LPS

FIRST TERM EXAMINATION 2014 – 2015
CLASS XII – MATHEMATICS

Time allowed: 3 hours — MM 100

General Instructions

- All questions are compulsory. The question paper consists of 26 questions divided into 3 sections A, B & C. Section A comprises of 6 questions of one mark each. Section B comprises of 13 questions of 4 marks each. Section C comprises of 7 questions of 6 marks each. Use of calculators is not permitted.

SECTION A

1 Using principal value, evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)+\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

2 If $f'(x) = 4x^3 - 6$ and $f(0) = 3$ find $f(x)$

3 If $y = 25^{\log_5 x}$ show that $\frac{dy}{dx} = 2x$

4 If $\begin{bmatrix}2x & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix} = 0$ find x

5 If A={a, b}
- Find number of relation on A
- Find number of binary operations on A

6 Evaluate $\int\frac{1}{\sin^2 x\cdot\cos^2 x}dx$

SECTION B

7 Let A = N x N and * be the binary operation on A defined by (a, b) * (c, d) = (a+c, b+d) Show that '*' is commutative and associative. Find the identity element for '*' on A, if any.

8 Prove that $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

9 It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3] Rolle's theorem holds with $c = 2+\frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'

OR

If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

10 Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$

11 Find the value of 'a' for which the function 'f' defined as $f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0\\ \frac{\tan x-\sin x}{x^3}, & x>0\end{cases}$ is continuous at x=0

12 Using properties of determinants, prove $\begin{vmatrix} a & b & c\\ a-b & b-c & c-a\\ b+c & c+a & a+b\end{vmatrix}=a^3+b^3+c^3-3abc$

13 Evaluate $\int(x-3)\sqrt{x^2+3x-18}\,dx$ OR $\int_0^{\pi}\frac{4x\sin x}{1+\cos^2 x}dx$

14 Find the intervals in which the function 'f' given by $f(x)=\sin x-\cos x,\ 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

OR

A man of height 2 meters walks at a uniform speed of 5km/h away from a lamppost which is 6 meters high. Find the rate at which the length of the shadow decreases.

15 Find the area bounded by the curves $\{(x, y) : y \ge x^2$ and $y= |x|\}$

16 Evaluate as limit of sums : $\int_1^4 (x^2 - x)dx$ $x^{1/4}$ $-3/4$

OR $\frac{1}{4}x$

Evaluate, using properties of definite integrals : $\int_0^{\pi/4} \log(1+\tan x)dx$

17 Using differentials, evaluate approximate value of $(15)^{\frac{1}{4}}$

18 Find $\int \frac{x^4}{(x-1)(x^2+1)}dx$ OR $\int \sqrt{\tan x}\,dx$

19 Using elementary operations find A^{-1} if $A=\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

SECTION C

20 Two schools 'A' and 'B' want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school 'A' wants to award ₹x each, ₹y each and ₹z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹1600. School 'B' wants to spend ₹2300 to award its 4, 1 and 3 students on the respective values. If the total amount of award for one prize on each value is ₹900, using matrices, find the award money for each value.

Apart from these values, suggest one more value which should be considered for award.

21 Find $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}dx$ OR $\int_0^{\pi/2} \log\cos x\,dx$

22 Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius 'r' is $\frac{4r}{3}$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of volume of the sphere.

23 Show that the function 'f' in the set $R - \left\{\frac{2}{3}\right\}$ defined as $f(x)=\frac{4x+3}{6x-4}$ is one one and onto. Hence find f^{-1} also find fof

24 Find the equations of tangents to the curve $y = x^3 + 2x + 6$ which are

a) perpendicular to the line $x + 14y + 4 = 0$
b) parallel to the line $5x - y + 1 = 0$

25 If $\sin^{-1}y = 2\log(x+1)$ then show $(x+1)^2.y_2 + (x+1).y_1 + 4y = 0$

OR

Find $\frac{dy}{dx}$ if $x^x + x^y + y^x = a^b$.

26 Using integrals, find the area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$.

OR

Using integrals, find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$