EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.

Sol. We have 2 women and 3 men

First women choose the chairs amongst the chairs 1 to 4 i.e. total number of chairs = 4

So, the number of arrangements = ${}^4P_2$ ways

Now 3 men choose from the remaining 6 chairs

So, the number of arrangements = ${}^6P_3$ ways

$\therefore$ Total number of arrangements = ${}^4P_2 \times {}^6P_3$

$$= \frac{4!}{(4-2)!} \times \frac{6!}{(6-3)!} = \frac{4!}{2!} \times \frac{6!}{3!}$$

$$= \frac{4 . 3 \cdot 2!}{2!} \times \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 12 \times 120 = 1440$$

Hence, the total number of possible arrangements are 1440.

Q2. If the letter of the word 'RACHIT' are arranged in all possible ways as listed in dictionary, then what is the rank of the word 'RACHIT'?

Sol. The alphabetical order of RACHIT is A, C, H, I, R and T

Number of words beginning with A = 5!

Number of words beginning with C = 5!

Number of words beginning with H = 5!

Number of words beginning with I = 5!

and Number of word beginning with R *i.e.* RACHIT = 1

$\therefore$ The rank of the word 'RACHIT' in the dictionary

$= 5! + 5! + 5! + 5! + 1 = 4 \times 5! + 1$

$= 4 \times 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 1 = 4 \text{ x } 120 + 1 = 480 + 1 = 481$

Q3. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.

Sol. Total number of questions = 12

Number of questions in each group = 6

7 questions are to be attempted but not more than 5 questions from either group

$\therefore$ Total number of ways $= {}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3 + {}^6C_3 \times {}^6C_4 + {}^6C_2 \times {}^6C_5$

$= 2[{}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3]$

$$= 2\left[6 \times \frac{6.5}{2.1} + \frac{6.5.4.3}{4.3.2.1} \times \frac{6.5.4}{3.2.1}\right]$$

$= 2[6 \times 15 + 15 \times 20] = 2[90 + 300] = 2 \times 390 = 780$

Hence, the total number of ways = 780

Q4. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the points.

Sol. Total number of points = 18

Out of 18 numbers, 5 are collinear and we get a straight line by joining any two points.

$\therefore$ Total number of straight line formed by joining 2 points out of 18 points $= {}^{18}C_2$

Number of straight lines formed by joining 2 points out of 5 points $= {}^5C_2$

But 5 points are collinear and we get only one line when they are joined pairwise.

So, the required number of straight lines are

$$= {}^{18}C_2 - {}^5C_2 + 1 = \frac{18 \cdot 17}{2 \cdot 1} - \frac{5 \cdot 4}{2 \cdot 1} + 1 = 153 - 10 + 1 = 144$$

Hence, the total number of straight lines = 144.

Q5. We wish to select 6 person from 8 but, if the person A is choosen, then B must be choosen. In how many ways can selections be made?

Sol. Total number of persons = 8

Number of persons to be selected = 6

Condition is that if A is choosen, B must be choosen

Case I: When A is choosen, B must be choosen

Number of ways $= {}^6C_4$

[$\because$ A and B are set to be choosen]

Case II: When A is not choosen, then B may be choosen

$\therefore$ Number of ways $= {}^7C_6$

So, the total number of ways = ${}^6C_4 + {}^7C_6$

[$\because$ There are two cases]

$$= {}^6C_2 + {}^7C_1 \qquad [{}^nC_r = {}^nC_{n-r}]$$

$$= \frac{6.5}{2.1} + 7 = 15 + 7 = 22 \text{ ways}$$

Hence, the required number of ways = 22.

Q6. How many committee of five persons with a chairperson can be selected from 12 persons?

Sol. Total number of Persons = 12

Number of persons to be selected = 5

Out of 5, there is a chairperson

$\therefore$ Number of ways of selecting a chairperson = ${}^{12}C_1 = 12$

Number of ways of selecting other 4 numbers out of remaining 11 persons = ${}^{11}C_4$

$\therefore$ Total number of ways = ${}^{12}C_1 \times {}^{11}C_4$

$$= 12 \times \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} = 12 \times 330 = 3960$$

Hence, the required number of ways = 3960.

Q7. How many automobile license plates can be made, if each plate contains two different letters followed by three different digits?

Sol. We have 26 English alphabet and 10 digits (0 to 9)

Since, it is given that each plate contains 2 different letters followed by 3 different digits.

$\therefore$ Number of arrangement of 26 letters taken 2 at a time

$$= {}^{26}P_2 = \frac{26!}{(26-2)!} = \frac{26!}{24!} = \frac{26 \cdot 25 \cdot 24!}{24!} = 650$$

Three digit number can be formed out of 10 digit = ${}^{10}P_3$

$$= \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 720$$

$\therefore$ Total number of license plates = $650 \times 720 = 468000$

Hence, the required number of plates = 468000.

Q8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.

Sol. Given that bag contains 5 black and 6 red balls.

Number of ways of selecting 2 black balls out of 5 black balls = ${}^5C_2$

and number of ways of selecting 3 red balls out of 6 red balls = ${}^6C_3$

$\therefore$ Total number of ways of selecting 2 black and 3 red balls $= {}^5C_2 \times {}^6C_3$

$$= \frac{5 \cdot 4}{2 \cdot 1} \times \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 10 \times 20 = 200 \text{ ways}$$

Hence, the required ways of selecting the balls = 200.

Q9. Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

Sol. Total number of things = n

3 things must be together

$\therefore$ the number of remaining things = $n - 3$

Number of things to be selected = r

Out of r, 3 are always together

$\therefore$ Number of ways of selection = ${}^{n-3}C_{r-2}$

Now permutation of 3 things which are always together = 3!

Number of permutations of $(r - 2)$ things = $(r - 2)!$

$\therefore$ Total number of arrangements = ${}^{n-3}C_{r-2} \times (r - 2)! \times 3!$

Hence the required arrangements = ${}^{n-3}C_{r-2} \times (r - 2)! \times 3!$

Q10. Find the number of different words that can be formed from the letters of the 'TRIANGLE' so that no vowels are together.

Sol. Total number of words in 'TRIANGLE' = 8

Out of 5 are consonants and 3 are vowels

If vowels are not together, taken we have the following arrangement

V | C | V | C | V | C | V | C | V | C | V

Consonant can be arranged in 5! = 120 ways

Vowel occupy 6 places

$\therefore$ 3 vowels can be arranged in 6 places = ${}^6P_3$

$$= \frac{6!}{(6-3)!} = \frac{6!}{3!} = 120 \text{ ways}$$

So, the total arrangement = 120×120 = 14400 ways

Here, the required arrangement = 14400 ways.

Q11. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

Sol. Any number divisible by 5, its unit place must have 0 or 5

We have to find 4-digit number greater than 6000 and less than 7000.

So, the unit place can be filled with 2 ways (0 or 5) since, repetition is not allowed

$\therefore$ tens place can be filled with 7 ways and hundreds place can be filled with 8 ways.

But the required number is greater than 6000 and less than 7000. So, thousand place can be filled with 1 digits i.e. 6

Th	H	T	O
1	8	7	2

So, the total number of integers $= 1 \times 8 \times 7 \times 2 = 112$

Hence, the required number of integers = 112

Q12. There are 10 persons named $P_1, P_2, P_3, P_4, \ldots, P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

Sol. Given that $P_1, P_2, P_3, P_4, \ldots P_{10}$ are 10 persons out of which 5 persons are to be arranged but P_1 must occur and P_4 and P_5 never occur

$\therefore$ selection is to be done only for $10 - 3 = 7$ persons

$\therefore$ Number of selection $= {}^7C_4 = \dfrac{7!}{4!(7-4)!} = \dfrac{7!}{4!3!}$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1} = 35$$

5 people can be arranged as 5!

So, the number of arrangement $= 35 \times 5! = 35 \times 120 = 4200$

Hence, the required arrangement = 4200.

Q13. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.

Sol. Total number of lamps = 10

The total number of ways in which hall can be illuminated is equal to the number of selection of one or more items out of n different items.

i.e. $${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \cdots + {}^nC_n = 2^n - 1$$

From Binomial expansion, we have

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$$

So total number of ways $= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \cdots + {}^{10}C_{10}$

$$= 2^{10} - 1 = 1024 - 1 = 1023$$

Hence, the required number of possible ways = 1023.

Q14. A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw?

Sol. We have 2 white, 3 black and 4 red balls in a box. 3 balls are to be drawn out of 9 balls atleast one black ball is to be included

So, the possible selection is

(1 black and 2 other balls) or (2 black and 1 other ball) or (3 black and no other ball)

So, the number of possible selection is

$$= {}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3 \times {}^6C_0$$
$$= 3 \times 15 + 3 \times 6 + 1 \times 1 = 45 + 18 + 1 = 64$$

Hence, the required selection = 64.

Q15. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find the value of ${}^rC_2$.

Sol. Given that

$$ {}^nC_{r-1} = 36 \quad ...(i)$$
$$ {}^nC_r = 84 \quad ...(ii)$$
$$ {}^nC_{r+1} = 126 \quad ...(iii)$$

Dividing eq. (*i*) by eq. (*ii*) we get

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{36}{84}$$

$$\Rightarrow \quad \frac{\dfrac{n!}{(r-1)!\,(n-r+1)!}}{\dfrac{n!}{r!(n-r)!}} = \frac{3}{7} \quad \left[\because \ {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$\Rightarrow \quad \frac{n!}{(r-1)!\,(n-r+1)!} \times \frac{r!\,(n-r)!}{n!} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{r\cdot(r-1)!\,(n-r)!}{(r-1)!\,(n-r+1)\,(n-r)!} = \frac{3}{7} \quad \Rightarrow \quad \frac{r}{n-r+1} = \frac{3}{7}$$

$$\Rightarrow \quad 3n - 3r + 3 = 7r \ \Rightarrow \ 3n - 10r = -3 \quad ...(iv)$$

Now dividing eq. (*ii*) by eq. (*iii*), we get

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{84}{126} \Rightarrow \frac{\dfrac{n!}{r!(n-r)!}}{\dfrac{n!}{(r+1)!\,(n-r-1)!}} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{n!}{r!(n-r)!} \times \frac{(r+1)!\,(n-r-1)!}{n!} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{(r+1)\cdot r!\,(n-r-1)!}{r!(n-r)\,(n-r-1)!} = \frac{2}{3} \Rightarrow \frac{r+1}{n-r} = \frac{2}{3}$$

$$\Rightarrow \quad 2n - 2r = 3r + 3 \ \Rightarrow \ 2n - 5r = 3 \quad ...(v)$$

Solving eq. (*iv*) and (*v*) we have

$$3n - 10r = -3$$
$$2n - 5r = 3$$

$$\begin{array}{rcrcr} 3n & - & 10r & = & -3 \\ 4n & - & 10r & = & 6 \\ (-) & & (+) & & (-) \\ \hline -n & & & = & -9 \end{array} \Rightarrow n = 9$$

$$\therefore \quad 2 \times 9 - 5r = 3 \Rightarrow 18 - 5r = 3$$

$\Rightarrow \qquad r = \dfrac{15}{5} = 3$

So, $\qquad {}^rC_2 = {}^3C_2 = \dfrac{3!}{2!(3-2)!} = 3$

Hence, the value of ${}^rC_2 = 3$

Q16. Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.

Sol. Given that all the 5 digit numbers are greater than 7000.
So, the ways of forming 5-digit numbers $= 5 \times 4 \times 3 \times 2 \times 1 = 120$
Now all the four digit number greater than 7000 can be formed as follows.
Thousand place can be filled with 3 ways
Hundred place can be filled with 4 ways
Tenths place can be filled with 3 ways
Units place can be filled with 2 ways
So, the total number of 4-digits numbers $= 3 \times 4 \times 3 \times 2 = 72$
$\therefore$ Total number of integers $= 120 + 72 = 192$
Hence, the required number of integers = 192

Q17. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?

Sol. Given that out of 20 lines, no two lines are parallel and no three lines are concurrent.
Therefore, number of point of intersection
$= {}^{20}C_2$ [$\because$ for any point of intersection, we need two lines]

$$= \frac{20 \cdot 19}{2 \cdot 1} = 190$$

Hence, the required number of points = 190.

Q18. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six-digits distinct?

Sol. If first two digits is 41, then the remaining 4 digits can be arranged in ${}^8P_4$ ways

$$= \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 1680$$

Similarly first two digits can be 42 or 46 or 62 or 64.
$\therefore$ Total number of telephone numbers have all digits distinct $= 5 \times 1680 = 8400$
Hence, the required telephone numbers = 8400

Q19. In an examinations, a student has to answer 4 questions out of 5 questions, question 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.

Sol. Given that question number 1 and 2 are compulsory.

$\therefore$ The remaining questions are $5 - 2 = 3$

Total number of questions to be attempted = 4 questions 1 and 2 are compulsory

So only 2 questions are to be done out of 3 questions

Therefore number of ways $= {}^3C_2 = {}^3C_{3-2} = {}^3C_1 = 3$

$[\therefore \quad {}^nC_r = {}^nC_{n-r}]$

Hence, the required number of ways = 3.

Q20. If a convex polygon has 44 diagonals, then find the number of its sides.

Sol. Let n be the number of sides in a polygon.

Since, Polygon of n sides has $({}^nC_2 - n)$ number of diagonals

$$\therefore {}^nC_2 - n = 44 = \frac{n!}{2!(n-2)!} - n = 44$$

$$= \frac{n(n-1)(n-2)!}{2 \cdot (n-2)!} - n = 44 \quad \Rightarrow \quad \frac{n(n-1)}{2} - n = 44$$

$$= \frac{n^2 - n - 2n}{2} = 44 \quad \Rightarrow \quad n^2 - 3n = 88 \quad \Rightarrow \quad n^2 - 3n - 88 = 0$$

$$= n^2 - 11n + 8n - 88 = 0 \quad \Rightarrow \quad n(n-11) + 8(n-11) = 0$$

$$= (n-11)(n+8) = 0 \quad \therefore n = 11 \text{ and } n = -8 \qquad [\because \quad n \neq -8]$$

So $n = 11$

Hence, the required number of sides = 11.

LONG ANSWER TYPE QUESTIONS

Q21. 18 mice were placed in two experimental groups and one control group with all groups equally large. In how many ways can the mice be placed into three groups?

Sol. Given that 18 mice were placed equally in two experimental groups and one control group i.e. 3 groups

$\therefore$ The required number of arrangements

$$= \frac{\text{Total arrangements}}{\text{Equally likely arrangements}} = \frac{18!}{6!\,6!\,6!} = \frac{18!}{(6!)^3}$$

Hence, the required arrangements $= \dfrac{18!}{(6!)^3}$

Q22. A bag contains six white marbles and five red marbles. Find the number of ways in white four marbles can be drawn from the bag, if (i) they can be of any colour. (ii) two must be white and two red (iii) they must all be of the same colour.

Sol. Total number of marbles = 6 white + 5 red = 11 marbles

(*i*) Since, we have to draw 4 marbles of any colour from the 11 marbels

$\therefore$ Required number of ways = $^{11}C_4$

(*ii*) If 2 must be white and 2 must be red, then the required number of ways = $^6C_2 \times {}^5C_2$

(*iii*) If all the 4 marbles are of the same colour, then, the required number of ways = $^6C_4 + {}^5C_4$

Hence the required number of ways are

(*i*) $^{11}C_4$ (*ii*) $^6C_2 \times {}^5C_2$ (*iii*) $^6C_4 + {}^5C_4$

Q23. In how many ways can a football team of 11 players be selected from 16 players? How many of them will

(*i*) include 2 particular players?

(*ii*) exclude 2 particular players?

Sol. Given that the total number of players = 16

We have to select 11 players out of 16 players.

(*i*) If 2 players are included, then

then number of ways of selection = $^{16-2}C_{11-2} = {}^{14}C_9$

(*ii*) If 2 players are excluded then,

the number of ways of selection = $^{16-2}C_{11} = {}^{14}C_{11}$

Hence, the required number of ways of selection

(*i*) $^{14}C_9$ (*ii*) $^{14}C_{11}$

Q24. A sports team of 11 students is to be constituted, choosing atleast 5 from class XI and atleast 5 from class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

Sol. Total number of students in each class = 20

We have to select atleast 5 students from each class.

We have the following cases.

(*i*) 5 students from XI class and 6 students from XII class

(*ii*) 6 students from XI class and 5 students from XII class

So, number of ways of selection of a team of 11 players

$$= {}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5 = 2[{}^{20}C_5 \times {}^{20}C_6]$$

Hence, the required ways of selection = $2[{}^{20}C_5 \times {}^{20}C_6]$

Q25. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has

(*i*) no girls (*ii*) atleast one boy and one girl

(*iii*) at least three girls.

Sol. We have 4 girls and 7 boys and a team of 5 members is to be selected.

(*i*) If no girl is selected, then all the 5 members are to be selected out of 7 boys *i.e.* $^7C_5 = \frac{7!}{5!2!} = \frac{7\times 6.5!}{5!\times 2} = 21$ ways

(*ii*) When at least one by and one girl are to be selected, then

$$\text{Number of ways} = {}^4C_1 \times {}^7C_4 + {}^4C_2 \times {}^7C_3 + {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$$

$$= 4\times\frac{7\times 6\times 5\times 4}{4\times 3\times 2\times 1} + \frac{4\times 3}{2\times 1}\times\frac{7\times 6\times 5}{3\times 2\times 1} + 4\times\frac{7\times 6}{2\times 1} + 1\times 7$$

$= 4 \times 35 + 6 \times 35 + 4 \times 21 + 7 = 140 + 210 + 84 + 7 = 441$ ways

(*iii*) When atleast 3 girls are included, then

$$\text{Number of ways} = {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$$

$$= 4\times\frac{7\times 6}{2\times 1} + 1\times 7 = 84 + 7 = 91 \text{ ways}$$

Hence the required number of ways are

(*i*) 21 ways (*ii*) 441 ways (*iii*) 91 ways

OBJECTIVE TYPE QUESTIONS

Q26. If $^nC_{12} = {}^nC_8$, then n is equal to

(*a*) 20 (*b*) 12 (*c*) 6 (*d*) 30

Sol. Given that $^nC_{12} = {}^nC_8$ $[\because {}^nC_r = {}^nC_{n-r}]$

$^nC_{12} = {}^nC_{n-8}$

$\therefore$ $n - 8 = 12 \Rightarrow n = 12 + 8 = 20$

Hence, the correct option is (*a*)

Q27. The number of possible outcomes when a coin is tossed 6 times is

(*a*) 36 (*b*) 64 (*c*) 12 (*d*) 32

Sol. We know that a coin has Head and Tail (H, T)

$\therefore$ When a coin is tossed 6 times, then the

Possible outcome = $2^6 = 64$

Hence, the correct option is (*b*).

Q28. The number of different four-digit numbers that can be formed with the digit 2, 3, 4, 7 and using each digit only once is

(*a*) 120 (*b*) 96 (*c*) 24 (*d*) 100

Sol. Four-digit numbers are to be formed from the digits 2, 3, 4, 7 without repetition

So, the required 4-digit numbers = $^4P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$

Hence, the correct option is (*c*).

Q29. The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taking all at a time is

(*a*) 432 (*b*) 108 (*c*) 36 (*d*) 18

Sol. If we fix 3 at unit place, then the total possible numbers = 3!
If we fix 4, 5 and 6 at unit place, this is each case, total possible numbers are 3!
Required sum of unit digits of all such numbers is

$$= 3 \times 3! + 4 \times 3! + 5 \times 3! + 6 \times 3! = (3 + 4 + 5 + 6) \times 3!$$
$$= 18 \times 3! = 18 \times 3 \times 2 \times 1 = 108$$

Hence, the correct option is (*b*).

Q30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to

(*a*) 60 (b) 120 (c) 7200 (d) 720

Sol. Given that total numbers of vowels = 4
and total numbers of consonants = 5
Total number of words formed by 2 vowels and 3 consonants

$$= {}^4C_2 \times {}^5C_3 = \frac{4!}{2!2!} \times \frac{5!}{3!2!} = \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} \times \frac{5 \times 4 \times 3!}{3! \times 2}$$
$$= 6 \times 10 = 60$$

Now permutation of 2 vowels and 3 consonants = 5!

$$= 5 \times 4 \times 3 \times 2 \times 1 = 120$$

So, the total number of words = 60 × 120 = 7200.
Hence, the correct option is (*c*).

Q31. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is

(*a*) 216 (*b*) 600 (*c*) 240 (*d*) 3125

Sol. We know that a number is divisible by 3 when the sum of its digits is divisible by 3.
If we take the digits 0, 1, 2, 4, 5, then the sum of the digits = 0 + 1 + 2 + 4 + 5 = 12 which is divisible by 3
So, the 5 digit numbers using the digits 0, 1, 2, 4, and 5

TTh	Th	H	T	O
4	4	3	2	1

$$= 4 \times 4 \times 3 \times 2 \times 1 = 96$$

and if we take the digits 1, 2, 3, 4, 5, then their sum

$$= 1 + 2 + 3 + 4 + 5 = 15 \text{ divisible by } 3$$

So, five digit numbers can be formed using the digits 1, 2, 3, 4, 5 is 5! ways = 5 × 4 × 3 × 2 × 1 = 120 ways
Total number of ways = 96 + 120 = 216
Hence, the correct option is (*a*).

Q32. Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

(*a*) 11 (*b*) 12 (*c*) 13 (*d*) 14

Sol. Let the total number of persons in a room be n since, two persons make 1 hand shake

$\therefore$ The number of hand shakes = ${}^nC_2$

So ${}^nC_2 = 66$

$$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \Rightarrow \frac{n(n-1)(n-2)!}{2\times 1\times (n-2)!} = 66$$

$$\Rightarrow \frac{n(n-1)}{2} = 66 \Rightarrow n^2 - n = 132$$

$$\Rightarrow n^2 - n - 132 = 0 \Rightarrow n^2 - 12n + 11n - 132 = 0$$

$$\Rightarrow n(n-12) + 11(n-12) = 0 \Rightarrow (n-12)(n+11) = 0$$

$$\Rightarrow n - 12 = 0, n + 11 = 0 \Rightarrow n = 12, n = -11$$

$$\therefore n = 12 \qquad (\because n \neq -11)$$

Hence, the correct option is (*b*).

Q33. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(*a*) 105 (*b*) 15 (*c*) 175 (*d*) 185

Sol. Total number of triangles formed from 12 points taking 3 at a time = ${}^{12}C_3$

But given that out of 12 points, 7 are collinear

So, these seven points will form no triangle.

$\therefore$ The required number of triangles = ${}^{12}C_3 - {}^7C_3$

$$= \frac{12!}{3!\cdot 9!} - \frac{7!}{3!4!} = \frac{12\times 11\times 10\times 9!}{3\times 2\times 1\times 9!} - \frac{7\times 6\times 5\times 4!}{3\times 2\times 1\times 4!}$$

$$= \frac{12\times 11\times 10}{3\times 2} - \frac{7\times 6\times 5}{3\times 2} = 220 - 35 = 185$$

Hence, the correct option is (*d*).

Q34. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

(*a*) 6 (*b*) 18 (*c*) 12 (*d*) 9

Sol. We know that to form a parallelogram, we require a pair of lines from a set of 4 lines and another pair of lines from another set of 3 lines

$\therefore$ Required numbers of parallelograms = ${}^4C_2 \times {}^3C_2 = 6\times 3 = 18$

Hence, the correct option is (*b*).

Q35. The number of ways in which a team of 11 players can be selected from 22 players always including 2 of them and excluding 4 of them is

(*a*) ${}^{16}C_{11}$ (*b*) ${}^{16}C_5$ (*c*) ${}^{16}C_9$ (*d*) ${}^{20}C_9$

Sol. Total number of players = 22
2 players are always included and 4 are always excluding or never included = 22 – 2 – 4 = 16
$\therefore$ Required number of selection = ${}^{16}C_9$
Hence, the correct option is (*c*).

Q36. The number of 5-digit telephone numbers having atleast one of their digits repeated is
(*a*) 90,000 (*b*) 10,000 (*c*) 30,240 (*d*) 69,760

Sol. Total number of 5-digit telephone number if all the digits are repeated = $(10)^5$ [$\because$ digits are from 0 to 9]
If digits are not repeated, then 5-digit telephones, can be formed in ${}^{10}P_5$ ways
$\therefore$ Required number of ways $=(10)^5 - {}^{10}P_5$

$$= 100000 - \frac{10!}{(10-5)!} = 100000 - \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!}$$
$$= 100000 - 30240 = 69760$$

Hence, the correct option is (*d*).

Q37. The number of ways in which we can choose committee from four men and six women so that the committee includes atleast two men and exactly twice as many women as men is
(*a*) 94 (*b*) 126 (*c*) 128 (*d*) None

Sol. Number of men = 4
Number of women = 6
We are given that the committee includes 2 men and exactly twice as many women as men.
Thus, the possible selection can be
2 men and 4 women and 3 men and 6 women.
So, the number of committee = ${}^4C_2 \times {}^6C_4 + {}^4C_3 \times {}^6C_6$
$= 6 \times 15 + 4 \times 1 = 90 + 4 = 94$
Hence, the correct option is (*a*).

Q38. The total number of 9 digit numbers which have all different digits is
(*a*) 10! (*b*) 9! (*c*) $9 \times 9!$ (*d*) $10 \times 10!$

Sol. We have to form 9 digit numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and we know that 0 can not be put on extremely left place.
So, first place from the left can be filled in 9 ways.
Now repetition is not allowed. So, the remaining 8 places can be filled in 9!
$\therefore$ The required number of ways = $9 \times 9!$
Hence, the correct option is (*c*).

Q39. The number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place is

(*a*) 1440 (*b*) 144 (*c*) 7! (*d*) $^4C_4 \times {}^3C_3$

Sol. Total number of letters in the 'ARTICLE' is 7 out which A, E, I are vowels and R, T, C, L are consonants

Given that vowels occupy even place

$\therefore$ possible arrangement can be shown as below

C, V, C, V, C, V, C i.e. on 2nd , 4th and 6th places

Therefore, number of arrangement = $^3P_3 = 3! = 6$ ways

Now consonants can be placed at 1, 3, 5 and 7th place

$\therefore$ Number of arrangement = $^4P_4 = 4! = 24$

So, the total number of arrangements = $6 \times 24 = 144$

Hence, the correct option is (*b*).

Q40. Given 5 different green dyes, four different blue dyes and 3 different red dyes, the number of combinations of dyes which can be choosen taking atleast one green and one blue dye is

(*a*) 3600 (*b*) 3720 (*c*) 3800 (*d*) 3600

Sol. Possible number of choosing 5 different green dyes = 2^5

Possible number of choosing 4 blue dyes = 2^4

and possible number of choosing 3 red dyes = 2^3

If atleast one blue and one green dyes are selected then the total number of selection

$$= (2^5 - 1) \times (2^4 - 1) \times 2^3 = 31 \times 15 \times 8 = 3720$$

Hence, the correct option is (*b*).

FILL IN THE BLANK

Q41. If $^nP_r = 840$, $^nC_r = 35$, then r = ____________.

Sol. Given that $^nP_r = 840$ and $^nC_r = 35$

$$\Rightarrow \quad = \frac{n!}{(n-r)!} = 840 \quad ...(i) \quad \text{and} \quad \frac{n!}{r!(n-r)!} = 35 \quad ...(ii)$$

Dividing eq. (*i*) by eq. (*ii*) we get

$$\frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{840}{35} \Rightarrow \frac{n!}{(n-r)!} \times \frac{r!(n-r)!}{n!} = 24$$

$$\Rightarrow \quad r! = 24 \quad \Rightarrow \quad r! = 4 \times 3 \times 2 \times 1$$

$$\Rightarrow \quad r! = 4! \quad \therefore \quad r = 4$$

Hence the value of the filler is 4.

Q42. $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ = ____________.

Sol. $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 = {}^{15}C_{15-8} + {}^{15}C_{15-9} - {}^{15}C_6 - {}^{15}C_7$

$[\because \quad {}^nC_r = {}^nC_{n-r}]$

$$= {}^{15}C_7 + {}^{15}C_6 - {}^{15}C_6 - {}^{15}C_7 = 0$$

Hence, the value of the filler is 0.

Q43. The number of permutations of n different objects, taken r at a time, when repetitions is allowed is ________.

Sol. Number of permutation of n different objects, taken r at a time is n^r.

Q44. The number of different words that can be formed from the letters of the word 'INTERMEDIATE' such that two vowels never come together is ________.

Sol. Total number of words is INTERMEDIATE = 12
which have 6 vowels and 6 consonants
If two vowels never come together then we can arrange as under

V C V C V C V C V C V C V

Here, vowels are IEEIAE where 2 I's and 3 E's are there.

$\therefore$ Number of ways of arranging vowels $= \dfrac{7!}{3!2!} = 420$

Consonants are NTRMDT where 2T's are there

$\therefore$ Number of ways arranging consonants $= \dfrac{6!}{2!}$

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 360$$

So, the total number of words are

$$= 420 \times 360 = 151200$$

Hence, the value of the filler is 151200.

Q45. Three balls are drawn from a bag containing 5 red, 4 white and 3 black balls. The number of ways is which this can be done if atleast 2 are red is ________.

Sol. We have 5 red, 4 white and 3 black balls out of which atleast 2 red balls are to be drawn

$\therefore$ Number of ways $= {}^5C_2 \times {}^7C_1 + {}^5C_3$

$$= 10 \times 7 + 10 = 70 + 10 = 80$$

Hence, the value of the filler = 80.

Q46. The number of 6-digit numbers, all digits of which are odd is ________.

Sol. Out of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 the odd digits are 1, 3, 5, 7, 9.
Therefore number of 6 digit numbers = $(5)^6$
Hence, the value of the filler is $(5)^6$.

Q47. In a football championship, 153 matches were played, Every two teams played one match with each other. The number of teams, participating in the championship is ________.

Sol. Let the number of participating teams be n

Given that every two teams played one match with each other.

$\therefore$ Total number of matches played = ${}^nC_2$

So ${}^nC_2 = 153$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 153 \quad \Rightarrow \quad n^2 - n = 306$$
$$\Rightarrow \quad n^2 - n - 306 = 0 \quad \Rightarrow \quad n^2 - 18n + 17n - 306 = 0$$
$$\Rightarrow \quad n(n-18) + 17(n-18) = 0 \quad \Rightarrow \quad (n-18)(n+17) = 0$$
$$\Rightarrow \quad n - 18 = 0 \quad \text{and} \quad n + 17 = 0$$
$$\Rightarrow \quad n = 18, \quad n \neq -17$$

Hence, the value of the filler is 18.

Q48. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two signs '–' occur together is ________.

Sol. The following may be the arrangement of (–) and (+)

(–) (+) (–) (+) (–) (+) (–) (+) (–) (+) (–) (+) (–)

Therefore, '+' sign can be arranged only is 1 way because all are identical.

and 4(–) signs can be arranged at 7 places in ${}^7C_4$ ways

$\therefore$ Total number of ways $= {}^7C_4 \times 1 = \dfrac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} \times 1 = 35$ ways

Hence, the value of the filler is 35.

Q49. A committee of 6 is to be choosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. In how many different ways can this be done if two particular women refuse to serve on the same committee?

Sol. We have 10 men and 7 women out of which a committee of 6 is to be formed which contain atleast 3 men and 2 women

Therefore, Number of ways = ${}^{10}C_3 \times {}^7C_3 + {}^{10}C_4 \times {}^7C_2$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} + \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times \frac{7 \times 6}{2 \times 1}$$
$$= 120 \times 35 + 210 \times 21 = 4200 + 4410 = 8610$$

If 2 particular women to be always present, then the number of ways = ${}^{10}C_4 \times {}^5C_0 + {}^{10}C_3 \times {}^5C_1$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times 1 + \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \times 5 = 210 + 120 \times 5$$
$$= 210 + 600 = 810$$

$\therefore$ Total number of committee = 8610 – 810 = 7800

Hence, the value of the filler is 7800.

Q50. A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways three balls be drawn from the box if atleast one black ball is to be included in the draw is ________.

Sol. We have 2 white, 3 black and 4 red balls

It is given that atleast 1 black ball is to be included.

$\therefore$ Required number of ways $= {}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3$

$= 3 \times 15 + 3 \times 6 + 1 = 45 + 18 + 1 = 64$

Hence, the value of the filler is 64.

TRUE/FALSE STATEMENT

Q51. There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^5C_2$

Sol. Required number of lines $= {}^{12}C_2 - {}^5C_2 + 1$

Hence, the given statement is 'False'

Q52. Three letters can be posted in five letter boxes in 3^5 ways.

Sol. Given that 3 letters are to be posted in 5 letter boxes

$\therefore$ Required number of ways $= 5^3 = 125$

Hence, the given statement is 'False'

Q53. In the permutation of n things, r taken together, the number of permutations in which m particular things occur together is ${}^{n-m}P_{r-m} \times {}^rP_m$.

Sol. Arrangement of n things, r taken at a time in which m things occur together.

So, number of object excluding m object $= (r - m)$

Here, we first arrange $(r - m + 1)$ object

$\therefore$ Number of arrangements $= (r - m + 1)!$

m objects can be arranged in $m!$ ways

So, the required number of arrangements $= (r - m + 1)! \times m!$

Hence, the given statement is 'False'.

Q54. In a steamer there are stalls for 12 animals, and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in 3^{12} ways.

Sol. There are 3 types of animals horses, cows and calves not less than 12 each.

So, number of ways of loading $= 3^{12}$

Hence, the given statement is 'True'.

Q55. If some or all of n objects are taken at a time, then the number of combination is $2^n - 1$.

Sol. When some or all objects, taken at a time, then the number of selection will be

$${}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n = 2^n - 1$$

$$[\because {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n]$$

Hence, the given statement is 'True'.

Q56. There will be only 24 selections containing at least one red ball out of a bag containing 4 red and 5 black balls. It is being given that the balls of the same colour are identical.

Sol. We have 4 red and 5 black balls in a box and atleast one red ball is to be drawn

$\therefore$ Number of selection $= [(4 + 1)(5 + 1) - 1] - 5 = [5 \times 6 - 1] - 5$
$29 - 5 = 24$

Hence, the given statement is 'True'.

Q57. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. The number of ways in which the seating arrangements can be made is

$$= \frac{11!}{5!6!}(9!)(9!)$$

Sol. When 4 guests sit can one side and 3 on the other side, we have to select out of 11. 5 sit one one side and 6 sit on the other side.

Now, remaining selecting on one half side $= {}^{18-4-3}C_5$
$= {}^{11}C_5$

and the other half side $= {}^{(11-5)}C_6 = {}^{6}C_6$

So, the total arrangements $= {}^{11}C_5 \times 9! \times {}^{6}C_6 \times 9!$

$$= \frac{11!}{5!6!} \times 9! \times 1 \times 9! = \frac{11!}{5!6!}(9!)(9!)$$

Hence, the given statement is 'True'.

Q58. A candidate is required to answer 7 questions, out of 12 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. He can choose the seven questions in 650 ways.

Sol. The candidate may attempt in following manner

Group A	2	3	4	5
Group B	5	4	3	2

So, the number of attempts of 7 questions

$= {}^6C_2 \times {}^6C_5 + {}^6C_3 \times {}^6C_4 + {}^6C_4 \times {}^6C_3 + {}^6C_5 \times {}^6C_2$

$= 2[{}^6C_2 \times {}^6C_5 + {}^6C_3 \times {}^6C_4]$

$= 2\,[15 \times 6 + 20 \times 15] = 2[90 + 300] = 2 \times 390 = 780.$

Q59. To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, the number of ways in which the selection can be made is ${}^5C_3 \times {}^{20}C_9$

Sol. Number of ways to select 3 scheduled caste candidate out of $5 = {}^5C_3$

We have to select 9 other candidates out of 22.

So the number of ways $= {}^{22}C_9$

Required number of selection $= {}^5C_3 \times {}^{22}C_9$

Hence, the given statement is False.

MATCH THE COLUMNS

Q60. There are 3 books in Mathematics, 4 on physics and 5 on English. How many different collections can be made such that each collection consists of:

C_1	C_2
(*a*) One book of each subject	(*i*) 3968
(*b*) At least one book of each subject	(*ii*) 60
(*c*) At least one book of English	(*iii*) 3255

Sol. We have 3 books of Mathematics, 4 of Physics and 5 on English

(*a*) One book of each subject $= {}^3C_1 \times {}^4C_1 \times {}^5C_1 = 3 \times 4 \times 5 = 60$

(*b*) Atleast one book of each subject $= (2^3 - 1) \times (2^4 - 1) \times (2^5 - 1)$

$$= 7 \times 15 \times 31 = 3255$$

(*c*) Atleast one book of English $= (2^5 - 1) \times 2^7 = 31 \times 128 = 3968.$

Hence the required matching is

(*a*) ↔ (*ii*), (*b*) ↔ (*iii*) and (*c*) ↔ (*i*)

Q61. Five boys and 5 girls form a line. Find the number of ways of making the seating arrangement under the following condition.

C_1	C_2
(*a*) Boys and girls sit alternate	(*i*) $5! \times 6!$
(*b*) No two girls sit together	(*ii*) $10! - 5!\ 6!$
(*c*) All the girls sit together	(*iii*) $(5!)^2 + (5!)^2$
(*d*) All the girls are never together	(*iv*) $2!\ 5!\ 5!$

Sol. (*a*) Total number of arrangement when boys and girls alternate: $= (5!)^2 + (5!)^2$

(*b*) No two girls sit together: $= 5!\ 6!$

(*c*) All the girls sit together $= 2!\ 5!\ 5!$

(*d*) All the girls sit never together $= 10! - 5!\ 6!$

Hence, the required matching is

(*a*) ↔ (*iii*), (*b*) ↔ (*i*), (*c*) ↔ (*iv*), (*d*) ↔ (*ii*)

Q62. There are 10 professors and 20 lecturers out of whom a committee of 2 professors and 3 lecturers is to be formed. Find

C_1	C_2
(*a*) In how many ways committee can be formed	(*i*) ${}^{10}C_2 \times {}^{19}C_3$
(*b*) In how many ways a particular professor is included	(*ii*) ${}^{10}C_2 \times {}^{19}C_2$
(*c*) In how many ways a particular lecturer is included	(*iii*) ${}^{9}C_1 \times {}^{20}C_3$
(*d*) In how many ways a particular lecturer is excluded	(*iv*) ${}^{10}C_2 \times {}^{20}C_3$

Sol. (*a*) We have to select 2 professor out of 10 and 3 lecturers out of 20

$\therefore$ Number of ways of selection = ${}^{10}C_2 \times {}^{20}C_3$

(*b*) When a particular professor is included taken the number of ways = ${}^{10-1}C_1 \times {}^{20}C_3 = {}^{9}C_1 \times {}^{20}C_3$

(*c*) When a particular lecturer is included then number of ways = ${}^{10}C_2 \times {}^{19}C_2$

(*d*) When a particular lecturer is excluded, then number of ways = ${}^{10}C_2 \times {}^{19}C_3$

Hence the required matching is

(*a*) $\leftrightarrow$ (*iv*), (*b*) $\leftrightarrow$ (*iii*), (*c*) $\leftrightarrow$ (*ii*), (*d*) $\leftrightarrow$ (*i*)

Q63. Using the digits 1, 2, 3, 4, 5, 6, 7 a number of 4 different digits is formed. Find

C_1	C_2
(*a*) How many numbers are formed?	(*i*) 840
(*b*) How many number are exactly divisible by 2?	(*ii*) 200
(*c*) How many numbers are exactly divisible by 25?	(*iii*) 360
(*d*) How many of these are exactly divisible by 4?	(*iv*) 40

Sol. (*a*) Total of 4 digit number formed with 1, 2, 3, 4, 5, 6, 7

$$= {}^{7}P_4 = \frac{7!}{(7-4)!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840$$

(*b*) When a number is divisible by 2 = $4 \times 5 \times 6 \times 3 = 360$

(*c*) Total numbers which are divisible by 25 = 40 (*d*) Total numbers which are divisible by 4 (last two digits is divisible by 4) = 200

Hence, the required matching is

(*a*) ↔ (*i*), (*b*) ↔ (*iii*), (*c*) ↔ (*iv*), (*d*) ↔ (*ii*)

Q64. How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

C_1	C_2
(*a*) 4 letters are used at a time	(*i*) 720
(*b*) All letters are used at a time	(*ii*) 240
(*c*) All letters are used but the first is a vowel	(*iii*) 360

Sol. (*a*) 4 letters are used at a time = $^6P_4 = \dfrac{6!}{2!} = 360$

(*b*) All letters are used at a time = $^6P_6 = 6! = 720$

(*c*) All letters are used but first letter is vowel = $2 \times 5! = 2 \times 120 = 240$

Hence, the required matching is

(*a*) ↔ (*iii*), (*b*) ↔ (*i*), (*c*) ↔ (*ii*)