EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Find the equation of the straight line which passes through the point (1, – 2) and cuts off equal intercepts from axes.

Sol. Intercept form of straight line

$\frac{x}{a}+\frac{y}{b}=1$, where a and b are the intercepts on the axis

Given that $a = b$

$$\therefore \quad \frac{x}{a}+\frac{y}{a}=1 \quad ...(i)$$

If eq. (i) passes through the point (1, – 2), we get

$$\frac{1}{a}-\frac{2}{a}=1 \Rightarrow -\frac{1}{a}=1 \Rightarrow a=-1$$

So, equation of the straight line is

$$\frac{x}{-1}+\frac{y}{-1}=1 \Rightarrow x+y=-1 \Rightarrow x+y+1=0$$

Hence, the required equation is $x + y + 1 = 0$.

Q2. Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, – 1).

Sol. Slope of the line joining the points (2, 3) and (3, – 1) is

$$\frac{-1-3}{3-2}=-4$$

Slope of the required line which is perpendicular to it

$$=\frac{-1}{-4}=\frac{1}{4} \qquad [\because m_1m_2=-1]$$

Equation of the line passing through the point (5, 2) is

$$y-2=\frac{1}{4}(x-5) \qquad [y-y_1=m(x-x_1)]$$

$$\Rightarrow \quad 4y-8=x-5$$

$$\Rightarrow \quad x-4y+3=0$$

Hence, the required equation is $x - 4y + 3 = 0$.

Q3. Find the angle between the lines $y = (2-\sqrt{3})(x+5)$ and $y = (2+\sqrt{3})(x-7)$.

Sol. The given equations are $y = (2-\sqrt{3})(x+5)$ ...(*i*)

and $y = (2+\sqrt{3})(x-7)$ ...(*ii*)

Slope of eq. (*i*) m_1 (say) $= (2-\sqrt{3})$

and slope of eq. (*ii*) m_2 (say) $= (2+\sqrt{3})$

Let θ be the angle between the two given lines

$$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2-\sqrt{3}-2-\sqrt{3}}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|$$

$$= \left|\frac{-2\sqrt{3}}{1+4-3}\right| = \left|\frac{-2\sqrt{3}}{2}\right| = \left|-\sqrt{3}\right|$$

$$\Rightarrow \quad \tan\theta = \sqrt{3} \text{ or } -\sqrt{3}$$

$$\therefore \quad \theta = 60° \text{ or } 120°$$

Hence, the required angle is 60° or 120°.

Q4. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinate axes such that their sum is 14.

Sol. Equation of line having *a* and *b* intercepts on the axis is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{...}(i)$$

Given that $a + b = 14 \Rightarrow b = 14 - a$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{14-a} = 1 \quad \text{...}(ii)$$

If eq. (*ii*) passes through the point (3, 4) then

$$\frac{3}{a} + \frac{4}{14-a} = 1$$

$$\Rightarrow \quad \frac{3(14-a)+4a}{a(14-a)} = 1$$

$$\Rightarrow \quad 42 + a = 14a - a^2$$

$$\Rightarrow \quad a^2 + a - 14a + 42 = 0$$

$$\Rightarrow \quad a^2 - 13a + 42 = 0$$

$$\Rightarrow \quad a^2 - 7a - 6a + 42 = 0$$

$$\Rightarrow \quad a(a-7) - 6(a-7) = 0$$

$$\Rightarrow \quad (a-6)(a-7) = 0$$

$$\Rightarrow \quad a = 6, 7$$

$$\therefore \quad b = 14 - 6 = 8, \; b = 14 - 7 = 7$$

Hence, the required equation of lines are

$$\frac{x}{6} + \frac{y}{8} = 1 \quad \Rightarrow \quad 4x + 3y = 24$$

and $$\frac{x}{7} + \frac{y}{7} = 1 \quad \Rightarrow \quad x + y = 7$$

Q5. Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.

Sol. Let (x_1, y_1) be any point lying in the equation $x + y = 4$

$\therefore$ $$x_1 + y_1 = 4 \qquad ...(i)$$

Distance of the point (x_1, y_1) from the equation $4x + 3y = 10$

$$\frac{4x_1 + 3y_1 - 10}{\sqrt{(4)^2 + (3)^2}} = 1$$

$$\left|\frac{4x_1 + 3y_1 - 10}{5}\right| = 1$$

$$4x_1 + 3y_1 - 10 = \pm 5$$

Taking (+) sign $$4x_1 + 3y_1 - 10 = 5$$

$\Rightarrow$ $$4x_1 + 3y_1 = 15 \qquad ...(ii)$$

From eq. (i) we get $y_1 = 4 - x_1$

Putting the value of y_1 in eq. (ii) we get

$$4x_1 + 3(4 - x_1) = 15$$

$\Rightarrow$ $$4x_1 + 12 - 3x_1 = 15$$

$\Rightarrow$ $$x_1 + 12 = 15$$

$\Rightarrow$ $$x_1 = 3 \quad \text{and} \quad y_1 = 4 - 3 = 1$$

So, the required point is (3, 1)

Now taking (–) sign, we have

$$4x_1 + 3y_1 - 10 = -5$$

$\Rightarrow$ $$4x_1 + 3y_1 = 5 \qquad ...(iii)$$

From eq. (i) we get $y_1 = 4 - x_1$

$\Rightarrow$ $$4x_1 + 3(4 - x_1) = 5$$

$\Rightarrow$ $$4x_1 + 12 - 3x_1 = 5$$

$\Rightarrow$ $$x_1 = 5 - 12 = -7$$

and $$y_1 = 4 - (-7) = 11$$

So, the required point is (– 7, 11)

Hence, the required points on the given line are (3, 1) and (–7, 11).

Q6. Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.

Sol. Given that: $\frac{x}{a}+\frac{y}{b} = 1$ ...(*i*)

and $\frac{x}{a}-\frac{y}{b} = 1$ ...(*ii*)

Slope of eq. (*i*) m_1 (say) $= -\frac{b}{a}$

and slope of eq. (*ii*) m_2 (say) $= \frac{b}{a}$

Let θ be the angle between the equation (*i*) and (*ii*)

$$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right| = \left|\frac{-\frac{b}{a}-\frac{b}{a}}{1+\left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-\frac{2b}{a}}{1-\frac{b^2}{a^2}}\right| = \left|\frac{-2ab}{a^2-b^2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{2ab}{a^2-b^2}.$$ Hence proved.

Q7. Find the equation of lines passing through (1, 2) and making angle 30° with y-axis.

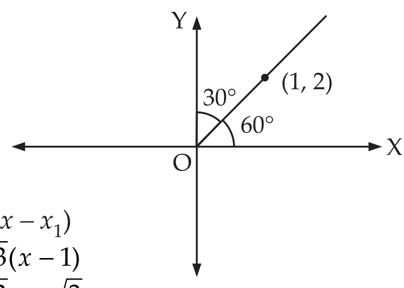
Sol. Given that the line makes angle 30° with y-axis

∴ Angle made by the line with x-axis is 60°

∴ Slope of the line

$m = \tan 60°$

$\Rightarrow \quad m = \sqrt{3}$



So, the equation of the line passing through the point (1, 2) and slope $\sqrt{3}$ is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 2 = \sqrt{3}(x-1)$$

$$\Rightarrow \quad y - 2 = \sqrt{3}x - \sqrt{3}$$

$$\Rightarrow \quad y - \sqrt{3}x + \sqrt{3} - 2 = 0$$

Hence, the required equation of line is $y - \sqrt{3}x + \sqrt{3} - 2 = 0$.

Q8. Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 7$.

Sol. Given that: $2x + y = 5$ ...(*i*)

$x + 3y + 8 = 0$ ...(*ii*)

$3x + 4y = 7$ ...(*iii*)

Equation of any line passing through the point of intersection of eq. (*i*) and eq. (*ii*) is

$$(2x + y - 5) + \lambda(x + 3y + 8) = 0 \quad ...(iv) \qquad (\lambda = \text{constant})$$

$$\Rightarrow \quad 2x + y - 5 + \lambda x + 3\lambda y + 8\lambda = 0$$

$$\Rightarrow \quad (2 + \lambda)x + (1 + 3\lambda)y - 5 + 8\lambda = 0$$

$$\text{Slope of line } m_1 \text{ (say)} = \frac{-(2+\lambda)}{1+3\lambda} \qquad \left[\because m = \frac{-a}{b}\right]$$

Now slope of line $3x + 4y = 7$ is

$$m_2 \text{ (say)} = -\frac{3}{4}$$

If eq. (*iii*) is parallel to eq. (*iv*) then

$$m_1 = m_2$$

$$\Rightarrow \quad \frac{-(2+\lambda)}{1+3\lambda} = -\frac{3}{4}$$

$$\Rightarrow \quad \frac{2+\lambda}{1+3\lambda} = \frac{3}{4} \quad \Rightarrow \quad 8 + 4\lambda = 3 + 9\lambda$$

$$\Rightarrow \quad 9\lambda - 4\lambda = 5 \quad \Rightarrow \quad 5\lambda = 5 \quad \Rightarrow \quad \lambda = 1$$

On putting the value of λ in eq. (*iv*) we get

$$(2x + y - 5) + 1(x + 3y + 8) = 0$$

$$\Rightarrow \quad 2x + y - 5 + x + 3y + 8 = 0$$

$$\Rightarrow \quad 3x + 4y + 3 = 0$$

Hence, the required equation is $3x + 4y + 3 = 0$.

Q9. For what value of a and b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes?

Sol. The given equation are $ax + by + 8 = 0$ ...(*i*)

and $2x - 3y + 6 = 0$ ...(*ii*)

From eq. (*i*) we get,

$$ax + by + 8 = 0 \quad \Rightarrow \quad ax + by = -8$$

$$\Rightarrow \quad \frac{a}{-8}x + \frac{b}{-8}y = 1$$

$$\Rightarrow \quad \frac{x}{\frac{-8}{a}} + \frac{y}{\frac{-8}{b}} = 1$$

So, the intercepts on the axes are $\dfrac{-8}{a}$ and $\dfrac{-8}{b}$

From eq. (*ii*), we get

$$2x - 3y + 6 = 0 \Rightarrow 2x - 3y = -6$$

$$\Rightarrow \frac{2x}{-6} - \frac{3y}{-6} = 1$$

$$\Rightarrow \frac{x}{-3} + \frac{y}{2} = 1$$

So, the intercepts are – 3 and 2.

According to the question

$$\frac{-8}{a} = +3 \Rightarrow a = -\frac{8}{3}$$

and $$\frac{-8}{b} = -2 \Rightarrow b = +4$$

Hence, the required values of a and b are $\frac{-8}{3}$ and 4.

Q10. If the intercept of a line between the coordinate axes is divided by the point (– 5, 4) in the ratio 1 : 2, then find the equation of the line.

Sol. Let a and b be the intercepts on the given line.

$\therefore$ Coordinates of A and B are $(a, 0)$ and $(0, b)$ respectively

$$\therefore \quad -5 = \frac{1 \times 0 + 2 \times a}{1+2}$$

$$\left[\because \quad X = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } Y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right]$$

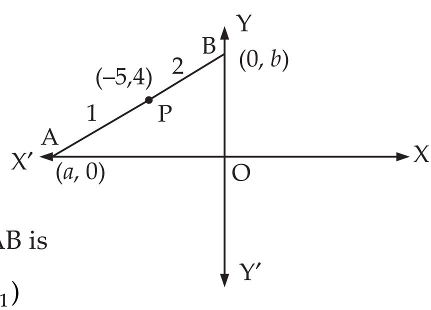
$$\Rightarrow \quad 2a = -15$$

$$\Rightarrow \quad a = \frac{-15}{2}$$

$$\therefore \quad A = \left(\frac{-15}{2}, 0\right)$$

and $$4 = \frac{1 \times b + 0 \times 2}{1+2}$$

$$\Rightarrow \quad 4 = \frac{b}{3} \Rightarrow b = 12$$



$$\therefore \quad B = (0, 12)$$

So, the equation of line AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \left(\frac{12-0}{0+\frac{15}{2}}\right)\left(x+\frac{15}{2}\right)$$

$\Rightarrow$ $$y = \frac{12\times 2}{15}\left(x+\frac{15}{2}\right)$$

$\Rightarrow$ $$y = \frac{8}{5}\left(x+\frac{15}{2}\right)$$

$\Rightarrow$ $$5y = 8x + 60$$

$\Rightarrow$ $$8x - 5y + 60 = 0$$

Hence, the required equation is $8x - 5y + 60 = 0$.

Q11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of X-axis.

Sol. Given that:

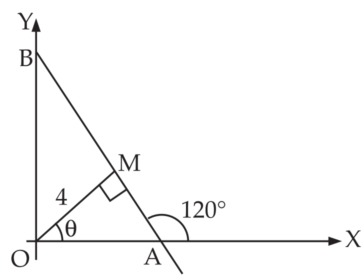
$$OM = 4 \text{ units}$$
$$\angle BAX = 120°$$

$\therefore$ $\angle BAO = 180° - 120°$ or $\angle MAO = 60°$

$\angle MOA + \angle MAO = 90°$ [$\because OM \perp AB$]

$$\theta + 60° = 90°$$

$\therefore$ $$\theta = 30°$$



So, equation of AB in its normal form

$$x\cos\theta + y\sin\theta = p$$

$\Rightarrow$ $$x\cos 30° + y\sin 30° = 4$$

$\Rightarrow$ $$x\times\frac{\sqrt{3}}{2} + y\times\frac{1}{2} = 4$$

$\Rightarrow$ $$\sqrt{3}x + y = 8$$

Hence, the required equation is $\sqrt{3}x + y = 8$.

Q12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x + 4y = 4$ and the opposite vertex of the hypotenuse is (2, 2).

Sol. Given that equation of the hypotenuse is $3x + 4y = 4$ and opposite vertex is (2, 2)

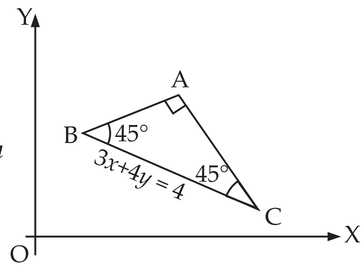
Slope BC = $\frac{-3}{4}$

Let slope of AC be m

$\therefore$ $$\tan 45° = \left|\frac{m+\frac{3}{4}}{1+\left(\frac{-3}{4}\right)m}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{4m+3}{4-3m}\right|$$

$$\Rightarrow \quad \frac{4m+3}{4-3m} = \pm 1$$



Taking (+) sign, $\frac{4m+3}{4-3m} = 1$

$$\Rightarrow \quad 4m + 3 = 4 - 3m$$

$$\Rightarrow \quad 7m = 1$$

$$\Rightarrow \quad m = \frac{1}{7}$$

Taking (–) sign, $\frac{4m+3}{4-3m} = -1$

$$\Rightarrow \quad 4m + 3 = -4 + 3m$$

$$\Rightarrow \quad 4m - 3m = -3 - 4 \quad \Rightarrow \quad m = -7$$

$\therefore$ Equation of AC with slope $\left(\frac{1}{7}\right)$ is

$$y - 2 = \frac{1}{7}(x - 2)$$

$$\Rightarrow \quad 7y - 14 = x - 2$$

$$\Rightarrow \quad x - 7y + 12 = 0$$

Equation of AC with slope (– 7) is

$$y - 2 = -7(x - 2)$$

$$\Rightarrow \quad y - 2 = -7x + 14$$

$$\Rightarrow \quad 7x + y - 16 = 0$$

Hence, the required equation are $x - 7y + 12 = 0$ and $7x + y - 16 = 0$.

LONG ANSWER TYPE QUESTIONS

Q13. If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is (2, – 1), then find the length of the side of the triangle.

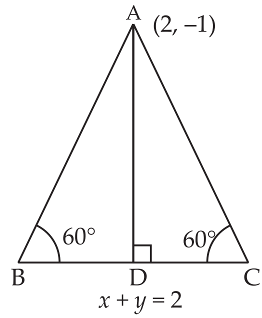
Sol. Equation of the base AB of a ΔABC is $x + y = 2$

In ΔABD,

$$\sin 60° = \frac{AD}{AB}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AD}{AB} \quad \Rightarrow \quad AD = \frac{\sqrt{3}}{2} AB$$

Length of perpendicular from A(2, – 1) to the line $x + y = 2$ is



$$AD = \left|\frac{1\times 2 + 1\times -1 - 2}{\sqrt{(1)^2 + (1)^2}}\right|$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}AB = \left|\frac{2-1-2}{\sqrt{2}}\right| = \left|\frac{-1}{\sqrt{2}}\right|$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}AB = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad AB = \frac{\sqrt{2}}{\sqrt{3}}$$

Hence, the required length of side = $\sqrt{\dfrac{2}{3}}$.

Q14. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P.

Sol. Let (x_1, y_1) be the coordinates of the given point P and m be the slope of the line.

$\therefore$ Equation of the line is $y - y_1 = m(x - x_1)$ ...(*i*)

Given points are A(2, 0), B(0, 2) and C(1, 1).

Perpendicular distance from A(2, 0) to the line (*i*) d_1 (say)

$$d_1 = \frac{0 - y_1 - m(2 - x_1)}{\sqrt{1+m^2}}$$

Perpendicular distance from B(0, 2) d_2 (say)

$$d_2 = \frac{2 - y_1 - m(0 - x_1)}{\sqrt{1+m^2}}$$

Similarly, perpendicular distance from C(1, 1) d_3 (say)

$$d_3 = \frac{1 - y_1 - m(1 - x_1)}{\sqrt{1+m^2}}$$

According to the question, we have

$$d_1 + d_2 + d_3 = 0$$

$$\therefore \frac{0-y_1-m(2-x_1)}{\sqrt{1+m^2}} + \frac{2-y_1-m(0-x_1)}{\sqrt{1+m^2}} + \frac{1-y_1-m(1-x_1)}{\sqrt{1+m^2}} = 0$$

$$\Rightarrow \quad -y_1 - 2m + mx_1 + 2 - y_1 + mx_1 + 1 - y_1 - m + mx_1 = 0$$

$$\Rightarrow \quad 3mx_1 - 3y_1 - 3m + 3 = 0$$

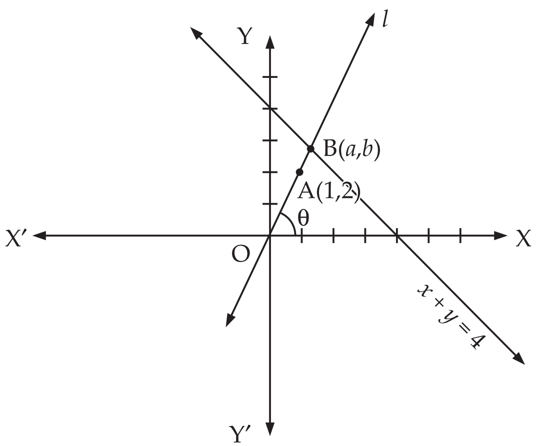
$$\Rightarrow \quad mx_1 - y_1 - m + 1 = 0$$

Since the point (1, 1) satisfies the above equation.

Hence, the point (1, 1) lies on the line.

Q15. In what direction should a line be drawn through the point (1, 2), so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

Sol.



Let the given line $x + y = 4$ and required line 'l' intersect at B(a, b).

Slope of line 'l' is given by $m = \frac{b-2}{a-1} = \tan\theta$ ...(i)

Given that AB $= \frac{\sqrt{6}}{3}$

So, by distance formula for point A(1, 2) and B(a, b), we get

$$\sqrt{(a-1)^2 + (b-2)^2} = \frac{\sqrt{6}}{3}$$

On squaring both the side

$$a^2 + 1 - 2a + b^2 + 4 - 4b = \frac{6}{9}$$

$$a^2 + b^2 - 2a - 4b + 5 = \frac{2}{3} \qquad ...(ii)$$

Point B(a, b) also satisfies the eqn. $x + y = 4$

$$\therefore \quad a + b = 4 \qquad ...(iii)$$

On solving (*ii*) and (*iii*), we get $a = \dfrac{3\sqrt{3}+1}{2\sqrt{3}}$, $b = \dfrac{5\sqrt{3}-1}{2\sqrt{3}}$

Putting values of a and b in eqn. (*i*), we have

$$\tan\theta = \frac{\dfrac{5\sqrt{3}-1}{2\sqrt{3}} - 2}{\dfrac{3\sqrt{3}+1}{2\sqrt{3}}} = \frac{5\sqrt{3}-1-4\sqrt{3}}{3\sqrt{3}+1-2\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$\therefore \tan\theta = \tan 15° \quad \Rightarrow \quad \theta = 15°$

Q16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

Sol. Intercepts form of a straight line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the intercepts made by the line on the axes.

Given that: $\quad \dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{k}$ (say)

$\Rightarrow \quad \dfrac{k}{a} + \dfrac{k}{b} = 1$

which shows that the line is passing through the fixed point (k, k).

Q17. Find the equation of the line which passes through the point (– 4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point.

Sol. Let AB be a line passing through a point (– 4, 3) and meets x-axis at A(a, 0) and y-axis at B(0, b).

$\therefore \quad -4 = \dfrac{5 \times 0 + 3a}{5+3}$

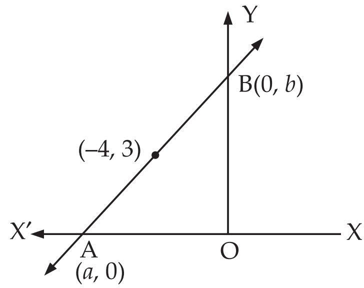
$\Rightarrow \quad -4 = \dfrac{3a}{8}$ $\quad \left[\because \quad X = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } Y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right]$

$\Rightarrow \quad 3a = -32$

$\therefore \quad a = \dfrac{-32}{3}$

and $\quad 3 = \dfrac{5.b + 3.0}{5+3}$

$\Rightarrow \quad 3 = \dfrac{5.b}{8} \quad \Rightarrow \quad 5b = 24$



$\Rightarrow \qquad b = \dfrac{24}{5}$

Intercept form of the line is

$$\frac{x}{\frac{-32}{3}} + \frac{y}{\frac{24}{5}} = 1$$

$$\Rightarrow \quad \frac{-3x}{32} + \frac{5y}{24} = 1$$

$\Rightarrow \quad -9x + 20y = 96 \quad \Rightarrow \quad 9x - 20y + 96 = 0$

Hence, the required equation is $9x - 20y + 96 = 0$.

Q18. Find the equation of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point (3, 2) is $\frac{7}{5}$.

Sol. Given equations are

$$x - y + 1 = 0 \qquad ...(i)$$

and
$$2x - 3y + 5 = 0 \qquad ...(ii)$$

Solving eq. (*i*) and eq. (*ii*) we get

$$\begin{array}{rrrl} 2x & -2y & +2 & = 0 \\ 2x & -3y & +5 & = 0 \\ (-) & (+) & (-) & \\ \hline & y & -3 & = 0 \quad \therefore y = 3 \end{array}$$

From eq. (*i*) we have

$$x - 3 + 1 = 0 \quad \Rightarrow \quad x = 2$$

So, (2, 3) is the point of intersection of eq. (*i*) and eq. (*ii*).

Let m be the slope of the required line

$\therefore$ Equation of the line is

$$y - 3 = m(x - 2)$$

$$\Rightarrow \quad y - 3 = mx - 2m$$

$$\Rightarrow \quad mx - y + 3 - 2m = 0$$

Since, the perpendicular distance from (3, 2) to the line is $\frac{7}{5}$ then

$$\frac{7}{5} = \left|\frac{m(3) - 2 + 3 - 2m}{\sqrt{m^2 + 1}}\right|$$

$$\Rightarrow \quad \frac{49}{25} = \frac{(3m - 2 + 3 - 2m)^2}{m^2 + 1}$$

$$\Rightarrow \quad \frac{49}{25} = \frac{(m+1)^2}{m^2+1}$$
$$\Rightarrow \quad 49m^2 + 49 = 25m^2 + 50m + 25$$
$$\Rightarrow \quad 49m^2 - 25m^2 - 50m + 49 - 25 = 0$$
$$\Rightarrow \quad 24m^2 - 50m + 24 = 0$$
$$\Rightarrow \quad 12m^2 - 25m + 12 = 0$$
$$\Rightarrow \quad 12m^2 - 16m - 9m + 12 = 0$$
$$\Rightarrow \quad 4m(3m-4) - 3(3m-4) = 0$$
$$\Rightarrow \quad (3m-4)(4m-3) = 0$$
$$\Rightarrow \quad 3m - 4 = 0 \text{ and } 4m - 3 = 0$$
$$\therefore \quad m = \frac{4}{3}, \frac{3}{4}$$

Equation of the line taking $m = \frac{4}{3}$ is

$$y - 3 = \frac{4}{3}(x-2)$$
$$\Rightarrow \quad 3y - 9 = 4x - 8 \quad \Rightarrow \quad 4x - 3y + 1 = 0$$

Equation of the line taking $m = \frac{3}{4}$ is

$$y - 3 = \frac{3}{4}(x-2)$$
$$\Rightarrow \quad 4y - 12 = 3x - 6 \quad \Rightarrow \quad 3x - 4y + 6 = 0$$

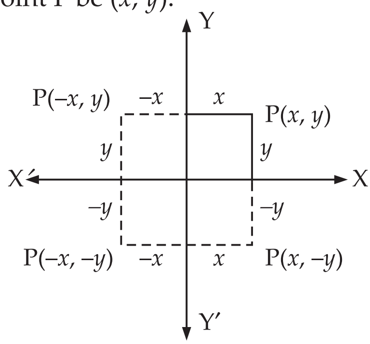
Hence, the required equations are $4x - 3y + 1 = 0$ and $3x - 4y + 6 = 0$

Q19. If the sum of the distance of a moving point in a plane from the axes is 1, then find the locus of the point.

Sol. Let coordinates of a moving point P be (x, y).

Given that the sum of the distances from the axes to the point is always 1

$$\therefore \quad |x| + |y| = 1$$
$$\Rightarrow \quad x + y = 1$$
$$\Rightarrow \quad -x - y = 1$$
$$\Rightarrow \quad -x + y = 1$$
$$\Rightarrow \quad x - y = 1$$



Hence, these equations gives us the locus of the point P which is a square.

Q20. P_1, P_2 are points on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines.

Sol. Given lines are $y - \sqrt{3}|x| = 2$

$$\Rightarrow \quad y - \sqrt{3}x = 2, \text{ if } x \ge 0 \quad ...(i)$$

$$\text{and} \quad y + \sqrt{3}x = 2, \text{ if } x < 0 \quad ...(ii)$$

Slope of eq. (*i*) is $\tan\theta = \sqrt{3}$ $\therefore \theta = 60°$

Slope of eq. (*ii*) is $\tan\theta = -\sqrt{3}$ $\therefore \theta = 120°$

Solving eq. (*i*) and eq. (*ii*) we get

$$\begin{array}{rcl} y - \sqrt{3}x &=& 2 \\ y + \sqrt{3}x &=& 2 \\ \hline 2y &=& 4 \Rightarrow y = 2 \end{array}$$

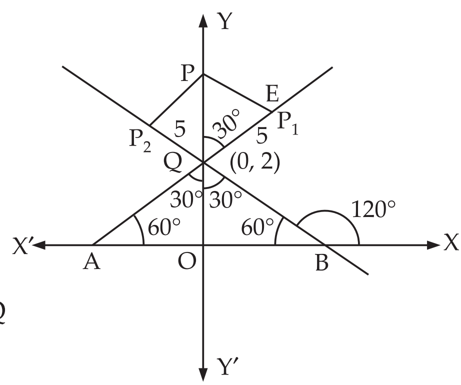
Putting the value of y is eq. (*i*) we get

$x = 0$

$\therefore$ Point of intersection of line (*i*) and (*ii*) is Q(0, 2)

$\therefore$ QO = 2

In ΔPEQ,



$$\cos 30° = \frac{PQ}{QE}$$

$$\frac{\sqrt{3}}{2} = \frac{PQ}{5}$$

$$\therefore \quad PQ = \frac{5\sqrt{3}}{2}$$

$$\therefore \quad OP = OQ + PQ$$

$$= 2 + \frac{5\sqrt{3}}{2}$$

Hence, the coordinates of the foot of perpendicular

$$= \left(0, 2 + \frac{5\sqrt{3}}{2}\right).$$

Q21. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P. then show that $a^4 + b^4 = 0$.

Sol. Given equation is $\frac{x}{a} + \frac{y}{b} = 1$

Since, p is the length of perpendicular drawn from the origin to the given line

$$\therefore \qquad p = \left|\frac{\frac{0}{a}+\frac{0}{b}-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

Squaring both sides, we have

$$p^2 = \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$$

$$\Rightarrow \qquad \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2} \qquad ...(i)$$

Since a^2, p^2, b^2 are in A.P.

$$\therefore \qquad 2p^2 = a^2+b^2$$

$$\Rightarrow \qquad p^2 = \frac{a^2+b^2}{2} \quad \Rightarrow \quad \frac{1}{p^2} = \frac{2}{a^2+b^2}$$

Putting the value of $\frac{1}{p^2}$ is eq. (i) we get,

$$\frac{1}{a^2}+\frac{1}{b^2} = \frac{2}{a^2+b^2}$$

$$\Rightarrow \qquad \frac{a^2+b^2}{a^2b^2} = \frac{2}{a^2+b^2}$$

$$\Rightarrow \qquad (a^2+b^2)^2 = 2a^2b^2$$

$$\Rightarrow \qquad a^4+b^4+2a^2b^2 = 2a^2b^2$$

$$\Rightarrow \qquad a^4+b^4 = 0.$$ Hence proved.

OBJECTIVE TYPE QUESTIONS

Q22. A line cutting off intercept – 3 from the y-axis and the tangent at angle to the x-axis is $\frac{3}{5}$, its equation is

(a) $5y - 3x + 15 = 0$ (b) $3y - 5x + 15 = 0$

(c) $5y - 3x - 15 = 0$ (d) None of these

Sol. Since the lines cut off intercepts – 3 on y-axis then the line is passing through the point (0, – 3).

Given that: $\tan\theta = \frac{3}{5} \Rightarrow$ Slope of the line $m = \frac{3}{5}$

So, the equation of the line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \qquad y + 3 = \frac{3}{5}(x-0)$$

$\Rightarrow \quad 5y + 15 = 3x$

$\Rightarrow \quad 3x - 5y - 15 = 0 \quad \Rightarrow \quad 5y - 3x + 15 = 0$

Hence, the correct option is (*a*).

Q23. Slope of a line which cuts off intercepts of equal length on the axis is

(*a*) -1 (*b*) -0

(*c*) 2 (*d*) $\sqrt{3}$

Sol. Intercept form of a line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{a} = 1 \qquad (\because a = b)$$

$\Rightarrow \quad x + y = a$

$\Rightarrow \quad y = -x + a$

$\therefore$ Slope is -1

Hence, the correct option is (*a*).

Q24. The equation of the straight line passing through the point (3, 2) and perpendicular to the line $y = x$ is

(*a*) $x - y = 5$ (*b*) $x + y = 5$

(*c*) $x + y = 1$ (*d*) $x - y = 1$

Sol. Eqn of line '*l*' is given by

$y - y_1 = m(x - x_1)$.

Since *l* passing through the point P(3, 2).

$\therefore \quad y - 2 = m(x - 3)$

$\Rightarrow \quad y = mx + 2 - 3m \quad ...(i)$

Since it is given that lines $y = x$ and '*l*' are perpendicular to each other,

$\therefore \quad m \times 1 = -1$

$[\because m_1 \times m_2 = -1]$

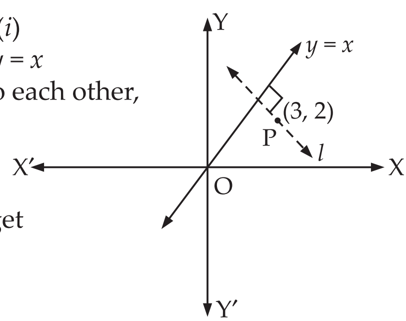
$m = -1$

Put $m = -1$ in eqn. (*i*), we get

$y = -x + 2 - 3(-1)$

$y = -x + 5$

$x + y = 5$



Hence, correct option is (*b*).

Q25. The equation of the line passing through the point (1, 2) and perpendicular to the line $x + y + 1 = 0$ is

(*a*) $y - x + 1 = 0$ (*b*) $y - x - 1 = 0$

(*c*) $y - x + 2 = 0$ (*d*) $y - x - 2 = 0$

Sol. Equation of any line perpendicular to the given line $x + y + 1 = 0$ is $x - y + k = 0$ ...(*i*)

If eq. (*i*) passes through the point (1, 2) then

$$1 - 2 + k = 0 \quad \Rightarrow \quad k = 1$$

Putting the value of k is eq. (i) we have

$$x - y + 1 = 0 \quad \Rightarrow \quad y - x - 1 = 0$$

Hence, the correct option is (b).

Q26. The tangent of angle between the line whose intercepts on the axes are $a, -b$ and $b, -a$ respectively are

(a) $\dfrac{a^2 - b^2}{ab}$ (b) $\dfrac{b^2 - a^2}{2}$

(c) $\dfrac{b^2 - a^2}{2ab}$ (d) None of these

Sol. First equation of line having intercepts on the axes

$a, -b$ is $\dfrac{x}{a} - \dfrac{y}{b} = 1 \quad \Rightarrow \quad bx - ay = ab$ ...(i)

Second equation of line having intercepts on the axes

$$b, -a \text{ is } \frac{x}{b} - \frac{y}{a} = 1 \quad \Rightarrow \quad ax - by = ab \qquad ...(ii)$$

Slope of eq. (i) $m_1 = \dfrac{b}{a}$

Slope of eq. (ii) $m_2 = \dfrac{a}{b}$

$$\therefore \qquad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{a}{b}\frac{b}{a}} = \frac{b^2 - a^2}{2ab}$$

Hence, the correct option is (c).

Q27. If the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ passes through the points $(2, -3)$ and $(4, -5)$ then (a, b) is

(a) (1, 1) (b) (– 1, 1)

(c) (1, – 1) (d) (– 1, – 1)

Sol. Equation of line passing through the points $(2, -3)$ and $(4, -5)$ is

$$y + 3 = \frac{-5 + 3}{4 - 2}(x - 2)$$

$$\Rightarrow \qquad y + 3 = \frac{-2}{2}(x - 2)$$

$$\Rightarrow \qquad y + 3 = -(x - 2)$$

$$\Rightarrow \qquad y + 3 = -x + 2$$

$$\Rightarrow \qquad x + y = -1$$

$\Rightarrow \quad \dfrac{x}{-1} + \dfrac{y}{-1} = 1$ (Intercept form)

$\therefore\ a = -1,\ b = -1$

Hence, the correct option is (*d*).

Q28. The distance of the point of intersection of the lines $2x - 3y + 5 = 0$ and $3x + 4y = 0$ from the line $5x - 2y = 0$ is

(*a*) $\dfrac{130}{17\sqrt{29}}$ (*b*) $\dfrac{13}{7\sqrt{29}}$

(*c*) $\dfrac{130}{7}$ (*d*) None of these

Sol. Given equations are:

$$2x - 3y + 5 = 0 \quad \text{...}(i)$$

$$3x + 4y = 0 \quad \text{...}(ii)$$

From eq. (*ii*) we get,

$$4y = -3x \Rightarrow y = \frac{-3}{4}x \quad \text{...}(iii)$$

Putting the value of y in eq. (*i*) we have

$$2x - 3\left(\frac{-3}{4}x\right) + 5 = 0$$

$$\Rightarrow \quad 8x + 9x + 20 = 0$$

$$\Rightarrow \quad 17x + 20 = 0$$

$$\Rightarrow \quad x = \frac{-20}{17}$$

Putting the value of x in eq. (*iii*) we get

$$y = \frac{-3}{4}\left(\frac{-20}{17}\right)$$

$$\Rightarrow \quad y = \frac{15}{17}$$

$\therefore$ Point of intersection is $\left(-\dfrac{20}{1`7}, \dfrac{15}{17}\right)$.

Now perpendicular distance from the point $\left(-\dfrac{20}{1`7}, \dfrac{15}{17}\right)$ to the given line $5x - 2y = 0$ is

$$\left|\frac{5\left(-\frac{20}{17}\right) - 2\left(\frac{15}{17}\right)}{\sqrt{25+4}}\right| = \left|\frac{\frac{-100}{17} - \frac{30}{17}}{\sqrt{29}}\right| = \frac{130}{17\sqrt{29}}$$

Hence, the correct option is (*a*).

Q29. The equation of the lines which pass through the point (3, – 2) and are inclined at 60° to the line $\sqrt{3}x + y = 1$ is

(*a*) $y + 2 = 0,\ \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(*b*) $x - 2 = 0,\ \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$

(*c*) $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(*d*) None of these

Sol. Equation of line is given by

$$\sqrt{3}x + y + 1 = 0$$

$$\Rightarrow \quad y = -\sqrt{3}x - 1$$

$\therefore$ Slope of this line, $m_1 = -\sqrt{3}$

Let m_2 be the slope of the required line

$$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \quad \tan 60^\circ = \left|\frac{-\sqrt{3} - m_2}{1 + (-\sqrt{3})m_2}\right|$$

$$\Rightarrow \quad \sqrt{3} = \pm\left(\frac{-\sqrt{3} - m_2}{1 - \sqrt{3}m_2}\right)$$

$$\Rightarrow \quad \sqrt{3} = \frac{-\sqrt{3} - m_2}{1 - \sqrt{3}m_2} \qquad \text{[taking (+) sign]}$$

$$\Rightarrow \quad \sqrt{3} - 3m_2 = -\sqrt{3} - m_2$$

$$\Rightarrow \quad 2m_2 = 2\sqrt{3} \quad \Rightarrow \quad m_2 = \sqrt{3}$$

$$\text{and} \quad \sqrt{3} = -\left(\frac{-\sqrt{3} - m_2}{1 - \sqrt{3}m_2}\right) \qquad \text{[taking (–) sign]}$$

$$\Rightarrow \quad \sqrt{3} = \frac{\sqrt{3} + m_2}{1 - \sqrt{3}m_2}$$

$$\Rightarrow \quad \sqrt{3} - 3m_2 = \sqrt{3} + m_2$$

$$\Rightarrow \quad 4m_2 = 0 \quad \Rightarrow \quad m_2 = 0$$

$\therefore$ Equation of line passing through (3, – 2) with slope $\sqrt{3}$ is

$$y + 2 = \sqrt{3}(x - 3)$$

$$\Rightarrow \quad y + 2 = \sqrt{3}x - 3\sqrt{3}$$

$$\Rightarrow \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$$

and the equation of line passing through (3, –2) with slope 0 is

$$y + 2 = 0(x - 3) \quad \Rightarrow \quad y + 2 = 0$$

Hence, the correct option is (*a*).

Q30. The equation of the lines passing through the point (1, 0) and at a distance $\frac{\sqrt{3}}{2}$ from the origin are

(*a*) $\sqrt{3}x + y - \sqrt{3} = 0, \sqrt{3}x - y - \sqrt{3} = 0$
(*b*) $\sqrt{3}x + y + \sqrt{3} = 0, \sqrt{3}x - y + \sqrt{3} = 0$
(*c*) $x + \sqrt{3}y - \sqrt{3} = 0, x - \sqrt{3}y - \sqrt{3} = 0$
(*d*) None of these

Sol. Equation of any line passing through (1, 0) is

$$y - 0 = m(x - 1) \quad \Rightarrow \quad mx - y - m = 0$$

Distance of the line from origin is $\frac{\sqrt{3}}{2}$

$$\therefore \quad \frac{\sqrt{3}}{2} = \left|\frac{m \times 0 - 0 - m}{\sqrt{1 + m^2}}\right|$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \left|\frac{-m}{\sqrt{1 + m^2}}\right|$$

Squaring both sides, we get

$$\frac{3}{4} = \frac{m^2}{1 + m^2}$$

$$\Rightarrow \quad 4m^2 = 3 + 3m^2 \quad \Rightarrow \quad 4m^2 - 3m^2 = 3$$

$$\Rightarrow \quad m^2 = 3 \quad \therefore \quad m = \pm\sqrt{3}$$

$\therefore$ Required equations are

$$\pm\sqrt{3}x - y \mp \sqrt{3} = 0$$

i.e., $\sqrt{3}x - y - \sqrt{3} = 0$ and $-\sqrt{3}x - y + \sqrt{3} = 0$

$$\Rightarrow \quad \sqrt{3}x + y - \sqrt{3} = 0$$

Hence, the correct option is (*a*).

Q31. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is

(*a*) $\frac{c_1 - c_2}{\sqrt{1 + m^2}}$ (*b*) $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$

(*c*) $\frac{c_2 - c_1}{\sqrt{1 + m^2}}$ (*d*) 0

Sol. Given equations are $y = mx + c_1$ ...(*i*)
and $y = mx + c_2$ ...(*ii*)
Slopes of eq. (*i*) and eq. (*ii*) are same i.e., m
So, they are parallel lines.

$\therefore$ Distance between the two lines $= \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$.

Hence, the correct option is (*b*).

Q32. The coordinates of the foot of perpendiculars from the point (2, 3) on the line $y = 3x + 4$ is given by

(*a*) $\left(\dfrac{37}{10}, \dfrac{-1}{10}\right)$ (*b*) $\left(\dfrac{-1}{10}, \dfrac{37}{10}\right)$

(*c*) $\left(\dfrac{10}{37}, -10\right)$ (*d*) $\left(\dfrac{2}{3}, -\dfrac{1}{3}\right)$

Sol. Given equation is $\quad y = 3x + 4$

$\Rightarrow \quad 3x - y + 4 = 0 \qquad ...(i)$

$\text{Slope} = 3$

Equation of any line passing through the point (2, 3) is

$y - 3 = m(x - 2) \qquad ...(ii)$

If eq. (*i*) is perpendicular to eq. (*ii*) then

$m \times 3 = -1 \qquad [\because m_1 \times m_2 = -1]$

$\Rightarrow \quad m = -\dfrac{1}{3}$

Putting the value of m in eq. (*ii*) we get

$$y - 3 = -\frac{1}{3}(x - 2)$$

$\Rightarrow \quad 3y - 9 = -x + 2$

$\Rightarrow \quad x + 3y = 11 \qquad ...(iii)$

Solving eq. (*i*) and eq. (*iii*) we get

$3x - y = -4 \quad \Rightarrow \quad y = 3x + 4 \qquad ...(iv)$

Putting the value of y in eq. (*iii*) we get

$x + 3(3x + 4) = 11$

$\Rightarrow \quad x + 9x + 12 = 11$

$\Rightarrow \quad 10x = -1 \quad \Rightarrow \quad x = \dfrac{-1}{10}$

From eq. (*iv*) we get, $\quad y = 3\left(\dfrac{-1}{10}\right) + 4$

$\Rightarrow \quad y = \dfrac{-3}{10} + 4 \quad \Rightarrow \quad y = \dfrac{37}{10}$

So the required coordinates are $\left(\dfrac{-1}{10}, \dfrac{37}{10}\right)$.

Hence, the correct option is (*b*).

Q33. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be

(*a*) $2x + 3y = 12$ (*b*) $3x + 2y = 12$

(*c*) $4x - 3y = 6$ (*d*) $5x - 2y = 10$

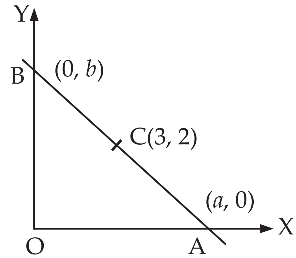
Sol. Let the given line meets the axes at A(a, 0) and B(0, b).

Given that C(3, 2) is the mid-point of AB

$$\therefore \quad 3 = \frac{a+0}{2} \Rightarrow a = 6$$

and

$$2 = \frac{0+b}{2} \Rightarrow b = 4$$



Intercept form of the line AB

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{6} + \frac{y}{4} = 1$$

$$\Rightarrow \quad 2x + 3y = 12$$

Hence, the correct option is (a).

Q34. Equation of the line passing through (1, 2) and parallel to the line $y = 3x - 1$ is

(a) $y + 2 = x + 1$ (b) $y + 2 = 3(x + 1)$
(c) $y - 2 = 3(x - 1)$ (d) $y - 2 = x - 1$

Sol. Given equation is $y = 3x - 1$

Slope = 3

Slope of the line passing through the given point (1, 2) and parallel to the given line = 3

So, the equation of the required line is

$$y - 2 = 3(x - 1)$$

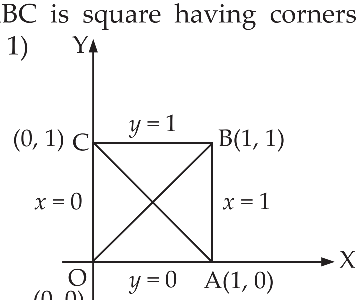
Hence, the correct option is (c).

Q35. Equations of diagonals of the squares formed by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 1$ are

(a) $y = x, y + x = 1$ (b) $y = x, x + y = 2$
(c) $2y = x,\ y + x = \frac{1}{3}$ (d) $y = 2x, y + 2x = 1$

Sol. Given equation $x = 0$, $y = 0$, $x = 1$ and $y = 1$ form a square of side 1 unit

From figure, we get that OABC is square having corners O(0, 0), A(1, 0), B(1, 1) and C(0, 1)



Equation of diagonal AC

$$y - 0 = \frac{1-0}{0-1}(x - 1)$$

$$\Rightarrow \quad y = -(x - 1)$$

$$\Rightarrow \quad y = -x + 1$$

$$\Rightarrow \quad y + x = 1$$

Equation of diagonal OB is

$$y - 0 = \frac{1-0}{1-0}(x - 0)$$

$\Rightarrow \quad y = x$

Hence, the correct option is (*a*).

Q36. For specifying a straight line, how many geometrical parameters should be known?

(*a*) 1 (*b*) 2

(*c*) 4 (*d*) 3

Sol. Different form of equation of straight lines are

Slope intercept form, $y = mx + c$, Parameter = 2

Intercept form, $\frac{x}{a} + \frac{y}{b} = 1$, Parameter = 2

One-point form, $y - y_1 = m(x - x_1)$, Parameter = 2

Normal form, $x \cos w + y \sin w = P$, Parameter = 2

Hence, the correct option is (*b*).

Q37. The point (4, 1) undergoes the following two successive transformation:

(*i*) Reflection about the line $y = x$

(*ii*) Translation through a distance 2 units along the positive x-axis

(*a*) (4, 3) (*b*) (3, 4)

(*c*) (1, 4) (*d*) $\left(\frac{7}{2}, \frac{7}{2}\right)$

Sol. Let the reflection of A(4, 1) in $y = x$ be B(a, b) mid-point of AB

$= \left(\frac{4+a}{2}, \frac{1+b}{2}\right)$ which lies on $y = x$

$$\Rightarrow \quad \frac{4+a}{2} = \frac{1+b}{2}$$

$$\Rightarrow \quad 4 + a = 1 + b$$

$$\Rightarrow \quad a - b = -3 \quad \text{...}(i)$$

The slope of the line $y = x$ is 1 and slope of AB $= \frac{b-1}{a-4}$

$$\therefore \quad 1\left(\frac{b-1}{a-4}\right) = -1$$

$$\Rightarrow \quad b - 1 = -a + 4$$

$$\Rightarrow \quad a + b = 5 \quad \text{...}(ii)$$

Solving eq. (*i*) and eq. (*ii*) we get

$$a = 1 \text{ and } b = 4$$

$\therefore$ The point after translation is (1 + 2, 4) or (3, 4).

Hence, the correct option is (*b*).

Q38. A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is

(*a*) $(1, -1)$ (*b*) $(1, 1)$

(*c*) $(0, 0)$ (*d*) $(0, 1)$

Sol. Given equations are

$$4x + 3y + 10 = 0 \quad \text{...}(i)$$

$$5x - 12y + 26 = 0 \quad \text{...}(ii)$$

and $$7x + 24y - 50 = 0 \quad \text{...}(iii)$$

Let (x_1, y_1) be any point equidistant from eq. (*i*), eq. (*ii*) and eq. (*iii*).

Distance of (x_1, y_1) from eq. (*i*)

$$= \left|\frac{4x_1 + 3y_1 + 10}{\sqrt{16+9}}\right| = \left|\frac{4x_1 + 3y_1 + 10}{5}\right|$$

Distance of (x_1, y_1) from eq. (*ii*)

$$= \left|\frac{5x_1 - 12y_1 + 26}{\sqrt{25+144}}\right| = \left|\frac{5x_1 - 12y_1 + 26}{13}\right|$$

Distance of (x_1, y_1) from eq. (*iii*)

$$= \left|\frac{7x_1 + 24y_1 - 50}{\sqrt{49+576}}\right| = \left|\frac{7x_1 + 24y_1 - 50}{25}\right|$$

If the point (x_1, y_1) is equidistant from the given lines, then

$$\left|\frac{4x_1 + 3y_1 + 10}{5}\right| = \left|\frac{5x_1 - 12y_1 + 26}{13}\right| = \left|\frac{7x_1 + 24y_1 - 50}{25}\right|$$

We see that putting $x_1 = 0$ and $y_1 = 0$, the above relation is satisfied i.e.,

$$\frac{10}{5} = \frac{26}{13} = \frac{50}{25} = 2$$

Hence, the correct option is (*c*).

Q39. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$

(*c*) 1 (*d*) $\frac{4}{3}$

Sol. Any line perpendicular to $3x + y = 3$

$$x - 3y = \lambda \quad (\lambda = \text{constant})$$

If it passes through the point (2, 2) then

$$2 - 3(2) = \lambda \Rightarrow \lambda = -4$$

$\therefore$ Required equation is $x - 3y = -4$

$\Rightarrow \qquad -3y = -x - 4$

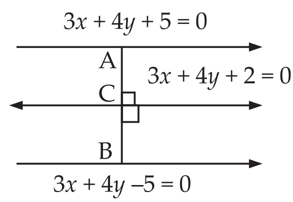
$\Rightarrow \qquad y = \frac{1}{3}x + \frac{4}{3} \qquad [\because y = mx + c]$

So, the y-intercept is $\frac{4}{3}$. Hence, the correct option is (d).

Q40. The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ is

(a) 1 : 2 (b) 3 : 7
(c) 2 : 3 (d) 2 : 5

Sol. The given equations are



$$3x + 4y + 5 = 0 \qquad ...(i)$$
$$3x + 4y - 5 = 0 \qquad ...(ii)$$
and $$3x + 4y + 2 = 0 \qquad ...(iii)$$

Clearly, eq. (i), (ii) and (iii) are parallel to each other as the coefficients of x and y are same.

Distance between parallel lines (i) and (iii) we get

$$\left|\frac{5-2}{\sqrt{(3)^2 + (4)^2}}\right| = \frac{3}{5} \qquad \left[\because \text{Distance between two parallel lines} = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|\right]$$

Distance between parallel lines (ii) and (iii) we get

$$\left|\frac{-5-2}{\sqrt{(3)^2 + (4)^2}}\right| = \frac{7}{5}$$

$$\therefore \text{Ratio between the distances} = \frac{3}{5} : \frac{7}{5} = 3 : 7$$

Hence, the correct option is (b).

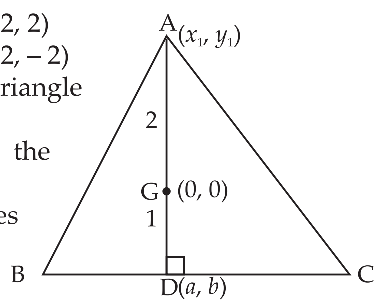
Q41. One vertex of the equilateral triangle with centroid at the origin and one side as $x + y - 2 = 0$ is

(a) $(-1, -1)$ (b) $(2, 2)$
(c) $(-2, -2)$ (d) $(2, -2)$

Sol. Let ABC be an equilateral triangle with vertex (x_1, y_1).

AD $\perp$ BC and let (a, b) be the coordinates of D.

Given that the centroid G lies at the origin i.e., (0, 0)



Since, the centroid of a triangle,divides the median in the ratio 1 : 2

So, $0 = \dfrac{1 \times x_1 + 2 \times a}{1+2}$

$\Rightarrow \quad x_1 + 2a = 0$ ...(*i*)

and $0 = \dfrac{1 \times y_1 + 2 \times b}{1+2} \Rightarrow y_1 + 2b = 0$ ...(*ii*)

Equations of BC is given by $x + y - 2 = 0$ ...(*iii*)

Point D(a, b) lies on the line $x + y - 2 = 0$

So $a + b - 2 = 0$ ...(*iv*)

Slope of eq. (*iii*) is = – 1

and the slope of AG $= \dfrac{y_1 - 0}{x_1 - 0} = \dfrac{y_1}{x_1}$

Since, they are perpendicular to each other

$\therefore \quad -1 \times \dfrac{y_1}{x_1} = -1 \Rightarrow y_1 = x_1$

From eq. (*i*) and (*ii*) we get

$x_1 + 2a = 0 \Rightarrow 2a = -x_1$

$y_1 + 2b = 0 \Rightarrow 2b = -y_1$

$\therefore \quad a = b$

From eq. (*iv*) we get

$a + b - 2 = 0$

$\Rightarrow \quad a + a - 2 = 0$

$\Rightarrow \quad 2a - 2 = 0 \Rightarrow a = 1$ and $b = 1$ $[\because a = b]$

$\therefore \quad x_1 = -2 \times 1 = -2$

and $y_1 = -2 \times 1 = -2$

Hence, the correct option is (*c*).

Fill in the Blanks:

Q42. If a, b, c are in A.P. then the straight line $ax + by + c = 0$ will always pass through ....

Sol. Given equation is $ax + by + c = 0$ ...(*i*)

Since a, b and c are in A.P.

$\therefore \quad b = \dfrac{a+c}{2}$

$\Rightarrow \quad a + c = 2b$

$\Rightarrow \quad a - 2b + c = 0$ ...(*ii*)

Comparing eq. (*i*) with eq. (*ii*) we get,

$x = 1, y = -2$

So, the line will pass through (1, – 2)

Hence, the value of the filler is **(1, – 2)**.

Q43. The line which cuts off equal intercepts from the axes and pass through the point (1, – 2) is ....

Sol. Intercept form of the line is

$$\frac{x}{a}+\frac{y}{b} = 1 \qquad ...(i)$$

Given that $a = b$

$$\therefore \qquad \frac{x}{a}+\frac{y}{a} = 1 \quad \Rightarrow \quad x + y = a \qquad ...(ii)$$

If the line (*i*) passes through (1, – 2) we get

$$1 - 2 = a \quad \Rightarrow \quad a = -1$$

So, the required equation is $x + y = -1 \quad \Rightarrow \quad x + y + 1 = 0.$

Hence, the value of the filler is $\boldsymbol{x + y + 1 = 0}$.

Q44. Equations of the lines through the point (3, 2) and making an angle of 45° with the line $x - 2y = 3$ are ....

Sol. Given line is $x - 2y = 3$ and the point is (3, 2)

Equation of a line passing through the point (3, 2) is

$$y - 2 = m(x - 3) \qquad ...(i)$$

Angle between eq. (*i*) and the given line $x - 2y = 3$ whose slope is $\frac{1}{2}$

$$\therefore \qquad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \qquad \tan 45° = \left|\frac{m - \frac{1}{2}}{1 + m \times \frac{1}{2}}\right|$$

$$\Rightarrow \qquad 1 = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right| \quad \Rightarrow \quad \frac{m - \frac{1}{2}}{1 + \frac{m}{2}} = \pm 1$$

Taking (+) sign,

$$\frac{m - \frac{1}{2}}{1 + \frac{m}{2}} = 1 \quad \Rightarrow \quad m - \frac{1}{2} = 1 + \frac{m}{2}$$

$$\Rightarrow \quad m - \frac{m}{2} = 1 + \frac{1}{2}$$

$$\Rightarrow \quad \frac{m}{2} = \frac{3}{2} \quad \Rightarrow \quad m = 3$$

Taking (–) sign,

$$\frac{m-\frac{1}{2}}{1+\frac{m}{2}} = -1 \Rightarrow \quad m-\frac{1}{2} = -1-\frac{m}{2}$$

$$\Rightarrow \quad m+\frac{m}{2} = -1+\frac{1}{2}$$

$$\Rightarrow \quad m = -\frac{1}{3}$$

So, the required equations are,

When $m = 3$,

$$y-2 = 3(x-3)$$

$$\Rightarrow \quad y-2 = 3x-9$$

$$\Rightarrow \quad 3x-y-7 = 0$$

When $m = -\dfrac{1}{3}$

$$y-2 = -\frac{1}{3}(x-3)$$

$$\Rightarrow \quad 3y-6 = -x+3$$

$$\Rightarrow \quad x+3y-9 = 0$$

Hence, the value of the filler is **$3x - y - 7 = 0$ and $x + 3y - 9 = 0$**.

Q45. The points (3, 4) and (2, – 6) are situated on the .... of the line $3x - 4y - 8 = 0$.

Sol. Given line is $3x - 4y - 8 = 0$ ...(*i*)

and the given points are (3, 4) and (2, – 6).

For point (3, 4), line becomes $= 3(3) - 4(4) - 8$

$= 9 - 16 - 8$

$= 9 - 24 = -15 < 0$

For the point (2, – 6), line becomes $= 3(2) - 4(-6) - 8$

$= 6 + 24 - 8 = 30 - 8$

$= 22 > 0$

So, the points (3, 4) and (2, – 6) are situated on the opposite sides of $3x - 4y - 8 = 0$.

Hence, the value of the filler is **opposite**.

Q46. A point moves so that square of its distance from the point (3, – 2) is numerically equal to its distance from the line $5x - 12y = 3$. The equation of its locus is ....

Sol. The given equation of line is $5x - 12y = 3$ and the given point is (3, – 2).

Let (a, b) be any moving point

$\therefore$ Distance between (a, b) and the point $(3, -2)$

$$= \sqrt{(a-3)^2 + (b+2)^2}$$

and the distance of (a, b) from the line $5x - 12y = 3$

$$= \left|\frac{5a - 12b - 3}{\sqrt{25+144}}\right| = \left|\frac{5a-12b-3}{13}\right|$$

According to the question, we have

$$\left[\sqrt{(a-3)^2 + (b+2)^2}\right]^2 = \left|\frac{5a-12b-3}{13}\right|$$

$$\Rightarrow \quad (a-3)^2 + (b+2)^2 = \frac{5a-12b-3}{13}$$

Taking numerical values only, we have

$$(a-3)^2 + (b+2)^2 = \frac{5a-12b-3}{13}$$

$$\Rightarrow \quad a^2 - 6a + 9 + b^2 + 4b + 4 = \frac{5a-12b-3}{13}$$

$$\Rightarrow \quad a^2 + b^2 - 6a + 4b + 13 = \frac{5a-12b-3}{13}$$

$$\Rightarrow \quad 13a^2 + 13b^2 - 78a + 52b + 169 = 5a - 12b - 3$$

$$\Rightarrow 13a^2 + 13b^2 - 83a + 64b + 172 = 0$$

So, the locus of the point is $13x^2 + 13y^2 - 83x + 64y + 172 = 0$.

Hence, the value of the filler is $\mathbf{13x^2 + 13y^2 - 83x + 64y + 172 = 0}$.

Q47. Locus of the mid-points of the portion of the line $x \sin\theta + y\cos\theta = p$ intercepted between the axis is ....

Sol. Given equation of the line is

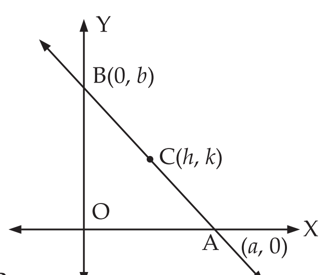
$$x\cos\theta + y\sin\theta = p \qquad ...(i)$$

Let $C(h, k)$ be the mid-point of the given line AB where it meets the two axis at $A(a, 0)$ and $B(0, b)$.

Since $(a, 0)$ lies on eq. (i) then

$$a\cos\theta + 0 = p$$

$$\Rightarrow \quad a = \frac{p}{\cos\theta} \qquad ...(ii)$$



B(0, b) also lies on the eq. (i) then

$$0 + b\sin\theta = p$$

$$\Rightarrow \quad b = \frac{p}{\sin\theta} \qquad ...(iii)$$

Since $C(h, k)$ is the mid-point of AB

$$\therefore \qquad h = \frac{0+a}{2} \Rightarrow a = 2h$$

and $$k = \frac{b+0}{2} \Rightarrow b = 2k$$

Putting the values of a and b is eq. (*ii*) and (*iii*) we get

$$2h = \frac{p}{\cos\theta} \Rightarrow \cos\theta = \frac{p}{2h} \qquad ...(iv)$$

and $$2k = \frac{p}{\sin\theta} \Rightarrow \sin\theta = \frac{p}{2k} \qquad ...(v)$$

Squaring and adding eq. (*iv*) and (*v*) we get

$$\Rightarrow \quad \cos^2\theta + \sin^2\theta = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow \quad 1 = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

So, the locus of the mid-point is

$$1 = \frac{p^2}{4x^2} + \frac{p^2}{4y^2}$$

$$\Rightarrow \quad 4x^2y^2 = p^2(x^2 + y^2)$$

Hence, the value of the filler is $\mathbf{4x^2y^2 = p^2(x^2 + y^2)}$.

State True or False:

Q48. If the vertices of a triangle have integral coordinates, then the triangle cannot be equilateral.

Sol. We know that if the vertices of triangle has integral coordinates, then the triangle can not be equilateral.
So, the given statement is **True**.

Q49. The points A(– 2, 1), B(0, 5) and C(– 1, 2) are collinear.

Sol. Given points are A(– 2, 1), B(0, 5), C(– 1, 2)

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 0 & 5 & 1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left| -2\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 5 \\ -1 & 2 \end{vmatrix} \right|$$

$$= \frac{1}{2}\left| -2(5-2) - 1(0+1) + 1(0+5) \right|$$

$$= \frac{1}{2}\left| -2 \times 3 - 1 \times 1 + 1 \times 5 \right|$$

$$= \frac{1}{2}|-6-1+5|$$

$$= \frac{1}{2}|-2| = 1 \text{ sq. unit}$$

So, the given points are not collinear.

Hence, the given statement is **False**.

Q50. Equation of the line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta + y\,\text{cosec}\,\theta = a$ is $x\cos\theta - y\sin\theta = a\sin 2\theta$.

Sol. Equation of any line perpendicular to $x\sec\theta + y\,\text{cosec}\,\theta = a$ is

$$x\,\text{cosec}\,\theta - y\sec\theta = k \qquad ...(i)$$

If eq. (i) passes through $(a\cos^3\theta, a\sin^3\theta)$ then

$a\cos^3\theta.\text{cosec}\,\theta - a\sin^3\theta.\sec\theta = k$

$$\Rightarrow \frac{a\cos^3\theta}{\sin\theta} - \frac{a\sin^3\theta}{\cos\theta} = k$$

$\therefore$ Required equation is

$$x\,\text{cosec}\,\theta - y\sec\theta = \frac{a\cos^3\theta}{\sin\theta} - \frac{a\sin^3\theta}{\cos\theta}$$

$$\Rightarrow \frac{x}{\sin\theta} - \frac{y}{\cos\theta} = a\left[\frac{\cos^4\theta - \sin^4\theta}{\sin\theta\cos\theta}\right]$$

$$\Rightarrow \frac{x\cos\theta - y\sin\theta}{\sin\theta\cos\theta} = a\left[\frac{(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta}\right]$$

$$\Rightarrow x\cos\theta - y\sin\theta = a(\cos^2\theta - \sin^2\theta)$$

$$\Rightarrow x\cos\theta - y\sin\theta = a\cos 2\theta.$$

Hence, the given statement is **False**.

Q51. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$.

Sol. Given equations are $x + 2y - 10 = 0$ ...(i)

and $2x + y + 5 = 0$ ...(ii)

From eq. (i) $x = 10 - 2y$ ...(iii)

Putting the value of x in eq. (ii) we get

$$2(10 - 2y) + y + 5 = 0$$

$$\Rightarrow 20 - 4y + y + 5 = 0$$

$$\Rightarrow -3y + 25 = 0$$

$$\Rightarrow y = \frac{25}{3}$$

Putting the value of y in eq. (iii) we get

$$x = 10 - 2\left(\frac{25}{3}\right)$$

$$= \frac{30-50}{3} = \frac{-20}{3}$$

$$\therefore \qquad \text{Point} = \left(\frac{-20}{3}, \frac{25}{3}\right)$$

If the given line $5x+4y=0$ passes through the point $\left(\frac{-20}{3}, \frac{25}{3}\right)$ then

$$5\left(\frac{-20}{3}\right) + 4\left(\frac{25}{3}\right) = 0$$

$$\Rightarrow \qquad \frac{-100}{3} + \frac{100}{3} = 0$$

$$\Rightarrow \qquad 0 = 0 \text{ satisfied.}$$

So, the given line passes through the point of intersection of the given lines.

Hence, the given statement is **True**.

Q52. The vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is $x + y = 2$. Then, the other two sides are $y - 3 = (2 \pm \sqrt{3})(x - 2)$.

Sol. Let ABC be an equilateral triangle with vertex (2, 3) and the opposite side is $x + y = 2$ with slope –1. Suppose slope of line AB is m.

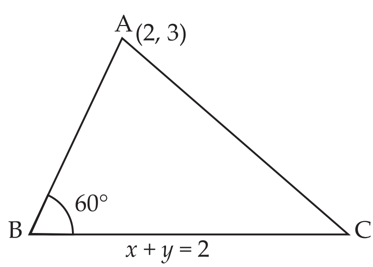
Since each angle of equilateral triangle is 60°.

$\therefore$ Angle between AB and BC

$$\tan 60^\circ = \left|\frac{-1-m}{1+(-1)m}\right|$$

$$\Rightarrow \qquad \sqrt{3} = \left|\frac{1+m}{1-m}\right|$$

$$\Rightarrow \qquad \sqrt{3} = \pm\left(\frac{1+m}{1-m}\right)$$



Taking (+) sign,

$$\sqrt{3} = \frac{1+m}{1-m} \quad \Rightarrow \quad \sqrt{3} - \sqrt{3}m = 1 + m$$

$$\Rightarrow \quad \sqrt{3}m + m = \sqrt{3} - 1$$

$$\Rightarrow \quad m(\sqrt{3}+1) = \sqrt{3} - 1$$

$$\Rightarrow \quad m = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow \qquad m = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow \qquad m = \frac{3+1-2\sqrt{3}}{3-1} = 2-\sqrt{3}$$

Taking (–) sign, $m = 2+\sqrt{3}$

So, the equations of other two lines are

$$y - 3 = (2 \pm \sqrt{3})(x-2)$$

Hence, the statement is **True**.

Q53. The equation of the line joining the point (3, 5) to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$ is equidistant from the points (0, 0) and (8, 34).

Sol. Given equations are

$$4x + y - 1 = 0 \qquad \text{...}(i)$$

and $$7x - 3y - 35 = 0 \qquad \text{...}(ii)$$

From eq. (*i*) $$y = 1 - 4x \qquad \text{...}(iii)$$

Putting the value of y in eq. (*ii*) we get

$$7x - 3(1-4x) - 35 = 0$$

$$\Rightarrow \qquad 7x - 3 + 12x - 35 = 0$$

$$\Rightarrow \qquad 19x - 38 = 0$$

$$\Rightarrow \qquad x = 2$$

From eq. (*iii*) we get, $y = 1 - 4 \times 2 \quad \Rightarrow \quad y = -7$

The point of intersection is (2, – 7).

Equation of line joining the point (3, 5) to the point (2, – 7) is

$$y - 5 = \frac{-7-5}{2-3}(x-3)$$

$$\Rightarrow \qquad y - 5 = 12(x-3)$$

$$\Rightarrow \qquad y - 5 = 12x - 36$$

$$\Rightarrow \qquad 12x - y - 31 = 0 \qquad \text{...}(iv)$$

Distance of eq. (*iv*) from the point (0, 0)

$$= \left| \frac{-31}{\sqrt{(12)^2 + (-1)^2}} \right| = \frac{31}{\sqrt{145}}$$

Distance of eq. (*iv*) from the point (8, 34) is

$$= \left| \frac{12 \times 8 - 34 - 31}{\sqrt{(12)^2 + (-1)^2}} \right|$$

$$= \left| \frac{96-65}{\sqrt{145}} \right| = \frac{31}{\sqrt{145}}$$

Hence, the given statement is **True**.

Q54. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ where c is a constant. The locus of the foot of the perpendicular from the origin on the given line is $x^2 + y^2 = c^2$.

Sol. The given equation is $\frac{x}{a}+\frac{y}{b}=1$ ...(*i*)

Equation of the line passing through (0, 0) and perpendicular to eq. (*i*) is

$$\frac{x}{b}-\frac{y}{a} = 0 \qquad ...(ii)$$

Squaring and adding eq. (*i*) and (*ii*) we get

$$\left(\frac{x}{a}+\frac{y}{b}\right)^2+\left(\frac{x}{b}-\frac{y}{a}\right)^2 = 1+0$$

$$\Rightarrow \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{2xy}{ab}+\frac{x^2}{b^2}+\frac{y^2}{a^2}-\frac{2xy}{ab} = 1$$

$$\Rightarrow \quad x^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+y^2\left(\frac{1}{b^2}+\frac{1}{a^2}\right) = 1$$

$$\Rightarrow \quad (x^2+y^2)\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = 1$$

$$\Rightarrow \quad (x^2+y^2)\left(\frac{1}{c^2}\right) = 1 \quad \left[\because \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}\right]$$

$$\Rightarrow \quad x^2+y^2 = c^2$$

Hence, the given statement is **True**.

Q55. The lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent, if a, b and c are in G.P.

Sol. Given equations are

$$ax + 2y + 1 = 0 \qquad ...(i)$$
$$bx + 3y + 1 = 0 \qquad ...(ii)$$
$$cx + 4y + 1 = 0 \qquad ...(iii)$$

Solving eq. (*i*) and (*ii*) we get

$$ax + 2y + 1 = 0 \quad \Rightarrow \quad y = \frac{-ax-1}{2}$$

Putting the value of y in eq. (*ii*) we have

$$bx + 3\left(\frac{-ax-1}{2}\right)+1 = 0$$

$$\Rightarrow \quad 2bx - 3ax - 3 + 2 = 0$$

$$\Rightarrow \quad (2b-3a)x = 1$$

$$\Rightarrow \qquad x = \frac{1}{2b-3a}$$

$$\therefore \qquad y = \frac{-a\left(\dfrac{1}{2b-3a}\right)-1}{2}$$

$$= \frac{-a-2b+3a}{2(2b-3a)}$$

$$= \frac{2a-2b}{2(2b-3a)} = \frac{a-b}{2b-3a}$$

So, the point of intersection of eq. (*i*) and (*ii*) is

$$\left(\frac{1}{2b-3a}, \frac{a-b}{2b-3a}\right).$$

If eq. (*i*), (*ii*) and (*iii*) are concurrent, then the above point must lie on eq. (*iii*)

$$cx + 4y + 1 = 0$$

$$\Rightarrow \qquad c\left[\frac{1}{2b-3a}\right] + 4\left[\frac{a-b}{2b-3a}\right] + 1 = 0$$

$$\Rightarrow \qquad \frac{c+4a-4b+2b-3a}{2b-3a} = 0$$

$$\Rightarrow \qquad c + a - 2b = 0$$

$$\Rightarrow \qquad 2b = a + c$$

So, a, b and c are in A.P. and not in G.P.

Hence, the given statement is **False**.

Q56. Line joining the points (3, – 4) and (– 2, 6) is perpendicular to the line joining the points (– 3, 6) and (9, – 18).

Sol. The given points are (3, – 4) and (– 2, 6), (– 3, 6) and (9, – 18).

Slope of the line joining the points (3, – 4) and (– 2, 6)

$$m_1 = \frac{6+4}{-2-3} = \frac{10}{-5} = -2$$

Slope of the line joining the points (– 3, 6) and (9, – 18)

$$m_2 = \frac{-18-6}{9+3} = \frac{-24}{12} = -2$$

Since $m_1 = m_2 = -2$

So, the lines are parallel and not perpendicular.

Hence, the given statement is **False**.

Q57. Match the Column:

	Column I		Column II
(*a*)	The coordinate of the points P and Q on the line $x + 5y = 13$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$ are	(*i*)	$(3, 1), (-7, 11)$
(*b*)	The coordinates of the point on the line $x + y = 4$, which are at a unit distance from the line $4x + 3y - 10 = 0$ are	(*ii*)	$\left(\frac{-1}{3}, \frac{11}{3}\right), \left(\frac{4}{3}, \frac{7}{3}\right)$
(*c*)	The coordinates of the point on the line joining A(– 2, 5) and (3, 1) such that AP = PQ = QB are	(*iii*)	$\left(1, \frac{12}{5}\right), \left(-3, \frac{16}{5}\right)$

Sol. (*a*) Let $P(x_1, y_1)$be any point on the given line

$$x + 5y = 13 \quad \therefore x_1 + 5y_1 = 13$$

Distance of line $12x - 5y + 26 = 0$ from the point $P(x_1, y_1)$

$$2 = \left|\frac{12x_1 - 5y_1 + 26}{\sqrt{(12)^2 + (-5)^2}}\right|$$

$$\Rightarrow \quad 2 = \left|\frac{12x_1 - (13 - x_1) + 26}{13}\right|$$

$$\Rightarrow \quad 2 = \left|\frac{12x_1 - 13 + x_1 + 26}{13}\right|$$

$$\Rightarrow \quad 2 = \left|\frac{13x_1 + 13}{13}\right|$$

$$\Rightarrow \quad 2 = \pm (x_1 + 1)$$

$$\Rightarrow \quad 2 = x_1 + 1 \Rightarrow x_1 = 1 \quad \text{(Taking (+) sign)}$$

$$\text{and} \quad 2 = -x_1 - 1 \Rightarrow x_1 = -3 \quad \text{(Taking (–) sign)}$$

Putting the values of x_1 in eq. $x_1 + 5y_1 = 13$.

We get $y_1 = \frac{12}{5}$ and $\frac{16}{5}$.

So, the required points are $\left(1, \frac{12}{5}\right)$ and $\left(-3, \frac{16}{5}\right)$.

Hence, (*a*) ↔ (*iii*).

(*b*) Let $P(x_1, y_1)$ be any point on the given line

$$x + y = 4 \quad \therefore x_1 + y_1 = 4 \qquad ...(i)$$

Distance of the line $4x + 3y - 10 = 0$ from the point $P(x_1, y_1)$

$$1 = \left|\frac{4x_1 + 3y_1 - 10}{\sqrt{(4)^2 + (3)^2}}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{4x_1 + 3(4 - x_1) - 10}{5}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{4x_1 + 12 - 3x_1 - 10}{5}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{x_1 + 2}{5}\right|$$

$$\Rightarrow \quad 1 = \pm\left(\frac{x_1 + 2}{5}\right)$$

$$\Rightarrow \quad \frac{x_1 + 2}{5} = 1 \qquad \text{(Taking (+) sign)}$$

$$\Rightarrow \quad x_1 + 2 = 5 \quad \Rightarrow \quad x_1 = 3$$

$$\text{and} \quad \frac{x_1 + 2}{5} = -1 \qquad \text{(Taking (–) sign)}$$

$$\Rightarrow \quad x_1 + 2 = -5 \quad \Rightarrow \quad x_1 = -7$$

Putting the values of x_1 in eq. (*i*) we get

$$x_1 + y_1 = 4$$

at $x_1 = 3$, $\quad y_1 = 1$

at $x_1 = -7$, $\quad y_1 = 11$

So, the required points are (3, 1) and (– 7, 11).

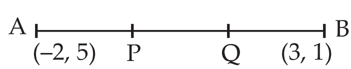
Hence, (*b*) ↔ (*i*).

(*c*) Given that AP = PQ = QB

Equation of line joining A(– 2, 5) and B(3, 1) is

$$y - 5 = \frac{1 - 5}{3 + 2}(x + 2)$$

$$\Rightarrow \quad y - 5 = \frac{-4}{5}(x + 2)$$



$$\Rightarrow \quad 5y - 25 = -4x - 8$$

$$\Rightarrow \quad 4x + 5y - 17 = 0$$

Let P(x_1, y_1) and Q(x_2, y_2) be any two points on the line AB

P(x_1, y_1) divides the line AB in the ratio 1 : 2

$$\therefore \quad x_1 = \frac{1.3 + 2(-2)}{1 + 2} = \frac{3 - 4}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1.1 + 2.5}{1 + 2} = \frac{1 + 10}{3} = \frac{11}{3}$$

So, the coordinates of $P(x_1, y_1) = \left(\frac{-1}{3}, \frac{11}{3}\right)$.

Now point $Q(x_2, y_2)$ is the mid-point of PB

$$\therefore \qquad x_2 = \frac{3 - \frac{1}{3}}{2} = \frac{4}{3}$$

$$y_2 = \frac{1 + \frac{11}{3}}{2} = \frac{7}{3}$$

Hence, the coordinates of $Q(x_2, y_2) = \left(\frac{4}{3}, \frac{7}{3}\right)$

Hence, $(c) \leftrightarrow (ii)$.

Q58. The value of the λ, if the lines $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$ are

	Column I		**Column II**
(*a*)	Parallel to y-axis is	(*i*)	$\lambda = -\frac{3}{4}$
(*b*)	Perpendicular to $7x + y - 4 = 0$	(*ii*)	$\lambda = -\frac{1}{3}$
(*c*)	Passes through (1, 2) is	(*iii*)	$\lambda = -\frac{17}{41}$
(*d*)	Parallel to x-axis is	(*iv*)	$\lambda = 3$

Sol. (*a*) Given equation is

$$(2x + 3y + 4) + \lambda(6x - y + 12) = 0$$

$$\Rightarrow \qquad (2 + 6\lambda)x + (3 - \lambda)y + 4 + 12\lambda = 0 \qquad ...(i)$$

If eq. (*i*) is parallel to y-axis, then

$$3 - \lambda = 0 \quad \Rightarrow \quad \lambda = 3$$

Hence, $(a) \leftrightarrow (iv)$

(*b*) Given lines are

$$(2x + 3y + 4) + \lambda(6x - y + 12) = 0 \qquad ...(i)$$

$$\Rightarrow \qquad (2 + 6\lambda)x + (3 - \lambda)y + 4 + 12\lambda = 0$$

$$\text{Slope} = -\left(\frac{2 + 6\lambda}{3 - \lambda}\right)$$

Second equation is $\quad 7x + y - 4 = 0 \qquad ...(ii)$

$$\text{Slope} = -7$$

If eq. (*i*) and eq. (*ii*) are perpendicular to each other

$$\therefore \quad (-7)\left[-\left(\frac{2+6\lambda}{3-\lambda}\right)\right] = -1$$

$$\Rightarrow \quad \frac{14+42\lambda}{3-\lambda} = -1$$

$$\Rightarrow \quad 14+42\lambda = -3+\lambda$$

$$\Rightarrow \quad 42\lambda-\lambda = -17$$

$$\Rightarrow \quad 41\lambda = -17$$

$$\Rightarrow \quad \lambda = -\frac{17}{41}$$

Hence, (*b*) ↔ (*iii*).

(*c*) Given equation is $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$ ...(*i*)

If eq. (*i*) passes through the given point (1, 2) then

$$(2\times 1+3\times 2+4)+\lambda(6\times 1-2+12) = 0$$

$$\Rightarrow \quad (2+6+4)+\lambda(6-2+12) = 0$$

$$\Rightarrow \quad 12+16\lambda = 0$$

$$\Rightarrow \quad \lambda = \frac{-12}{16} = \frac{-3}{4}$$

Hence, (*c*) ↔ (*i*).

(*d*) The given equation is $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$

$$\Rightarrow \quad (2+6\lambda)x+(3-\lambda)y+4+12\lambda = 0 \qquad ...(i)$$

If eq. (*i*) is parallel to *x*-axis, then

$$2+6\lambda = 0 \quad \Rightarrow \quad \lambda = \frac{-1}{3}$$

Hence, (*d*) ↔ (*ii*).

Q59. The equation of the line through the intersection of the lines $2x - 3y = 0$ and $4x - 5y = 2$ and

	Column I		**Column II**
(*a*)	Through the point (2, 1) is	(*i*)	$2x - y = 4$
(*b*)	Perpendicular to the line $x + 2y + 1 = 0$ is	(*ii*)	$x + y - 5 = 0$
(*c*)	Parallel to the line $3x - 4y + 5 = 0$ is	(*iii*)	$x - y - 1 = 0$
(*d*)	Equally inclined to the axis is	(*iv*)	$3x - 4y - 1 = 0$

Sol. (*a*) Given equations are $2x - 3y = 0$ ...(*i*)

and $4x - 5y = 2$ ...(*ii*)

Equations of line passing through eq. (*i*) and (*ii*) we get

$$(2x-3y)+k(4x-5y-2) = 0 \qquad ...(iii)$$

If eq. (*iii*) passes through (2, 1), we get

$$(2 \times 2 - 3 \times 1) + k(4 \times 2 - 5 \times 1 - 2) = 0$$
$$\Rightarrow \quad (4 - 3) + k(8 - 5 - 2) = 0$$
$$\Rightarrow \quad 1 + k(8 - 7) = 0$$
$$\Rightarrow \quad k = -1$$

So, the required equation is

$$(2x - 3y) - 1(4x - 5y - 2) = 0$$
$$\Rightarrow \quad 2x - 3y - 4x + 5y + 2 = 0$$
$$\Rightarrow \quad -2x + 2y + 2 = 0$$
$$\Rightarrow \quad x - y - 1 = 0$$

Hence, (*a*) ↔ (*iii*).

(*b*) Equation of any line passing through the point of intersection of the line $2x - 3y = 0$ and $4x - 5y = 2$ is

$$(2x - 3y) + k(4x - 5y - 2) = 0 \qquad ...(i)$$
$$\Rightarrow \quad (2 + 4k)x + (-3 - 5k)y - 2k = 0$$
$$\text{Slope} = \frac{-(2 + 4k)}{-3 - 5k} = \frac{2 + 4k}{3 + 5k}$$

Slope of the given line $x + 2y + 1 = 0$ is $-\frac{1}{2}$.

If they are perpendicular to each other then

$$-\frac{1}{2}\left(\frac{2 + 4k}{3 + 5k}\right) = -1$$
$$\Rightarrow \quad \frac{1 + 2k}{3 + 5k} = 1$$
$$\Rightarrow \quad 1 + 2k = 3 + 5k$$
$$\Rightarrow \quad 3k = -2 \quad \Rightarrow \quad k = \frac{-2}{3}$$

Putting the value of k is eq. (*i*) we get

$$(2x - 3y) - \frac{2}{3}(4x - 5y - 2) = 0$$
$$\Rightarrow \quad 6x - 9y - 8x + 10y + 4 = 0$$
$$\Rightarrow \quad -2x + y + 4 = 0$$
$$\Rightarrow \quad 2x - y = 4$$

Hence, (*b*) ↔ (*i*)

(*c*) Given equations are

$$2x - 3y = 0 \qquad ...(i)$$
$$4x - 5y = 2 \qquad ...(ii)$$

Equation of line passing through eq. (*i*) and (*ii*) we get

$$(2x - 3y) + k(4x - 5y - 2) = 0$$

$$\Rightarrow \quad (2 + 4k)x + (-3 - 5k)y - 2k = 0$$

$$\text{Slope} = \frac{-(2 + 4k)}{-3 - 5k} = \frac{2 + 4k}{3 + 5k}$$

Slope of the given line $3x - 4y + 5 = 0$ is $\frac{3}{4}$.

If the two equations are parallel, then

$$\frac{2 + 4k}{3 + 5k} = \frac{3}{4}$$

$$\Rightarrow \quad 8 + 16k = 9 + 15k$$

$$\Rightarrow \quad 16k - 15k = 9 - 8$$

$$\Rightarrow \quad k = 1$$

So, the equation of the required line is

$$(2x - 3y) + 1(4x - 5y - 2) = 0$$

$$2x - 3y + 4x - 5y - 2 = 0$$

$$\Rightarrow \quad 6x - 8y - 2 = 0$$

$$\Rightarrow \quad 3x - 4y - 1 = 0$$

Hence, (c) $\leftrightarrow$ (iv)

(d) Given equations are

$$2x - 3y = 0 \quad \quad ...(i)$$

$$4x - 5y - 2 = 0 \quad \quad ...(ii)$$

Equation of line passing through the intersection of eq. (i) and (ii) we get

$$(2x - 3y) + k(4x - 5y - 2) = 0$$

$$\Rightarrow \quad (2 + 4k)x + (-3 - 5k)y - 2k = 0$$

$$\text{Slope} = \frac{2 + 4k}{3 + 5k}$$

Since the equation is equally inclined with axes

$\therefore$ Slope = $\tan 135° = \tan (180° - 45°) = -\tan 45° = -1$

So $\frac{2 + 4k}{3 + 5k} = -1 \Rightarrow \quad 2 + 4k = -3 - 5k$

$$\Rightarrow \quad 4k + 5k = -3 - 2$$

$$\Rightarrow \quad 9k = -5 \quad \Rightarrow \quad k = \frac{-5}{9}$$

Required equation is

$$(2x - 3y) - \frac{5}{9}(4x - 5y - 2) = 0$$

$$\Rightarrow \quad 18x - 27y - 20x + 25y + 10 = 0$$

$$\Rightarrow \quad -2x - 2y + 10 = 0$$

$$\Rightarrow \quad x + y - 5 = 0$$

Hence, (d) $\leftrightarrow$ (ii)