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MODERN SCHOOL, BARAKHAMBA.

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HALF YEARLY EXAMINATION, 2015–16

MATHEMATICS

Time : 3 hrs. **Class : XII** **M.M. : 100**

General Instructions :

(1) All questions are compulsory.

(2) The question paper consists of 26 questions divided into sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 7 questions of 6 marks each.

(3) There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(4) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(5) Use of calculators is not permitted. You may ask for logarithmic table if required.

26.

SECTION—(A)

1. Show that the binary operation * defined by :

$$a * b = ab + 1$$

on Q is commutative.

2. Write the value of :

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

3. What is the principal value of :

$$\cos^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$$

4. A square matrix A, of order 3 has $|A| = 5$, find $|A.adj\ A|$.

5. Find the rate of change of the area of a square of side x cm, when the sides vary.

6. Let $f : R \to R$ be the function defined by :

$$f(x) = 2x - 3 \ \forall\ x \in R$$

Write f^{-1}.

SECTION—(B)

7. Find :

$$\int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$$

8. Prove that :

$$\int_0^{\pi/2} \frac{dx}{1 + \tan x} = \frac{\pi}{4}$$

OR

$$\int_0^{\pi} \frac{x \sin x \, dx}{1 + \cos^2 x} = \frac{\pi^2}{4}$$

9. Evaluate :

$$\int \sqrt{7x - 10 - x^2} \, dx$$

10. Evaluate :

$$\int \frac{(2x - 3) \, dx}{(x^2 - 1)(2x + 3)}$$

OR

$$\int \frac{dx}{\sin x + \sin 2x}$$

26.

11. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1 - a^3)^2$$

12. Examine the following function $f(x)$ for continuity at $x = 1$ and differentiability at $x = 2$

$$f(x) = \begin{cases} 5x - 4 & ; \quad 0 < x < 1 \\ 4x^2 - 3x & ; \quad 1 < x < 2 \\ 3x + 4 & ; \quad x \geq 2 \end{cases}$$

13. If $\dfrac{x}{x - y} = \log \dfrac{\alpha}{a - y}$; then prove that :

$$\frac{dy}{dx} = 2 - \frac{x}{y}$$

14. Find the derivative of the following function $f(x)$ w.r.t. x at $x = 1$

$$f(x) = \cos^{-1}\left[\sin \sqrt{\frac{1 + x}{2}}\right] + x^x$$

OR

If $f(x) = \sqrt{1 + x^2}$, $g(x) = \dfrac{x + 1}{x^2 + 1}$ and $h(x) = 2x - 3$, then find $f'[h'(g'(x))]$.

15. If $x = a\cos\theta + b\sin\theta$,

$y = a\sin\theta - b\cos\theta$, then prove that :

$$y^2 y_2 - xy_1 + y = 0$$

16. If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, $\theta \neq 0$, then find the value of θ.

17. Prove the following :

$$\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1.\ 0 < x < 1$$

OR

If :

$$\tan^{-1}\left(\frac{x-5}{x-6}\right) + \tan^{-1}\left(\frac{x+5}{x+6}\right) = \frac{\pi}{4} ;$$

then find the value of x.

18. In the first five months, the performance of a student in x months is governed by the relation $f(x) = 2x^3 - 9x^2 + 12x + 1$. Find the months in which the performance of the student is increasing or decreasing. What life skills should the student develop to improve his performance ?

19. Find the percentage error in calculating the volume of a cubical box if an error of 2% is made in measuring the lengths of edges of the cube.

26.

SECTION—(C)

20. Check whether the operation * defined on the set $A = R \times R$ as :

$$(a, b) * (c, d) = (a + c, b + d)$$

Is a binary operation or not, where R is the set of all real numbers ? If it is a binary operation, is it commutative and associative too ? Also find the identity element of *.

OR

Let $A = \{-1, 0, 1, 2\}$,

$B = \{-4, -2, 0, 2\}$ and

$f, g : A \to B$ be functions defined by

$f(x) = x^2 - x, x \in A$ and

$g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A.$

Find $gof(x)$ and hence show that $f = g = gof$.

21. Draw a rough sketch of the region enclosed between the circles :

$$x^2 + y^2 = 4 \text{ and}$$

$$(x - 1)^2 + y^2 = 4$$

Using method of integration, find the area of this enclosed region.

22. Show that the normal at any point θ to the curve :

$$x = a \cos \theta + a\theta \sin \theta,$$

$$y = a \sin \theta - a\theta \cos \theta$$

is at a constant distance from the origin.

OR

Find the value of p for when the curves $x^2 = 9p(p - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

23. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

24. It is given that for the function f given by :

$$f(x) = x^3 + bx^2 + ax;\ x \in [1, 3],$$

Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$.

Find the values of a and b.

25. Evaluate :

$$\int_0^2 (x^2 + 2x + 1)\, dx$$

as limit of sums.

26. Two schools A and B want to award their students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 1,600. School B wants to spend Rs. 2,300 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs. 900, using matrices, find the award money for each value.

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$