JEE Main 2023 (Memory based)

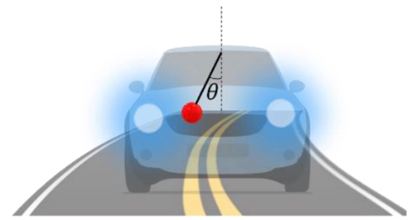
25 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. A car moving with constant speed of $2\ m/s$ in circle having radius R. A pendulum is suspended from the ceiling of car. Find the angle made by the pendulum with the vertical. Take $R = 8/15\ m$ and $g = 10\ m/s^2$.

A. $30°$
B. $53°$
C. $37°$
D. $60°$



Answer (C)

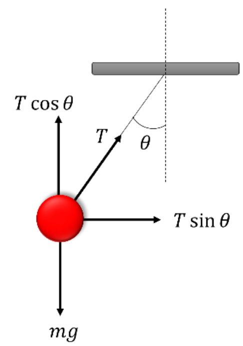
Solution:

$$T \sin\theta = \frac{mv^2}{R}$$

$$T \cos\theta = mg$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{4}{\frac{8}{15} \times 10} = \frac{3}{4}$$

$$\theta = 37°$$



2. A particle is dropped inside a tunnel of the earth about any diameter. Particle starts oscillating, with time period T. (R = Radius of earth, g = acceleration due to gravity on earth's surface). Then find T.

A. $T = 2\pi\sqrt{\frac{R}{g}}$

B. $T = \pi\sqrt{\frac{R}{g}}$

C. $T = 2\pi\sqrt{\frac{2R}{g}}$

D. $T = 2\pi\sqrt{\frac{3R}{g}}$

Answer (A)

Solution:

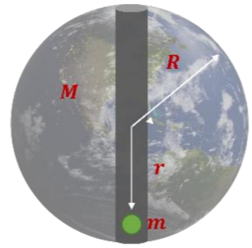
$$Restoring\ force, F = -\frac{GMmr}{R^3}$$

$$m\frac{dv}{dt} = -\left(\frac{GMm}{R^3}\right)r$$

$$\frac{dv}{dt} = -\left(\frac{GM}{R^3}\right)r = -\left(\frac{g}{R}\right)r$$

$$\omega = \sqrt{\frac{g}{R}}$$

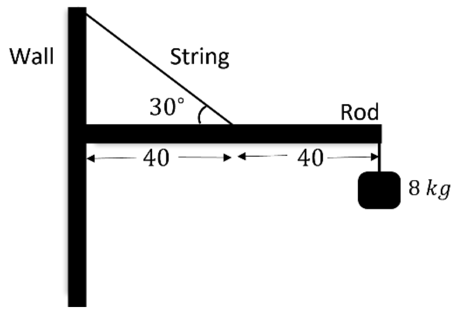
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$



3. A massless rod is arranged as shown:

Find the tension in the string. (Take $g = 10\ m/s^2$.)

A. $320\ N$
B. $640\ N$
C. $160\ N$
D. $480\ N$



Answer (A)

Solution:

Balancing the torque on the rod about the point of contact with the wall:

$(T\sin 30°) \times 40 = (mg) \times (40 + 40)$

$T = 320\ N$

4. A Carnot engine working between a source and a sink at $200\ K$ has efficiency of $50\ \%$. Another Carnot engine working between the same source and another sink with unknown temperature T has efficiency of $75\ \%$. The value of T is equal to

A. $400\ K$
B. $300\ K$
C. $200\ K$
D. $100\ K$

Answer (D)

Solution:

Let the source temperature of first engine is T.

$$\eta = 1 - \frac{200}{T} = \frac{50}{100}$$

$\Rightarrow T = 400\ K$

Let the source temperature of second engine is T.

$$\eta = 1 - \frac{T'}{400} = \frac{75}{100}$$

$\Rightarrow T' = 100\ K$

5. Mark the option correctly matching the following columns with appropriate dimensions.

Column–1	Column–2
A-Surface Tension	$P - [ML^{-1}T^{-2}]$
B-Pressure	$Q - [MT^{-2}]$
C-Viscosity	$R - [MLT^{-1}]$
D-Impulse	$S - [ML^{-1}T^{-1}]$

A. $A - Q, B - P, C - R, D - S$
B. $A - Q, B - P, C - S, D - R$
C. $A - S, B - Q, C - P, D - R$
D. $A - R, B - P, C - Q, D - S$

Answer (B)

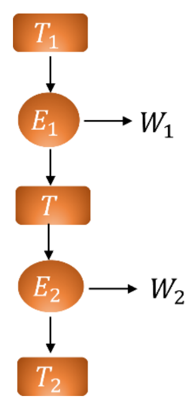
Solution:

$$[Surface\ tension] = \left[\frac{F}{L}\right] = [MT^{-2}]$$
$$[Pressure] = \left[\frac{F}{A}\right] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$
$$[Viscosity] = \left[\frac{F}{rv}\right] = \frac{[MLT^{-2}]}{[L.LT^{-1}]} = [ML^{-1}T^{-1}]$$
$$[Impulse] = [Ft] = [MLT^{-1}]$$

6. In the series sequence of two engines E_1 and E_2 as shown. $T_1 = 600K$ and $T_2 = 300K$. It is given that both the engines working on Carnot principle have same efficiency, then temperature T at which exhaust of E_1 is fed into E_2 is equal to $300\sqrt{n}\,K$. Value of n is equal to ________.

Answer (2.0)

Solution:



$$\eta_1 = 1 - \frac{T_1}{600}$$
$$\eta_2 = 1 - \frac{300}{T}$$
Given: $\eta_1 = \eta_2$
$$\Rightarrow \frac{T}{600} = \frac{300}{T}$$
$$\Rightarrow T = \sqrt{180000}\,K = 300\sqrt{2}\,K$$
$$\Rightarrow n = 2$$

7. A solenoid of length $2\ m$, has $1200\ turns$. The magnetic field inside the solenoid, when $2\ A$ current is passed through it is $N \times \pi \times 10^{-5}\ T$. Find the value of N. (Diameter of solenoid is $0.5\ m$)

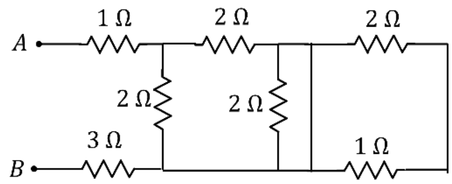
Answer (48.0)

Solution:

Magnetic field inside solenoid $= \mu_o ni$
where $n =$ Number of turns per unit length $= 1200/2 = 600\ turns/m$

$$B_{solenoid} = \mu_o ni = (4\pi \times 10^{-7} \times 600 \times 2)\ T$$
$$= 8\pi \times 10^{-7} \times 600\ T$$
$$= 48\pi \times 10^{-5}\ T$$

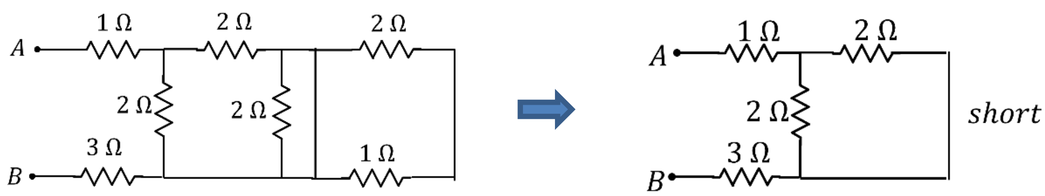
8. Consider a network of resistors as shown. Find the effective resistance $(in\ \Omega)$ across A and B.



Answer (5.0)

Solution:

Effectively, the network is



$$R_{AB} = 1\,\Omega + \frac{2 \times 2}{2+2}\,\Omega + 3\,\Omega$$
$$= 5\Omega$$

9. Find the ratio of density of $Oxygen(\mathrm{O}_8^{16})$ to the density of $Helium(He_2^4)$ at STP.

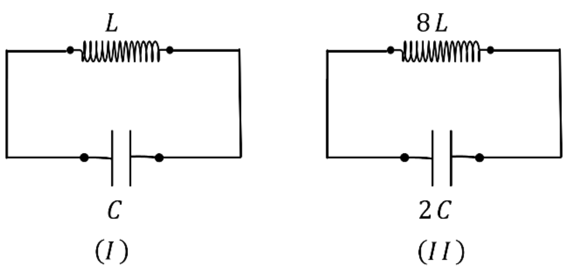
Answer (8.0)

Solution:

We know,

$$\frac{P}{\rho} = \frac{RT}{M_0}$$
$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{M_1}{M_2} = \frac{32}{4} = 8$$

10. Consider the following two LC circuit.



Then find ω_1/ω_2, where ω_1 and ω_2 are resonance frequencies of the two circuits.

Answer (4.0)

Solution:

$$\omega_1 = \frac{1}{\sqrt{LC}}$$

$$\omega_2 = \frac{1}{\sqrt{8L \times 2C}} = \frac{1}{4\sqrt{LC}}$$

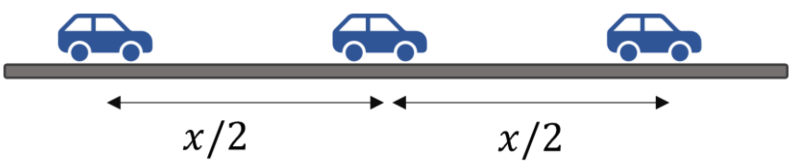
$$\frac{\omega_1}{\omega_2} = 4$$

11. A car moving on a straight-line travels in same direction half of the distance with uniform velocity v_1 and other half of the distance with uniform velocity v_2. Average velocity of the car is equal to

A. $2v_1v_2/(v_1 + v_2)$
B. $(v_1 + v_2)/2$
C. $v_1 + v_2$
D. $\sqrt{(v_1 + v_2)}$

Answer (A)

Solution:



Time to travel:

$$t_1 = \frac{x}{2v_1} \quad and \quad t_2 = \frac{x}{2v_2}$$

So,

$$v_{avg} = \frac{\text{Total distance}}{\text{Total Time}}$$

$$v_{avg} = \frac{x}{t_1 + t_2}$$

$$v_{avg} = \frac{x}{\frac{x}{2v_1} + \frac{x}{2v_2}}$$

$$v_{avg} = \frac{2v_1v_2}{v_1 + v_2}$$

12. If T is the temperature of a gas, then RMS velocity of the gas molecules is proportional to

A. $T^{1/2}$
B. $T^{-1/2}$
C. T
D. T^2

Answer (A)

Solution:

We know that:

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

So,

$$v_{rms} \propto \sqrt{T}$$

13. The period of a pendulum at earth's surface is T. Find the time period of the pendulum at distance (from centre) which is twice the radius of earth.

A. $T/4$
B. $4T$
C. $T/2$
D. $2T$

Answer (D)

Solution:

We know that :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Case 1:

$$T = 2\pi\sqrt{\frac{l}{GM/R^2}}$$

Case 2:

$$T' = 2\pi\sqrt{\frac{l}{GM/4R^2}}$$

So,

$$\frac{T'}{T} = \frac{2}{1} \Rightarrow T' = 2T$$

14. Let I_{cm} be the moment of Inertia of disc passing through center and perpendicular to its plane. I_{AB} be the moment of inertia about axis AB that is in the plane of disc and $\frac{2r}{3}$ distance from center. Find $\frac{I_{cm}}{I_{AB}}$?

A. $1/4$
B. $18/25$
C. $9/17$
D. $1/2$

Answer (B)

Solution:

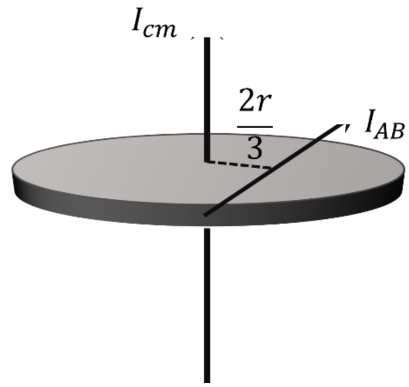
Moment of Inertia, $I_{cm} = \frac{Mr^2}{2}$ (Perpendicular to plane)

$I_{cm}(in\ plane) = \frac{Mr^2}{4}$

$I_{AB} = \frac{Mr^2}{4} + M\left(\frac{2}{3}r\right)^2$

$I_{AB} = \frac{(9+16)Mr^2}{36} = \frac{25}{36}Mr^2$

$\frac{I_{cm}(\text{Perpendicular})}{I_{AB}} = \frac{\frac{1}{2}Mr^2}{\frac{25}{36}Mr^2} = \frac{18}{25}$



15. Temperature of hot soup in a bowl goes 98^0C to 86^0C in $2\ min$. The temperature of surrounding is 22^0C. Find the time taken for the temperature of soup to go from 75^0C to 69^0C. (Assume Newton's law of cooling is valid)

A. $1\ min$
B. $1.4\ min$
C. $2\ min$
D. $3.2\ min$

Answer (B)

Solution:

We have,

$$\frac{\Delta\theta}{\Delta t} = -K\left(\frac{\theta_1+\theta_2}{2} - \theta_0\right)$$

Given, $\theta_0 = 22°C$

$$\frac{98-86}{2} = -K\left(\frac{98+86}{2} - 22\right) \dots .(1)$$

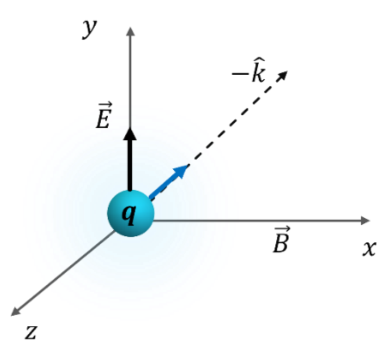
$$\frac{75-69}{t_2} = -K\left(\frac{75+69}{2} - 22\right) \dots (2)$$

From (1) and (2)

$$t_2 = \frac{70}{50} = 1.4\ min$$

16. Electric field is applied along $+y$ direction. A charged particle is travelling along $-\hat{k}$, undeflected. Then magnetic field in the region will be along?

A. $\hat{i}$
B. $-\hat{i}$
C. $\hat{j}$
D. $-\hat{k}$



Answer (A)

Solution:

If the charged particle is moving in both uniform electric and magnetic field with no deflection than force will be zero on charged particle.

$$q(\vec{E} + \vec{v} \times \vec{B}) = 0$$

$$(\vec{v} \times \vec{B}) = -\vec{E}$$

$$(v_0(-\hat{k}) \times \vec{B}) = -E_0\hat{j}$$

$\vec{B}$ should be in $\hat{\imath}$ direction to balance the electrostatic force on the charge particle. (Assuming the given charge to be positive.)

17. When an electron is accelerated by $20\ kV$, its de-broglie wavelength is λ_0. If the electron is accelerated by $40\ kV$, find its de-Broglie wavelength.

A. $2\lambda_0$
B. $\frac{\lambda_0}{2}$
C. $\sqrt{2}\lambda_0$
D. $\frac{\lambda_0}{\sqrt{2}}$

Answer (D)

Solution:

We know,

$$\lambda_0 = \frac{h}{p}$$

$$\lambda_0 = \frac{h}{\sqrt{2mK}}$$

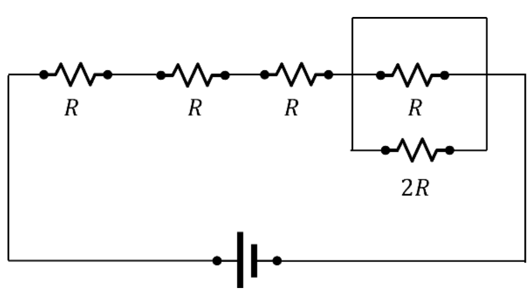
$$\lambda_0 = \frac{h}{\sqrt{2meV}}$$

Since V doubles.

$$\frac{\lambda'}{\lambda_0} = \sqrt{\frac{V}{2V}} = \frac{1}{\sqrt{2}}$$

$$\lambda' = \frac{\lambda_0}{\sqrt{2}}$$

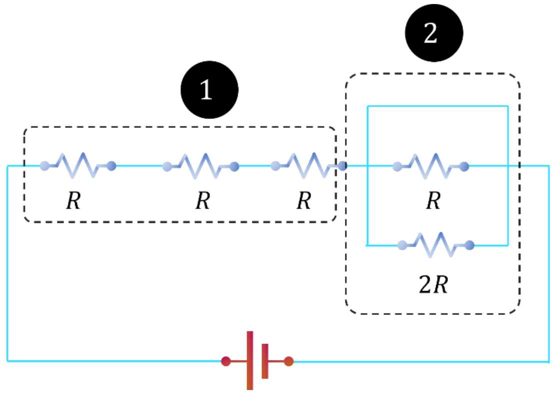
18. Find the equivalent resistance of the given circuit across the terminals of ideal battery.



A. $2R$
B. $3R$
C. $4R$
D. $5R$

Answer (B)

Solution:



In 2^{nd} part of diagram a connecting wire is nullifying the resistance of parallel resistance thus their new resistance is zero. So, net resistance of circuit is $3R$

19. For an AM signal, it is given that $f_{carrier} = 10\ MHz$ & $f_{signal} = 5\ kHz$. Find the bandwidth of the transmitted signal.

A. $5\ kHz$
B. $10\ kHz$
C. $2.5\ kHz$
D. $20\ MHz$

Answer (B)

Solution:

Bandwidth of amplitude modulated wave is:

$\Delta f = 2f_m = 10\ kHz$

20. Let nuclear densities of $^{4}_{2}He$ and $^{40}_{20}Ca$ be ρ_1 and ρ_2 respectively. Find the ratio $\frac{\rho_1}{\rho_2}$.

A. $1:10$
B. $10:1$
C. $1:1$
D. $1:2$

Answer (C)

Solution:

We know radius,

$R = R_o A^{\frac{1}{3}}$

Density = $\frac{\text{Mass}}{\text{Volume}}$

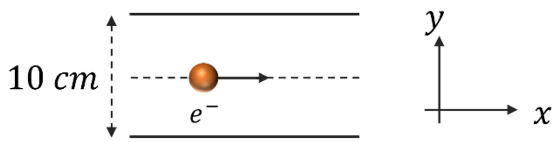
$$Density = \frac{A}{\frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3} = \frac{1}{\frac{4}{3}\pi R_o^3}$$

Density is independent of A

$\frac{\rho_1}{\rho_2} = 1 \Rightarrow \rho_1 : \rho_2 = 1:1$

21. A particle is projected with $0.5\ eV$ kinetic energy in a uniform electric field $\vec{E} = -10\frac{N}{C}\hat{j}$ as shown in the figure. Find the angle particle made from the $x-$axis when it leaves $\vec{E}$.

A. $\theta = 45°$
B. $\theta = 60°$
C. $\theta = 30°$
D. $\theta = 37°$



Answer (A)

Solution:

In x –direction:

$v_x = v_0$

In y –direction:

$a_y = \left(\frac{eE}{m_e}\right)$

$s_y = 5 \times 10^{-2}\ m$

$v_y^2 = 2a_y s_y$

$v_y = \sqrt{\frac{2eE}{m_e}s_y}$

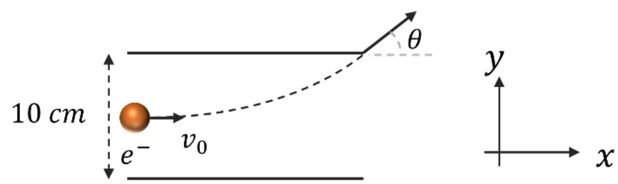
$\tan\theta = \left(\frac{v_y}{v_x}\right)$

$K_i = 0.5\ eV = \frac{1}{2}\frac{m_e v_x^2}{e}$

$v_x = \sqrt{\frac{0.5 \times 2e}{m_e}} = \sqrt{\frac{e}{m_e}}$

$\tan\theta = \frac{\sqrt{\frac{2eE}{m_e}s_y}}{\sqrt{\frac{e}{m_e}}} = \sqrt{2Es_y} = \sqrt{2 \times 10 \times 5 \times 10^{-2}} = 1$

$\theta = \tan^{-1} 1 = 45°$



22. Find the ratio of acceleration due to gravity at an altitude $h = R$ to the value at the surface of earth (where R=radius of earth)

A. 1/2
B. 1/4
C. 1/8
D. 1/6

Answer (B)

Solution:

We have,

$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2$

$\frac{g_h}{g} = \left(\frac{R}{R+R}\right)^2 = \frac{1}{4}$

23. Statement 1: Photodiodes are operated in reverse biased.

Statement 2 : Current in forward biased is more than current in reverse bias in $p-n$ diode.

A. Both the statements are true and 2 is the correct explanation of 1.

B. Both the statements are true and 2 is not the correct explanation of 1.

C. Statement 1 is true and statement 2 is false.

D. Statement 2 is true and statement 1 is false.

Answer (B)

Sol. Statement 1 is true as photodiode is used in reverse bias to increase the sensitivity of diode current.

Statement 2 is true as diode provides greater resistance in reverse bias.

1. Radius of 2nd orbit of Li^{2+} ion is x, radius of 3rd orbit of Be^{3+} will be

A. $\frac{27x}{16}$
B. $\frac{16x}{27}$
C. $\frac{4x}{3}$
D. $\frac{3x}{4}$

Answer (A)

Solution:

$$r_{Li^{2+}} = r_o \times \frac{2^2}{3} = \frac{4r_o}{3} = x \Rightarrow r_o = \frac{3x}{4}$$

$$r_{Be^{3+}} = r_o \times \frac{3^2}{4} = \frac{9r_o}{4} = \frac{9\times3\times x}{4\times4} = \frac{27x}{16}$$

2. If X-atoms are present at alternate corners and at body centre of a cube and Y-atoms are present at 1/3rd of face centers then what will be the empirical formula?

A. $X_{2.5}Y$
B. X_5Y_2
C. $X_{1.5}Y$
D. X_3Y_2

Answer (D)

Solution:

No. of X – atoms per unit cell = $1 + 4 \times \frac{1}{8} = \frac{3}{2}$

No. of Y – atoms per unit cell = $2 \times \frac{1}{2} = 1$

Therefore, the empirical formula of the solid is X_3Y_2.

3. Which of the following option contains the correct match

Table – I (Elements)	Table – II (Flame colour)
A. K	P. Violet
B. Ca	Q. Brick Red
C. Sr	R. Apple Green
D. Ba	S. Crimson Red

A. A – P, B – Q, C – S, D – R
B. A – Q, B – P, C – S, D – R
C. A – R, B – S, C – P, D – Q
D. A – S, B – R, C – Q, D – P

Answer (A)

Solution:

K – Violet

Ca – Brick Red

Sr – Crimson Red

Ba – Apple Green

4. Match the following

List - I	List - II
A. Pb^{2+}, Cu^{2+}	1. $H_2S\ in\ dil\ HCl$
B. Fe^{3+}, Al^{3+}	2. $NH_4Cl\ with\ (NH_4)_2CO_3$
C. Ni^{2+}, Co^{2+}	3. $H_2S\ in\ dil\ NH_4OH$
D. Ca^{2+}, Ba^{2+}	4. $NH_4Cl\ with\ NH_4OH$

A. A – 1, B – 2, C – 3, D – 4
B. A – 1, B – 4, C – 3, D - 2
C. A – 4, B – 3, C – 2, D – 1
D. A – 2, B – 1, C – 4, D – 3

Answer (B)

Solution:

Pb^{2+} and Cu^{2+} will precipitate as *PbS* and *CuS* respectively by passing H_2S gas in presence of $dil. HCl$.

$Fe^{3+}\ and\ Al^{3+}$ will precipitate as $Fe(OH)_3$ and $Al(OH)_3$ respectively by adding NH_4Cl and NH_4OH

Ni^{2+} and Co^{2+} will precipitate as *NiS* and *CoS* respectively by passing H_2S in presence of $dil\ NH_4OH$.

$Ca^{2+}\ and\ Ba^{2+}$ will precipitate as $CaCO_3$ and $BaCO_3$ respectively by adding $NH_4Cl\ and\ (NH_4)_2CO_3$.

5. Which of the following is correct about antibiotics

A. Antibiotics are the substances that promote the growth of micro-organisms
B. Penicillin has bacteriostatic effect
C. Erythromycin has bactericidal effect
D. They are synthesised artificially

Answer (D)

Solution: Antibiotics are synthesised artificially.

6. Consider the following sequences of the reactions

$NO_2 \xrightarrow{hv} A + B$
$B + O_2 \rightarrow O_3(g)$
A can be?

A. N_2O
B. NO
C. N_2O_3
D. N_2

Answer (B)

Solution:

$$NO_2 \xrightarrow{hv} NO(g) + O(g)$$

(A) (B)

$$O(g) + O_2(g) \rightarrow O_3(g)$$

(B)

7. Correct order of basic strength in aqueous solution for

1. $CH_3 - NH_2$
2. $CH_3 - NH - CH_3$
3. $CH_3 - N(CH_3) - CH_3$
4. NH_3

A. 2 > 1 > 3 > 4

B. 3 > 2 > 1> 4

C. 4 > 2 > 1> 3

D. 2 > 4 > 3 > 1

Answer (A)

Solution:

Basic strength $\propto$ Availability of lone pairs on Nitrogen atom

The correct order of basic strength in aqueous medium is

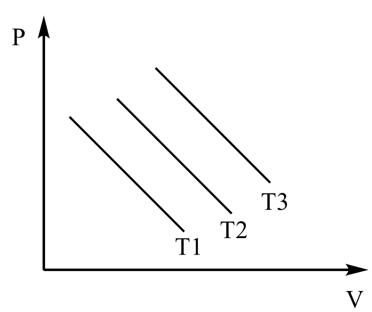
$$CH_3 - NH - CH_3 > CH_3 - NH_2 > CH_3 - N(CH_3) - CH_3 > NH_3$$

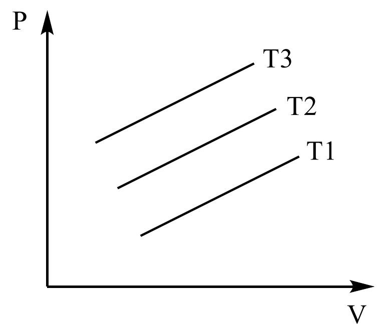
(2) (1) (3) (4)

The availability of lone pair on N-atom in case of ammonia and alkyl amines in aqueous medium depend on three factors

1) Electron donating effects: + I effect is present in case of alkyl amines but not in case of ammonia and availability of electrons on N – atom $\propto$ +I effect
2) Solvation: More is the solvation less will be the availability of electrons on N-atom. Extent of solvation $\propto$ no. of H-atoms directly attach to N-atom
3) Steric Crowding: More is no. of alkyl groups more is the steric crowding and less will be the availability of electrons on N-atom

8. Which Graph graph is correct for Isothermal process at T_1, T_2 & T_3 if ($T_3 > T_2 > T_1$)

A. 



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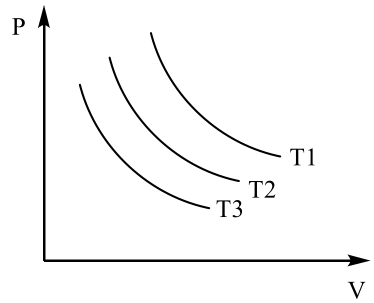
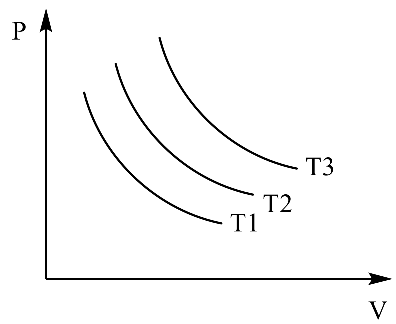


C.



D.

Answer (D)

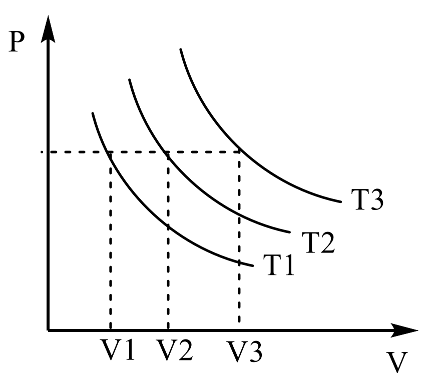
Solution:

According to Boyle Law $P \propto \frac{1}{V}$

The graph must be hyperbola.

As we know, $PV = nRT$

So as increase the Temperature the PV graph area increases



As $(V_3 > V_2 > V_1)$ for fixed P

$= (T_3 > T_2 > T_1)$

And the correct option is (D)

9. An athlete is given 100g of glucose energy equivalent to 1560KJ to utilise 50% of this gained energy in an event. Enthalpy of evaporation of H_2O is 44KJ/mol. In order to avoid storage of energy in the body the mass of water (in g) he would perspire is: (Round off the nearest Integer)

Answer (319)

Solution:

Given 100 g of glucose yields 1560 KJ of energy.

50% of 1560 KJ that is 780 KJ is used to perspire water

To perspire 1 mol of water that is 18 g of water 44 KJ energy is required

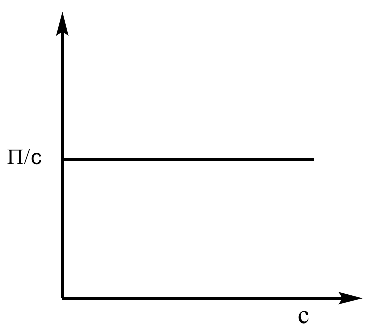
Therefore,

Moles of water evaporated = $\frac{780}{44}\ mol$

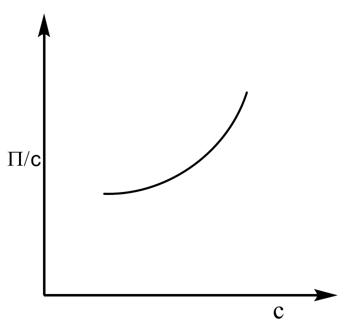
Weight of water evaporated = $\frac{780}{44} \times 18 = 319\ g$

(Assuming water is contained in the body)

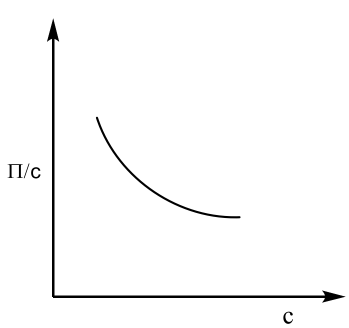
10. Which of the following option contains the correct graph between π/c and c at constant temperature (Where π is osmotic pressure and c is concentration of the solute)



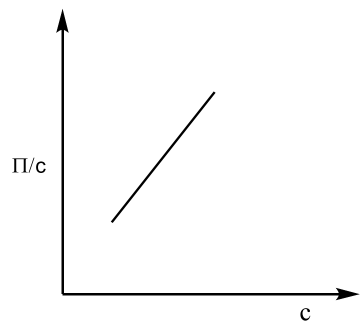
A.



B.



C.



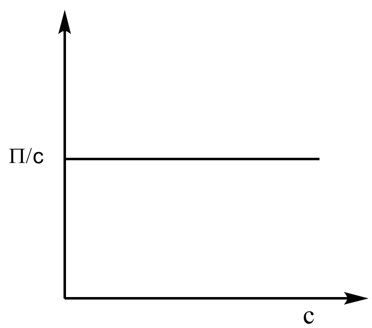
D.

Answer (A)

Solution:

$\pi = cRT$

$\frac{\pi}{c} = RT$



The value of $\frac{\pi}{c}$ is constant at constant temperature

11. How many of the following ions/elements has the same value of spin magnetic moment?

V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{2+}

Answer (2)

Solution:

V^{3+} - d^2 – 2 unpaired electrons

Cr^{3+} - d^3 – 3 unpaired electrons

Fe^{2+} - d^6 – 4 unpaired electrons

Ni^{2+} - d^8 – 2 unpaired electrons

V^{3+} and Ni^{2+} has the same number of unpaired electrons and hence has the same value of spin magnetic Moment.

12. How many of the following complexes is (are) paramagnetic?

$[Fe(CN)_6]^{3-}$, $[Fe(CN)_6]^{4-}$, $[NiCl_4]^{2-}$, $[Ni(CN)_4]^{2-}$, $[CuCl_4]^{2-}$, $[Cu(CN)_4]^{3-}$, $[Cu(H_2O)_4]^{2+}$

Answer (4)

Solution:

$[Fe(CN)_6]^{3-}$ - d^5 - paramagnetic

$[Fe(CN)_6]^{4-}$ - d^6 - diamagnetic

$[NiCl_4]^{2-}$ - d^8 - paramagnetic

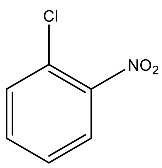
$[Ni(CN)_4]^{2-}$ - d^8 - diamagnetic

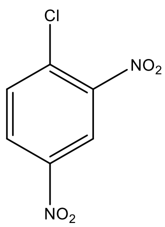
$[CuCl_4]^{2-}$ - d^9 - paramagnetic

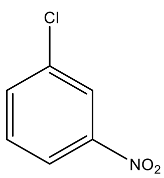
$[Cu(CN)_4]^{3-}$ - d^{10} - diamagnetic

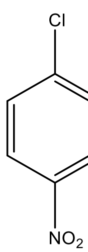
$[Cu(H_2O)_4]^{2+}$ - d^9 – paramagnetic

13. Which of the following shows least reactivity towards nucleophilic substitution reaction?

A. 

B. 

C. 

D. 

Answer (C)

Solution:

Aryl halides containing EWG at ortho or para position are more reactive towards nucleophilic substitution. reaction than meta isomer.

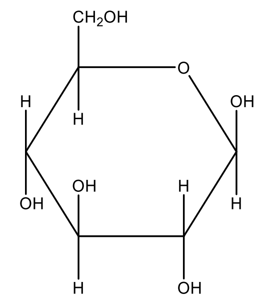
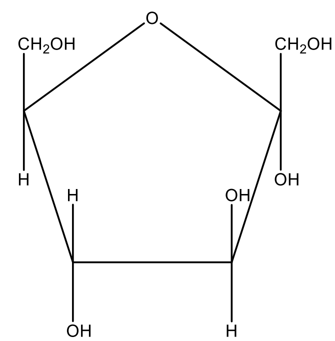
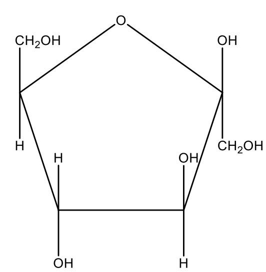
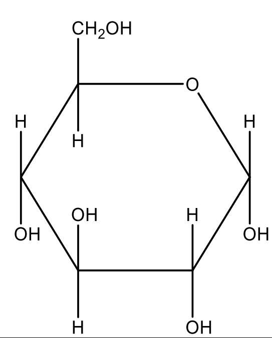
14. For a first order reaction, A → B; $t_{1/2}$ is 30 minutes. Then find the time in minutes required for 75% completion of reaction?

Answer (60 minutes)

Solution:

$t_{75\%} = t_{1/4} = 2 \times t_{1/2} = 2 \times 30\ minutes = 60\ minutes$

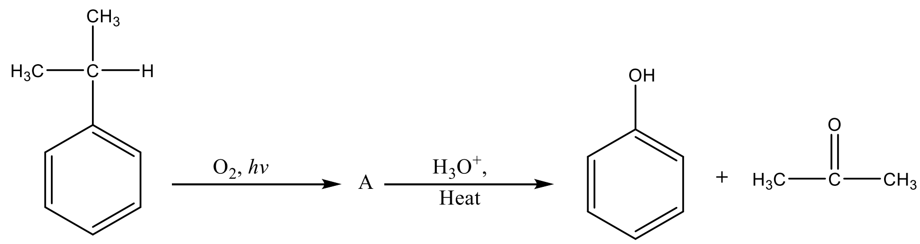
15. Match List – I with List – II.

List - I	List - II
$A. \alpha - D - Glucopyranose$	1. 
$B.\ \beta - D - Glucopyranose$	2. 
$C.\ \alpha - D - Fructofuranose$	3. 
$D.\ \beta - D - Fructofuranose$	4. 

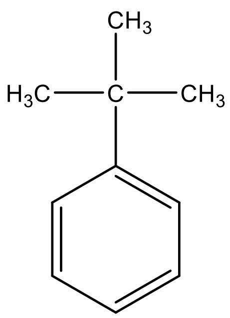
A. A – 4 ; B – 1 ; C – 2 ; D – 3
B. A – 1; B – 4 ; C – 3 ; D – 2
C. A – 2 ; B – 3 ; C – 4 ; D – 1
D. A – 1 ; B – 3 ; C – 2 ; D – 4

Answer (A)

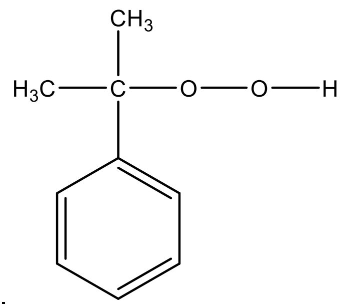
16. Consider the following conversion.



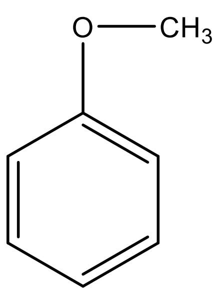
Which of the following option contains the correct structure of 'A'.



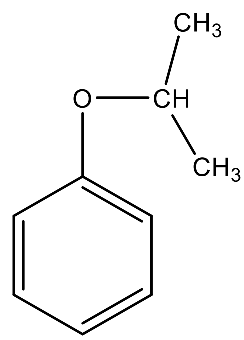
A.



B.



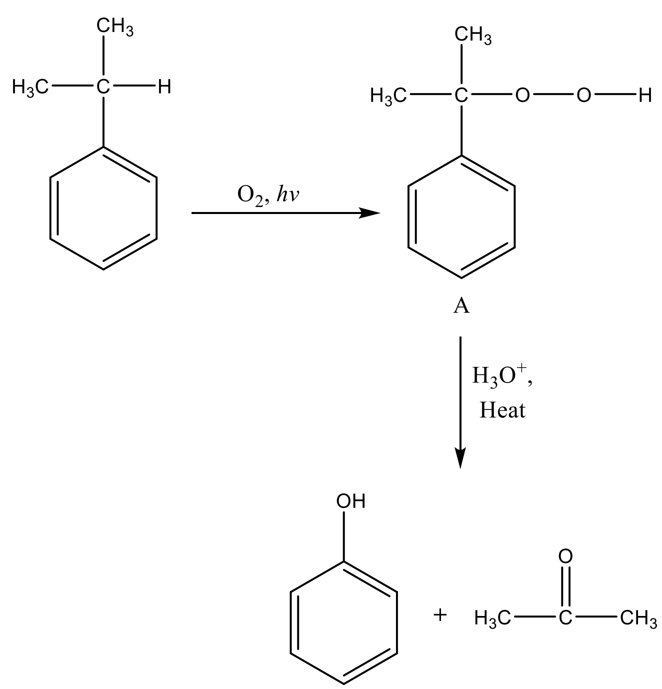
C.



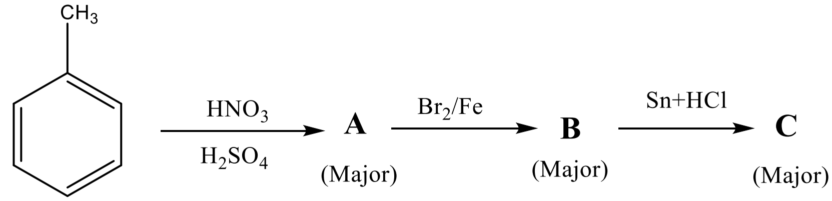
D.

Answer (B)

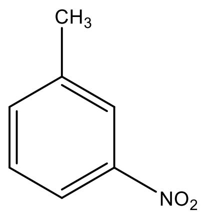
Solution:

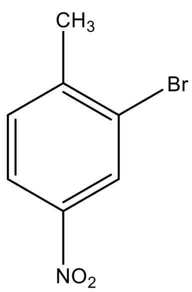


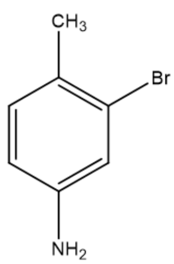
17. Consider the following sequence of reaction.

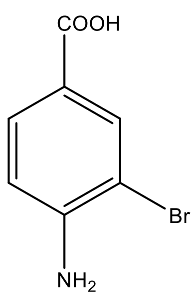


Which of the following option contains the correct structure?

A. A is 

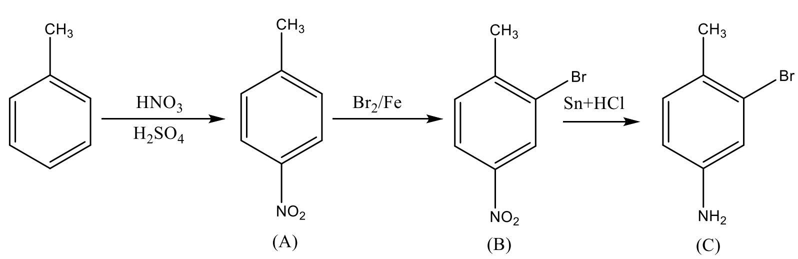
B. B is 

C. C is 

D. D is 

Answer (C)

Solution:



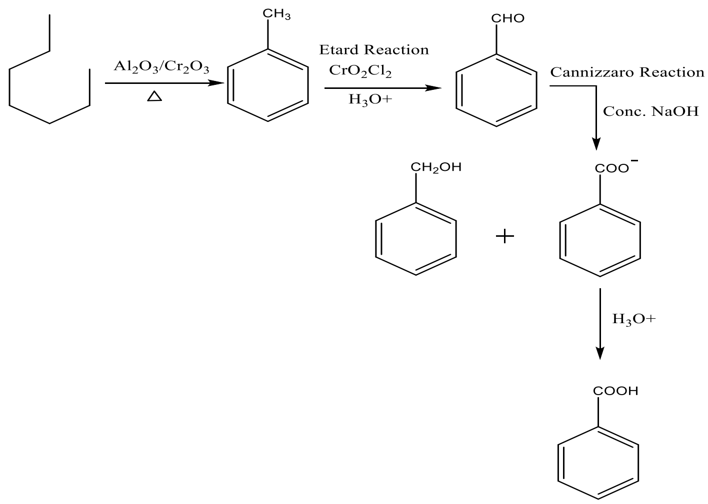
18. Identify the correct sequence of reactants for the following conversion.

n-Heptane $\longrightarrow \longrightarrow \longrightarrow \longrightarrow$ PhCOOH + $PhCH_2OH$

A. Al_2O_3/Cr_2O_3, CrO_2Cl_2/H_3O^+, Conc. NaOH, H_3O^+
B. Al_2O_3/Cr_2O_3, CrO_2Cl_2/H_3O^+, H_3O^+ ,Conc. NaOH
C. CrO_2Cl_2, Al_2O_3 ,Conc. NaOH, H_3O^+
D. Sn/HCl, Conc. NaOH, CrO_2Cl_2, HNO_3

Answer (A)

Solution:



19. Thionyl chloride on reaction with white phosphorous gives compound A. A on hydrolysis gives compound B which is dibasic. Identify A and B.

A. $A - PCl_5, B = H_3PO_2$
B. $A - P_4\,O_6,\;\; B = H_3PO_4$
C. $A - POCl_3, B = H_3PO_4$
D. $A - PCl_3, B = H_3PO_3$

Answer (D)

Solution:

$$P_4 + 8SOCl_2 \rightarrow \underset{(A)}{4PCl_3} + 4SO_2 + 2S_2Cl_2$$

$$PCl_3 + H_2\,O \rightarrow \underset{(B)}{H_3\,PO_3}$$

20. The correct decreasing order of positive electron gain enthalpy for the following inert gases.
He, Ne, Kr, Xe

A. $He > Ne > Kr > Xe$
B. $He > Ne > Xe > Kr$
C. $He > Xe > Ne > Kr$
D. $Ne > Kr > Xe > He$

Answer (D)

Solution: The correct order is, $Ne > Kr > Xe > He$

21. Consider the following cell represent:

$Pt/H_2/H^+ // \ Fe^{+3}/Fe^{+2}$
(1 atm) (1 M)

Then Find the ratio of concentration of concentration of Fe^{+2} to Fe^{+3}? [Given E_{cell} = 0.712, E^0_{cell} = 0.771]

Answer (10)

Solution:

$$E_{Cell} = E^0_{cell} - \frac{0.059}{n}\log\left[\frac{[Fe^{2+}][H^+]}{[Fe^{3+}]}\right]^2$$

$$\Rightarrow 0.712 = 0.771 - \frac{0.059}{2} \times 2\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\Rightarrow -0.059 = -0.059\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\Rightarrow \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

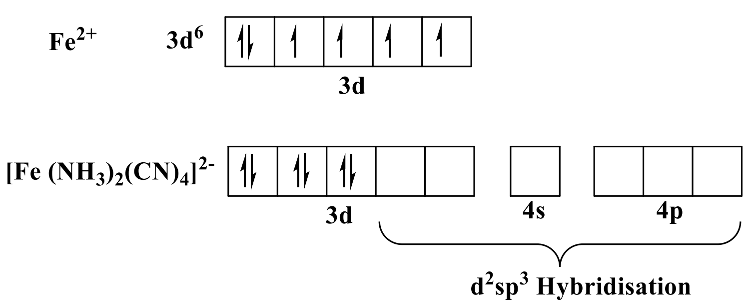
22. Which of the following complexes is paramagnetic in nature?

A. $[Fe(NH_3)_2(CN)_4]^{2-}$
B. $[Ni(CN)_4]^{2-}$
C. $[Ni(H_2O)_6]^{2+}$
D. $[Co(NH_3)_4Cl_2]^+$

Answer (C)

Solution:

1. $[Fe(NH_3)_2(CN)_4]^{2-}$



Complex is diamagnetic.

2. $[Ni(CN)_4]^{2-}$ dsp^2 hybridisation, so it is diamagnetic
3. $[Ni(H_2O)_6]^{2+}$ sp^3d^2 hybridisation, so it is paramagnetic
4. $[Co(NH_3)_4Cl_2]^+$ d^2sp^3 hybridisation, so it is diamagnetic

So correct answer is option (C)

1. $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$, $x \in [-1,\ 1]$ sum of all solutions is $\alpha - \frac{4}{\sqrt{3}}$, then α is:

A. 1
B. 2
C. -2
D. $\sqrt{3}$

Answer (B)

Solution:

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$

for $-1 < x < 0$, $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$ and $\cot^{-1}\left(\frac{1-x^2}{2x}\right) = \pi + 2\tan^{-1}x$

$2\tan^{-1}x + \pi + 2\tan^{-1}x = \frac{\pi}{3}$

$4\tan^{-1}x = -\frac{2\pi}{3}$

$x = -\frac{1}{\sqrt{3}}$

for $0 < x < 1$, $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$ and $\cot^{-1}\left(\frac{1-x^2}{2x}\right) = 2\tan^{-1}x$

$4\tan^{-1}x = \frac{\pi}{3}$

$x = \tan\frac{\pi}{12} = 2 - \sqrt{3}$

sum $= 2 - \sqrt{3} - \frac{1}{\sqrt{3}} = 2 - \frac{4}{\sqrt{3}}$

$\therefore \alpha = 2$

2. Mean of a data set is 10 and variance is 4. If one entry of data set changes from 8 to 12, then new mean becomes 10.2. Then now variance is:

A. 3.92
B. 3.96
C. 4.04
D. 4.08

Answer (B)

Solution:

Let number of observations be n

$10n - 8 + 12 = (10.2)n$

$10n + 4 = (10.2)n$

$\Rightarrow n = 20$

For earlier set of observations

$\frac{\sum x_i^2}{20} - (10)^2 = 4$

$\Rightarrow \sum x_i^2 = (104)(20) = 2080$

After change

$(\sum x_i^2)_{\text{new}} = 2080 - 8^2 + 12^2$

$= 2160$

New variance $= \frac{2160}{20} - (10.2)^2$

$= 108 - (10.2)^2$

$= 3.96$

3. If $y = (1+x)(x^2+1)(x^4+1)(x^8+1)(x^{16}+1)$, then find the value of $y'' - y'$ at $x = -1$:

A. 496
B. 946
C. -496
D. -946

Answer (C)

Solution:

$y = (1+x)(x^2+1)(x^4+1)(x^8+1)(x^{16}+1)$
Multiply and divide by $(x-1)$ we get,
$y = \frac{(1+x)(x^2+1)(x^4+1)(x^8+1)(x^{16}+1)(x-1)}{(x-1)}$
$\Rightarrow y = \frac{(x^2-1)(x^2+1)(x^4+1)(x^8+1)(x^{16}+1)}{(x-1)}$
$\Rightarrow y = \frac{(x^4-1)(x^4+1)(x^8+1)(x^{16}+1)}{(x-1)}$
$\Rightarrow y = \frac{(x^8-1)(x^8+1)(x^{16}+1)}{(x-1)}$
$\Rightarrow y = \frac{(x^{16}-1)(x^{16}+1)}{(x-1)}$
$\Rightarrow y = \frac{(x^{32}-1)}{(x-1)}$
At $x = -1$ we get $y = 0$
$y(x-1) = x^{32} - 1$
Differentiate on both sides,
$y'(x-1) + y = 32x^{31} \quad \cdots (1)$
At $x = -1$
$y'(-1) = \frac{-32}{-2} = 16$
Differentiate equation (1) on both sides we get,
$y''(x-1) + y' + y' = 32 \times 31x^{30}$
At $x = -1$
$y''(-1) = \frac{32\times31-16-16}{-2} = -480$
$\therefore y''(-1) - y'(-1) = -480 - 16 = -49$

4. The logical statement $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$ is a:

A. Tautology
B. Fallacy
C. Equivalent to $p \vee \sim q$
D. Equivalent to $p \wedge \sim q$

Answer (A)

Solution:

$(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$
$= (p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$
$= \sim(p \wedge \sim q) \vee (\sim p \vee \sim q)$
$= (\sim p \vee q) \vee (\sim p \vee \sim q)$
$= \sim p \wedge T = T$ (Tautology)

5. If a_r is the coefficient of x^{10-r} in expansion of $(1+x)^{10}$ then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is:

A. 390
B. 1210
C. 485
D. 220

Answer (B)

Solution:

Coefficient of x^{10-r} in $(1+x)^{10}$ is ${}^{10}C_{10-r}$

$\therefore a_r = {}^{10}C_{10-r}$

$\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = \sum_{r=1}^{10} r^3 \cdot \left(\frac{10!}{r!(10-r)!} \cdot \frac{(11-r)!(r-1)!}{10!}\right)^2$

$= \sum_{r=1}^{10} r^3 \cdot \left(\frac{11-r}{r}\right)^2 = \sum_{r=1}^{10} r(11-r)^2$

$\sum_{r=1}^{10} r(11-r)^2 = 1 \times 10^2 + 2 \times 9^2 + \cdots + 9 \times 2^2 + 10 \times 1^2$

Which is same as $\sum_{r=1}^{10} r^2(11-r)$

$\sum_{r=1}^{10} r^2(11-r) = 1^2 \times 10 + 2^2 \times 9 + \cdots + 9^2 \times 2 + 10^2 \times 1$

$\Rightarrow \sum_{r=1}^{10} r(11-r)^2 = \sum_{r=1}^{10} r^2(11-r)$

$\Rightarrow \sum_{r=1}^{10} r^2(11-r) = 11\sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} r^3$

$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = 11\left(\frac{10\times11\times21}{6}\right) - \left(\frac{10\times11}{2}\right)^2$

$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = 11^2 \times 35 - 11^2 \times 25$

$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = 11^2 \times 10 = 1210$

6. $\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots(3n-2)+(3n-1)-3n}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$

A. $\frac{3}{2}(\sqrt{2}+1)$
B. $\frac{2}{3}(\sqrt{2}+1)$
C. $\frac{2}{3\sqrt{2}}$
D. $2\sqrt{2}$

Answer (A)

Solution:

$\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots(3n-2)+(3n-1)-3n}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$

$= \lim_{n\to\infty} \frac{\sum_{r=1}^{n}\left((3r-2)+(3r-1)-3r\right)}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$

$= \lim_{n\to\infty} \frac{\sum_{r=1}^{n} 3(r-1)}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$

$= \lim_{n\to\infty} \frac{3\frac{n(n-1)}{2}}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$

$= \lim_{n\to\infty} \frac{3\frac{n(n-1)}{2}}{n^2\left(\sqrt{2+\frac{3}{n^3}+\frac{1}{n^4}}-\sqrt{1+\frac{1}{n^3}+\frac{3}{n^4}}\right)}$

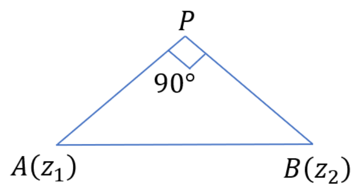
$= \frac{3}{2}\left(\frac{1}{\sqrt{2}-1}\right) = \frac{3}{2}(\sqrt{2}+1)$

7. If $|z-z_1|^2 + |z-z_2|^2 = |z_1-z_2|^2$ when $z_1 = 2+3i$ and $z_2 = 3+4i$, then locus of z is:

A. Straight line with slope $-\frac{1}{2}$
B. Circle with radius $\frac{1}{\sqrt{2}}$
C. Hyperbola with eccentricity $\sqrt{2}$
D. Hyperbola with eccentricity $\frac{5}{2}$

Answer (B)

Solution:



So, locus of P is circle whose diameter is AB

$AB = \sqrt{2}$

$\therefore$ radius of circle $= \frac{1}{\sqrt{2}}$

8. $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)} dx$ if $f(3) = \frac{1}{2}[\ln 5 - \ln 6]$, then $f(4)$ is:

A. $\frac{1}{2}[\ln 17 - \ln 19]$
B. $\frac{1}{2}[\ln 19 - \ln 17]$
C. $\ln 19 - \ln 17$
D. $\ln 17 - \ln 19$

Answer (A)

Solution:

$f(x) = \int \frac{2x}{(x^2+1)(x^2+3)} dx$

Let $x^2 = t$

$2xdx = dt$

$\Rightarrow \int \frac{dt}{(t+1)(t+3)}$

$= \frac{1}{2} \int \frac{(t+3)-(t+1)}{(t+1)(t+3)} dt$

$= \frac{1}{2}[\ln|t+1| - \ln|t+3|] + \frac{c}{2}$

$= \frac{1}{2}[\ln|x^2+1| - \ln|x^2+3|] + \frac{c}{2}$

Now $f(3) = \frac{1}{2}[\ln 5 - \ln 6]$

$\Rightarrow \frac{1}{2}[\ln 5 - \ln 6] = \frac{1}{2}[\ln 10 - \ln 12] + \frac{c}{2}$

$\Rightarrow c = 0$

$\therefore f(x) = \frac{1}{2}[\ln|x^2+1| - \ln|x^2+3|]$

$\therefore f(4) = \frac{1}{2}[\ln 17 - \ln 19]$

9. If $f(x) = \int_0^2 e^{|x-t|} dt$, then the minimum value of $f(x)$ is equal to:

A. $2(e-1)$
B. $2(e+1)$
C. $2e-1$
D. $2e+1$

Answer (A)

Solution:

For $x > 2$

$f(x) = \int_0^2 e^{x-t} dt \Rightarrow e^x(-e^{-t})|_0^2 \Rightarrow e^x(1-e^{-2})$

For $x < 0$

$f(x) = \int_0^2 e^{t-x} dt \Rightarrow e^{-x}e^t|_0^2 \Rightarrow e^{-x}(e^2-1)$

For $0 \le x \le 2$

$f(x) = \int_0^x e^{x-t} dt + \int_x^2 e^{t-x} dt$

$= -e^x e^{-t}|_0^x + e^{-x}e^t|_x^2$
$= -e^x(e^{-x} - 1) + e^{-x}(e^2 - e^x)$
$= -1 + e^x + e^{2-x} - 1$
$= e^{2-x} + e^x - 2$

$$f(x) = \begin{cases} e^x(1-e^{-2})\ x > 2 \\ e^{2-x} + e^x - 2\ 0 \le x \le 2 \\ e^{-x}(e^x - 1)\ x < 0 \end{cases}$$

For $x > 2$
$f(x)_{\min} = e^2 - 1$
For $0 \le x \le 2$
$f'(x) = -e^{2-x} + e^x = 0$
$\Rightarrow e^x = e^{2-x}$
$\Rightarrow e^{2x} = e^2$
$\Rightarrow x = 1$
$f(x)_{min} = 2e - 2 = 2(e-1)$

10. If $f(x) = x^b + 3,\ g(x) = ax + c$. If $\left(g\left(f(x)\right)\right)^{-1} = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$, then $fog(ac) + gof(b)$ is:

A. 189
B. 195
C. 194
D. 89

Answer (A)

Solution:

$g\left(f(x)\right) = a(x^b + 3) + c$
$\left(g\left(f(x)\right)\right)^{-1} = \left(\frac{x-3a-c}{a}\right)^{\frac{1}{b}} = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$
$\Rightarrow a = 2$
$\Rightarrow b = 3$
$\Rightarrow c = 1$
$g(x) = 2x + 1$
$f(x) = x^3 + 3$
Now $fog(2) + gof(3) = 128 + 61 = 189$

11. The term independent of x in the expansion of $\left(2x + \frac{1}{x^7} - 7x^2\right)^5$ is :

A. 1372
B. 2744
C. -13720
D. 13720

Answer (C)

Solution:

Using multinomial theorem,
$\left(2x + \frac{1}{x^7} - 7x^2\right)^5$
$= \frac{5!}{\alpha!\beta!\gamma!}(2x)^\alpha \left(\frac{1}{x^7}\right)^\beta (-7x^2)^\gamma$, where $\alpha + \beta + \gamma = 5 \cdots (i)$
$= \frac{5!}{\alpha!\beta!\gamma!} 2^\alpha . (-7)^\gamma x^{\alpha - 7\beta + 2\gamma}$
For independent term,
$\alpha - 7\beta + 2\gamma = 0 \cdots (ii)$
From (i) and (ii), $\beta = \frac{\gamma+5}{8}$
Since α, β, γ are integers from $[1,5]$
$\Rightarrow\ \gamma = 3,\ \beta = 1, \alpha = 1$
$\therefore$ independent term $= \frac{5!}{1!1!3!} 2^1 . (-7)^3$
$= -13720$

12. The value of $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$ then $|adj(adj\ A^2)|$ is:

A. 6^4
B. 4^8
C. 4^5
D. 2^8

Answer (D)

Solution:

$$A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$$
$$|A| = \frac{1}{\log x \log y \log z}\begin{bmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{bmatrix}$$
$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$
$$\Rightarrow |A| = 2$$
$$|adj(adj\ A^2)| = |A|^8$$
$$= 2^8$$

13. Sum of two positive integers is 66 and μ is the maximum value of their product $S = \left\{x \in \mathbb{Z},\ x(66-x) \geq \frac{5\mu}{9}\right\},\ x \neq 0$, then probability of A when $A = \{x \in S; x = 3k,\ x \in \mathbb{N}\}$ is:

A. $\frac{1}{4}$
B. $\frac{2}{3}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$

Answer (C)

Solution:

Let the two numbers be α and β
$\alpha + \beta = 66$
$A.M. \geq G.M.$
$\frac{\alpha+\beta}{2} \geq \sqrt{\alpha\beta}$
$\mu = 33 \times 33 = 1089$
$x(66-x) \geq \frac{5\mu}{9}$
$x(66-x) \geq 605$
$x^2 - 66x + 605 \leq 0$
$x \in [11,\ 55]$
Favourable set of values of x for event $A = \{12,\ 15,\ 18,\ \cdots,\ 54\}$
$P(A) = \frac{15}{45} = \frac{1}{3}$

14. Let $L_1 = \frac{x-3}{1} = \frac{y-2}{2} = \frac{z-1}{3}$ and $L_2 = \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and direction ratios of line L_3 are $< 1,\ -1,\ 3 >$. P and Q are points of intersection of L_1 and L_3 and L_2 & L_3, respectively. Then, distance between P and Q is:

A. $\frac{10}{3}\sqrt{6}$
B. $\frac{8}{3}\sqrt{11}$
C. $\frac{4}{3}\sqrt{11}$
D. $\frac{11}{3}\sqrt{6}$

Answer (B)

Solution:

Let $PQ = AB$
Let $A(3,\ 2,\ 1)$
Equation of line AB:
$\frac{x-3}{1} = \frac{y-2}{-1} = \frac{z-1}{3} = k$ (let)
$\Rightarrow x = kx + 3,\ y = -k + 2,\ z = 3k + 1$
Let coordinates of $B(k+3,\ -k+2,\ 3k+1)$
B lies on L_2
$B(\lambda+1,\ 2\lambda+2,\ 3\lambda+3)$
$k + 3 = \lambda + 1 \Rightarrow \lambda - k = 2$
$2 - k = 2\lambda + 2 \Rightarrow 2\lambda + k = 0 \Rightarrow k = -2\lambda$
$\Rightarrow 3\lambda = 2 \Rightarrow \lambda = \frac{2}{3}$
$B\left(\frac{5}{3}, \frac{10}{3}, 5\right)$
$AB = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + 16}$
$= \frac{4}{3}\sqrt{11} = PQ$

15. If $\vec{a} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ is rotated by $90°$ about origin passing through y-axis. If new vector is $\vec{b}$ then projection of $\vec{b}$ on $\vec{c} = 5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ is equal to:

A. $\frac{6}{5}$
B. $\frac{3}{5}$
C. $\frac{6}{5\sqrt{3}}$
D. $\frac{6\sqrt{3}}{5}$

Answer (A)

Solution:

$\vec{b} = \lambda\vec{a} + \mu\hat{\jmath}$

$b = \lambda(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \mu\hat{\jmath}$

$\vec{b} \cdot \vec{a} = 0$

$(\lambda\vec{a} + \mu\hat{\jmath})\vec{a} = 0$

$6\lambda + 2\mu = 0$

$\Rightarrow \mu = -3\lambda$

$\vec{b} = \lambda(\vec{a} - 3\hat{\jmath}) = \lambda(-\hat{\imath} - \hat{\jmath} + \hat{k})$

$\lambda = \pm\sqrt{2}$

Projection of $\vec{b}$ on $\vec{c} = \left|\vec{b} \cdot \hat{c}\right|$

$= \left|(-\hat{\imath} - \hat{\jmath} + \hat{k})\frac{(5\hat{\imath}+4\hat{\jmath}+3\hat{k})}{5\sqrt{2}}\right| = \frac{6}{5}$

16. Given $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2(1 + \ln x)\right)$. If $y(1) = 3$, then the value of $\frac{y^2(3)}{9}$ is:

A. $-\frac{1}{43+27\ln 3}$
B. $\frac{1}{43+27\ln 3}$
C. $\frac{9}{59-162(1+\ln 3)}$
D. $\frac{1}{27-43\ln 3}$

Answer (B)

Solution:

$\frac{dy}{dx} - \frac{y}{x} = y^3(1 + \ln x)$

$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} - \frac{1}{xy^2} = (1 + \ln x)$

Taking $\frac{1}{y^2} = t$

$\Rightarrow -\frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore -\frac{1}{2}\frac{dt}{dx} - \frac{t}{x} = (1 + \ln x)$

$\Rightarrow \frac{dt}{dx} + \frac{2t}{x} = -2(1 + \ln x)$

I.F. $= e^{\int \frac{2}{x}dx} = x^2$

$\therefore tx^2 = \int -2(1 + \ln x)x^2 dx$

$\Rightarrow tx^2 = -2\left[\frac{(1+\ln x)x^3}{3} - \int \frac{x^2}{3}dx\right] + c$

$\frac{x^2}{y^2} = -2\left[\frac{x^3}{3}(1 + \ln x) - \frac{x^3}{9}\right] + c \cdots (i)$

$y(1) = 3 \Rightarrow \frac{1}{9} = -2\left(\frac{1}{3} - \frac{1}{9}\right) + c$

$\therefore c = \frac{5}{9}$

Now putting $x = 3,\ c = \frac{5}{9}$ in (i)

$\frac{9}{y^2} = -2(9(1 + \ln 3) - 3) + \frac{5}{9}$

$= \frac{59}{9} - 18(1 + \ln 3)$

$\Rightarrow \frac{y^2}{9} = \frac{9}{59-162(1+\ln 3)}$

17. If $a,\ b \in [1,\ 25],\ a, b \in \mathbb{N}$ such that $a + b$ is multiple of 5, then the number of ordered pair $(a,\ b)$ is __________.

Answer (125)

Solution:

TYPE	NUMBERS
$5k$	$5,\ 10,\ 15,\ 20,\ 25$
$5k + 1$	$1,\ 6,\ 11,\ 16,\ 21$
$5k + 2$	$2,\ 7,\ 12,\ 17,\ 22$
$5k + 3$	$3,\ 8,\ 13,\ 18,\ 23$
$5k + 4$	$4,\ 9,\ 14,\ 19,\ 24$

$(a,\ b)$ can be selected as

$I.$ 1 of $5k + 1$ and 1 of $5k + 4 = 2 \times 25 = 50$

$II.$ 1 of $5k + 2$ and 1 of $5k + 3 = 2 \times 25 = 50$

$III.$ Both of the type $5k = 25$

Total $= 125$

18. If $\log_2(9^{2\alpha-4} + 13) - \log_2\left(3^{2\alpha-4}.\frac{5}{2} + 1\right) = 2$, then maximum integral value of β for which equation, $x^2 - ((\sum \alpha)^2 x) + \sum(\alpha + 1)^2\beta = 0$ has real roots is ______.

Answer (6)

Solution:

$\log_2(9^{2\alpha-4}+13)-\log_2\left(3^{2\alpha-4}.\frac{5}{2}+1\right)=2$

$\therefore \frac{9^{2\alpha-4}+13}{3^{2\alpha-4}.\frac{5}{2}+1}=4$

Let $3^{2\alpha-4}=t$

$\Rightarrow t^2+13=10t+4$

$\Rightarrow t^2-10t+9=0$

$\Rightarrow t=9,1$

$\Rightarrow\ \alpha=3,2$

Now equation will become:

$x^2-25x+25\beta=0$ has real roots

$\therefore D\geq 0$

$\Rightarrow 25^2-4\cdot 25\beta\geq 0$

$\Rightarrow \beta\leq \frac{25}{4}$

Maximum integral value $=6$