

JEE Main 2023 (Memory based)

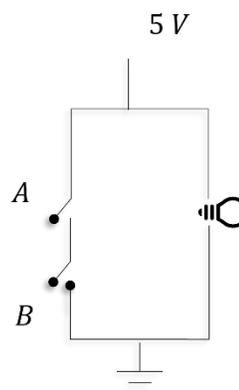
24 January 2023 - Shift 2

Answer & Solutions

PHYSICS

1. The corresponding gate for the given circuit diagram is

- A. OR
- B. AND
- C. NAND
- D. XOR



Answer (C)

Sol.

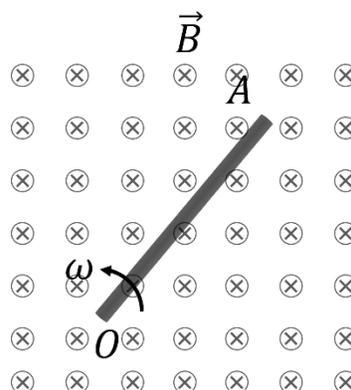
When both A and B are open or any one of them is open, bulb will glow.
When both switches are closed, due to short circuit, bulb will not glow.

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

$$\text{Output} = \overline{A \cdot B}$$

2. A rod 'OA' of length 'L' is rotating with constant angular speed ' ω ' about an axis passing through point O in a perpendicular uniform magnetic field 'B'. Find the emf induced between points O and A of the rod.

- A. $\frac{1}{2} B\omega L^2$
- B. $B\omega L^2$
- C. $\frac{3}{2} B\omega L^2$
- D. $2 B\omega L^2$



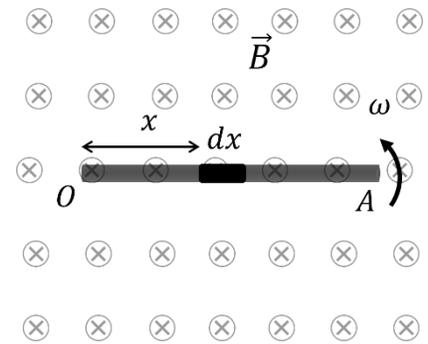
Answer (A)

Sol.

$$d\mathcal{E} = Bvdx = B\omega x dx$$

$$\int d\mathcal{E} = \int_0^L B\omega x dx$$

$$\mathcal{E} = \frac{B\omega L^2}{2}$$



3. Let γ_1 be the ratio of molar specific heat at constant pressure and constant volume for a monoatomic gas and let γ_2 be this ratio for a diatomic gas. Find $\frac{\gamma_1}{\gamma_2}$

- A. 21/25
- B. 7/3
- C. 25/21
- D. 3/7

Answer (C)

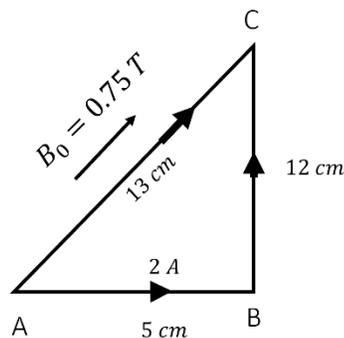
Sol.

We know that $\gamma_1 = \frac{5}{3}$ and $\gamma_2 = \frac{7}{5}$

$$\frac{\gamma_1}{\gamma_2} = \frac{25}{21}$$

4. A right-angled triangle has current of 2 A. The edge length are shown in the diagram. Magnetic field is acting in the plane of the triangle. The magnetic force acting on wire AB is

- A. $\frac{5}{130} N$
- B. $\frac{15}{2} N$
- C. $\frac{3}{40} N$
- D. $\frac{9}{130} N$



Answer (D)

Sol.

$$B = 0.75 T$$

$$|\vec{F}_{AB}| = i \times (l_{AB})B \sin \theta$$

$$= 2 \times \left(\frac{5}{100}\right) \times 0.75 \times \frac{12}{13}$$

$$= \frac{9}{130} N$$

5. Assertion (A): Steel is used to build big structures.

Reason (R): Steel has more elastic modulus as compared to other materials.

- A. Both A and R are true and R is the correct explanation of A.
- B. Both A and R are true but R is not the correct explanation of A.
- C. A is true but R is false.
- D. Both A and R are false.

Answer (A)

Sol.

Large elastic modulus implies deformations will stay in elastic region and permanent deformations will not occur for small magnitude of force.

6. If $\vec{a} = \hat{i} + 2\hat{j} + m\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other, then $m = \sqrt{x}$. Find x .

Answer (3)

Sol.

$$\vec{a} = \hat{i} + 2\hat{j} + m\hat{k} \quad \text{and} \quad \vec{b} = \hat{i} - 2\hat{j} + m\hat{k}$$

$$\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$1 - 4 + m^2 = 0$$

$$m = \sqrt{3}$$

$$x = 3.00$$

7. The distance of earth from the sun is $1.5 \times 10^8 \text{ km}$. If the time period of an imaginary planet is 2.83 years, then its distance from the sun comes out to be $n \times 10^8 \text{ km}$. Find n .

Answer (3)

Sol.

$$T^2 \propto a^3$$

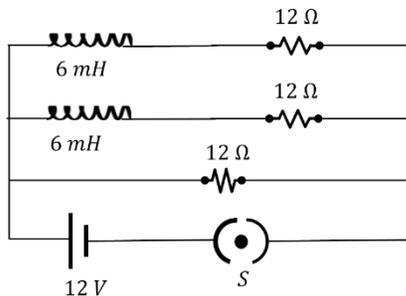
$$\left[\frac{T^2}{a^3} \right]_{\text{earth}} = \left[\frac{T^2}{a^3} \right]_{\text{any planet}}$$

$$\frac{1}{(1.5 \times 10^8)^3} = \frac{2.83^2}{(x)^3}$$

$$x = 3 \times 10^8$$

$$n = 3$$

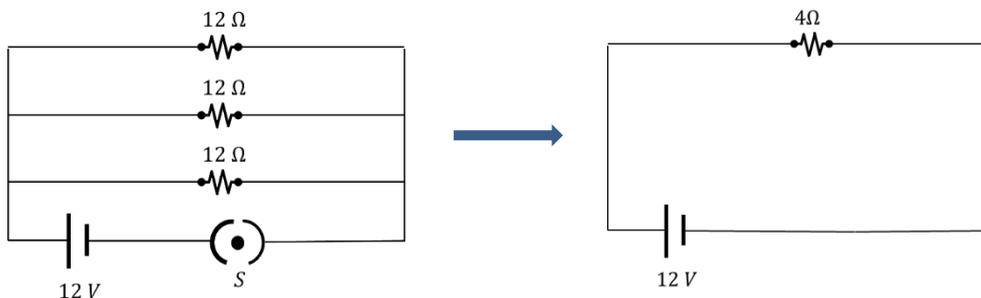
8. For the given electrical circuit, the switch 's' is closed at $t = 0$. Find the current (in A) drawn through the battery at steady state.



Answer (3)

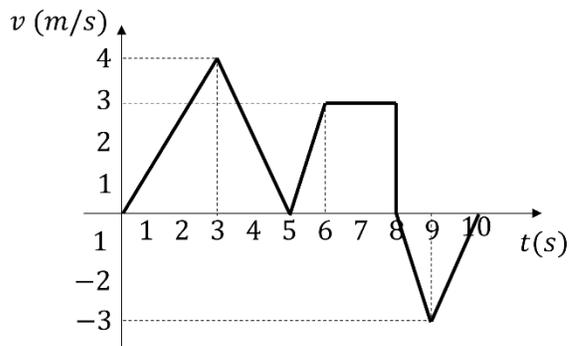
Sol.

At steady state, inductors behave as shorted.



$$i = \frac{12}{4} = 3A$$

9. The velocity-time ($v - t$) graph for a particle moving along a straight path is shown. Find the time taken (in seconds) by the particle to cover the first 10 m of distance.



Answer (5)

Sol.

Distance = Area under $v - t$ graph

Area in first 5 seconds,

$$A = \frac{1}{2} \times t \times 4 = 10 m$$

$$t = 5 s$$

10. An electron jumps from 3^{rd} excited state to the ground state in a hydrogen atom. The wavelength of photon emitted in nm to the nearest integer is _____ ($hc = 1240 eV - nm$)

Answer (97)

Sol.

$$n_i = 4, n_f = 1$$

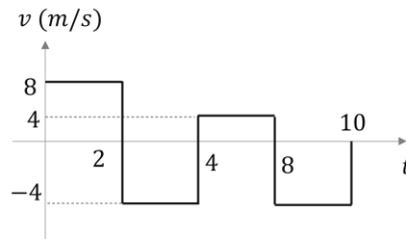
$$\Delta E = 13.6 \left(1 - \frac{1}{16}\right) = \frac{13.6 \times 15}{16} = 12.75 eV$$

$$\lambda(nm) = \frac{1240}{12.75} \approx 97.25 nm$$

$$\lambda(nm) = 97 nm$$

11. The velocity-Time graph of a body moving along straight line is given as shown. The ratio of displacement and distance is:

- A. 1:1
- B. 1:2
- C. 1:3
- D. 1:4



Answer (C)

Sol.

$$\text{Distance} = 8 \times 2 + 4 \times 2 + 4 \times 4 + 4 \times 2 = 48 m$$

$$\text{Displacement} = 8 \times 2 - 4 \times 2 + 4 \times 4 - 4 \times 2 = 16 m$$

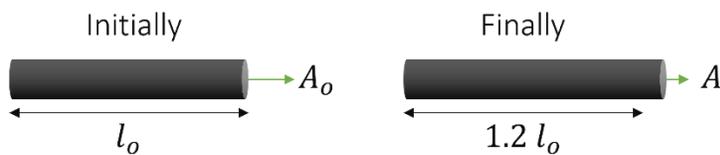
$$\text{Ratio} = 1:3$$

12. A copper wire is elongated such that its length is increased by 20%. Then the percentage increase in the resistance is

- A. 20%
- B. 30%
- C. 44%
- D. 50%

Answer (C)

Sol.



Before Heating

$$R_o = \left(\frac{\rho l_o}{A_o}\right)$$

After Heating

Volume = Constant

$$A_0 l_0 = 1.2 l_0 A$$

$$A = \left(\frac{A_0}{1.2} \right)$$

So,

$$R = \frac{\rho l}{A} = \frac{\rho \times (1.2 l_0)}{\left(\frac{A_0}{1.2} \right)} = \frac{\rho l_0}{A_0} \times 1.44 = 1.44 R_0$$

$$\text{Percentage increase in } = \frac{R - R_0}{R_0} \times 100 = 44\%$$

13. Assertion (A): Acceleration due to gravity decreases with both height and depth from earth surface.

Reason (R): If height and depth are equal for two points from surface of the earth, then acceleration due to gravity will be same at those points.

- A. Both A and R are true and R is the correct explanation of A
- B. Both A and R are true and R is not the correct explanation of A
- C. A is true but R is false.
- D. Both A and R are false

Answer (C)

Sol.

Acceleration due to gravity is maximum on the surface.

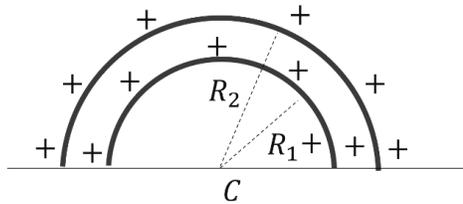
It increases linearly from centre to surface and then decreases by inverse square law.

So, Assertion is true, and Reason is false.

14. Two concentric semi-circular rings (radii R_1 and R_2) have equal linear charge density (λ each) as shown:

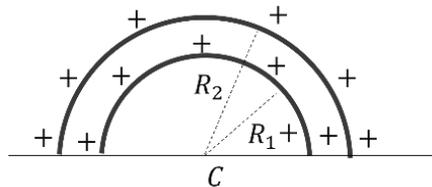
Find the potential at center C .

- A. $\frac{\lambda}{\epsilon_0}$
- B. $\frac{2\lambda}{\epsilon_0}$
- C. $\frac{\lambda}{4\epsilon_0}$
- D. $\lambda/2\epsilon_0$



Answer (D)

Sol.



At point C:

$$V_c = V_1 + V_2$$

$$V_c = \frac{1}{4\pi\epsilon_0} \times \frac{\lambda_1(\pi R_1)}{R_1} + \frac{1}{4\pi\epsilon_0} \times \frac{\lambda_1(\pi R_2)}{R_2} = \frac{\lambda_1 \text{ (or } \lambda)}{2\epsilon_0}$$

15. Electric field vector and magnetic field vector of an electromagnetic wave is given as $\vec{E} = E_0 \sin(\omega t - kx)\hat{j}$ and $\vec{B} = B_0 \sin(\omega t - kx)\hat{k}$, then choose the correct option.

- A. $\omega E_0 = kB_0$
- B. $kE_0 = \omega B_0$
- C. $k\omega = E_0 B_0$
- D. None of these

Answer (B)

Sol.

Given: $\vec{E} = E_0 \sin(\omega t - kx)\hat{j}$ and $\vec{B} = B_0 \sin(\omega t - kx)\hat{k}$

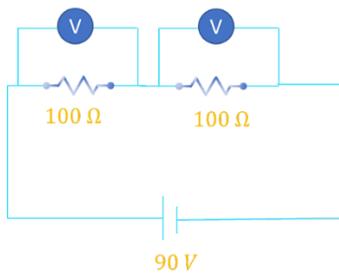
$$E_0 = vB_0$$

$$v = \frac{\omega}{k}$$

$$E_0 = \frac{\omega}{k} B_0 \Rightarrow kE_0 = \omega B_0$$

16. Consider the circuit shown:

Resistance of each voltmeter is 400Ω . Find the reading of any one voltmeter.



- A. $60 V$
- B. $45 V$
- C. $80 V$
- D. $30 V$

Answer (B)

Sol.

By symmetry, voltage would be divided equally.

$$\Rightarrow \text{Voltage across a voltmeter} = \frac{90}{2} V = 45 V$$

17. A long solenoid has 70 turns per cm and carries current $2 A$. The magnetic field inside the solenoid is _____ ($\mu_0 = 4\pi \times 10^{-7}$ in SI units)

- A. $125.2 \times 10^{-4} T$
- B. $835.2 \times 10^{-4} T$
- C. $176.0 \times 10^{-4} T$
- D. $880 \times 10^{-4} T$

Answer (C)

Sol.

Magnetic field inside solenoid = $\mu_0 ni$

where n = Number of turns per unit length

$n = 7000 \text{ turns/m}$

$$B_{\text{solenoid}} = \mu_0 ni = (4\pi \times 10^{-7} \times 7000 \times 2) \text{ T}$$

$$= 8 \times \frac{22}{7} \times 10^{-7} \times 7000 \text{ T}$$

$$= 176 \times 10^{-4} \text{ T}$$

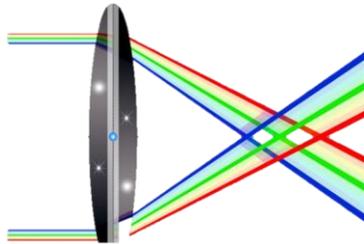
18. A ray of parallel white light incidence on convex lens and colour gets separated, this phenomenon is called

- A. Chromatic aberration
- B. Polarization
- C. Scattering
- D. Spherical aberration

Answer (A)

Sol.

This phenomenon is known as chromatic aberration.



19. **Assertion (A):** The weight of an object at Mount Everest is lesser than its weight at sea level.

Reason (R): The value of g decreases as height increases.

- A. Assertion and Reason both are correct, and Reason is the correct explanation of Assertion.
- B. Assertion and Reason both are correct, but Reason is not the correct explanation.
- C. Assertion is correct reason is incorrect.
- D. Assertion is incorrect reason is correct.

Answer (A)

Sol.

As we know that:

$$g = \frac{GM}{(R + h)^2}, \text{ Weight} = mg$$

As height from surface of earth increases g decreases, so weight on Mount Everest is less than the weight on the surface of earth.

20. A convex lens of refractive index 1.5 has focal length 20 cm in air. When this lens is immersed in water (refractive index = 4/3), the new focal length (in cm) is _____.

Answer (80)

Sol.

$$f_{air} = 20 \text{ cm}, \mu_{water} = \frac{4}{3}, \mu = 1.5$$

When convex lens is in air,

$$\frac{1}{f_{air}} = (\mu - 1) \left(\frac{2}{R} \right) = 0.5 \left(\frac{2}{R} \right) \quad \text{-----(1)}$$

When lens is immersed in water,

$$\frac{1}{f} = \left(\frac{\mu}{\mu_{water}} - 1 \right) \left(\frac{2}{R} \right) = 0.125 \left(\frac{2}{R} \right) \quad \text{-----(2)}$$

From (1) and (2),

$$\frac{f}{f_{air}} = \frac{0.5}{0.125}$$

$$f = 0.5 \times \frac{20}{0.125} = 80 \text{ cm}$$

CHEMISTRY

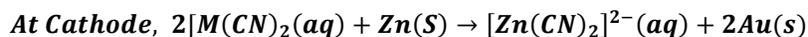
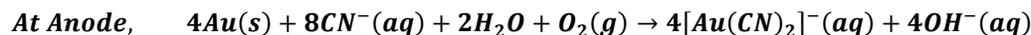
1. In which of the following metal extraction both Oxidation and Reduction process are involved?

- A. Au
- B. Cu
- C. Fe
- D. Al

Answer (A)

Sol.

In the extraction of Gold (Au) first oxidation of gold takes place and then it is reduced into gold.



2. α – particle, proton and electron have same kinetic energy. Select the correct order of de Broglie wavelength

- A. $\lambda_p = \lambda_\alpha = \lambda_e$
- B. $\lambda_e > \lambda_p > \lambda_\alpha$
- C. $\lambda_\alpha > \lambda_e > \lambda_p$
- D. $\lambda_p > \lambda_e = \lambda_\alpha$

Answer (B)

Sol.

As we know,

$$\lambda = \frac{h}{\sqrt{2m(K.E)}}$$

λ = De Broglie wavelength

m = Mass of the Particles, kE = Kinetic Energy of the Particle

h = Planck's Constant = 6.6×10^{-34} Joule Second

Mass of α – particle > mass of proton > mass of electron

λ is inversely proportional to the mass of the particle.

So, higher the mass of the particle less will be the de Broglie wavelength associated with it so option (B) is the correct option.

3. Find out order of reaction of decomposition of $AB_3(g)$ using the given information

Initial pressure (mmHg) of $AB_3(g)$	50	100	200	400
$t_{\frac{1}{2}}$ (sec)	4	2	1	5

- A. 0
- B. 1
- C. 2
- D. 3

Answer (C)

Sol.

As we know,

$$(t_1) \propto (P_0)^{1-n}$$

By observation,

$$\frac{4}{2} = \left(\frac{50}{100}\right)^{1-n}$$

$$2 = \left(\frac{1}{2}\right)^{1-n}$$

$$2^1 = 2^{(n-1)}$$

$$1 = (n-1)$$

$$\text{So, } n=2$$

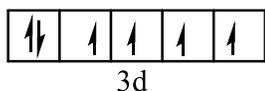
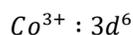
second order reaction so correct option is (C).

4. Predict the hybridization state of the central metal ion and magnetic nature of the complex $[Co(NH_3)_6]^{3+}$.
- A. sp^3d^2 , Paramagnetic
 - B. sp^3d^2 , Diamagnetic
 - C. d^2sp^3 , Paramagnetic
 - D. d^2sp^3 , Diamagnetic

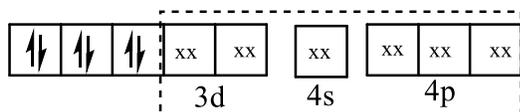
Answer (D)

Sol.

Oxidation state of cobalt in the given complex is +3



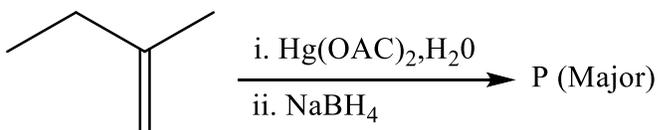
As (NH_3) act as strong field ligand so pairing will be take place here,



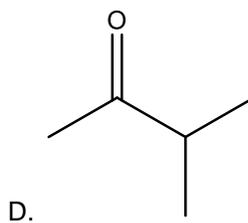
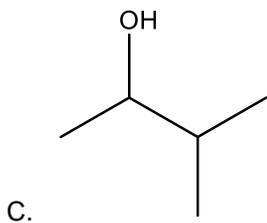
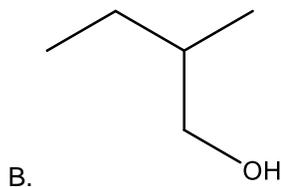
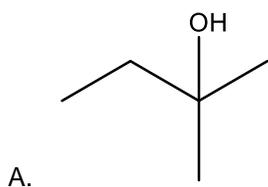
Hybridisation state of $Co^{3+} : d^2sp^3$

Since no unpaired electron is present here so Magnetic nature is Diamagnetic and Correct option is (D)

5. Consider the following reaction

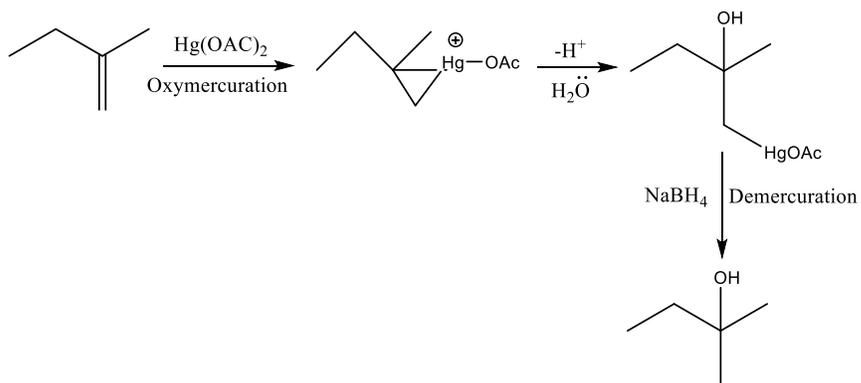


P is ?

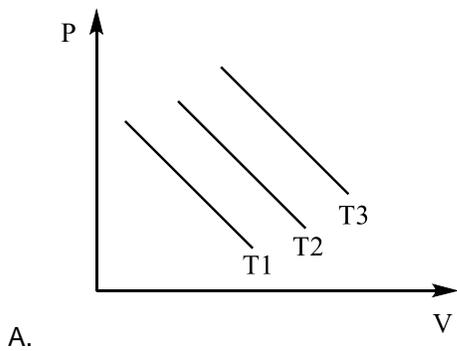


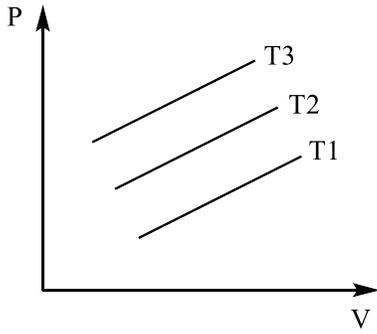
Answer (A)

Sol.

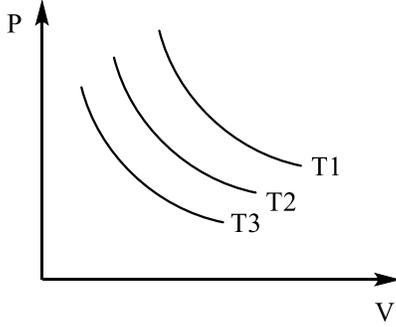


6. Which graph is correct for Isothermal process at T_1, T_2 & T_3 if ($T_3 > T_2 > T_1$)

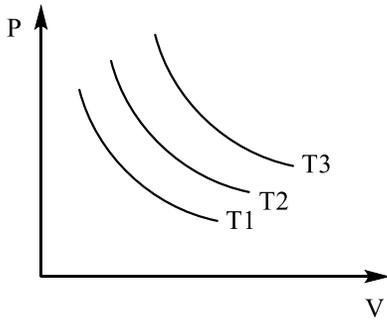




B.



C.



D.

Answer (D)

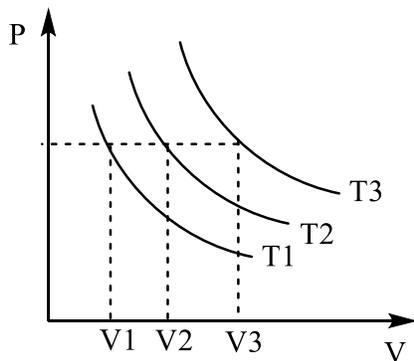
Sol.

According to Boyle Law $P \propto \frac{1}{V}$

The graph must be hyperbola.

As we know, $PV = nRT$

So as increase the Temperature the PV graph area increases.



As $(V_3 > V_2 > V_1)$ for fixed P

$$= (T_3 > T_2 > T_1)$$

And the correct option is (D)

7. The number of Peptide bonds present in Tripeptide VAL – PRO – GLY is
- A. 1
 - B. 2
 - C. 3
 - D. 4

Answer (B)

Sol.

In between two Amino Acids there are 1 peptide bond in question 3 Amino acids are given so Tripeptide has two peptide bonds.

8. Which of the following options contains correct match of the following

A. Antifertility Drugs	P. Arsphenamine
B. Antibiotics	Q. Norethindrone
C. Tranquilizers	R. Seldane
D. Antihistamines	S. Meprobamate

- A. A – Q, B – P, C – S, D – R
- B. A – P, B – Q, C – R, D – S
- C. A – S, B – R, C – Q, D – P
- D. A – S, B – R, C – P, D – Q

Answer (B)

Sol.

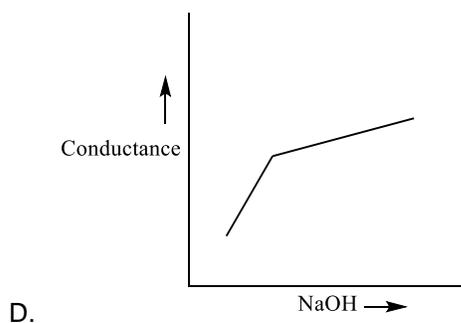
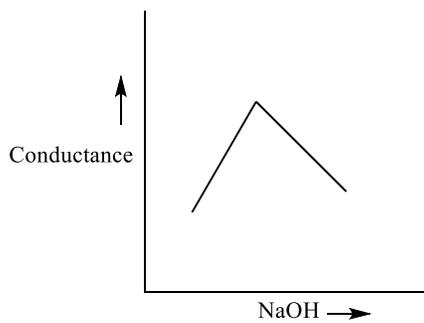
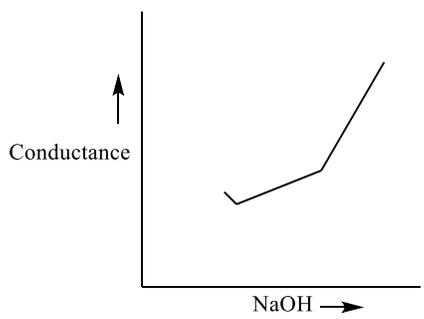
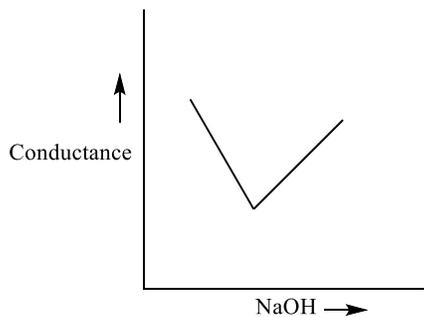
Antifertility Drugs - Norethindrone

Antibiotics - Arsphenamine

Tranquilizers - Meprobamate

Antihistamines – Seldane

9. When NaOH is added slowly to Benzoic acid, then which of the following plot of conductance Vs amount of NaOH will be correct



Answer (B)

Sol.

First there are only Benzoic acid is present Which is weak acid

So , $\text{PhCOOH} \leftrightarrow \text{PhCOO}^- + \text{H}^+$

From Beginning the Conductance increases Slightly

After adding NaOH in that it is make PhCOONa which is a salt and the conductance increases

When all Na^+ reacts with PhCOO^- then Only OH^- Present in the solution and the conductance increases very highly

So, option B is the correct answer.

$$= (T_3 > T_2 > T_1)$$

And the correct option is (D)

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- A. 1
 - B. 2
 - C. 3
 - D. 4

Answer (B)

Sol.

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A. Antifertility Drugs	P. Arsphenamine
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C. Tranquilizers	R. Seldane
D. Antihistamines	S. Meprobamate

- A. A – Q, B – P, C – S, D – R
- B. A – P, B – Q, C – R, D – S
- C. A – S, B – R, C – Q, D – P
- D. A – S, B – R, C – P, D – Q

Answer (B)

Sol.

Antifertility Drugs - Norethindrone

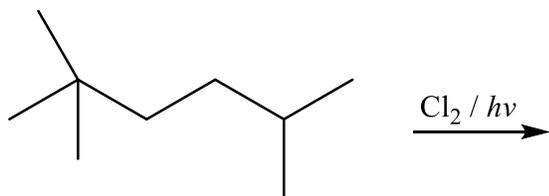
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Tranquilizers - Meprobamate

Antihistamines – Seldane

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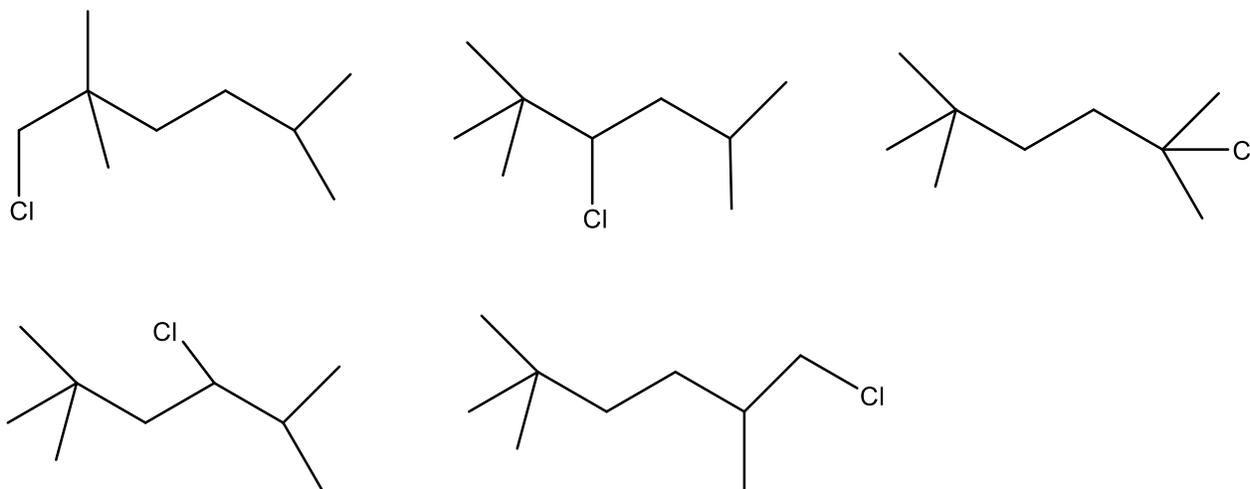
12. Find the number of mono chloro products (structural only) which are obtained in the following reaction is



Answer (5)

Sol.

Possible Mono chloro Products are



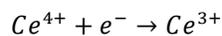
13. Which of the following Lanthanoid ions is the best oxidising agent?

- A. Lu^{2+}
- B. Ce^{2+}
- C. Ce^{4+}
- D. Sn^{2+}

Answer (C)

Sol.

The most stable oxidation state of lanthanoids is +3. The Lu^{2+} , Ce^{2+} and Sn^{2+} will function as reducing agent because they will easily get oxidised to +3 oxidation state in each case. But Ce^{4+} will function as as oxidising agent and get reduced to Ce^{3+}



14. The number of 's' electrons in unipositive state of an element having 55 protons in its nucleus is

- A. 10
- B. 8
- C. 11
- D. 12

Answer (A)**Sol.**

The element is Cs. Cs⁺ has 10 's' electrons in its nucleus

15. Select the correct statement about physisorption

- A. Physisorption is highly specific
- B. Physisorption is always monolayer
- C. Physisorption doesn't require activation energy
- D. Physisorption is associated with very high enthalpy of adsorption

Answer (C)**Sol.**

Physisorption doesn't require activation energy

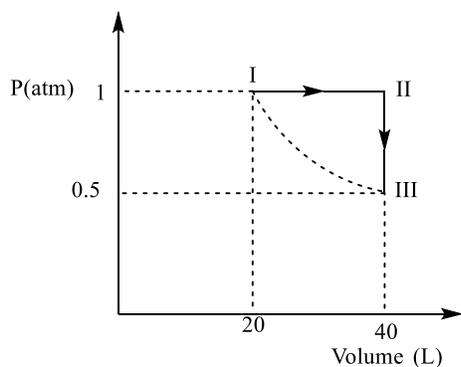
16. How many s-electrons are there in a Br-atom (Atomic No: of Br = 35)

Answer (8)**Sol.**

Electronic configuration of Br atom is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^5$

Therefore No of s – electrons in a Br atom is 8.

17.



I → II : Isobaric

II → III : Isochoric

III → I : Isothermal

All process are reversible

Find out the work done by the Gas for complete cyclic process (In atm.Lit)

(Report your answer to closest integer)

Answer (6.00)

Sol.

$$W_{I \rightarrow II} = -P\Delta V = -1 \times (40 - 20) = -20 \text{ atm.lit}$$

$$W_{II \rightarrow III} = 0$$

$$W_{III \rightarrow I} = 2.303nRT \cdot \log \frac{V_2}{V_1} = 2.303PV \log \frac{V_2}{V_1} = 2.303(1 \times 20) \times \log 22 \\ = +13.818$$

$$W_{I \rightarrow II \rightarrow III \rightarrow IV} = -20 + 13.818 \\ = -6.182 \text{ atm.lit}$$

Work done by the gas = + 6.182 atm.lit

18. Find the sum of number of unpaired electrons in the following diatomic molecules: N_2 , N_2^+ , O_2 , O_2^+ ?

Answer (4.00)

Sol.

No. of unpaired electrons in $N_2 = 0$

No. of unpaired electrons in $N_2^+ = 1$

No. of unpaired electrons in $O_2 = 2$

No. of unpaired electrons in $O_2^+ = 1$

Sum = $0+1+2+1 = 4$

19. pK_a of lactic acid is 4, Find the pH of 0.005 M calcium lactate at 27°C is?

Answer (8)

Sol.

$$p^H = \frac{1}{2}(pK_w + pK_a + \log C)$$

$$= \frac{1}{2}(14 + 4 + \log(0.01)) = \frac{1}{2}(18 - 2) = \frac{1}{2}(16) = 8$$

20. Find the sum of number of π bonds in peroxydisulphuric acid and pyrosulphuric acid ?

Answer (8)

Sol.

Peroxydisulphuric acid ($H_2S_2O_8$) has 4 π bonds

Pyrosulphuric ($H_2S_2O_7$) acid has 4 π bonds

Total number of π bonds = 8

21. How many of the following concentration terms are temperature independent?

Mole fraction,

Mass percent (% w/w),

Molarity (M)

Molality (m)

ppm

volume percent (%V/V)

Answer (4:00)

Sol.

Temperature independent concentration terms are :

Mole fraction

Molality (m)

Parts Per Million (ppm)

Mass percentage (%w/w)

22. One atom of X has 25 MeV energy. The energy in 102 g of X is $P \times 10^{25}$ MeV. Then find the value of 'P'?
Given : X has molar mass = 61 g and $N_A = 6 \times 10^{23}$

Answer (3)

Sol.

$$\text{Total energy} = 25 \times \frac{102}{61} \times 6 \times 10^{23} = 3 \times 10^{25} \text{ MeV}$$

23. If the ratio $\frac{C_p}{C_v}$ for monoatomic gases is r_1 and similar ratio for diatomic gas is r_2 . Then the value of $\frac{r_1}{r_2} \times 21$ is

Answer (25)

Sol.

$$r_1 = \frac{5}{3}$$

$$r_2 = \frac{7}{5}$$

$$\frac{r_1}{r_2} = \frac{25}{21}$$

$$\frac{r_1}{r_2} \times 21 = 25$$

1. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$ is equal to:

- A. π
- B. 2π
- C. -2π
- D. 3π

Answer (B)

Sol.

$$I = \int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{3 \times \sqrt{1 - \frac{4x^2}{9}}} dx$$

$$\text{Let } \frac{2x}{3} = t$$

$$I = 16 \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} \frac{\frac{3}{2}}{\sqrt{1-t^2}} dt$$

$$I = 24 \sin^{-1} t \Big|_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}}$$

$$I = 24 \left(\frac{\pi}{3} - \frac{\pi}{4} \right) = 2\pi$$

2. $\left(\frac{1 + \cos\left(\frac{2\pi}{9}\right) + i \sin\left(\frac{2\pi}{9}\right)}{1 + \cos\left(\frac{2\pi}{9}\right) - i \sin\left(\frac{2\pi}{9}\right)} \right)^3$ is equal to:

- A. $-\frac{1}{2} + \frac{i\sqrt{3}}{2}$
- B. $-\frac{1}{2} - \frac{i\sqrt{3}}{2}$
- C. $\frac{1}{2} - \frac{i\sqrt{3}}{2}$
- D. $\frac{1}{2} + \frac{i\sqrt{3}}{2}$

Answer (A)

Sol.

$$\begin{aligned} \left(\frac{1 + \cos\left(\frac{2\pi}{9}\right) + i \sin\left(\frac{2\pi}{9}\right)}{1 + \cos\left(\frac{2\pi}{9}\right) - i \sin\left(\frac{2\pi}{9}\right)} \right)^3 &= \left(\frac{2 \cos\left(\frac{\pi}{9}\right) \left(\cos\frac{\pi}{9} + i \sin\left(\frac{\pi}{9}\right) \right)}{2 \cos\left(\frac{\pi}{9}\right) \left(\cos\frac{\pi}{9} - i \sin\left(\frac{\pi}{9}\right) \right)} \right)^3 \\ &= \left(e^{\frac{i2\pi}{9}} \right)^3 \\ &= e^{\frac{i2\pi}{3}} \\ &= -\frac{1}{2} + \frac{i\sqrt{3}}{2} \end{aligned}$$

3. If $\vec{a} = \hat{i} + 2\hat{j} + m\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + m\hat{k}$ if \vec{a} & \vec{b} are perpendicular to each other then m equals:

- A. $\pm\sqrt{2}$
- B. $\pm\sqrt{3}$
- C. ± 2
- D. $\pm\sqrt{5}$

Answer (B)

Sol.

$$\begin{aligned} \text{If } \vec{a} \text{ \& } \vec{b} \text{ are perpendicular to each other then } \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\hat{i} + 2\hat{j} + m\hat{k}) \cdot (\hat{i} - 2\hat{j} + m\hat{k}) &= 0 \\ \Rightarrow 1 - 4 + m^2 &= 0 \\ \Rightarrow m^2 &= 3 \\ \Rightarrow m &= \pm\sqrt{3} \end{aligned}$$

4. The sum of the coefficients of first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$ is 376. The coefficient of x^4 is equal to:

- A. 695
- B. 410
- C. 405
- D. 395

Answer (C)

Sol.

$$\begin{aligned} \text{The first three of } \left(x - \frac{3}{x^2}\right)^n &= {}^nC_0x^n - {}^nC_1 \cdot 3x^{n-3} + {}^nC_2 \cdot 9x^{n-6} \\ {}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 \cdot 9 &= 376 \\ \Rightarrow 1 - 3n + \frac{n(n-1)}{2} \times 9 &= 376 \\ \Rightarrow 3n^2 - 5n - 250 &= 0 \\ \Rightarrow n &= 10 \\ \text{Now,} \\ T_{r+1} &= {}^{10}C_r x^{10-r} \cdot \left(-\frac{3}{x^2}\right)^r \\ \Rightarrow {}^{10}C_r (-3)^r \cdot x^{10-3r} \\ \Rightarrow 10 - r - 2r &= 4 \\ \Rightarrow r &= 2 \\ \text{Coefficient of } x^4 &= {}^{10}C_2 (-3)^2 \\ &= 45 \times 9 = 405 \end{aligned}$$

5. Let $A = \{a, b, c, d\}$ and a relation $A \rightarrow A$ be $R = \{(a, b), (b, d), (b, c), (b, a)\}$ then minimum number of elements required to make R equivalent is:

- A. 7
- B. 10
- C. 12
- D. 14

Answer (C)**Sol.**

Adding $(a, a), (b, b), (c, c), (d, d)$ makes it reflexive.

Adding (d, b) and (c, b) makes it symmetric.

And adding $(a, d), (a, c)$ makes it transitive.

So further (d, a) & (c, a) to be added to maintain symmetry of relation.

further (c, d) & (d, c) also be added.

Hence total of 12 elements to be added to mole equivalence.

6. 3 urns A, B, C contain 4 red, 6 black; 5 red, 5 black; λ red, 4 black balls. A ball is drawn and found to be red. If probability that ball was drawn from urn C is 0.4, then the square of side of equilateral triangle inscribed in parabola $y^2 = \lambda x$ with one vertex at vertex of parabola is

- A. 144
 B. 432
 C. 368
 D. 284

Answer (B)**Sol.**

$$P(\text{Red ball from urn C}) = \frac{\frac{1 \cdot \lambda}{3\lambda+4}}{\frac{1 \cdot 4}{3 \cdot 10} + \frac{1 \cdot 5}{3 \cdot 10} + \frac{1 \cdot \lambda}{3 \cdot \lambda+4}} = \frac{4}{10}$$

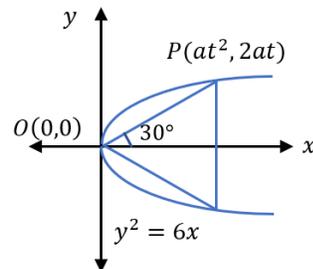
$$\Rightarrow \lambda = 6$$

$$m_{op} = \frac{1}{\sqrt{3}}$$

$$\frac{2}{t} = \frac{1}{\sqrt{3}}$$

$$\text{Length of side} = 4at = 12\sqrt{3} \text{ units}$$

$$\text{Square of side length} = 432$$



7. Total number of numbers formed using digits 3, 5, 6, 7, 8 (without repetition) which are greater than 7000, is equal to:

- A. 148
 B. 168
 C. 144
 D. 124

Answer (B)**Sol.**

Number using all the 5 digits = $5! = 120$

Number using 4 digits

Case I:

When 7 is fixed at 1000's place

$$7 \text{ ___ ___ ___ } = 24 \text{ ways}$$

Case II:

When 8 is fixed at 1000's place

$$8 \text{ --- } = 24 \text{ ways}$$

$$\text{Total number} = 120 + 24 + 24$$

$$= 168$$

8. A 5×5 matrix whose each entry is either 0 or 1, is such that sum of entries of each column as well as each row is 1. Number of such matrices is :

- A. 30
- B. 60
- C. 90
- D. 120

Answer (D)

Sol.

In first column, 1 can be placed in any of the 5 places = 5
 In second column, 1 can be placed in any of the 4 places = 4
 In third column, 1 can be placed in any of the 3 places = 3
 In fourth column, 1 can be placed in any of the 2 places = 2
 In fifth column, 1 can be placed in only 1 place = 1

$$\text{Total} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

9. If the shortest distance between the lines $\frac{x-\sqrt{6}}{1} = \frac{y+\sqrt{6}}{2} = \frac{z-\sqrt{6}}{3}$ and $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z-3\sqrt{6}}{5}$ is 6, then the sum of squares of all possible values of λ is equal to:

- A. 1024
- B. 732
- C. 416
- D. 312

Answer (B)

Sol.

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = (\lambda - \sqrt{6})\hat{i} + 3\sqrt{6}\hat{j} + 2\sqrt{6}\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \Rightarrow 6 = \frac{|-2(\lambda - \sqrt{6}) + 12\sqrt{6} - 4\sqrt{6}|}{\sqrt{24}}$$

$$\Rightarrow \lambda = 11\sqrt{6}, -\sqrt{6}$$

$$\Rightarrow \lambda_1^2 + \lambda_2^2 = 732$$

10. The proposition $\sim(p \wedge (p \rightarrow \sim q))$ is equivalent to :

- A. $p \wedge (p \vee q)$
- B. $\sim p \vee q$
- C. $p \vee q$
- D. $\sim p \wedge q$

Answer (B)**Sol.**

$$\begin{aligned}
& \sim(p \wedge (p \rightarrow \sim q)) \\
& = \sim(p \wedge (\sim p \vee \sim q)) \\
& = \sim(f \vee (p \wedge \sim q)) \\
& = \sim(p \wedge \sim q) \\
& = \sim p \vee q
\end{aligned}$$

11. $({}^{30}C_1)^2 + 2 \cdot ({}^{30}C_2)^2 + \dots + 30 \cdot ({}^{30}C_{30})^2 = \frac{\alpha \cdot 60!}{(30!)^2} - 1$ then α is:

- A. 12
- B. 15
- C. 10
- D. 13

Answer (B)**Sol.**

$$\begin{aligned}
\sum_{r=1}^{30} r \cdot {}^{30}C_r \cdot {}^{30}C_r &= 30 \sum_{r=1}^{30} {}^{29}C_{r-1} \cdot {}^{30}C_{30-r} \\
&= 30 \cdot \text{coefficient of } x^{29} \text{ in } (1+x)^{29} \cdot (1+x)^{30} \\
&= 30 \cdot {}^{59}C_{29} \\
&= 30 \cdot \frac{59!}{29!30!} \\
&\Rightarrow \frac{900 \cdot 59!}{(30!)^2} \\
&\Rightarrow \frac{15 \cdot 60!}{(30!)^2} \\
&\Rightarrow \alpha = 15
\end{aligned}$$

12. If $\lim_{x \rightarrow \alpha} |x - 5| - [2x + 2]| = 0$ then:

- A. $\alpha \in (-7.5, -6.5)$
- B. $\alpha \in [-7.5, -6.5)$
- C. $\alpha \in (-7.5, -6.5]$
- D. $\alpha \in [-7.5, -6.5]$

Answer (B)**Sol.**

$$\begin{aligned}
& \lim_{x \rightarrow \alpha} |x - 5| - [2x + 2]| \\
& \lim_{x \rightarrow \alpha} |[x] - 5 - [2x] - 2| \\
& \Rightarrow \lim_{x \rightarrow \alpha} |[x] - [2x] - 7| \\
& \lim_{x \rightarrow -7.5^+} |[x] - [2x] - 7| \\
& \Rightarrow |-8 + 15 - 7| = 0
\end{aligned}$$

$$\text{At } x = -7.5, |-8 + 15 - 7| = 0$$

$$\lim_{x \rightarrow -6.5^-} |[x] - [2x] - 7|$$

$$\Rightarrow |-7 + 14 - 7| = 0$$

$$\text{At } x = -6.5, |[x] - [2x] - 7|$$

$$\Rightarrow |-7 + 13 - 7| \neq 0$$

$$\therefore \alpha \in [-7.5, -6.5)$$

13. The locus of the mid points of chords of the circle $(x - 4)^2 + (y - 5)^2 = 4$ which subtends angle θ_i at the centre of this circle is a circle of radius r_i . If $\theta_1 = \frac{\pi}{3}$, $\theta_2 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$ then θ_3 is equal to:

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{12}$
- C. $\frac{\pi}{6}$
- D. $\frac{\pi}{4}$

Answer (A)

Sol.

If a chord of circle of radius R subtends angle θ_i at the centre then locus of the midpoint of this chord is a

circle of radius $r_i = R \cdot \cos\left(\frac{\theta_i}{2}\right)$

Given,

$$r_1^2 = r_2^2 + r_3^2$$

$$\Rightarrow \cos^2 \frac{\theta_1}{2} = \cos^2 \frac{\theta_2}{2} + \cos^2 \frac{\theta_3}{2}$$

$$\Rightarrow \cos^2 \frac{\pi}{6} = \cos^2 \frac{\pi}{3} + \cos^2 \frac{\theta_3}{2}$$

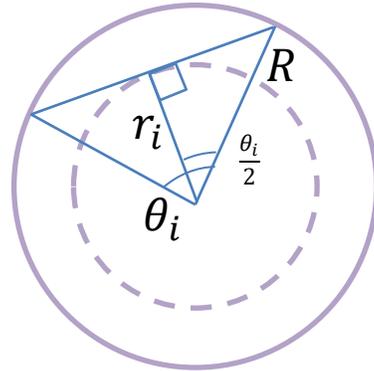
$$\Rightarrow \frac{3}{4} = \frac{1}{4} + \cos^2 \frac{\theta_3}{2}$$

$$\Rightarrow \cos^2 \frac{\theta_3}{2} = \frac{1}{2}$$

$$\Rightarrow \cos \frac{\theta_3}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\theta_3}{2} = \frac{\pi}{4}$$

$$\therefore \theta_3 = \frac{\pi}{2}$$



14. Let $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$, then:

- A. $f(0) = f(1) + f(2) + f(3)$
- B. $f(3) + 2f(0) = f(2) + f(1)$
- C. $2f(0) = f(1) - f(2)$
- D. $f(3) - f(1) = 2f(2)$

Answer (B)

Sol.

$$\text{Let } f(x) = x^3 - Ax^2 + Bx - C$$

$$\Rightarrow f'(1) = 3 - 2A + B = A$$

$$\Rightarrow f''(2) = 12 - 2A = B$$

$$\Rightarrow f'''(3) = 6 = C$$

$$\text{Solving, } f(x) = x^3 - 3x^2 + 6x - 6$$

$$f(0) = -6, f(1) = -2$$

$$f(2) = 8 - 12 + 12 - 6 = 2$$

$$f(3) = 27 - 27 + 18 - 6 = 12$$

$$\therefore f(3) + 2f(0) = f(2) + f(1)$$

15. $\frac{1^3+2^3+3^3+\dots\text{up to } n \text{ terms}}{1\cdot 3+2\cdot 5+3\cdot 7+\dots\text{up to } n \text{ terms}} = \frac{9}{5}$ then n is equal to:

- A. 5
- B. 7
- C. 12
- D. 9

Answer (B)

Sol.

$$\frac{1^3+2^3+3^3+\dots\text{up to } n \text{ terms}}{1\cdot 3+2\cdot 5+3\cdot 7+\dots\text{up to } n \text{ terms}} =$$

$$\frac{1^3+2^3+3^3+\dots+n^3}{\sum n(2n+1)}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\cdot\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4} - 6}{2(2n+1)+3} = \frac{9}{5}$$

$$\Rightarrow 5n^2 - 19n - 30 = 0$$

$$\Rightarrow 5n^2 - 25n + 6n - 30 = 0$$

$$\Rightarrow n = 5, -\frac{6}{5} \Rightarrow n = 5$$

16. If $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$ then $|A^{-1}(\text{adj}A)|$ equals: (where A is matrix of order 3×3)

- A. $2\sqrt{3}$
- B. 1
- C. 6
- D. 12

Answer (A)

Sol.

$$|\text{adj}(\text{adj}(\text{adj}A))| = |A|^{2^3} = |A|^8 = 12^4$$

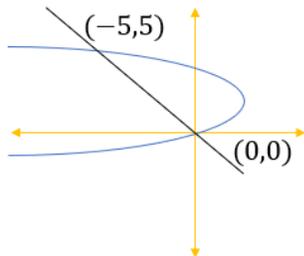
$$\therefore |A| = 2\sqrt{3}$$

$$|A^{-1}\text{adj}(A)| = |A^{-1}| \cdot |\text{adj}(A)| = \frac{1}{|A|} \cdot |A|^2 = |A| = 2\sqrt{3}$$

17. If area bound between $y^2 - 4y = -x$ and $x + y = 0$ is A then $6A$ equals.

Answer (125/6)

Sol.



Point of intersection of $y^2 - 4y = -x$ and $x + y = 0$ is

$$\begin{aligned} y^2 - 4y &= y \\ \Rightarrow y^2 - 5y &= 0 \\ \Rightarrow y &= 5 \text{ or } y = 0 \\ \therefore x &= -5 \text{ or } x = 0 \end{aligned}$$

$$\begin{aligned} \text{Required Area} &= \int_0^5 (4y - y^2) - (-y) dy \\ &= \int_0^5 (5y - y^2) dy \\ &= \left[\frac{5y^2}{2} - \frac{y^3}{3} \right]_0^5 \\ \Rightarrow \frac{125}{2} - \frac{125}{3} &= \frac{125}{6} \end{aligned}$$

18. Let a_1, a_2, \dots, a_6 be in A.P. such that $a_1 + a_3 = 10$ & mean of a_1, a_2, \dots, a_6 is $\frac{19}{2}$. Then $8\sigma^2$ is equal to _____.

Answer (210)

Sol.

$$\begin{aligned} a_1 + a_3 &= 10 \\ \Rightarrow 2a + 2d &= 10 \end{aligned}$$

$$\Rightarrow a + d = 5$$

$$\text{Also, } \frac{a+(a+d)+\dots+(a+5d)}{6} = \frac{19}{2}$$

$$\Rightarrow 2a + 5d = 19$$

$$\therefore a = 2 \text{ \& } d = 3$$

\therefore Given A.P. is 2, 5, 8, 11, 14, 17

$$\Rightarrow \sigma^2 = \frac{\left(2 - \frac{19}{2}\right)^2 + \left(5 - \frac{19}{2}\right)^2 + \dots + \left(17 - \frac{19}{2}\right)^2}{6} = 26.25$$

$$\therefore 8\sigma^2 = 8 \times 26.25 = 210$$