2. POLYNOMIALS

POLYNOMIALS

An algebraic expression in one variable x, of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots\dots\dots\dots + a_2x^2 + a_1x + a_0$, where $a_0, a_1, a_2, \dots\dots\dots, a_n$ are real numbers (constants), $a_n \neq 0$ and all the exponents of x are non-negative integers is called a polynomial in x. a_nx^n, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$ .... a_2x^2, a_1x, a_0 are known as terms of polynomials and $a_0, a_1, a_2, \dots\dots, a_n$ are also known as coefficients of polynomials.

EXAMPLE: $f(x) = 4x + 3$, $g(x) = 3x^2 + 9x - 3$, $p(x) = \frac{1}{2}x^3 + \frac{3}{4}x^2 - x + 3$

POLYNOMIALS IN ONE VARIABLE

All the polynomials which have only one variable are called polynomials in one variable.

EXAMPLE: $x^3 - x^2 + 4x + 7$ is a polynomial in x. Similarly, $3y^2 + 5y$ is a polynomial in the variable y and $t^2 + 4$ is a polynomial in the variable t.

CHECK WHETHER ALGEBRAIC EXPRESSION IS POLYNOMIAL OR NOT

If the exponent of any term of the algebraic expression is not a whole number then the expression is not a polynomial.

EXAMPLE: $\sqrt{x} + 3$ can be written as $x^{\frac{1}{2}} + 3$. the exponent of x is $\frac{1}{2}$, which is not a whole number. So it is not a polynomial. Also, $x^{-3} + x^2 - x + 4$ has a term having negative exponent –3, which is not a whole number. So it is not a polynomial.

PRACTICE PROBLEMS

1. Which of the following expressions are polynomials in one variable and which are not? State reasons

(i) $x^2 + 5x - 8$ (ii) $7u^3 - 4u^5 - 5\sqrt{2}$ (iii) $5t + \frac{1}{t}$ (iv) $\frac{(x + x^2)}{x}$ (v) $x + \sqrt{7}x^3 + x^2$

(vi) $\sqrt{r^4} + 5$ (vii) $\frac{\sqrt{3}}{x^2} + 8$ (viii) $x^3 + \frac{1}{x^2} + \frac{1}{x} + 1$ (ix) $y^3 - 5y + \sqrt{y}$ (x) $x^{10} + y^3 + t^{50}$

TERMS & COEFFICIENT OF TERMS

Terms: In the polynomial $3y^2 + 5y$, the expression $3y^2$ and $5y$ are called the terms of the polynomial.

Coefficient: In $-x^3 + 4x^2 + 7x - 2$, the coefficient of x^3 is –1, the coefficient of x^2 is 4, the coefficient of x is 7 and –2 is the coefficient of x^0.

ILLUSTRATION

Q.1 Write the coefficient of x^3 in each of the following polynomials :

(i) $\frac{2}{3} - \frac{5}{4}x + x^3 + \frac{1}{2}x^2$ (ii) $2x^3 - 7x + x^2 + 3x^4$ (iii) $\frac{x}{2} - \frac{x^2}{3} + \frac{x^3}{4}$

Sol. (i) Given polynomial is $\frac{2}{3} - \frac{5}{4}x + x^3 + \frac{1}{2}x^2$ Coefficient of $x^3 = 1$.

(ii) $2x^3 - 7x + x^2 + 3x^4$ Coefficient of $x^3 = 2$.

(iii) Given polynomial is $\frac{x}{2} - \frac{x^2}{3} + \frac{x^3}{4}$ Coefficient of $x^3 = \frac{1}{4}$

PRACTICE PROBLEMS

2. If $p(x) = 9x^5 - 5x^3 + 6x^2 - 7x - 21$

(i) What is the coefficient of x^3? (ii) What is the coefficient of x^2? (iii) Write the constant term.

• TYPES OF POLYNOMIALS (ON THE BASIS OF TERMS) •

MONOMIALS: Polynomials having only one term are called monomials.

EXAMPLE: $2x$, 2, $5x^3$, y and u^4.

BINOMIALS: Polynomials having only two terms are called binomials.

EXAMPLE: $p(x) = x + 1$, $r(y) = y^{30} + 1$...

TRINOMIALS: Polynomials having only three terms are called trinomials.

EXAMPLE: $p(x) = x + x^2 + \pi$, $q(x) = \sqrt{2} + x - x^2$, $r(y) = y^4 + y + 5$, etc.

PRACTICE PROBLEMS

3. In the following, identify the monomials, binomials and trinomials:

(i) $7x^2 + 2x$ (ii) 7 (iii) $x^2 + x + 1$ (iv) 3 (v) $3x + 7x + 4x^2$ (vi) $(2t + 1)(5t - 7)$ (vii). $3m^2 - 5m + 7m^4$

• DEGREE OF POLYNOMIAL •

The highest power of the variable in a polynomial, is the degree of the polynomial.

REMARK: If a polynomial involves two or more variables, then the sum of the powers of all the variables in each term is taken up and the highest sum so obtained is the degree of the polynomial.

EXAMPLE: Degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7 and the degree of the polynomial $5y^6 - 4y^2 - 6$ is 6.

ILLUSTRATION

Q.2 Find the degree of each of the polynomials given below :

(i) $x^5 - x^4 + 3$ (ii) $2 - y^2 - y^3 + 2y^8$ (iii) 2

Sol. (i) The highest power of the variable of 5. So, the degree of the polynomial is 5.

(ii) The highest power of the variable is 8. So, the degree of the polynomial is 8.

(iii) The only term here is 2 which can be written as $2x^0$. So the power of x is 0. Therefore, the degree of the polynomial is 0.

PRACTICE PROBLEMS

4. Write the degree of polynomial p(x) gives as:

(i) $p(x) = x^2 + \frac{3}{2}x + 1$ (ii) $p(x) = 3x^3 - \frac{7}{2}x + \sqrt{3}$ (iii) $p(x) = \sqrt{7}x^4 + 2x^3 + \sqrt{9}x + 4$

(iv) $p(x) = \frac{3}{4}x - 5$ (v) $p(x) = -4x^3 - \frac{1}{\sqrt{3}}x^2 + x^4$ (vi) $p(x) = -\sqrt{5}$

• TYPES OF POLYNOMIALS (ON THE BASIS OF DEGREE) •

0. Constant Polynomial: A polynomial of degree zero is called a constant polynomial.

   EXAMPLE: $f(x) = 5$, $q(x) = \frac{5}{2}$, $r(x) = -\frac{7}{5}$

   The constant polynomials 0 (zero) is known as the zero polynomial. The degree of zero polynomial is ***not defined*** because $f(x) = 0$, $g(x) = 0x^4$, $h(x) = 0x^6$, $p(x) = 0x^{12}$ are all equal to zero polynomials.

1. Linear polynomial: A polynomial of degree 1 is called a linear polynomial.

EXAMPLE: f (x) = 7x, q (y) = $\frac{4}{3}$ y + 8, r(t) = $-\frac{4}{7}$ t – 7, h(x) = $\sqrt{3}x + 7$

2. Quadratic polynomial: A polynomial of degree 2 is called a quadratic polynomial.

EXAMPLE: f (y) = $4y^2$, q (s) = $\frac{2}{5}s^2 + 8$, r(x) = $\sqrt{3}x^2 + 7x + 9$

3. Cubic polynomial: A polynomial of degree 3 is called a cubic polynomial.

EXAMPLE: f (t) = $7t^3$, p (r) = $\frac{2}{5}r^3 + 9r^2 + 3r + 8$, h(x) = $\sqrt{5}x^2 + 7x + 9x^3 + 2$

4. Bi-Quadratic polynomial: A polynomial of degree 4 is called a bi-quadratic polynomial.

EXAMPLE: p(x) = $3x^4 + 5x^3 + 2x^2 - 6x - 3$, q(t) = $5t^4 + 9t^2 + 4$

PRACTICE PROBLEMS

5. Find the types of the polynomials on the basis of degree:

(i) $\sqrt{5}x^2 + 6x + 3$ (ii) $x^4 + 2x^2 + 2$ (iii) 5 (iv) 4y + 8 (v) 12 s^3

• STANDARD FORM OF POLYNOMIAL •

A polynomial is in its standard form if its terms are arranged in ascending or descending order of their degrees.

EXAMPLE: $x^2 + x + 3$, $2y^5 + 3y^4 + 2y + 7$, $2y^6 + 11y^2 + 2y$

PRACTICE PROBLEMS

6. Write the following polynomials in standard form:

(i) $x^6 - 3x^4 + \sqrt{2}x + 5x^2 + 7x^5 + 4$ (ii) $m^7 + 8m^5 + 4m^6 + 6m - 3m^2 - 11$

• VALUE OF POLYNOMIAL •

The value of a polynomial f(x) at x = a is obtained by substituting x = a in the given polynomial and is denoted by f(a).

EXAMPLE: Consider the polynomial p (x) = $5x^3 - 2x^2 + 3x - 2$

If we replace x by 1 everywhere in p (x), we get p (1) = $5 \times (1)^3 - 2 \times (1)^2 + 3(1) - 2 = 5 - 2 + 3 - 2 = 4$

So, we say that the value of p (x) at x = 1 is 4. Similarly, p (0) = $5(0)^3 - 2(0)^2 + 3(0) - 2 = -2$

ILLUSTRATION

Q.3 Find the value of p(x) = $x^2 + 5x + 7$ at x = 5 and x = –2

Sol. p(5) = $(5)^2 + 5(5) + 7 = 25 + 25 + 7 = 57$ Ans.

p(–2) = $(-2)^2 + 5(-2) + 7 = 4 - 10 + 7 = -6 + 7 = 1$ Ans.

Q.4 Evaluate p(x) = $x^3 + 4x^2 - 3x + 5$ for x = $\frac{1}{2}$

Sol. $$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 5 = \frac{1}{8} + 4\times\frac{1}{4} - 3\times\frac{1}{2} + 5 = \frac{1}{8} + 1 - \frac{3}{2} + 5 = \frac{1+8-12+40}{8} = \frac{37}{8}$$

PRACTICE PROBLEMS

7. Find the value of the polynomial p(x) = $2x^3 - 13x^2 + 17x + 12$ at x = 2 , x = –3, x = 0

8. Evaluate $p\left(-\frac{3}{2}\right)$ for $p(x) = (x)^3 + 5(x)^2 - 7(x) + 5$.

•ZEROES (ROOTS) OF A POLYNOMIAL•

Zero of a polynomial p(x) is a number c such that p(c) = 0. It is also called the root or solution of the polynomial.

Method of finding a Zero of the Polynomial: Let the given polynomial in variable x is p(x). To find its zero, equate p(x) to 0 to get an equation p(x) = 0. Now, solve this equation and get the value/values of x. These value/values of x gives the zero/zeroes of polynomial.

POINTS TO NOTE

* A linear polynomial can have at most one zero.
* A quadratic or cubic polynomial can have at most two and three zeroes respectively.
* In general, a polynomial of degree n has atmost n zeroes.
* A polynomial can have minimum 0 (zero) zeroes.

ILLUSTRATION

Q.5 Check whether –2 and 2 are zeroes of the polynomial x + 2.

Sol. Let r(x) = x + 2. Then r (2) = 2 + 2 = 4, r (–2) = –2 + 2 = 0.
Therefore, –2 is a zero of the polynomial x + 2, but 2 is not.

Q.6 Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$.

Sol. Let $r(x) = x^2 - 2x$ be the given polynomial. Then $r(2) = 2^2 - 4 = 4 - 4 = 0$ and $r(0) = 0 - 0 = 0$
Hence, 2 and 0 are both zeroes of the polynomial $x^2 - 2x$.

Q.7 Find the zero of the polynomial of (i) x – 7 (ii) x + 3 (iii) 3x – 9 (iv) 4x + 20 (v) 7x

Sol. (i) $x - 7 = 0 \Rightarrow x = 7$ (ii) $x + 3 = 0 \Rightarrow x = -3$ (iii) $3x - 9 = 0 \Rightarrow x = \frac{9}{3} = 3$

(iv) $4x + 20 = 0 \Rightarrow x = \frac{-20}{4} = -5$ (v) $7x = 0 \Rightarrow x = \frac{0}{7} = 0$

Q.8 If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$; $\frac{4}{3}$ is a zero of polynomial, $\therefore f\left(\frac{4}{3}\right) = 0$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0 \Rightarrow 6\times\frac{64}{27} - 11\times\frac{16}{9} + \frac{4k}{3} - 20 = 0 \Rightarrow 128 - 176 + 12k - 180 = 0$$

$\Rightarrow 12k + 128 - 356 = 0 \Rightarrow 12k = 228 \Rightarrow k = 9$

Q.9 If x = 2 and x = 0 are roots of the polynomials $f(x) = 2x^3 - 5x^2 + ax + b$. Find the values of a and b.

Sol. If $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$ $\Rightarrow 16 - 20 + 2a + b = 0$ $\Rightarrow 2a + b = 4$
$\Rightarrow f(0) = 2(0)^3 - 5(0)2 + a(0) + b = 0$ $\Rightarrow$ b = 0, since b = 0, 2a = 4 $\Rightarrow$ a = 2, b = 0

PRACTICE PROBLEMS

9. Verify whether the indicated numbers are zeros of the polynomial corresponding to them:
(i) $p(x) = x^2 + 5x - 6$, x = –6, 1 (ii) $p(x) = x^3 - 1$, x = 1, 2
(iii) $p(x) = 3x^2 - 15x$, x = 0, 3 (iv) p(x) = (x + 7) (x – 5), x = –7, 5

10. Find the zero of the polynomial:
(i) p(x) = x + 4 (ii) p(x) = x – 6 (iii) p(x) = 2x – 7 (iv) p(x) = 5x

•DIVISION ALGORITHM FOR POLYNOMIAL•

On dividing a polynomial p (x) by a polynomial d (x), let the quotient be q (x) and the remainder be r(x), then p(x) = g (x) . q (x) + r(x), where either r (x) = 0 or deg. r (x) < deg.g (x)

Here, Dividend = p (x), Divisor = g (x), Quotient = q (x) and Remainder = r (x).

DIVISION OF A POLYNOMIAL BY A POLYNOMIAL

STEP 1: Arrange the terms of the dividend and the divisor in descending order of their degrees.

STEP 2: Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

STEP 3: Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

STEP 4: Consider the remainder as new dividend and proceed as before.

STEP 5: Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the divisor.

ILLUSTRATION

Q.10 Divide $3x^2 + x - 1$ by $x + 1$. Find quotient and remainder and verify division algorithm.

Sol. Let Dividend $p(x) = 3x^2 + x - 1$, divisor $g(x) = x + 1$

$$\begin{array}{r|l} & 3x - 2 \\ \hline x+1 & 3x^2 + x - 1 \\ & 3x^2 + 3x \\ & - \quad - \\ \hline & \quad -2x - 1 \\ & \quad -2x - 1 \\ & \quad + \quad + \\ \hline & \qquad\quad 1 \end{array}$$

Hence quotient $q(x) = 3x - 2$, remainder $r(x) = 1$.

Using, division algorithm i.e. $p(x) = g(x) \cdot q(x) + r(x)$

$p(x) = 3x^2 + x - 1$ and $g(x) \cdot q(x) + r(x) = (x + 1) \cdot (3x - 2) + 1 = 3x^2 - 2x + 3x - 2 + 1 = 3x^2 + x - 1$

Hence verified.

Q.11 Divide $y^3 + y^2 + 2y + 3$ by $y + 2$. Find quotient and remainder and verify division algorithm.

Sol. Let Dividend $p(x) = y^3 + y^2 + 2y + 3$, divisor $g(x) = y + 2$

$$\begin{array}{r|l|l} y+2 & y^3 + y^2 + 2y + 3 & y^2 - y + 4 \\ & -(y^3 + 2y^2) & \\ \hline & \quad -y^2 + 2y + 3 & \\ & \quad -(-y^2 - 2y) & \\ \hline & \qquad 4y + 3 & \\ & \qquad -(4y + 8) & \\ \hline & \qquad\quad -5 & \end{array}$$

Hence quotient $q(x) = y^2 - y + 4$, remainder $r(x) = -5$.

Using, division algorithm i.e. $p(x) = g(x) \cdot q(x) + r(x)$ $p(x) = y^3 + y^2 + 2y + 3$ and $g(x) \cdot q(x) + r(x) = (y + 2).$ $(y^2 - y + 4) - 5 = y^3 - y^2 + 4y + 2y^2 - 2y + 8 - 5 = y^3 + y^2 + 2y + 3$ Hence verified.

Q.12 Divide $3x^4 - 4x^3 + 3x - 1$ by $x - 1$. Find quotient and remainder and verify division algorithm.

Sol. Let Dividend $p(x) = 3x^4 - 4x^3 + 3x - 1$, divisor $g(x) = x - 1$

$$\begin{array}{r|l} & 3x^3 - x^2 - x + 2 \\ \hline x-1 & 3x^4 - 4x^3 + 3x - 1 \\ & 3x^4 - 3x^3 \\ & - \quad + \\ \hline & \quad -x^3 \quad + 3x - 1 \\ & \quad -x^3 + x^2 \\ & \quad + \quad - \\ \hline & \qquad -x^2 + 3x - 1 \\ & \qquad -x^2 + x \\ & \qquad + \quad - \\ \hline & \qquad\quad 2x - 1 \\ & \qquad\quad 2x - 2 \\ & \qquad\quad - \quad + \\ \hline & \qquad\qquad 1 \end{array}$$

Hence quotient $q(x) = 3x^3 - x^2 - x + 2$, remainder $r(x) = 1$.

Using, division algorithm i.e. $p(x) = g(x) \cdot q(x) + r(x)$

$p(x) = 3x^4 - 4x^3 + 3x - 1$ and $g(x) \cdot q(x) + r(x) = (x - 1) \cdot (3x^3 - x^2 - x + 2) + 1$

$= 3x^4 - x^3 - x^2 + 2x - 3x^3 + x^2 + x - 2 + 1 = 3x^4 - 4x^3 + 3x - 1$ Hence verified.

PRACTICE PROBLEMS

11. Divide $4x^3 + 7x^2 + 8x - 5$ by $x + 3$. Find quotient and remainder and verify division algorithm.

12. Divide $x^4 - 3x^3 + 2x^2 + 4x - 5$ by $x - 2$. Find quotient and remainder and verify division algorithm.

• REMAINDER THEOREM •

REMAINDER THEOREM: Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$.

Proof: Let $p(x)$ be any polynomial with degree $n \geq 1$. Suppose that $p(x)$ is divided by $(x - a)$, the quotient and remainder are respectively $q(x)$ and $r(x)$.

Then, Dividend = (Divisor x Quotient) + Remainder. i.e. $p(x) = [(x - a) \times q(x)] + r(x)$ ....(1)

Since, degree of $r(x) <$ degree of $(x - a)$. $\Rightarrow$ degree of $r(x) < 1$ [degree of $(x - a) = 1$]

i.e. degree of $r(x) = 0$ $\Rightarrow$ $r(x)$ is constant, equal to r (let)

Therefore, $p(x) = [(x - a) \times q(x)] + r$

In particular, if $x = a$, this equation gives, $p(a) = [(a - a) \times q(a)] + r = r$, which proves the theorem.

METHOD OF FINDING REMAINDER

If a polynomial $p(x)$ is divided by a linear polynomial $g(x)$, then we can find the remainder without using division algorithm. For this, we use the following steps

STEP I: Find the zero of the given linear polynomial $g(x)$ by which we have to divide $p(x)$. Zero of $g(x)$ is $x = a$.

STEP II: Put the value zero i.e. $x = a$ in given polynomial $p(x)$, which have to be divided by $g(x)$.

STEP III: The value of $p(a)$ obtained from Step II will be required remainder.

ILLUSTRATION

Q.13 Find the remainder obtained on dividing $5x^2 - 3x + 2$ by $x - 2$ using long division and remainder theorem.

Sol. By long division:

$$
\begin{array}{r|l}
 & 5x + 7 \\
\hline
x - 2 & 5x^2 + 3x + 2 \\
 & 5x^2 - 10x \\
 & (-) \quad (+) \\
\hline
 & \quad 7x + 2 \\
 & \quad 7x - 14 \\
 & \quad - \quad + \\
\hline
 & \quad 16 \rightarrow \text{Remainder}
\end{array}
$$

By Remainder theorem,

$p(x) = 5x^2 - 3x + 2$; $g(x) = x - 2$; $x - 2 = 0 \Rightarrow x = 2$, Zero of $g(x) = 2$

So, Remainder $= p(2) = 5(2)^2 - 3(2) + 2 = 5 \times 4 - 6 + 2 = 20 - 6 + 2 = 16$. Remainder = 16

Q.14 Using remainder theorem, find the remainder when $x^3 + x^2 - 2x + 1$ is divided by $x - 3$.

Sol. Let $p(x) = x^3 + x^2 - 2x + 1$

By remainder theorem, we know that $p(x)$ when divided by $(x - 3)$ gives a remainder = to $p(3)$

Now, $p(3) = [3^3 + 3^2 - 2 \times 3 + 1] = (27 + 9 - 6 + 1) = 31$ Hence, the remainder is 31.

Q.15 Find the remainder when $4x^3 - 3x^2 + 2x - 4$ is divided by (a) $x - 2$ (b) $x + \frac{1}{2}$

Sol. (a) Given Divisor = $x - 2$

$x - 2 = 0 \Rightarrow x = 2$

$\therefore$ Zero of $x - 2$ is 2

Let $r(x) = 4x^3 - 3x^2 + 2x - 4$

$\therefore$ Remainder = $r(2) = 4(2)^3 - 3(2)^2 + 2(2) - 4 = 32 - 12 + 4 - 4 = 20$.

(b) Given Divisor = $x + \frac{1}{2}$. $x + \frac{1}{2} = 0 \Rightarrow x = -\frac{1}{2}$

$\therefore$ Zero of $x + \frac{1}{2}$ is $-\frac{1}{2}$

Let $r(x) = 4x^3 - 3x^2 + 2x - 4$

$\therefore$ Remainder = $r\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 4 = -\frac{1}{2} - \frac{3}{4} - 1 - 4 = -\frac{25}{4}$.

Q.16 The polynomial $p(x) = kx^3 + 9x^2 + 4x - 8$ when divided by $x + 3$ leaves a remainder -20. Find the value of k.

Sol. Divisor = $x + 3$; $x + 3 = 0 \Rightarrow x = -3$ $\therefore$ Zero of $x + 3$ is -3

Remainder is -20 [Given]

$\Rightarrow p(-3) = -20 \Rightarrow k(-3)^3 + 9(-3)^2 + 4(-3) - 8 = -20 \Rightarrow 27k = 81 \Rightarrow k = 3$

Q.17 Let R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$, find the value of a.

Sol. Let $p(x) = x^3 + 2x^2 - 5ax - 7$ and $q(x) = x^3 + ax^2 - 12x + 6$ be the given polynomials.

Now, R_1 = Remainder when p(x) is divided by $x + 1$

$\Rightarrow R_1 = p(-1)$

$\Rightarrow R_1 = (-1)^3 + 2(-1)^2 - 5a(-1) - 7$ [$p(x) = x^3 + 2x^2 - 5ax - 7$]

$\Rightarrow R_1 = -1 + 2 + 5a - 7$ $\Rightarrow R_1 = 5a - 6$

And, R_2 = Remainder when q(x) is divided by $x - 2$

$\Rightarrow R_2 = q(2)$

$\Rightarrow R_2 = (2)^3 + a(2)^2 - 12(2) + 6$ [$q(x) = x^3 + ax^2 - 12x + 6$]

$\Rightarrow R_2 = 8 + 4a - 24 + 6$ $\Rightarrow R_2 = 4a - 10$

Substituting the values of R_1 and R_2 in $2R_1 + R_2 = 6$, we get $2(5a - 6) + (4a - 10) = 6$

$\Rightarrow 10a - 12 + 4a - 10 = 6 \Rightarrow 14a - 22 = 6 \Rightarrow 14a = 28 \Rightarrow a = 2$

Q.18 If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x - 1$ and $x + 1$, the remainders are respectively 5 and 19. Determine the remainder when f(x) is divided by $(x - 2)$.

Sol. When f(x) is divided by $x - 1$ and $x + 1$ the remainders are 5 and 19 respectively.

$\therefore f(1) = 5$ and $f(-1) = 19$

$\Rightarrow 1^4 - 2 \times 1^3 + 3 \times 1^2 - a \times 1 + b = 5$ and $(-1)^4 - 2 \times (-1)^3 + 3 \times (-1)^2 - a \times -1 + b = 19$

$\Rightarrow 1 - 2 + 3 - a + b = 5$ & $1 + 2 + 3 + a + b = 19 \Rightarrow 2 - a + b = 5$ & $6 + a + b = 19 \Rightarrow -a + b = 3$ & $a + b = 13$

Adding these two equations, we get $(-a + b) + (a + b) = 3 + 13 \Rightarrow 2b = 16 \Rightarrow b = 8$

Putting $b = 8$ in $-a + b = 3$, we get $-a + 8 = 3 \Rightarrow a = -5 \Rightarrow a = 5$

Putting the values of a and b in $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

The remainder when f(x) is divided by $(x - 2)$ is equal to f(2).

So, Remainder = $f(2) = 2^4 - 2 \times 2^3 + 3 \times 2^2 - 5 \times 2 + 8 = 16 - 16 + 12 - 10 + 8 = 10$

PRACTICE PROBLEMS

13. Find the remainder when the polynomial $p(y) = y^3 + y^2 - 2y + 1$ is divided by $y + 3$.

14. Find the remainder when the polynomial $p(y) = y^4 - 3y^2 + 2y + 1$ is divided by $y - 1$.

15. $f(x)$ is the polynomial $4x^3 - 12x^2 + 11x - 3$. Use the remainder theorem to find the remainder when $f(x)$ is divided by (i) $x - \frac{1}{2}$ (ii) $x + \frac{1}{2}$ (iii) $x - \frac{3}{2}$

16. The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by $(x + 2)$. If the remainder in each case is the same, find the value of a.

17. When $px^3 + 5x^2 + x + q$ is divided by $(x + 1)$ the remainder is 7 and when it is divided by $(x - 2)$ the remainder is 37. Find 'p' and 'q'.

• FACTORISATION OF A POLYNOMIAL •

FACTOR OF A POLYNOMIAL: Let p (x) be any polynomial and let a, b, c be any real numbers such that $p(x) = (x - a)(x - b)(x - c)$.

Then, clearly each one of $(x - a)$, $(x - b)$, $(x - c)$ is a linear factor of p (x).

FACTOR THEOREM: Let f(x) be a polynomial of degree $n \geq 1$ and a be any real number. Then,

(i) if $f(a) = 0$, then $(x - a)$ is a factor of f(x). (ii) if $(x - a)$ is a factor of f(x), then $f(a) = 0$.

PROOF OF FACTOR THEOREM

(i) Given, $f(a) = 0$

Now, suppose that f(x) is divided by $(x - a)$, then quotient is g(x).

By remainder theorem, we know that, when f(x) is divided by $(x - a)$, then remainder is f(a).

$\therefore$ $f(x) = (x - a)\, g(x) + f(a)$

$\Rightarrow$ $f(x) = (x - a) \cdot g(x)$ [$f(a) = 0$, given]

So, $(x - a)$ is a factor of f(x).

(ii) Let $(x - a)$ be a factor of f(x).

On dividing f(x) by $(x - a)$, let g(x) be the quotient.

Then, $f(x) = (x - a) \cdot g(x)$.

On putting $x = a$, we get $f(a) = (a - a) \cdot g(a) = 0 \cdot g(a) \Rightarrow f(a) = 0$

Thus, if $(x - a)$ is a factor of f(x), then $f(a) = 0$

NOTE: • If $(x + a)$ is a factor of f(x), then $f(-a) = 0$. • If $(ax - b)$ is a factor of f(x), then $f\left(\frac{b}{a}\right) = 0$

ILLUSTRATION

Q.19 Use the Factor Theorem to determine whether q(x) is a factor of p(x) :

a. $p(x) = x^3 - 3x^2 + 4x - 4$, $q(x) = x - 2$ b. $p(x) = 7x^2 - 2\sqrt{8}x - 6$, $q(x) = x - \sqrt{2}$

c. $p(x) = 2\sqrt{2}\,x^2 - 5x + \sqrt{2}$, $q(x) = x + \sqrt{2}$

Sol. a. The zero p(x) is 2. Now, $p(2) = (2)^3 - 3(2)^2 + 4(2) - 4 = 8 - 12 + 8 - 4 = 0$

$\therefore$ By factor Theorem, q(x) is a factor of p(x).

b. The zero of q(x) is $\sqrt{2}$. Now, $p(\sqrt{2}) = 7(\sqrt{2})^2 - 2\sqrt{8}(\sqrt{2}) - 6 = 14 - 8 - 6 = 0$

$\therefore$ By Factor Theorem, q(x) is a factor of p(x).

c. The zero of q(x) is $-\sqrt{2}$. Now, $p(-\sqrt{2}) = 2\sqrt{2}(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2} = 4\sqrt{2} - 5\sqrt{2} + \sqrt{2} = 0$

$\therefore$ By factor thm, q(x) is a factor of p(x).

Q.20 Check whether the polynomial $q(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$.

Sol. q(t) will be a multiple of 2t + 1 only, if 2t + 1 divides q(t) leaving remainder zero.

Now, taking 2t + 1 = 0, we have $t = -\frac{1}{2}$. Also, $q\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1 = -\frac{1}{2} + 1 + \frac{1}{2} - 1 = 0$

So the remainder is 0.

So, 2t + 1 is a factor of the given polynomial q(t), i.e. q(t) is a multiple of 2t + 1.

Q.21 If x – 2 is a factor of each of the following polynomials, then find the value of a in each case:

a. $x^3 - 2ax^2 + ax - 1$ b. $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Sol. a. Let $p(x) = x^3 - 2ax^2 + ax - 1$

x – 2 is a factor of p(x)

$\therefore$ p(2) = 0 (By factor theorem)

$\Rightarrow (2)^3 - 2a(2)^2 + a(2) - 1 = 0 \Rightarrow 6a = 7 \Rightarrow a = \frac{7}{6}$.

b. Let $p(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

x – 2 is a factor of p(x)

$\therefore$ p(2) = 0 [By Factor Theorem]

$\Rightarrow (2)^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4 = 0$

$\Rightarrow 32 - 48 - 8a + 12a + 4a + 4 = 0 \Rightarrow a = \frac{12}{8} = \frac{3}{2}$.

Q.22 For what value of s is the polynomial $2x^4 - sx^3 + 4x^2 + 2x + 1$ divisible by (1 – 2x).

Sol. Let $r(x) = 2x^4 - sx^3 + 4x^2 + 2x + 1$ put $1 - 2x = 0 \Rightarrow x = \frac{1}{2}$

$\therefore$ Zero of 1 – 2x is $\frac{1}{2}$, If r (x) is divisible by 1 – 2x, then $r\left(\frac{1}{2}\right) = 0$

$\Rightarrow 2\left(\frac{1}{2}\right)^4 - s\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) + 1 = 0 \Rightarrow \frac{1}{8} - \frac{s}{8} + 1 + 1 + 1 = 0 \Rightarrow s = 25$.

Q.23 Find the values of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ has (x – 1) and (x + 3) as factors.

Sol. Let $p(x) = x^3 - ax^2 - 13x + b$ be the given polynomial. If (x – 1) and (x + 3) are factors of p(x), then p(1) = 0 and p(–3) = 0

$\Rightarrow 1^3 - a \times 1^2 - 13 \times 1 + b = 0$ and $(-3)^3 - a(-3)^2 - 13 \times -3 + b = 0$

$\Rightarrow 1 - a - 13 + b = 0$ and $-27 - 9a + 39 + b = 0$ $\Rightarrow -12 - a + b = 0$ and $-9a + b + 12 = 0$

$\Rightarrow a - b = -12$ and $9a - b = 12$

Subtracting second equation from first, we get $(a - b) - (9a - b) = -12 - 12$

$\Rightarrow a - b - 9a + b = -24 \Rightarrow -8a = -24 \Rightarrow a = 3$.

Putting a = 3 in a – b = –12, we get, $3 - b = -12 \Rightarrow b = 15$

Hence, a = 3 and b = 15

Q.24 Find the values of a and b so that the polynomial $x^3 + 10x^2 + ax + b$ is exactly divisible by x – 1 as well as x – 2.

Sol. Let $p(x) = x^3 + 10x^2 + ax + b$ be the given polynomial. If p(x) is exactly divisible by (x – 1) and (x – 2), then (x – 1) and (x – 2) are factors of p(x).

$\therefore$ p(1) and p(2) = 0

$\Rightarrow (1)^3 + 10(1)^2 + a(1) + b = 0$ and $(2)^3 + 10(2)^2 + a(2) + b = 0 \Rightarrow 1 + 10 + a + b = 0$ & $8 + 40 + 2a + b = 0$

$\Rightarrow a + b = -11$ and $2a + b = -48$

Subtracting second equation from first, we get $(a + b) - (2a + b) = -11 - (-48)$

$\Rightarrow a + b - 2a - b = -11 + 48 \Rightarrow -a = 37 \Rightarrow a = -37$

Putting, $a = -37$ in $a + b = -11$, we get ; $-37 + b = -11 \Rightarrow b = 26$.

Hence, $a = -37$ and $b = 26$.

PRACTICE PROBLEMS

18. In each of the following parts, use the Factor Theorem to check whether or not the polynomial g(x) is a factor of the polynomial f(x).

(i) $f(x) = x^4 + x^3 - x^2 - x - 18$; $g(x) = x - 2$
(ii) $f(x) = x^5 + 3x^4 + x^2 + 8x + 15$; $g(x) = x + 3$
(iii) $f(x) = x^3 + x^2 + x + 1$; $g(x)\ x - 1$
(iv) $f(x) = x^4 + 4x^2 + x + 6$; $g(x) = x + 2$
(v) $f(x) = x^3 + x^2 + 3x - 4$; $g(x) = x + 5$

19. For what value of a is $2x^3 + ax^2 + 11x + a + 3$ exactly divisible by $2x - 1$?

20. Find the value of k if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$.

21. Find the value of p and q, if $(x + 3)$ and $(x - 4)$ are factors of $x^3 - px^2 - qx + 24$.

22. Find the values of p and q so that $x + 2$ and $x - 1$ are factors of the polynomial $x^3 + 10x^2 + px + q$.

• FACTORISATION OF QUADRATIC POLYNOMIAL •

Quadratic polynomials of the type $ax^2 + bx + c$, where $a \neq 0$ and a, b and c are constants, can be factorised by different methods like splitting the middle term and by factor theorem.

I. **By Splitting the Middle Term**: Working steps for this method are given below

STEP I: Convert the given quadratic polynomial in the standard form $ax^2 + bx + c$ and find the values of a, b and c.

STEP II: Find two numbers p and q such that (a) sum of p and q is equal to b. i.e., $p + q = b$

(b) product of p and q is equal to ac. i.e., $pq = ac$

STEP III: Write the middle term bx as px + qx and ac as pq. Then, $ax^2 + bx + c = x^2 + px + qx + pq$

STEP IV: Now, factorise it. $ax^2 + bx + c = (x^2 + px) + (qx + pq) = x(x + p) + q(x + p) \Rightarrow ax^2 + bx + c = (x + p)(x + q)$

ILLUSTRATION

Q.25 Factorise $2x^2 + 7x + 3$ by splitting the middle terms.

Sol. Here, $ac = 2 \times 3 = 6$, so we try to split $b = 7$ into two parts whose sum is 7 and product is 6.

Clearly $6 + 1 = 7$ and $6 \times 1 = 6$.

Given polynomial is $2x^2 + 7x + 3$.

On comparing with $ax^2 + bx + c$, we get $a = 2$, $b = 7$ and $c = 3$

Now, $ac = 2 \times 3 = 6$

So, all possible pairs of factors of 6 are 1 and 6, 2 and 3.

Clearly, pair 1 and 6 gives $1 + 6 = 7 = b$

$\therefore\ 2x^2 + 7x + 3 = 2x^2 + (1 + 6)x + 3 = 2x^2 + x + 6x + 3 = x(2x + 1) + 3(2x + 1) = (2x + 1)(x + 3)$

II. **By Factor Theorem:**

Let the given quadratic polynomial be $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\,p(x)$

where $p(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$

Let α and β be two zeroes of p(x), then $\Rightarrow ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\Rightarrow ax^2 + bx + c = ax^2 - a(\alpha + \beta)x + a\alpha\beta$

On comparing constant term both sides, we get $a\,\alpha\beta = c \Rightarrow \alpha\beta = \frac{c}{a}$

Now, find all possible factors of $\frac{c}{a}$ by trial method and take that factor at which p(x) is zero.

ILLUSTRATION

Q.26 Factorise $x^2 - 5x + 6$ by using factor theorem.

Sol. Given polynomial is $f(x) = x^2 - 5x + 6$

Here, constant term is 6 and all factors of 6 are ±1, ±2 and ±3.

At x = 2, $f(2) = 2^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 10 - 10 = 0$ and at x = 3,

$f(3) = 3^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 15 - 15 = 0$

Hence, (x – 2) and (x – 3) are the factors of given polynomial.

• FACTORISATION OF CUBIC POLYNOMIAL •

The splitting of middle term method is not applicable for cubic polynomials. Cubic polynomial needs to finds atleast one factor first and then adjust the given polynomial such that it becomes the product of terms in which one of them is that factor and the other is a quadratic polynomial. Further, solve this quadratic polynomial by previous methods.

ILLUSTRATION

Q.27 Factorise $2x^3 - 5x^2 - 19x + 42$.

Sol. Let $p(x) = 2x^3 - 5x^2 - 19x + 42$

STEP I: Hit and trial Method: Here, constant term is 42 and its all factors are 1, 2, 3, 6, 7, 14, 21 and 42.

At x = 1, $p(1) = 2(1)^3 - 5(1)^2 - 19 + 42 = 2 - 5 - 19 + 42 = 20 \neq 0$

So, (x – 1) is not a factor of p(x).

At x = 2, $p(2) = 2(2)^3 - 5(2)^2 - 19(2) + 42 = 16 - 20 - 38 + 42 = 68 - 68 = 0$

So, (x – 2) is a factor of p(x).

STEP II: Long Division Method: Now, dividing p(x) by (x – 2),

$$
\begin{array}{r|l}
 & 2x^2 - x - 21 \\
x - 2 & 2x^3 - 5x^2 - 19x + 42 \\
 & 2x^3 - 4x^2 \\
 & -\quad + \\
\hline
 & -x^2 - 19x + 42 \\
 & -x^2 + 2x \\
 & +\quad - \\
\hline
 & -21x + 42 \\
 & -21x + 42 \\
 & +\quad - \\
\hline
 & 0
\end{array}
$$

So, $p(x) = (x - 2)(2x^2 - x - 21)$

STEP III: Splitting the middle term: $2x^2 - x - 21$ [by splitting method, ac = – 42 and b = –1

$\therefore 6 \times (-7) = -42$ and $6 - 7 = -1]$ = $x(2x - 7) + 3(2x - 7) = (2x - 7)(x + 3)$

$\therefore p(x) = (x - 2)(2x - 7)(x + 3)$

Q.28 Factorise $x^3 - 23x^2 + 142x - 120$.

Sol. Let $r(x) = x^3 - 23x^2 + 142x - 120$

Some of the factors of –120 are ±1, ±2, ±3, ±4, ±5, ±6, ±8, ±10, ±12, ±15, ±20, ±24, ±30, ±60.

By trial, we find that r(1) = 0. So x – 1 is a factor of r(x).

Now $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120 = x^2(x - 1) - 22x(x - 1) + 120(x - 1)$
$= (x - 1)(x^2 - 22x + 120)$ [Taking $(x - 1)$ common]
we could have also got this by dividing r(x) by x – 1.
Now, $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem.
By splitting the middle term, we have :
$x^2 - 22x + 120 = x^2 - 12x - 10x + 120 = x(x - 12) - 10(x - 12) = (x - 12)(x - 10)$
So, $x^3 - 23x^2 - 142x - 120 = (x - 1)(x - 10)(x - 12)$.

PRACTICE PROBLEMS

23. Factorise: (i) $11x^2 - 41x - 12$ (ii) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$ (iii) $6x^2 + 17x + 12$

24. Factorise: (i) $3x^3 - x^2 - 3x + 1$ (ii) $x^3 + 6x^2 + 11x + 6$ (iii) $x^3 - 6x^2 - 19x + 84$

ALGEBRAIC IDENTITIES

An algebraic identity is an algebraic expression that is true for all values of the variables occurring in it.
Some important Identities

Identity I : $(x + y)^2 = x^2 + 2xy + y^2$

Identity II : $(x - y)^2 = x^2 - 2xy + y^2$

Identity III : $x^2 - y^2 = (x + y)(x - y)$

Identity IV : $(x + a)(x + b) = x^2 + (a + b)x + ab$

Identity V : $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Identity VI : $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ or $(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$.

Identity VII : $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ or $(x + y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$.

Identity VIII : $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

Identity IX : $x^3 + y^3 = (x - y)(x^2 + y^2 + xy)$

Identity X : $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

Identity XI : If $(x + y + z) = 0$ in Identity X, then $x^3 + y^3 + z^3 - 3xyz = 0$ or $x^3 + y^3 + z^3 = 3xyz$

ILLUSTRATION

Q.29 Write the following in expanded form: (i) $(9x + 2y + z)^2$ (ii) $(3x + 2y - z)^2$

Sol. (i) We have $(9x + 2y + z)^2 = 9(x)^2 + (2y)^2 + z^2 + 2 \times 9x \times 2y + 2 \times 2y \times z + 2 \times 9x \times z$
$= 81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18xz$

(ii) We have, $(3x + 2y - z)^2 = \{3x + 2y + (-z)\}^2$
$= (3x)^2 + (2y)^2 + (-z)^2 + 2 \times 3x \times 2y + 2 \times 2y \times -z + 2 \times 3x \times -z$
$= 9x^2 + 4y^2 + z^2 + 12xy - 4yz - 6xz$

Q.30 If $a^2 + b^2 + c^2 = 20$ and $a + b + c = 0$, find $ab + bc + ca$.

Sol. We have, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \Rightarrow (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow 0^2 = 20 + 2(ab + bc + ca) \Rightarrow -20 = 2(ab + bc + ca)$

$$\Rightarrow -\frac{20}{2} = \left\{\frac{2(ab + bc + ca)}{2}\right\} \Rightarrow -10 = ab + bc + ca \Rightarrow ab + bc + ca = -10$$

Q.31 Write in expanded form: $(3x - 2y)^3$

Sol. Replacing a by 3x and b by 2y in the identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.
$(3x - 2y)^3 = (3x)^3 - (2y)^3 - 3 \times 3x \times 2y \times (3x - 2y)$
$\Rightarrow (3x - 2y)^3 = 27x^3 - 8y^3 - 18xy \times (3x - 2y) \Rightarrow (3x - 2y)^3 = 27x^3 - 8y^3 - 54x^2y + 36xy^2$

Q.32 If $x + y = 12$ and $xy = 27$, find the value of $x^3 + y^3$.

Sol. We know that, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Putting $x + y = 12$ and $xy = 27$ in the above identity, we get $12^3 = x^3 + y^3 + 3 \times 27 \times 12$

$\Rightarrow 1728 = x^3 + y^3 + 972 \Rightarrow x^3 + y^3 = 1728 - 972 \Rightarrow x^3 + y^3 = 756$

Q.33 If $x - y = 4$ and $xy = 21$, find the value of $x^3 - y^3$.

Sol. We know that, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

Putting, $x - y = 4$ and $xy = 21$, we get ; $4^3 = x^3 + y^3 - 3 \times 21 \times 4$

$\Rightarrow 64 = x^3 - y^3 - 252 \Rightarrow 64 + 252 = x^3 + y^3 \Rightarrow x^3 - y^3 = 316$

Q.34 Evaluate each of the following using suitable identities: (i) $(104)^3$ (ii) $(999)^3$

Sol. (i) We have $(104)^3 = (100 + 4)^3 = (100)^3 + (4)^3 + 3(100)(4)(100 + 4)$ (Using Identity VI)

$= 1000000 + 64 + 124800 = 1124864$

(ii) We have $(999)^3 = (1000 - 1)^3 = (1000)^3 - (1)^3 - 3(1000)(1)(1000-1)$ (Using Identity VII)

$= 1000000000 - 1 - 2997000 = 997002999$

Q.35 If $x^2 + \frac{1}{x^2} = 7$, find the value of $x^3 + \frac{1}{x^3}$

Sol. We have, $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} \Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 7 + 2$ $\left[\text{Putting } x^2 + \frac{1}{x^2} = 7\right]$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 9 \quad \Rightarrow \left(x + \frac{1}{x}\right)^2 = 3^2$

$\Rightarrow x + \frac{1}{x} = 3$ [Taking square root of both sides]

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = 3^3$ [Cubing both sides]

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27 \quad \Rightarrow \left(x^3 + \frac{1}{x^3}\right) + 3 \times 3 = 27 \Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 \Rightarrow x^3 + \frac{1}{x^3} = 18$

Q.36 If $x^2 + \frac{1}{x^2} = 83$. Find the value of $x^3 - \frac{1}{x^3}$

Sol. We know that, $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 83 - 2$ $\left[\text{Putting } x^2 + \frac{1}{x^2} = 83\right]$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 81 \Rightarrow \left(x - \frac{1}{x}\right)^2 = 9^2 \Rightarrow x - \frac{1}{x} = 9$ [Taking square root of both sides]

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 9^3$ [Cubing both sides]

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 729 \quad \Rightarrow x^3 - \frac{1}{x^3} - 3 \times 9 = 729 \Rightarrow x^3 - \frac{1}{x^3} = 729 + 27 \Rightarrow x^3 - \frac{1}{x^3} = 756$

Q.37 Evaluate: $30^3 + 20^3 - 50^3$

Sol. Let $a = 30$, $b = 20$ and $c = -50$. Then, $a + b + c = 30 + 20 - 50 = 0$

$\therefore a^3 + b^3 + c^3 = 3abc \Rightarrow 30^3 + 20^3 + (-50)^3 = 3 \times 30 \times 20 \times -50 \quad \Rightarrow \quad 30^3 + 20^3 + (-50)^3 = -90000$

PRACTICE PROBLEMS

25. If $p+\frac{1}{p}=3$, then find the value of $p^2+\frac{1}{p^2}$.

26. If a + b + c = 7 and ab + bc + ca = 20, then find the value of $a^2 + b^2 + c^2$.

27. If ab = 5 and a – b = 2, then find the value of $a^3 - b^3$.

28. If $x+\frac{1}{x}=7$, then find the value of $x^3+\frac{1}{x^3}$.

29. Expand $\left(4-\frac{1}{3x}\right)^3$.

30. Factorise $4x^2 + y^2 + 1 + 4xy + 2y + 4x$.

31. Factorise $(2x + 3y)^3 - (2x - 3y)^3$.

32. Evaluate: $(15)^3 - (8)^3 - (7)^3$

PRACTICE PROBLEMS ANSWERS

1. (i) yes (ii) yes (iii) no (iv) yes (v) yes (vi) yes (vii) no (viii) no (ix) no (x) no

2. (i) – 5 (ii) 6 (iii) – 21 **3.** (i) binomial (ii) monomial (iii) trinomial (iv) monomial (v) trinomial (vi) trinomial (vii) trinomial

4. (i) 2 (ii) 3 (iii) 4 (iv) 1 (v) 4 (vi) 0 **5.** (i) quadratic (ii) bi-quadratic (iii) constant (iv) linear (v) cubic

6. (i) $x^6 + 7x^5 - 3x^4 + 5x^2 + \sqrt{2}x + 4$ (ii) $m^7 + 4m^6 + 8m^5 - 3m^2 + 6m - 11$

7. 10, –210, 12 **8.** 187/8 **9.** (i) –6, 1 are zeroes (ii) 1 is a zero, 2 is not a zero (iii) 0, 3 are zeroes (iv) –7, 5 are zeroes

10. (i) –4 (ii) 6 (iii) 7/2 (iv) 0 **11.** $q(x) = 4x^2 - 5x + 23$, $r(x) = -74$ **12.** $q(x) = x^3 - x^2 + 4$, $r(x) = 3$ **13.** –11 **14.** 1

15. (i) 0 (ii) –12 (iii) 0 **16.** $\frac{5}{9}$ **17.** $p = \frac{4}{3}$, $q = \frac{13}{3}$ **18.** (i) yes (ii) yes (iii) no (iv) no (v) no **19.** a = –7

20. k = –3 **21.** p = 3, q = 10 **22.** p = 7, q = –18 **23.** (i) (11x + 3)(x – 4) (ii) $(x+3\sqrt{3})(\sqrt{3}x+2)$ (iii) (3x + 4)(2x + 3)

24. (i) (x –1) (x + 1) (3x –1) (ii) (x + 1) (x + 2) (x + 3) (iii) (x – 1) (x – 3) (x +3) (2x – 5).

25. 7 **26.** 9 **27.** 38 **28.** 322 **29.** $64-\frac{1}{27x^3}-\frac{16}{x}+\frac{4}{3x^2}$ **30.** $(2x + y + 1)^2$ **31.** $18y(4x^2 + 3y^2)$ **32.** 2520

EXERCISE

SECTION–A

TYPE I : FUNDAMENTALS OF POLYNOMIALS

1A. Which of the following expressions are polynomials in one variable?

i. $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+\frac{1}{2}$ **ii.** $x^2 (x-1)$ **iii.** $\frac{1}{x}(x-1)(x-2)$ **iv.** $\frac{(x^2+x+1)(x+1)}{(1+x)}$ **v.** $\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$

vi. $y^3-\sqrt{3}\,y$ **vii.** $t^2 - 2/5\, t + \sqrt{2}$ **viii.** $5\sqrt{z}-6$ **ix.** $x^{108}-1$ **x.** $x^{-2}+2x^{-1}+3$

xi. $\sqrt[3]{x}-27$ **xii.** $\frac{1}{\sqrt{2}}x^2-\sqrt{2x}+2$ **xiii.** $\sqrt[3]{2}y^2-8$ **xiv.** 1 **xv.** $-\frac{3}{5}$

1B. Write whether the following statements are true or false.

a. A binomial can have atmost 2 terms.

b. Every polynomialnis a binomial.

c. A binomial may have degree 6.

d. Zero of a polynomial is always 1.

e. A polynomial cannot have more than two zeroes.

2. Write:

i. Coefficient of x^3 in $2x + x^2 - 5x^3 + x^4$

ii. Coefficient of x in $\sqrt{3} - 2\sqrt{2}\,x + 4x^2$

iii. Coefficient of x^2 in $\pi/3\, x^2 + 7x - 3$

iv. Coefficient of x^2 in $3x - 5$

3. **i.** Give an example of a binomial of degree 27. **ii.** Give an example of a monomial of degree 16.

iii. Give an example of a trinomial of degree 3.

4. Which of the following are monomials(m), binomials(b) or trinomials(t)?

i. 1 **ii.** 2 **iii.** $x^3 + x - x + 2$ **iv.** $x^3 + 2x + x + 3$ **v.** $x^5 + 6x^4$ **vi.** $\frac{x^6}{4}+\frac{x^5}{5}+\frac{x^4}{4}$

5. Rewrite the following polynomials in the standard form and also find the degree of each of the following polynomials:

i. $4x^3 + 2x^2 + x - 1$ **ii.** $x + 1 + 2x^2 - x^4$ **iii.** 5 **iv.** x **v.** $x^2 + 4x + 5$

vi. $x - x^5 + 1$ **vii.** $x - 7 + 8x^2 + 9x^3$ **viii.** $-5x^2 + 6 - 3x^3 + 4x$ **ix.** $-4 + 6x^3 - x + 7x^4 - \sqrt{2}\,x^2$

x. $y^2 + 5y^3 - 11 - 7y + 9y^4$

6. Classify the following as linear(l), quadratic (q) or cubic(c) polynimials :

i. $x^2 + x$ **ii.** $x - x^3$ **iii.** $y + y^2 + 4$ **iv.** $1 + x$ **v.** 3t **vi.** r^2

vii. $7x^3$ **viii.** $2x^2 + 4x$ **ix.** $2 - y - y^2$ **x.** $-7 + z$ **xi.** p^3

TYPE II : VALUE OF POLYNOMIAL

7. Find the value of the polynomial :

i. If $p(x) = 4+x^3-x+2x^2$, find p(0), p(1), p(–2)

ii. If $p(x) = 5 - 4x + 2x^2$, find p(0), p(3), p(–2)

iii. If $p(y)=4+3y-y^2+5y^3$, find p(0), p(2), p(–1)

iv. If $f(t) = 4t^2 - 3t + 6$, find f(0), f(4), f(–5)

TYPE III : ZEROES OF POLYNOMIALS

8. Verify whether the indicated numbers are zeros of the polynomial corresponding to them in the following cases :

i. $p(x) = x^2 - 1$, x = 1, –1 **ii.** $p(x) = (x + 1)(x - 2)$, x = –1, 2 **iii.** $p(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

iv. $p(x) = (x - 1)(x - 2)$, x = 1, 2. **v.** $p(x) = x^2 - 3x$, x = 0, –3. **vi.** $p(x) = x^2 + x - 6$, x = 2, 3.

9. Find the zero of the polynomial in each of the following cases :

i. $p(x) = x + 9$ **ii.** $p(x) = 6x + 13$ **iii.** $p(x) = 4x - 2$ **iv.** $p(x) = 5x$ **v.** $p(x) = qx$

vi. $p(x) = x - 5$ **vii.** $p(t) = 2t - 3$ **viii.** $g(x) = 5 - 4x$ **ix.** $p(x) = ax + b, a \neq 0$

TYPE IV : REMAINDER THEOREM

10. Use remainder theorem, to find remainder when p(x) is divided by q(x) :

i. $p(x) = 2x^2 - 5x + 7$, $q(x) = x - 1$

ii. $p(x) = 2x^3 - 3x^2 + 4x - 1$, $q(x) = x + 2$

iii. $p(x) = x^9 - 5x^4 + 1$, $q(x) = x + 1$

iv. $p(x) = 4x^3 - 12x^2 + 11x - 5$, $q(x) = x - ½$

v. $p(x) = 4x^3 - 3x^2 + 2x - 4$, $q(x) = x + 2$

vi. $p(x) = x^3 - 6x^2 - 2x - 4$, $q(x) = 1 - 3x$

vii. $p(x) = x^3 - ax^2 + 6x - a$, $q(x) = x - a$

viii. $p(x) = x^3 + 3x^2 + 3x + 1$, $q(x) = 5 + 2x$

ix. $p(x) = x^3 - 6x^2 + 9x + 3$, $q(x) = x - 1$

x. $p(x) = 2x^3 - 5x^2 + 9x - 8$, $q(x) = x - 3$

xi. $p(x) = 3x^4 - 6x^2 - 8x + 2$, $q(x) = x - 2$

xii. $p(x) = x^3 - 7x^2 + 6x + 4$, $q(x) = x - 6$

xiii. $p(x) = 2x^4 + 6x^3 + 2x^2 + x - 8$, $q(x) = x + 3$

xiv. $p(x) = 81x^4 + 54x^3 - 9x^2 - 3x + 2$, $q(x) = 3x + 2$

xv. $p(x) = x^3 - ax^2 + 2x - a$, $q(x) = x - a$

xvi. $p(x) = x^{51} + 51$, $q(x) = x + 1$.

11. Find the value of k if the division of $kx^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder -22.

12. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + 7a$ are divided by $x - 2$. The remainder in each cases the same. Find the value of a.

13. The polynomial $kx^4 + 3x^3 + 6$ when divided by $x - 2$ leaves a remainder which is double the remainder left by the polynomial $2x^3 + 17x + k$ when divided by $x - 2$. Find the value of k.

14. Let R_1 and R_2 be the remainders when the polynomials $x^3 + 2x^2 - 5ax + 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $(x + 1)$ and $(x - 2)$ respectively. If $R_1 - R_2 = 20$, find the value of a.

15. If $x^3 + px^2 + qx + 6$ leaves remainder 3 when divided by $x - 3$ and leaves zero when divided by $x - 2$. Find the values of p and q.

TYPE V : FACTOR THEOREM

16. Use factor theorem to verify that q(x) is a factor of p(x) :

i. $p(x) = x^3 - 4x^2 + x + 6$, $q(x) = x - 3$

ii. $p(x) = x^4 - 4x^2 + 2x + 1$, $q(x) = x - 1$

iii. $p(x) = 3x^3 + x^2 - 20x + 12$, $q(x) = 3x - 2$

iv. $p(x) = 2x^3 - 9x^2 + x + 12$, $q(x) = 2x - 3$

v. $p(x) = 7x^2 - 2\sqrt{8}x - 6$, $q(x) = x - \sqrt{2}$

vi. $p(x) = 2\sqrt{2}x^3 + 5\sqrt{2}x^2 - 7\sqrt{2}$, $q(x) = x - 1$

vii. $q(x) = (x - 3)$, $p(x) = (2x^3 + 7x^2 - 24x - 45)$

viii. $q(x) = (x - 1)$, $p(x) = (2x^4 + 9x^3 + 6x^2 - 11x - 6)$

ix. $q(x) = (x + 2)$, $p(x) = (x^4 - x^2 - 12)$

x. $q(x) = (x + 5)$, $p(x) = (2x^3 + 9x^2 - 11x - 30)$

xi. $q(x) = (x - \sqrt{2})$, $p(x) = (7x^2 - 4\sqrt{2}x - 6)$

xii. $q(x) = (x + \sqrt{2})$, $p(x) = (2\sqrt{2}x^2 + 5x + \sqrt{2})$

17. Show that $(x - 1)$ is a factor of $(x^{10} - 1)$ and also of $(x^{11} - 1)$.

18. Show that $x - 2$, $x + 3$ and $x - 7$ are factors of $x^3 - 6x^2 - 13x + 42$.

19. Determine the value of 'a' if $(x - a)$ is a factor of the polynomial $x^3 - (a^2 - 1)x + 2$.

20. Use factor theorem to determine the value of k for which $x + 2$ is a factor of $(x + 1)^7 + (2x + k)^3$.

21. Find the value of k if q(x) is a factor of p(x)

i. $p(x) = 2x^3 - 6x^2 + 5x + k$, $q(x) = x - 2$

ii. $p(x) = 3x^2 + kx + 6$, $q(x) = x + 3$

iii. $p(x) = x^3 + kx + 2k - 2$, $q(x) = x + 1$

iv. $p(x) = 2x^3 + kx^2 + 11x + k + 3$, $q(x) = 2x - 1$

v. $p(x) = 2x^3 + 9x^2 - x - k$, $q(x) = 2x + 3$

vi. $p(x) = x^3 + kx^2 - 3x + k + 5$, $q(x) = x + k$

vii. $p(x) = x^4 - k^2x^2 + 4x - 2k$, $q(x) = x + k$

viii. $p(x) = x^5 - k^2x^3 + 2x + k + 1$, $q(x) = x - k$

TYPE VI : APPLICATIONS OF REMAINDER AND FACTOR THEOREM

22. For what value of a is the polynomial $2x^4 - ax^3 + 4ax^2 + 2x + 1$ is divisible by $1 - 2x$?

23. For what value of k is $y^3 + ky + 2k - 2$ exactly divisible by $y + 1$?

24. Using factor theorem, show that $a - b$ is a factor of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

25. Let A & B are the remainders when polynomials $y^3 + 2y^2 - 5ay - 7$ & $y^3 + ay^2 - 12y + 6$ are divided by $y + 1$ and $y - 2$ respectively. If $2A + B = 6$, find the value of a.

26. The polynomials $ax^3 + 3x^2 - 26$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leaves the remainder M and N respectively. Find the value of a if $M + N = 0$.

27. The polynomials $ay^3 + 3y^2 - 3$ & $2y^3 - 5y + 20a$ when divided by $(y - 4)$ leaves the remainder A and B respectively. Find the value of a if $2A - B = 0$.

28. If $ax^3 + bx^2 + x - 6$ has $(x + 2)$ as a factor and leaves a remainder 4 when divided by $(x - 2)$, find the values of ‘a’&‘b’.

29. Find the values of ‘a’ and ‘b’ if $x^2 - 4$ is a factor of $ax^4 + 2x^3 - 3x^2 + bx + 4$.

30. Without actual division, prove that $x^4 + 2x^3 - 2x^2 + 2x - 3$ is exactly divisible by $x^2 + 2x - 3$.

31. What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$?

32. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the result is exactly divisible by $x^2 + 2x - 3$.

33. If $a^2 - 1$ is a factor of $pa^4 + qa^3 + ra^2 + sa + t$, show that $p + r + t = q + s = 0$

34. Without actual division, prove that $(x - 2)$ is a factor of the polynomial $3x^3 - 13x^2 + 8x + 12$. Using division or otherwise factorise it completely.

35. Using factor theorem, show that $(p - q)$, $(q - r)$ and $(r - p)$ are factors of $p^2(q - r) + q^2(r - p) + r^2(p - q)$

36. Using factor theorem, show that $(x + y)$, $(y + z)$ and $(z + x)$ are factors of $(x + y + z)^3 - x^3 - y^3 - z^3$

TYPE VII : FACTORISATION OF POLYNOMIALS USING FACTOR THEOREM

37. Factorise the following polynomials using factor theorem

i. $x^2 - 5x + 6$ **ii.** $x^2 + 7x + 10$ **iii.** $x^2 + 25x + 126$ **iv.** $6x^2 + 17x + 5$ **v.** $5x^2 + 16x + 3$
vi. $2x^2 + 11x - 21$ **vii.** $x^3 + 2x^2 - x - 2$ **viii.** $2x^3 - x^2 - 13x - 6$ **ix.** $x^3 - 6x^2 + 3x + 10$ **x.** $x^3 + 13x^2 + 32x + 20$
xi. $2y^3 + y^2 - 2y + 1$ **xii.** $x^3 - 3x^2 - 9x - 5$ **xiii.** $x^3 - 2x^2 - 5x + 6$

38. Factorise $x^3 - 23x^2 + 142x - 120$ given that $(x - 1)$ is a factor.

39. Factorise $9x^3 - 27x^2 - 100x + 300$ given that $(3x + 10)$ is a factor.

SECTION–B : ALGEBRAIC IDENTITIES

TYPE VIII : ALGEBRAIC IDENTITIES

1. Use suitable identities to find the following products:

i. $(p - 9)(p + 2)$ **ii.** $(y - 9)(y - 2)$ **iii.** $(y + 7)(y + 5)$ **iv.** $(z - 3)(z - 5)$ **v.** $(z^2 + 4)(z^2 - 5)$
vi. $(z^2 - 2)(z^2 + 11)$ **vii.** $\left(x + \frac{4}{3}\right)\left(x + \frac{14}{3}\right)$ **viii.** $\left(x + \frac{5}{3}\right)\left(x + \frac{3}{5}\right)$

2. Evaluate using suitable identity:

i. 102 x 106 **ii.** 103 x 96 **iii.** 95 x 97 **iv.** 53 x 55 **v.** 34 x 36
vi. 98 x 99 **vii.** 46 x 48 **viii.** 105 x 95 **ix.** 104 x 95

3. Factorise using appropriate identities:

i. $4x^2 + 4xy + y^2$ **ii.** 4a2 + 1 + 4a **iii.** $36x^2 + 60x + 25$ **iv.** $x^2 - 2x + 1$ **v.** $25x^2 - 10x + 1$
vi. $49a^2 - 42ab + 9b^2$ **vii.** $x^2 - 4y^2$ **viii.** $25p^2 - 36q^2$

4. Expand using suitable identites:

i. $(x+2y+3z)^2$ **ii.** $(x+y-2z)^2$ **iii.** $(2p+2q-3r)^2$ **iv.** $(p-3q-2z)^2$ **v.** $(x-5y+2z)^2$

vi. $(-3m-5n+2p)^2$ **vii.** $\left[3x+\left(\frac{-1}{2}\right)+2q\right]^2$ **viii.** $\left[x-\frac{1}{2}y+\frac{1}{3}z\right]^2$

5. Factorise the following expression:

i. $x^2+4y^2+z^2+4xy-4yz-2xz$ **ii.** $4p^2+9q^2+4r^2+12pq+12qr+8pr$

iii. $4x^2+y^2+9z^2-4xy+6yz-12xz$ **iv.** $p^2+pq+\frac{q^2}{4}+1+2p+q$

6. Write the following cubes in exapnded form:

i. $(x+3y)^3$ **ii.** $(3x-2y)^3$ **iii.** $(px+2y)^3$ **iv.** $(x^2+y)^3$ **v.** $(2x-y^2)^3$

vi. $(2z-7y)^3$ **vii.** $\left(\frac{2}{3}x-\frac{5}{3}z\right)^3$ **viii.** $\left(\frac{x}{3}+\frac{y}{5}\right)^3$ **ix.** $\left(\frac{2}{5}-\frac{4}{3}x\right)^3$

7. Evaluate using suitable identities:

i. $(101)^3$ **ii.** $(9.8)^3$ **iii.** $(501)^3$ **iv.** $(995)^3$ **v.** $(399)^3$ **vi.** $(1.1)^3$

8. Factorise each of the following:

i. $x^3+8y^3+6xy(x+2y)$ **ii.** $x^3+3x+\frac{3}{x}+\frac{1}{x^3}$ **iii.** $\frac{x^3}{8}-64-3x^2+24x$ **iv.** $1+27z^3$ **v.** $64x^3-y^3$

vi. m^3-27n^3 **vii.** $p^3+8q^3+64r^3-24pqr$ **viii.** $8x^3-27y^3+z^3+18xyz$ **ix.** $-27x^3+y^3-z^3-9xyz$

9. Find the value using identity:

i. $55^3-25^3-30^3$ **ii.** $47^3+29^3-76^3$ **iii.** $35^3-27^3-8^3$ **iv.** $(9.8)^3-(11.3)^3+(1.5)^3$

v. $(2.7)^3-(1.6)^3-(1.1)^3$ **vi.** $(2.5)^3-(2.8)^3+(0.3)^3$

10. Factorise the following:

i. $(x-y)^3+(y-z)^3+(z-x)^3$ **ii.** $(x-3y)^3+(3y-7z)^3+(7z-x)^3$

TYPE IX : FACTORISE THE FOLLOWING EXPRESSION USING SUITABLE METHOD

11. $a^2+3a+2ab+6b$

12. $xy+3bx+2ay+6ab$

13. $3b^2+bc+15ab+5ac$

14. $4a^2-2a^2b-10ab+20a$

15. $ab(c^2+1)+c(a^2+b^2)$

16. $1+x+y+z+xy+yz+zx+xyz$

17. $(x+2)(x^2+25)+10x^2+20x$

18. $9x^4+24x^2y^2+16y^4$

19. $\left(x+\frac{1}{x}\right)^2+6\left(x+\frac{1}{x}\right)+9$

20. $\left(x-\frac{1}{x}\right)^2-14\left(x-\frac{1}{x}\right)+49$

21. $\frac{x^2}{4y^2}-\frac{2}{3}+\frac{4y^2}{9x^2}$

22. $(x-y+z)^2+2(x-y+z)(y-z+x)+(y-z+x)^2$

23. $16x^2-72xy+81y^2-12x+27y+9/4$

24. $16(x+3y)^2+24(x+3y)(p+q)+9(p+q)^2$

25. x^2-y^2+6y-9

26. $x^2-1-2a-a^2$

27. $3x^3y-243xy^3$

28. $(x+1)^4-(x-1)^4$

29. $x^2+4y^2-z^2-4xy$

30. $4x^2-z^2+9y^2-4p^2+4pz-12xy$

31. x^3-5x^2-x+5

32. x^4-4x^2+3

33. $m^2+8mn+16n^2$

34. $a^2+20a-69$

35. $p^2-26p+133$

36. $x^2-3x-28$

37. $a^2-26a-87$

38. $x^2+7\sqrt{2}x+24$

39. $x^2-2\sqrt{3}x-24$

40. $x^2-4\sqrt{5}x+20$

41. $x^2+7\sqrt{6}x+60$

42. x^4-x^2-12

43. $m^8-11m^4n^4-80n^8$

44. $(x^2-4x)(x^2-4x-1)-20$

45. $(3x-4)^2-(3x-4)-42$

46. $2x^2+13x+20$

47. $9x^2+27x+20$

48. $10x^2-9x-7$

49. $15+p(7-2p)$

50. $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$ **51.** $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$ **52.** $\frac{3x^2}{2} + 16x + 10$ **53.** $x^3 + 64$

54. $125x^3 - 343y^3$ **55.** $\frac{p^3}{343} + 8q^3$ **56.** $128x^3y^3 - 250z^3$ **57.** $x^3 - \frac{1}{x^3} - 2x + \frac{2}{x}$ **58.** $27x^3 + 5\sqrt{5}y^3$

59. $a^3 + b^3 + c(a^2 - ab + b^2)$ **60.** $(2a + 3b)^3 - 27c^3$ **61.** $(a + 2b)^3 - (a - 2b)^3$

62. $(3x + 4y)^3 + (3x - 4y)^3$ **63.** $(a - 2b)^3 + (2b - 3c)^3 + (3c - a)^3$ **64.** $(3p - q)^3 + (q + 2r)^3 - (2r + 3p)^3$

65. $(x + y - z)^3 + (x - y + z)^3 - (x - y - z)^3$ where $x + y + z = 0$. **66.** $(0.6a - 0.5b)^3 + (0.5b + 0.7c)^3 - (0.6a + 0.7c)^3$

SECTION–C (HOTS AND EXEMPLAR SECTION)

TYPE IX : MISCELLENOUS QUESTIONS ON POLYNOMIALS

1. Factorise : $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$.
2. If $a + b + c = 7$ and $ab + bc + ca = 20$, find the value of $a^2 + b^2 + c^2$.
3. Factorise $(x - 3y)^3 + (3y - 7z)^3 + (7z - x)^3$.
4. Find the value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$.
5. Factorise : $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$.
6. Factorise : $(ax + by)^2 + (ay - bx)^2$.
7. Find the value of $64x^3 + 125z^3$, if $4x + 5z = 19$ and $xz = 5$.
8. Multiply $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y - 4$ using suitable identity.
9. Factorise : $a^2 + b^2 - 2(ab - ac + bc)$.
10. If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, find $a + b + c$.
11. If $x + \frac{1}{x} = 7$ then find value of **i.** $x - \frac{1}{x}$ **ii.** $x^3 + \frac{1}{x^3}$
12. If $x - \frac{1}{x} = 3$ then find value of **i.** $x + \frac{1}{x}$ **ii.** $x^3 - \frac{1}{x^3}$
13. Prove that $\frac{0.75 \times 0.75 \times 0.75 + 0.25 \times 0.25 \times 0.25}{0.75 \times 0.75 - 0.75 \times 0.25 + 0.25 \times 0.25} = 1$
14. Simplify : $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$
15. Simplify : $\left(x - \frac{2}{3}y\right)^3 - \left(x + \frac{2}{3}y\right)^3$.
16. Factorise : $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$.
17. If $a + b = 12$ and $ab = 27$, find the value of $a^3 + b^3$.
18. If both $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ are factors of $px^2 + 5x + r$, show that $p = r$.
19. Without finding the cubes, factorise and find the value of : $\left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{7}{12}\right)^3$.
20. If $x + y + z = 1$, $xy + yz + zx = -1$ and $xyz = -1$. Find the value of $x^3 + y^3 + z^3$.
21. If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.
22. What are the possible expressions for the dimensions of cuboids whose volume are given below? Volume = $12ky^2 + 8ky - 20k$.

23. Prove: $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = 8y^3$.

24. Find the value of 'a' if $(x - a)$ is a factor of $x^5 - a^2x^3 + 2x + a + 3$, hence factorise $x^2 - 2ax - 3$.

25. If $x^2 + \frac{1}{x^2} = 51$, find : **i.** $x - \frac{1}{x}$ **ii.** $x^3 - \frac{1}{x^3}$ **iii.** $x + \frac{1}{x}$ **iv.** $x^3 + \frac{1}{x^3}$

26. Simplify using identity : $\frac{186 \times 186 \times 186 + 14 \times 14 \times 14}{186 \times 186 - 186 \times 14 + 14 \times 14}$.

27. If $2a = 3 + 2b$, prove that $8a^3 - 8b^3 - 36ab = 27$.

28. Prove that : $2x^3 + 2y^3 + 2z^3 - 6xyz = (x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$
Hence evaluate : $2(7)^3 + 2(9)^3 + 2(13)^3 - 6(7)(9)(13)$.

29. If $x + y + z = 2$, $xy + yz + zx = -2$ and $xyz = -2$, then find the value of $x^3 + y^3 + z^3$.

30. If $a + b + c = 6$, then find the value of $(2-a)^3 + (2-b)^3 + (2-c)^3 - 3(2-a)(2-b)(2-c)$

31. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x - 1$ and $x + 1$, the remainders are 5 and 19 respectively. Determine the remainder when $f(x)$ is divided by $x - 2$.

32. If $a^3 + b^3 + c^3 = 3abc$ and $a + b + c = 0$, prove that : $\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ac} + \frac{(a+b)^2}{3ab} = 1$.

33. If $\frac{x}{y} + \frac{y}{x} = -1$ $(x, y \neq 0)$, then find the value of **i .** $x^3 - y^3$ **ii.** $x^3 + y^3$.

34. Simplify : $7x^3 + 8y^3 - (4x + 3y) \cdot (16x^2 - 12xy + 9y^2)$.

35. What will be maximum number of zeroes of the polynomial $p(y) = my^a$

36. Write the coefficient of x^3 in the expansion of $m^3\left(1 - \frac{x}{m}\right)^3$.

37. Factorise : **i.** $(x^2 - 4x)(x^2 - 4x - 1) - 20$. **ii.** $m(m - 1) - n(n - 1)$

38. Prove that $x^2 + 6x + 15$ has no zero.

39. Find the coefficient of x^2 in $(3x + x^3)\left(x + \frac{1}{x}\right)$

40. For the polynomial $\frac{7x^5 - 5x + 6}{11} - \frac{3}{4}x^4 - x^{11}$, write

i. the degree of the polynomials **ii.** the coefficient of x^5. **iii.** the coefficient of x^{11}. **iv.** the constant term

41. If the polynomials $p(x) = 2x^3 + bx^2 + 3x - 5$ and $q(x) = x^3 + x^2 - 4x - b$ leaves the same remainder, when divided by $x - 2$, then prove that $b = -13/5$.

42. If $x - a$ is a factor of $4x^2 - mx - na$, then prove that $a = (m+n)/4$.

43. Simplify $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$.

44. The remainder of the polynomial $5 + bx - 2x^2 + ax^3$, when divided by $x - 2$ is twice the remainder when it is divided by $x + 1$. Show that $10a + 4b = 9$.

45. If $\sqrt{u} + \sqrt{v} - \sqrt{w} = 0$, then find the value of $(u + v - w)$.

46. If $a + b + c = 0$, then prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$

47. If $(x + a)$ is a factor of two polynomials $x^2 + px + q$ & $x^2 + mx + n$, then prove that $a = \frac{n-q}{m-p}$

48. Prasad sells x kg of apple at the rate of Rs. 80 per kg, some oranges at the rate of Rs. 40 per kg. Along that he sells 10 kg pomegrante at the rate of Rs. 120 per kg. The quantity of orange is equal to the square of the apple quantity.

i. Write an equation of the total cost of the quantity. **ii.** Write an equation of the total quantity.

iii. Which mathematical concept is used here?

iv. Find the degree of an equation of total cost of quantity.

iv. If Prasad sells 5 kg of apple, then how much money he earn?

49. Dhoni and Virat, students of class IX decided to collect donation for PM drought relief fund. The monry collected by Dhoni is 100 more than square of money collected by virat.

i. Express the total money collected by both of them in the form of polynomial.

ii. Find the degree of the polynomial so formed. **iii.**What values are exhibited by them?

50. During assembly election in a constituency of 18 lakhs people, 'x' number of people voted for candidate A, while root 16 times of x people voted for candidate B and 10 lakh people did not go for voting at all.

i. Find the value of x, using above information.

ii. What values are shown by the people casting their votes for either candidate A or B or none of them.

51. If a teacher divides a material of volume $(x^3 + 6x^2 + 12x + 8)$ cubic units among three students of his class equally. Is it possible, to find the quantity of material each get and which moral value is depicted?

52. Two friends start a business together. They decided to share their capitals depending upon a variable expenditure. The capital polynomial of the two partners together is given by $poly^n$ $6x^2 + 11x - 35$, which is the product of their share factors.

i. Find their factors **ii.** Are their capital shares same? **iii.** Write the value depicted by this question.

53. If $x = \sqrt{7} - \sqrt{5}$, $y = \sqrt{5} - \sqrt{3}$ and $z = \sqrt{3} - \sqrt{7}$ then find the value of $x^3 + y^3 + z^3$.

54. If $x = (2+\sqrt{5})^{\frac{1}{2}} + (2-\sqrt{5})^{\frac{1}{2}}$ and $y = (2+\sqrt{5})^{\frac{1}{2}} - (2-\sqrt{5})^{\frac{1}{2}}$ then evaluate $x^2 + y^2$.

55. If $a + b + c = 6$ & $ab + bc + ca = 15$, then find value of $(a + b)^3 + (b + c)^3 + (a + c)^3 - 3(a + b)(b + c)(c + a)$.

ANSWERS

SECTION–A

1A. All except (iii), (viii), (x), (xi) and (xii) **1B.** **a.** F **b.** F **c.** T **d.** F **e.** F **2.** (i) –5 (ii) $-2\sqrt{2}$ (iii) $\pi/3$ (iv) 0

3. (i) $x^{27} + 1$ (ii) x^{16} (iii) $x^3 + 4x + 9$ **4.** (i) m (ii) m (iii) b (iv) t (v) b (vi) t

5. (i) 3 (ii) 4 (iii) 0 (iv) 1 (v) 2 (vi) 5 (vii) 3 (viii) 3 (ix) 4 (x) 4

6. (i) q (ii) c (iii) q (iv) l (v) l (vi) q (vii) c (viii) q (ix) q (x) l (xi) c

7. (i) 4, 6, 6 (ii) 5, 11, 21 (iii) 4, 46, –5 (iv) 6, 58, 121

8. (i) x=1 yes, x=–1 yes (ii) x= –1 yes, x=2 yes, (iii) $x = -\frac{1}{\sqrt{3}}$ yes, $x = \frac{2}{\sqrt{3}}$ no. (iv) x = 1 yes , x = 2 yes
(v) x = 0 yes, x = –3 no (vi) x = 2 yes, x = 3 no

9. (i) –9, (ii) –13/6, (iii) 1/2, (iv) 0, (v) 0 (vi) 5 (vii) 3/2 (viii) 5/4 (ix) –b/a

10. (i) 4, (ii) –37, (iii) –5, (iv) –2, (v)–52, (vi) –143/27, (vii) 5a, (viii) –27/8
(ix) 7 (x) 28 (xi) 10 (xii) 4 (xii) 7 (xiv) 0 (xv) a (xvi) 50

11. 3 **12.** –3 **13.** 5 **14.** 2 **15.** p = –3, q = –1 **19.** –2 **20.** 3

21. (i) –2, (ii) 11, (iii) 3, (iv) –7, (v) 15, (vi) –5/4, (vii) 0, (viii) –1/3

22. –17/7 **23.** 3 **25.** 2 **26.** –2 **27.** 1/6 **28.** a = 0, b = 2 **29.** a = 1/2, b = –11

31. 4x – 4 **32.** x – 2 **34.** (x – 2)(x – 3)(x + 2)

37. (i) (x – 2)(x – 3), (ii) (x + 5)(x + 2) (iii) (x + 18)(x + 7) (iv) (2x + 5)(3x + 1) (v) (x + 3)(5x + 1) (vi) (x + 7)(2x – 3) (vii) (x – 1)(x + 2)(x + 1) (viii) (2x + 1)(x + 2)(x – 3) (ix) (x – 2)(x –5)(x + 1) (x) (x + 1) (x + 2) (x + 10) (xi) (y –1) (y + 1) (2y + 1) (xii) (x + 1) (x –1) (x –5) (xiii) (x –1) (x –3) (x + 2)

38. (x – 1)(x – 10)(x – 12) **39.** (3x + 10)(3x – 10)(x – 3)

SECTION–B

1. (i) $p^2 - 7p - 18$ (ii) $y^2 - 11y + 18$ (iii) $y^2 + 12y + 35$ (iv) $z^2 - 8z + 15$ (v) $z^4 - z^2 - 20$ (vi) $z^4 + 9z^2 - 22$

(vii) $x^2 + 6x + \frac{56}{9}$ (viii) $x^2 + \frac{34}{15}x + 1$ (ix) $x^2 - \frac{10}{15}x - \frac{75}{49}$

2. (i) 10812 (ii) 9888 (iii) 9215 (iv) 2915 (v) 1224 (vi) 9702 (vii) 2208 (viii) 9975 (ix) 9880

3. (i) (2x + y)(2x + y) (ii) (2a + 1)(2a + 1) (iii) (6x + 5)(6x + 5) (iv) (x – 1)(x – 1) (v) (5x – 1)(5x– 1) (vi) (7a–3b)(7a –3b) (vii) (x + 2y)(x–2y) (viii) (5p+6q)(5p– 6q) (ix) (x + 0.6)(x – 0.6)

4. (i) $x^2 + 4y^2 + 9z^2 + 4xy + 12yz + 6xz$ (ii) $x^2 + y^2 + 4z^2 + 2xy - 4yz - 4xz$

(iii) $4p^2 + 4q^2 + 9r^2 + 8pq - 12qr - 12rp$ (iv) $p^2 + 9q^2 + 4z^2 - 6pq + 12qz - 4zp$

(v) $x^2 + 25y^2 + 4z^2 - 10xy - 20yz + 4zx$ (vi) $9m^2 + 25n^2 + 4p^2 + 30mn - 20np - 12mp$

(vii) $9x^2 + \frac{1}{4} + 4q^2 - 3x - 2q + 12qx$ (viii) $x^2 + \frac{1}{4}y^2 + \frac{1}{9}z^2 - xy - \frac{1}{3}yz + \frac{2}{3}zx$

5. (i) (x + 2y – z)(x + 2y – z) (ii) (2p + 3q + 2r)(2p + 3q + 2r)

(iii) (–2x + y + 3z)(–2x + y + 3z) (iv) (p + q/2 + 1)(p + q/2 + 1)

6. (i) $x^3 + 9x^2y + 27xy^2 + 27y^3$ (ii) $27x^3 - 54x^2y + 36xy^2 - 8y^3$ (iii) $p^3x^3 + 6p^2x^2z + 12pxz^2 + 8z^3$

(iv) $x^6 + 3x^4y + 3x^2y^2 + y^3$ (v) $8x^3 - 12x^2y^2 + 6xy^4 - y^6$ (vi) $8z^3 - 84z^2y + 294zy^2 - 343y^3$

(vii) $\frac{8}{27}x^3 - \frac{20}{9}x^2z + \frac{50}{9}xz^2 - \frac{125}{27}z^3$ (viii) $\frac{x^3}{27} + \frac{x^2y}{15} + \frac{xy^2}{25} + \frac{y^3}{125}$ (ix) $\frac{8}{125} - \frac{16x}{25} + \frac{32x^2}{15} - \frac{64x^3}{27}$

7. (i) 1030301 (ii) 941.192 (iii) 125751501 (iv) 985074875 (v) 63521199 (vi) 1.331

8. (i) (x + 2y)(x + 2y)(x + 2y) (ii) $\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$ (iii) $\left(\frac{x}{2}-4\right)\left(\frac{x}{2}-4\right)\left(\frac{x}{2}-4\right)$ (iv) $(1 + 3z)(1 + 9z^2 - 3z)$

(v) $(4x - y)(16x^2 + y^2 + 4xy)$ (vi) $(m - 3n)(m^2 + 9n^2 + 3mn)$ (vii) $(p+2q+4r)(p^2+4q^2+16r^2-2pq-8qr-4rp)$

(viii) $(2x-3y+z)(4x^2+9y^2+z^2+6xy+3yz-2zx)$ (ix) $(-3x+y-z)(9x^2+y^2+z^2+3xy+yz-3zx)$

9. (i) 123750 (ii) –310764 (iii) 22680 (iv) –498.33 (v) 14.256 (vi) 6.300

10. (i) 3(x – y)(y – z)(z – x) (ii) 3(x – 3y)(3y – 7z)(7z – x)

11. (a + 3)(a + 2b) **12.** (y + 3b)(x + 2a) **13.** (3b + c)(b + 5a) **14.** 2a(2 – b)(a + 5)

15. (bc + a)(ac + b) **16.** (1 + x)(1 + y)(1 + z) **17.** (x+2)(x+5)(x +5) **18.** $(3x^2 + 4y^2)(3x^2 + 4y^2)$

19. $\left(x+\frac{1}{x}+3\right)\left(x+\frac{1}{x}+3\right)$ **20.** $\left(x-\frac{1}{x}-7\right)\left(x-\frac{1}{x}-7\right)$ **21.** $\left(\frac{x}{2y}-\frac{2y}{3x}\right)\left(\frac{x}{2y}-\frac{2y}{3x}\right)$ **22.** $4x^2$

23. $\left(-4x+9y+\frac{3}{2}\right)^2$ **24.** $(2x+6y+3p+3q)^2$ **25.** $(x+y-3)(x-y+3)$ **26.** $(x+1+a)(x-1-a)$

27. 3xy(x + 9y)(x – 9y) **28.** 8x(x^2 + 1) **29.** (x – 2y + z)(x – 2y – z)

30. (2x – 3y + z – 2p)(2x – 3y – z + 2p) **31.** (x + 1)(x –1)(x – 5) **32.** (x^2–3)(x+1)(x–1)

33. (m + 4n)(m + 4n) **34.** (a + 23)(a – 3) **35.** (p – 19)(p – 7) **36.** (x – 7)(x + 4)

37. (a – 29)(a + 3) **38.** $(x+4\sqrt{2})(x+3\sqrt{2})$ **39.** $(x-4\sqrt{3})(x+2\sqrt{3})$ **40.** $(x-2\sqrt{5})(x-2\sqrt{5})$

41. $(x+5\sqrt{6})(x+2\sqrt{6})$ **42.** $(x+2)(x-2)(x^2+3)$ **43.** $(m+2n)(m-2n)(m^2+4n^2)(m^4+5n^4)$

44. $(x-5)(x+1)(x-2)(x-2)$ **45.** (3x –11)(3x + 2) **46.** (x + 4)(2x + 5) **47.** (3x + 5)(3x + 4)

48. (5x – 7)(2x + 1) **49.** (3 + 2p)(5 – p) **50.** $(\sqrt{5}x+4)(5x+2\sqrt{5})$ **51.** $(x-\sqrt{2})(7\sqrt{2}x+4)$

52. 1/2(x – 10)(3x – 2) **53.** (x + 4)(x2 + 16 – 4x) **54.** (5x –7y)(25x2+49y2+35xy)

55. $\left(\frac{p}{7}+2q\right)\left(\frac{p^2}{49}+4q^2+\frac{2}{7}pq\right)$ **56.** $2\left[(4xy-5z)(16x^2y^2+25z^2+20xyz\right]$ **57.** $\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

58. $(3x+\sqrt{5}y)(9x^2+5y^2-3\sqrt{5}xy)$ **59.** $(a^2+b^2-ab)(a+b+c)$ **60.** (2a+3b–3c) (4a^2+9b^2+12ab+9c^2+ 6ac+9ac)

61. 4b(3a2 + 4b2) **62.** 6x(9x2 + 48y2) **63.** 3(a –2b)(2b –3c)(3c –a) **64.** –3(3p –q)(q + 2r)(2r + 3p)

65. –3(x + y –z)(x –y + z)(x –y –z) **66.** –3(0.6a –0.5b)(0.5b + 0.7c)(0.6a + 0.7c)

SECTION–C

1. $(x-\sqrt{2})(7\sqrt{2}x+4)$ **2.** 9 **3.** 3(x – 3y)(3y – 7z)(7z – x) **4.** 0

5. $\sqrt{2}a+2b-3c)\ (2a^2+4b^2+9c^2-2\sqrt{2}ab+6bc+3\sqrt{2}ac)$ **6.** (a^2 + b^2)(x^2 + y^2) **7.** 1159

8. 27x^3 – 125y^3 – 64 – 180xy **9.** (a – b)(a – b – c) **10.** 16 **11.** 322 **12.** 36

14. (a + b)(b + c)(c + a) **15.** –(16 / 27) y^3 – 4x^2y **16.** (x + 1/x)(x + 1/x – 2) **17.** 756 **19.** –7/48

20. 1 **21.** 370 **22.** 4k (3y + 5)(y – 1) **24.** a = – 1, (x + 3) (x – 1) **25.** 364 **26.** 200

29. 20 **30.** 0 **31.** 10 **33. i.** 0 **ii.** 0 **34.** –19(3x^3 + y^3) **35.** a **36.** –1

37. i. (x + 1) (x –5) (x –2)^2 **ii.** (m –n) (m+n –1) **39.** 4 **40. i.** 11, **ii.** 7/11, **iii.** –1, **iv.** 6/11.

43. $\sqrt{2}a+\sqrt{3}b$ **45.** $-2\sqrt{uv}$

48. i. p(x) = 80x + 40x2 + 10 x 20 **ii.** g(x) = x + x2 + 10 **iii.** Concept of polynomial formation. **iv.** degree = 2 **v.** ` 2600.

49. i. p(x) = x + x2 + 100 **ii.** degree = 2 **iii.** Students are generous, kind and socially aware of their responsibilities.

50. i. Rs. 1600000, **ii.** People who have casted their vote, know the value of vote and their duties for the democracy. While those who did not vote are not aware about the value of vote and their duties towards the nation.

51. (x + 2) (x + 2) (x + 2), It shows that teacher treat each student equally.

52. i. (2x + 7)(3x – 5), **ii.** No, **iii.** Friendship, business mindedness. **53.** $3(\sqrt{7}-\sqrt{5})(\sqrt{5}-\sqrt{3})(\sqrt{3}-\sqrt{7})$ **54.** 8 **55.** –108.