FA$$

Sol. Given: A circle inscribed in a hexagon ABCDEF. Sides, AB, BC, CD, DE and DF touches the circle at P, Q, R, S, T and U respectively.



To prove: AB + CD + EF = BC + DE + FA

Proof: We know that tangents from an external point to a circle are equal.

Here, vertices of hexagon are outside the circle so

$$AP = AU$$
$$BP = BQ$$
$$CQ = CR$$
$$DR = DS$$
$$ES = ET$$
$$FT = FU$$

$$LHS = AB + CD + EF = (AP + PB) + (DR + CR) + (ET + TF)$$

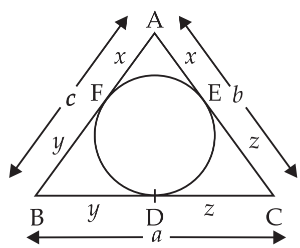
By using above results, we have

$$\begin{aligned} LHS = AB + CD + EF &= AU + BQ + DS + CQ + ES + FU \\ &= AU + FU + BQ + CQ + DS + ES \\ &= AF + BC + DE. \end{aligned}$$

Hence, proved.

Q2. Let s denotes the semi-perimeter of a ΔABC in which BC = a, CA = b, AB = c. If a circle touches the sides BC, CA, AB at D, E, F respectively, prove that BD = $s - b$.

Sol. Given: A circle inscribed in ΔABC touches the sides BC, CA and AB at D, E, F respectively.



To prove: BD = $s - b$

Proof: Tangents drawn from an external point to the circle are equal. Vertices of ΔABC are in the exterior of circle. So,

$$AF = AE = x$$
$$BF = BD = y$$
$$CD = CE = z$$

$$\begin{aligned} \text{Now,} \quad AB + BC + CA &= c + a + b \\ \Rightarrow \quad AF + BF + BD + DC + AE + CE &= a + b + c \\ \Rightarrow \quad x + y + y + z + x + z &= a + b + c \\ \Rightarrow \quad 2x + 2y + 2z &= a + b + c \\ \Rightarrow \quad 2(x + y + z) &= a + b + c \\ \Rightarrow \quad x + y + z &= \frac{a + b + c}{2} \end{aligned}$$

$\Rightarrow \quad x + y + z = s$ [Given]

$\Rightarrow \quad y = s - (x + z) \quad \Rightarrow \quad y = s - x - z$

$\Rightarrow \quad y = s - (AE + EC)$

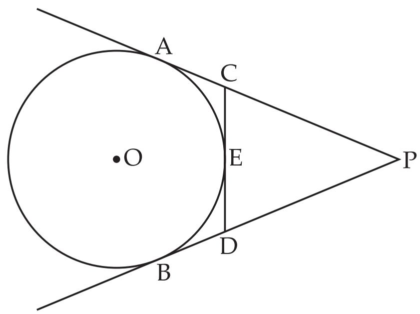
$\Rightarrow \quad = s - AC$

$\Rightarrow \quad BD = s - b$

Hence, proved.

Q3. From an external point P, two tangents PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of ΔPCD.

Sol. Given: A circle with centre O. PA, PB are tangents from an external point P. A tangent CD at E intersect AP and PB at C and D respectively.



To find: Perimeter of ΔPCD.

Method: Tangents drawn from an external point to a circle are equal.

$\therefore \quad PA = PB = 10$ cm [Given]

$CA = CE$

$DE = DB$

Perimeter of $\Delta PCD = PC + PD + CD$

$= PC + PD + CE + DE$

$= PC + CE + PD + DE$

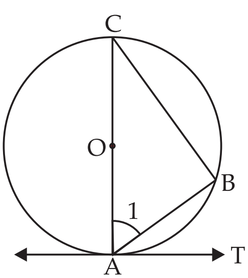
$= PC + CA + PD + DB$

$= PA + PB$

$= 10 + 10 = 20$ cm

$\therefore$ Perimeter of ΔPCD = 20 cm.

Q4. If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB.



Sol. Given: Chord AB, diameter AOC and tangent at A of a circle with centre O.

To prove: $\angle BAT = \angle ACB$

Proof: Radius OA and tangent AT at A are perpendicular.

$\therefore \quad \angle OAT = 90°$

$\Rightarrow \quad \angle BAT = 90° - \angle 1$ ...(*i*)

AOC is diameter.

$\therefore \quad \angle B = 90°$

$\Rightarrow \quad \angle C + \angle 1 = 90°$

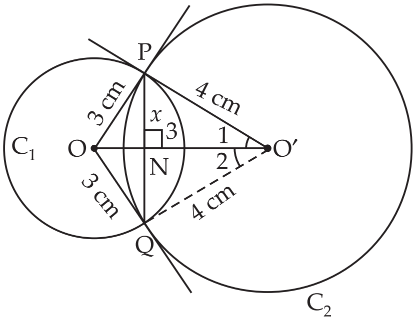
$\Rightarrow \quad \angle C = 90° - \angle 1$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\angle BAT = \angle ACB. \text{ Hence, proved.}$$

Q5. Two circles with centres O and O′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O′P are tangents to the two circles. Find the length of common chord PQ.

Sol. PO′ is tangent on circle C_1 at P. OP is tangent on circle C_2 at P. As radius OP and tangent PO′ are at a point of contact P



$\therefore \quad \angle P = 90°$

So, by Pythagoras theorem in right angled $\Delta OPO'$,

$OO'^2 = OP^2 + PO'^2 = 3^2 + 4^2 = 9 + 16$
$= 25$ cm

$\Rightarrow \quad OO' = 5$ cm

$\Delta OO'P \cong \Delta OO'Q$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle 1 = \angle 2$

$\Delta O'NP \cong \Delta O'NQ$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle 3 = \angle O'NQ$ [CPCT]

$\Rightarrow \quad \angle 3 = \angle O'NQ = 90°$ [Linear Pair axiom]

Let ON = y, then NO′ = $(5 - y)$

Let PN = x

By Pythagoras theorem in ΔPNO and $\Delta PNO'$, we have

$x^2 + y^2 = 3^2$ ...(*i*)

$x^2 + (5 - y)^2 = 4^2$

$x^2 + 25 + y^2 - 10y = 16$ ...(*ii*)

$\underline{-\ x^2 \quad - y^2 \quad = -9}$ [From (*i*)]

$25 - 10y = 7$ [Subtract (*i*) from (*ii*)]

$\Rightarrow \quad -10y = 7 - 25$

$\Rightarrow \quad -10y = -18$

$\Rightarrow \quad y = 1.8$

But, $\quad x^2 + y^2 = 3^2$ [From (*i*)]

$\Rightarrow \quad x^2 + (1.8)^2 = 3^2$

$\Rightarrow \quad x^2 = 9 - 3.24$

$\Rightarrow \quad x^2 = 5.76$

$\Rightarrow \quad x = 2.4$

$\therefore$ The perpendicular drawn from the centre bisects the chord.

$\therefore \quad PQ = 2PN = 2x$

$= 2 \times 2.4$

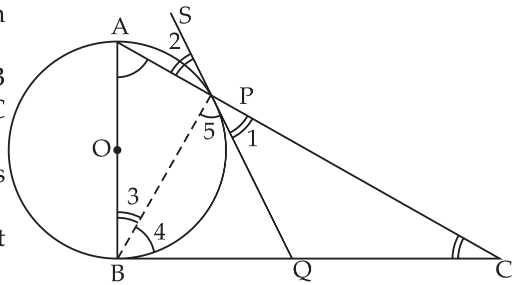
$\Rightarrow \quad PQ = 4.8$ cm

Q6. In a right triangle ABC in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.

Sol. Given: ΔABC in which ∠B = 90°

Circle with diameter AB intersect the hypotenuse AC at P.

A tangent SPQ at P is drawn to meet BC at Q.



To prove: Q is mid point of BC.

Construction: Join PB.

Proof: SPQ is tangent and AP is chord at contact point P.

∴	∠2 = ∠3	[Angles in alternate segment of circle]
	∠2 = ∠1	[Vertically opposite angles]
⇒	∠3 = ∠1 ...(*i*)	[From above two relations]
	∠ABC = 90°	[Given]

OB is radius so, BC will be tangent at B.

∴	∠3 = 90° – ∠4	...(*ii*)
	∠APB = 90°	[∠ in a semi circle]
⇒	∠C = 90° – ∠4	...(*iii*)

From (*ii*) and (*iii*), ∠C = ∠3

Using (*i*),	∠C = ∠1	
⇒	CQ = QP ...(*iv*)	[Sides opp. to = ∠s in ΔQPC]
	∠4 = 90° – ∠3	[From fig.]
	∠5 = 90° – ∠1	
	∠3 = ∠1	
∴	∠4 = ∠5	
⇒	PQ = BQ ...(*v*)	[Sides opp. to equal angles in ΔQPB]

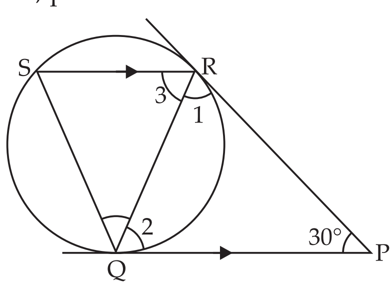
From (*iv*) and (*v*),

BQ = CQ

Therefore, Q is mid-point of BC. Hence, proved.

Q7. In the given figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to tangent PQ. Find the ∠RQS.

[**Hint:** Draw a line through Q and perpendicular to QP.]



Sol. In ΔPRQ, PQ and PR are tangents from an external point P to circle.

$\therefore$ PR = PQ

$\Rightarrow$ $\angle 2 = \angle 1$ [$\angle$s opp. to equal sides in ΔPRQ are equal]

$\angle 1 + \angle 2 + \angle RPQ = 180°$ [Int. $\angle$s of Δ]

$\Rightarrow$ $\angle 1 + \angle 1 + 30° = 180°$

$\Rightarrow$ $2\angle 1 = 180° - 30°$

$\Rightarrow$ $\angle 1 = \dfrac{150°}{2}$

$\therefore$ $\angle 1 = \angle 2 = 75°$

Tangent PQ || SR [Given]

$\therefore$ $\angle 2 = \angle 3 = 75°$ [Alternate interior angles]

PQ is tangent at Q and QR is chord at Q.

$\therefore$ $\angle S = \angle 2 = 75°$ [$\angle$s in alternate segment of circle]

In ΔSRQ,

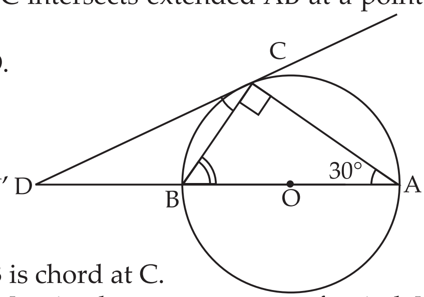
$\angle S + \angle 3 + \angle SQR = 180°$ [Angle sum property of a triangle]

$\Rightarrow$ $75° + 75° + \angle SQR = 180°$

$\Rightarrow$ $\angle SQR = 180° - 150°$

$\Rightarrow$ $\angle SQR = 30°$

Q8. AB is a diameter and AC is chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects extended AB at a point D. Prove that BC = BD.



Sol. Given: A circle with centre O.

A tangent CD at C.

Diameter AB is produced to D.

BC and AC chords are joined, $\angle BAC = 30°$.

To prove: BC = BD

Proof: DC is tangent at C and, CB is chord at C.

$\therefore$ $\angle DCB = \angle BAC$ [$\angle$s in alternate segment of a circle]

$\Rightarrow$ $\angle DCB = 30°$ ...(*i*) [$\because$ $\angle BAC = 30°$ (Given)]

AOB is diameter. [Given]

$\therefore$ $\angle BCA = 90°$ [Angle in a semi circle]

$\therefore$ $\angle ABC = 180° - 90° - 30° = 60°$

In ΔBDC,

Exterior $\angle ABC = \angle D + \angle BCD$

$\Rightarrow$ $60° = \angle D + 30°$

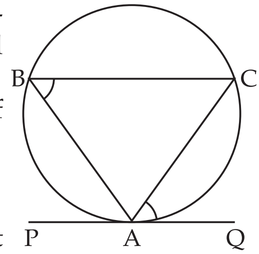
$\Rightarrow$ $\angle D = 30°$ ...(*ii*)

$\therefore$ $\angle DCB = \angle D = 30°$ [From (*i*), (*ii*)]

$\Rightarrow$ BD = BC [$\because$ Sides opposite to equal angles are equal in a triangle]

Hence, proved.

Q9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.



Sol. Given: arc BAC in which A is mid point of arc BAC.

PAQ is tangent at A.

To prove: BC || PAQ

Proof: PAQ is tangent and CAB is an arc at contact point A.

$\therefore \quad \angle CAQ = \angle B \quad ...(i)$

[Angles in alternate segment of a circle]

A is mid point of arc BAC.

$\therefore$ min. arc AB = min. arc AC

$\Rightarrow$ Chord AB = Chord AC [Equal arcs subtend equal chords]

$\Rightarrow \quad \angle C = \angle B \quad ...(ii)$ [Angles opp. to equal sides in ΔABC are equal]

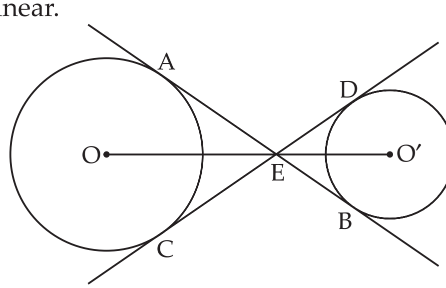
$\Rightarrow \quad \angle C = \angle CAQ$ [From (*i*) and (*ii*)]

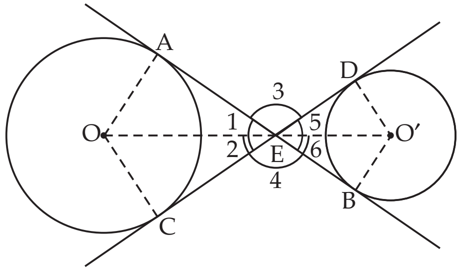
These are alternate interior angles and are equal.

$\therefore$ BC || PAQ.

Hence, proved.

Q10. In the given figure, the common tangents, AB and CD to two circles with centres O and O′ intersect at E. Prove that the points O, E and O′ are collinear.





Sol. Given: Two circles (non intersecting) with their centres O and O′.

Two common tangents AB and CD intersect at E between the circles.

To prove: O, E, O′ points are collinear.

Construction: Join OA, OC, O′D, O′B and EO and EO′

Proof: In ΔAEO and ΔCEO,

$OE = OE$ [Common]

$OA = OC$ [Radii of same circle]

$EA = EC$ [Tangents from an external point to a circle are equal in length]

$\therefore \quad \angle OEA \cong \angle OEC$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle OEA = \angle OEC$ [CPCT]

$\therefore \quad \angle 1 = \angle 2$ [CPCT]

Similarly, $\angle 5 = \angle 6$

and $\angle 3 = \angle 4$ [Vertically opposite angles]

Since sum of angles at a point = 360°

$\therefore \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

$\Rightarrow \quad 2(\angle 1 + \angle 3 + \angle 5) = 360°$

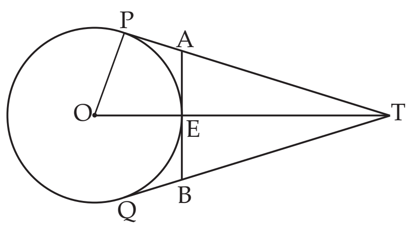
$\Rightarrow \quad \angle 1 + \angle 3 + \angle 5 = 180°$

$\Rightarrow \quad \angle OEO' = 180°$

$\therefore$ OEO′ is a straight line.

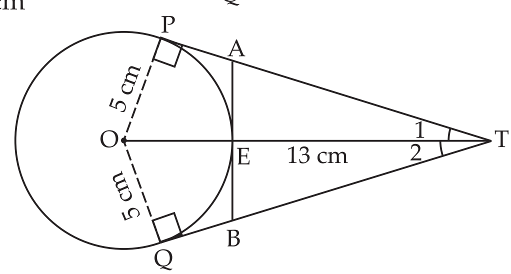
Hence, O, E and O′ are collinear.

Q11. In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



Sol. $OP = OQ = 5$ cm

$OT = 13$ cm



OP and PT are radius and tangent respectively at contact point P.

$\therefore \quad \angle OPT = 90°$

So, by Pythagoras theorem, in right angled ΔOPT,

$PT^2 = OT^2 - OP^2 = 13^2 - 5^2$

$= 169 - 25 = 144$

$\Rightarrow \quad PT = 12$ cm.

AP and AE are two tangents from an external point A to a circle.

$\therefore \quad AP = AE$

AEB is tangent and OE is radius at contact point E.

So, $AB \perp OT$ ...(*i*)

So, by Pythagoras theorem, in right angled ΔAET,

$AE^2 = AT^2 - ET^2$

$\Rightarrow \quad AE^2 = (PT - PA)^2 - [TO - OE]^2$

$$= (12 - AE)^2 - (13 - 5)^2$$

$\Rightarrow$ $AE^2 = (12)^2 + (AE)^2 - 2(12)(AE) - (8)^2$

$\Rightarrow$ $AE^2 - AE^2 + 24AE = 144 - 64$

$\Rightarrow$ $24AE = 80$

$\Rightarrow$ $AE = \frac{80}{24}$ cm

$\Rightarrow$ $AE = \frac{10}{3}$ cm

In ΔTPO and ΔTQO,

$OT = OT$ [Common]

$PT = QT$ [Tangents from T]

$OP = OQ$ [Radii of same circle]

$\therefore$ $\Delta TPO \cong \Delta TQO$ [By SSS criterion of congruence]

$\Rightarrow$ $\angle 1 = \angle 2$ ...(*ii*) [CPCT]

In ΔETA and ΔETB,

$ET = ET$ [Common]

$\angle TEA = \angle TEB = 90°$ [From (*i*)]

$\angle 1 = \angle 2$ [CPCT] [From (*ii*)]

$\therefore$ $\Delta ETA \cong \Delta ETB$ [By ASA criterion of congruence]

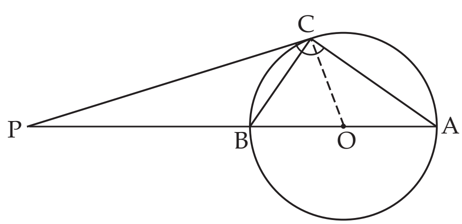
$\Rightarrow$ $AE = BE$ [CPCT]

$\Rightarrow$ $AB = 2AE = 2 \times \frac{10}{3}$

$\Rightarrow$ $AB = \frac{20}{3}$ cm.

Hence, the required length is $\frac{20}{3}$ cm.

Q12. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110°$, find $\angle CBA$.
[**Hint:** Join C with centre O].



Sol. OC and CP are radius and tangent respectively at contact point C.

So, $\angle OCP = 90°$

$\angle OCA = \angle ACP - \angle OCP$

$\Rightarrow$ $\angle OCA = 110° - 90°$

$\Rightarrow$ $\angle OCA = 20°$

In ΔOAC,

$OA = OC$ [Radii of same circle]

$\therefore$ $\angle OCA = \angle A = 20°$ [$\because$ Angles opposite to equal sides are equal]

CP and CB are tangent and chord of a circle.

$\therefore \quad \angle CBP = \angle A$ [Angles in alternate segments are equal]

In ΔCAP,

$\angle P + \angle A + \angle ACP = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad \angle P + 20° + 110° = 180°$

$\Rightarrow \quad \angle P = 180° - 130°$

$\Rightarrow \quad \angle P = 50°$

In ΔBPC,

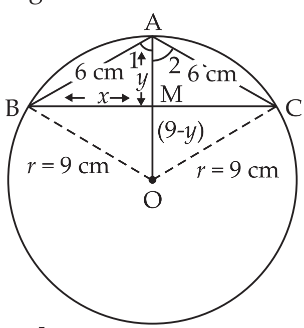
Exterior angle $\angle CBA = \angle P + \angle BCP$

$\Rightarrow \quad \angle CBA = 50° + 20°$

$\Rightarrow \quad \angle CBA = 70°$

Q13. If an isosceles ΔABC in which AB = AC = 6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

Sol. In figure, ΔABC has AB = AC = 6 cm.



In ΔOAB and ΔOAC,

AB = AC [Given]

OA = OA [Common]

OB = OC [Radii of same circle]

$\therefore \quad \Delta OAB \cong \Delta OAC$

[By SSS criterion of congruence]

$\Rightarrow \quad \angle 1 = \angle 2$ [CPCT]

In ΔAMC and ΔAMB,

$\angle 1 = \angle 2$ [Proved above]

AM = AM [Common]

AB = AC [Given]

$\therefore \quad \Delta AMB \cong \Delta AMC$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle AMB = \angle AMC = 90°$ [CPCT and Linear pair axiom]

Now, Area of $\Delta ABC = \frac{1}{2} BC \times AM$

Let BM = x and AM = y,

then MO = OA – AM

$\Rightarrow$ MO = OA – AM

$\Rightarrow$ MO = $9 - y$

In right angled ΔBMA and ΔBMO,

$x^2 + y^2 = 6^2$ ...(i) [By Pythagoras theorem]

$x^2 + (9 - y)^2 = 9^2$

$x^2 + (9)^2 + (y)^2 - 2(9)(y) = 81$

$\Rightarrow \quad x^2 + 81 + y^2 - 18y = 81$

$\Rightarrow \quad x^2 + y^2 - 18y = 0$ ...(ii)

Now, subtract (i) from (ii)

$$\begin{array}{rrrrl} x^2 & +\,y^2 & -\,18y & = & 0 \\ x^2 & +\,y^2 & & = & 36 \\ - & - & & & - \\ \hline & & -18y & = & -36 \end{array}$$

$$\Rightarrow \quad y = \frac{-36}{-18}$$

$$\Rightarrow \quad y = 2 \text{ cm} \quad \Rightarrow \quad AM = 2 \text{ cm}$$

But, $\quad x^2 + y^2 = 36 \qquad$ [From (*i*)]

$$\Rightarrow \quad x^2 + (-2) = 36$$

$$\Rightarrow \quad x^2 = 36 - 4 = 32$$

$$\Rightarrow \quad x = \sqrt{32} = 4\sqrt{2} \text{ cm}$$

$$\therefore \quad BC = 2x = 2 \times 4\sqrt{2} = 8\sqrt{2} \text{ cm}$$

($\because$ Perpendicular from centre to chord bisects the chord)

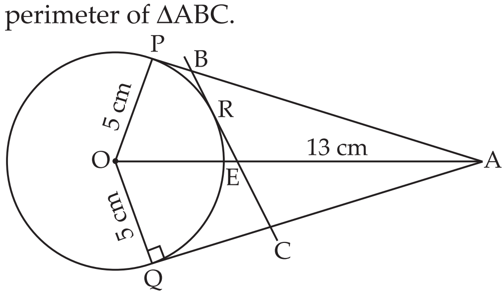
$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times 2 \times 8\sqrt{2}$$

$$\Rightarrow \quad \text{Area of } \Delta ABC = 8\sqrt{2} \text{ cm}^2$$

Q14. A is a point at a distance 13 cm from the centre 'O' of a circle of radius 5 cm. AP and AQ are the tangents to circle at P and Q. If a tangent BC is drawn at point R lying on minor arc PQ to intersect AP at B and AQ at C. Find the perimeter of ΔABC.

Sol. $\quad OA = 13$ cm

$\quad OP = OQ = 5$ cm



OP and PA are radius and tangent respectively at contact point P.

$$\therefore \quad \angle OPA = 90°$$

In right angled ΔOPA by Pythagoras theorem

$$PA^2 = OA^2 - OP^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$\Rightarrow \quad PA = 12 \text{ cm}$$

Points A, B and C are exterior to the circle and tangents drawn from an external point to a circle are equal so

$$PA = QA$$

$$BP = BR$$

$$CR = CQ$$

$$\begin{aligned} \text{Perimeter of } \Delta ABC &= AB + BC + AC \\ &= AB + BR + RC + AC \qquad \text{[From figure]} \\ &= AB + BP + CQ + AC = AP + AQ \\ &= AP + AP = 2AP = 2 \times 12 = 24 \text{ cm} \end{aligned}$$

So, the perimeter of ΔABC = 24 cm.