1. REAL NUMBER

• EUCLID'S DIVISION LEMMA •

Given two positive integers a and b, there exist unique integers q and r such that

$$a = bq + r, \ 0 \le r < b$$

Here, a = Dividend, b = Divisor, q = Quotient and r = Remainder,

i.e. Dividend = (Divisor x Quotient) + Remainder.

Lemma : A lemma is a proven statement used for proving another statement.

ILLUSTRATION

Q.1 Find number q and r for each pair (a, b) satisfying $a = bq + r, \ 0 \le r < b$

(i) (13, 5) (ii) 41, 9 (iii) 12, 75

Sol. (i) Let a = 13, b = 5 $\therefore$ $13 = 5 \times 2 + 3$ $\Rightarrow$ q = 2, r = 3

(ii) Let a = 41, b = 9 $\therefore$ $41 = 9 \times 4 + 5$ $\Rightarrow$ q = 4, r = 5

(iii) Let a = 12, b = 75 $\therefore$ $12 = 75 \times 0 + 12$ $\Rightarrow$ q = 0, r = 12

PRACTICE PROBLEMS

1. Find number q and r for each pair (a, b) satisfying $a = bq + r, \ 0 \le r < b$:

(i) 508, 27 (ii) 132, 13 (iii) 63, 8 (iv) 3007, 105

• EUCLID'S DIVISION ALGORITHM •

Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two positive integers.

Algorithm: An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.

To find the HCF of two positive integers (say c and d such that c > d),

We follows the steps given below,

Step I: Apply Euclid's division lemma to c and d, to find the whole numbers q and r, such that

$$c = dq + r, \ 0 \le r < d.$$

Step II: If r = 0, then d is the HCF of a and b. If $r \neq 0$, then again apply the Euclid's division lemma to d and r.

Step III: Continue this process till the remainder is zero, i.e. repeat the step II again and again, untill r = 0. Then, the divisor at this stage will be the required HCF.

ILLUSTRATION

Q.2 Find H.C.F of 564 and 192 using Euclid's Division Algorithm.

Sol. **Step 1:** As 564 > 192, we apply E.D.L to 564 and 192, to get, $564 = 192 \times 2 + 180$

Step 2: Since remainder $180 \neq 0$, we again apply E.D.L to 192 and 180, to get, $192 = 180 \times 1 + 12$

Step 3: Now, applying E.D.L. to 180 and 12, to get, $180 = 12 \times 15 + 0$

As, in this step remainder r = 0, so we stop this procedure here and divisor (b) = 12 is the required H.C.F.

HCF (564, 192) = 12

PRACTICE PROBLEMS

2. Find HCF of following numbers using Euclid's Division Algorithm :

(i) 20 and 188 (ii) 74 and 407 (iii) 190 and 2299

• APPLICATIONS OF EUCLID'S DIVISION LEMMA •

ILLUSTRATION

Q.3 Show that every positive even integer is of the form 2q and every positive odd integer is of the form 2q + 1, where q is some integer.

Sol. Let a be any positive integer and it is divided by 2, then by Euclid division lemma we have,

$$a = 2q + r,\ 0 \le r < 2 \text{ and } r = 0, 1$$

Let r = 0, a = 2q which is clearly even as it is a multiple of 2 and r = 1, a = 2q + 1 which is clearly odd.

Thus, any integer 'a' can be of form a = 2q or 2q + 1.

When a = 2q $\Rightarrow$ a is even number and when a = 2q + 1 $\Rightarrow$ a is odd.

Q.4 Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

Sol. Let a be any positive integer and it is divided by 4, then by Euclid division lemma we have,

$a = 4q + r,\ 0 \le r < 4$ and r = 0, 1, 2, 3

Let r = 0, a = 4q which is clearly even as it is a multiple of 2.

r = 1, a = 4q + 1 which is clearly odd.

r = 2, a = 4q + 2 which is even.

r = 3, a = 4q + 3 which is odd.

Thus, any odd integer is of the form 4q + 1 or 4q + 3.

Q.5 Show that any positive integer is of the form 3q, 3q + 1 or 3q + 2 for some integer q.

Sol. Let a be any positive integer and b = 3. Applying division Lemma with a and b = 3, we have

a = 3q + r, where $0 \le r < 3$ and q is some integer

$\Rightarrow$ a = 3q + 0 or, a = 3q + 1 or, a = 3q + 2

$\Rightarrow$ a = 3q or, a = 3q + 1 or, a = 3q + 2 for some integer q.

Q.6 Show than $n^2 - 1$ is divisible by 8, if n is an odd positive integer.

Sol. We know that any odd positive integer is of the form 4q + 1 or 4q + 3 for some integer q.

So, we have the following cases:

Case I: When n = 4q + 1

In this case, we have, $n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q + 1 - 1 = 16q^2 + 8q = 8q(2q + 1)$

$\Rightarrow n^2 - 1$ is divisible by 8 [$\because$ 8q (2q + 1) is divisible by 8]

Case II: When n = 4q + 3

In this case, we have, $n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 9 - 1 = 16q^2 + 24q + 8$

$\Rightarrow n^2 - 1 = 8(2q^2 + 3q + 1) = 8(2q + 1)(q + 1)$

$\Rightarrow n^2 - 1$ is divisible by 8 [$\because$ 8(2q + 1) (q + 1) is divisible by 8]

Hence, $n^2 - 1$ is divisible by 8.

Q.7 Show that the square of any positive integer is of the form 3m or 3m + 1 for some integer m.

Sol. Let a be any positive integer. Then, it is of the form 3q, 3q + 1 or 3q + 2.

So, we have the following cases:

Case I: When a = 3q

In this case, we have, $a^2 = (3q)^2 = 9q^2 = 3q(3q) = 3m$, where m = 3q

Case II: When a = 3q + 1

In this case, we have, $a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3q(3q + 2) + 1 = 3m + 1$, where m = q (3q + 2)

Case III: When $a = 3q + 2$

In this case, we have, $a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1 = 3m + 1$, where $m = 3q^2 + 4q + 1$

Hence, a is of the form 3m or 3m + 1.

Q.8 Show that the cube of any positive integer is either of the form 9m, 9m + 1 or 9m + 8 for some integer m.

Sol. Let x be any positive integer. Then, it is of the form 3q, 3q + 1 or 3q + 2. So, we have the following cases:

Case I: When $x = 3q$

In this case, we have $x^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where m = 3q2

Case II: When $x = 3q + 1$

In this case, we have $x^3 = (3q + 1)^3$

$\Rightarrow x^3 = 27q^3 + 27q^2 + 9q + 1 \Rightarrow x^3 = 9q(3q^2 + 3q + 1) + 1 \quad \Rightarrow x^3 = 9m + 1$, where $m = q(3q^2 + 3q + 1)$

Case III: When $x = 3q + 2$

In this case, we have, $x^3 = (3q + 2)^3$

$\Rightarrow x^3 = 27q^3 + 54q^2 + 36q + 8 \quad \Rightarrow x^3 = 9q(3q^2 + 6q + 4) + 8 \quad \Rightarrow x^3 = 9m + 8$, where $m = q(3q^2 + 6q + 4)$

Here, x^3 is either of the form 9m or, 9m + 1 or, 9m + 8.

Q.9 Find the HCF of 81 and 237 and express it as a linear combination of 81 and 237.

Sol. Given integers are 81 and 237 such that 81 < 237.

Applying division lemma to 81 and 237, we get

$237 = 81 \times 2 + 75$ ....(i) $\left[\because 81\overline{)237}(2 \quad \frac{162}{75}\right]$

Since the remainder $75 \neq 0$. So, consider the divisor 81 and the remainder 75 and apply division lemma to get

$81 = 75 \times 1 + 6$ ....(ii) $\left[\because 75\overline{)81}(1 \quad \frac{75}{6}\right]$

We consider the new divisor 75 and the new remainder 6 and apply division lemma to get

$75 = 6 \times 12 + 3$ ....(iii) $\left[\because 6\overline{)75}(12 \quad \frac{72}{3}\right]$

We consider the new divisor 6 and the new remainder 3 and apply division lemma to get

$6 = 3 \times 2 + 0$ ....(iv) $\left[\because 3\overline{)6}(2 \quad \frac{6}{0}\right]$

The remainder at this stage is zero. So, the divisor at this stage or the remainder at the earlier stage i.e. 3 is the HCF of 81 and 237.

To represent the HCF as a linear combination of the given two numbers, we start from the last but one step and successively eliminate the previous remainders as follows:

From (iii), we have ; $3 = 75 - 6 \times 12$

$\Rightarrow \quad 3 = 75 - (81 - 75 \times 1) \times 12$ [Substituting $6 = 81 - 75 \times 1$ obtained from (ii)]

⇒ $3 = 75 - 12 \times 81 + 12 \times 75$

⇒ $3 = 13 \times 75 - 12 \times 81$ [Substituting $75 = 237 - 81 \times 2$ obtained from (i)]

⇒ $3 = 13 \times (237 - 81 \times 2) - 12 \times 81$

⇒ $3 = 13 \times 237 - 26 \times 81 - 12 \times 81$

⇒ $3 = 13 \times 237 - 38 \times 81$

⇒ $3 = 237x + 81y$, where $x = 13$ and $y = -38$.

Q.10 Find the HCF of 65 and 117 and express it in the form 65 m + 117 n.

Sol. Given integers are 65 and 117 and 117 > 65.

Applying division lemma to 117 and 65, we get

$117 = 65 \times 1 + 52$ ...(i) $\left[\because 65\overline{)117}(1,\ \frac{65}{52}\right]$

Since the remainder $52 \neq 0$. So, we apply the division lemma to the divisor 65 and the remainder 52 to get

$65 = 52 \times 1 + 3$ ...(ii) $\left[\because 52\overline{)65}(1,\ \frac{52}{13}\right]$

We consider the new divisor 52 and the new remainder 13 and apply division lemma, to get

$52 = 13 \times 4 + 0$ ...(iii)

At this stage the remainder is zero. So, the last divisor or the non-zero remainder at the earlier stage i.e. 13 is the HCF of 65 and 117.

From (ii), we have ; $13 = 65 - 52 \times 1$

⇒ $13 = 65 - (117 - 65 \times 1)$ [Substituting $52 = 117 - 65 \times 1$ obtain from (i)]

⇒ $13 = 65 - 117 + 65 \times 1$

⇒ $13 = 65 \times 2 + 117 \times (-1)$

⇒ $13 = 65 - 117 + 65 \times 1$

⇒ $13 = 65\,m + 117n$, where $m = 2$ and $n = -1$.

PRACTICE PROBLEMS

3. Show that any positive odd integer is of the from 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

4. Prove that if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

5. Prove that one of the even three consecutive positive integers is divisible by 3.

6. Express the HCF of 468 and 222 in the linear combination of 468 and 222.

7. If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$, find y.

• FUNDAMENTAL THEOREM OF ARITHMETIC •

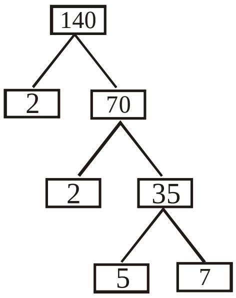
Every composite number can be expressed (factorise) as a product of primes and this factorisation is unique. (neglecting the order in which the prime factors occur).

EXAMPLE : factorizing 90, we get $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

ILLUSTRATION

Q.11 Express 140 as a product of prime factors using factor tree

Sol.



Factors of 140 = 2 x 2 x 5 x 7

Q.12 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. (i) $7 \times 11 \times 13 + 13 = 13 \times \{7 \times 11 + 1\} = 13 \times 78$ which is a composite number.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times \{7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1\} = 5 \times 1009$ which is a composite number.

Q.13 Check whether 6^n can end with the digit 0 for any natural number n.

Sol. Let (if possible) 6^n ends with digit 0 $\Rightarrow 6^n = 10 \times q$ $\Rightarrow 2^n \times 3^n = 2 \times 5 \times q$ $\Rightarrow$ 5 is a prime factor of $2^n \times 3^n$ which is not possible because $2^n \times 3^n$ can have only 2 and 3 prime factors. Hence, 6^n cannot end with the digit 0 for any natural number n.

PRACTICE PROBLEMS

8. Express the following as the product of prime number using factor tree: (i) 3825 (ii) 468

9. Prove that there is no natural number for which 4^n ends with the digit zero.

10. Explain why $5 \times 17 \times 23 + 5$ is a composite number.

• APPLICATIONS OF FUNDAMENTAL THEOREM OF A ARITHMETIC •

FINDING HCF AND LCM OF POSITIVE INTEGERS

Fundamental theorem of arithmetic can be used to find the HCF and LCM of two or more positive integers. This method is also called prime factorisation method. In this method, first express the given two or more numbers into the product of prime numbers seperately. Then,

HCF of two or more numbers = Product of smallest power of each common prime factor involved in the numbers.

LCM of two or more numbers = Product of greatest power of each prime factor involved in the numbers.

• RELATION BETWEEN HCF AND LCM OF TWO NUMBERS •

HCF × LCM = product of the two numbers

For any two positive integers a and b, we have

$HCF(a, b) \times LCM(a, b) = a \times b$

For any three positive integers a, b and c, we have

$$HCF(a, b, c) = \frac{a \times b \times c \times LCM(a,b,c)}{LCM(a,b) \times LCM(b,c) \times LCM(a,c)} \; ; \; LCM(a, b, c) = \frac{a \times b \times c \times HCF(a,b,c)}{HCF(a,b) \times HCF(b,c) \times HCF(a,c)}$$

ILLUSTRATION

Q.14 Find the L.C.M and H.C.F. of 60 and 45 by the prime factorisation method

Sol. We have $60 = 2 \times 2 \times 3 \times 5$ $\quad 45 = 3 \times 3 \times 5$

HCF (60, 45) = $3 \times 5 = 15$

LCM (60, 45) = $2 \times 2 \times 3 \times 3 \times 5 = 180$

LCM can also be found by using the below given formula.

LCM (a, b) = $\dfrac{a \times b}{HCF(a,b)}$ $\quad$ LCM (60, 45) = $\dfrac{60 \times 45}{HCF(60,45)} = \dfrac{60 \times 45}{15} = 180$

PRACTICE PROBLEMS

11. Find LCM and HCF of the following integers by using prime factorisation method.

(i) 24, 30, 54 $\quad$ (ii) 120, 144, 336 $\quad$ (iii) 28, 49, 84

12. Find the LCM of 66 & 486 by the Prime factorisation method. Hence find their HCF.

13. Find the HCF of 145 and 382 by the Prime factorisation method. Hence find their L.C.M.

14. HCF of two numbers is 113 and their LCM is 56952. If one number is 904, then find the other number.

15. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

16. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

IRRATIONAL NUMBER

Any number, which cannot be written in the form p/q (where p and q are integers and $q \neq 0$) is called irrational.

EXAMPLE : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, \dfrac{1}{\sqrt{7}}$ etc are irrational numbers.

PROVING IRRATIONAL NUMBERS

POINTS TO NOTE :

(i) Sum or difference of a rational and an irrational number is irrational.

(ii) The product and quotient of a non-zero rational and irrational number is irrational.

(iii) If p is a prime and p divides a^2, then p divides 'a' where a is a positive integer.

ILLUSTRATION

Q.15 Prove $\sqrt{2}$ is irrational.

Sol. Let us assume that $\sqrt{2}$ is a rational.

So $\sqrt{2} = \dfrac{r}{s}$ where r, s are integers and $s \neq 0$. r and s are co-prime (does not have any common factors)

On squaring, we get $2 = \dfrac{r^2}{s^2} \Rightarrow s^2 = \dfrac{r^2}{2} \Rightarrow 2$ divides $r^2 \Rightarrow 2$ divides r.

Now, let r = 2a for some integer a. Putting r = 2a in $2s^2 = r^2$, we get $2s^2 = (2a)^2$

$\Rightarrow s^2 = 2a^2 \Rightarrow \frac{s^2}{2} = a^2$ i.e., 2 divides $s^2 \Rightarrow$ 2 divides s.

$\Rightarrow$ 2 is divisor of both r and s that contradicts our assumption that r and s are co-prime.

So, our assumption of taking $\sqrt{2}$ as rational is incorrect. So, $\sqrt{2}$ is irrational.

Q.16 Show that $\sqrt{2}+5$ is irrational.

Sol. Let us assume $\sqrt{2}+5$ be rational. So, $\sqrt{2}+5=\frac{r}{s}$ where r, s are co-prime integers and $s \neq 0$

$\Rightarrow \sqrt{2}=\frac{r}{s}-5$ or $\sqrt{2}=\frac{r-5s}{s}$

since r, s are integers, So, $\frac{(r-5s)}{2}$ is a rational numbers. So, $\sqrt{2}$ should be rational number.

But we know the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}+5$ is rational.

So, $\sqrt{2}+5$ is irrational.

PRACTICE PROBLEMS

17. Prove that $\sqrt{3}$ is irrational.

18. Prove that $7+4\sqrt{5}$ is irrational.

DETERMINING THE NATURE OF DECIMAL EXPANSIONS OF RATIONAL NUMBERS

THEOREM 1: Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form p/q, where p and q are co-prime, and the prime factorisation of q is of the form $2^m 5^n$ where m, n are non-negative integers.

THEOREM 2: Let x = p/q be a rational number, such that the prime factorisation of q is of the form $2^m 5^n$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

THER, OREM 3: Let x = p/q be a rational number, such that the prime factorisation of q is not of the form $2^m 5^n$, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

Note: The decimal expansion of every rational number is either terminating or non-terminating repeating.

ILLUSTRATION

Q.17 Which rational numbers of the following will have a terminating decimal expansion.

(i) $\frac{13}{16}$ (ii) $\frac{221}{12}$ (iii) $\frac{239}{50}$ (iv) $\frac{1}{98}$

Sol. (i) We have the denominator = $16 = 2^4\,5^0$ which is in the form of $2^m 5^n$.

So, $\frac{13}{16}$ is terminating decimal expansion.

(ii) We have the denominator $12 = 2^2 \times 3$ which is not of the form $2^m 5^n$.

So, $\frac{221}{12}$ is non-terminating repeating decimal expansion.

(iii) We can write the denominator as 50 = 2 × 5^2 = 2^1 5^2 which is in the form of $2^m 5^n$.

So, $\frac{239}{50}$ has terminating decimal expansion.

(iv) We have the denominator, 98 = 2 × 7^2 which cannot be put in the form 2^m5^n. So, $\frac{1}{98}$ is non-terminating.

PRACTICE PROBLEMS

19. Without actually performing the long division, state whether the following numbers have a terminating or non-terminating repeating decimal expansion.

(i) $\frac{124}{125}$ (ii) $\frac{7}{75}$ (iii) $\frac{5}{64}$ (iv) $\frac{131}{2^2 5^3 7^4}$

(v) $\frac{157}{57}$ (vi) $\frac{109}{110}$ (vii) $\frac{113}{162}$ (viii) $\frac{709}{216}$

PRACTICE PROBLEMS ANSWERS

1. (i) q = 18, r = 22 (ii) q = 10, r = 2 (iii) q = 7, r = 7 (iv) q = 28, r = 67

2. (i) 4 (ii) 37 (iii) 19 **6.** 6 = 468 × –9 + 222 × 19 **7.** y = –19

8. 3825 = 3 × 3 × 5 × 5 × 17, 468 = 2 × 2 × 3 × 3 × 13 **11.** (i) 1080, 6 (ii) 5040, 24 (iii) 588, 7

12. LCM (66, 486) = 5346; HCF (66, 486) = 6 **13.** HCF(145, 382) = 1; LCM (145, 382) = 55390

14. 7119 **15.** 8 column **16.** 36 minutes

19. (i), (iii) have terminating decimal expansion while (ii), (iv), (v), (vi), (vii), (viii) have non-terminating repeating decimal expansion

EXERCISE

EXERCISE – A

TYPE I : EUCLID'S DIVISION ALGORITHM

Use Euclid's division algorithm to find the H.C.F. of : (Q. 1 to Q. 12)

1. 32 and 54　**2.** 18 and 24　**3.** 70 and 30　**4.** 210 and 55　**5.** 81 and 237

6. 65 and 117　**7.** 495 and 475　**8.** 240 and 6552　**9.** 196 and 38220　**10.** 144, 180 and 192

11. 391, 425 and 527　**12.** 84, 90 and 120

TYPE II : PRIME FACTORISATION METHOD

Using prime factorisation method find H.C.F. and L.C.M. of :

13. 26 and 91　**14.** 510 and 92　**15.** 336 and 54　**16.** 17, 23 and 29　**17.** 8, 9 and 25　**18.** 40, 36 and 126

TYPE III: LCM × HCF =PRODUCTS OF NUMBERS

Using prime factorisation method verify that LCM × HCF =Products of numbers(Q.19 - 21)

19. 90 & 144　**20.** 180 & 192　**21.** 18 & 12

22. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

23. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

24. The LCM and HCF of two numbers are 180 and 6 respectively. If one of the number is 30, find the other number.

25. The HCF of two numbers is 16 and their product is 3072. Find their LCM.

26. Given that HCF (1152, 1664) = 128, find LCM (1152, 1664).　**27.** If LCM (96, 404) is 9696, find HCF(96, 404).

TYPE IV: PROVING IRRATIONAL NUMBERS

Prove that following numbers are irrationals :

28. $\sqrt{3}$　**29.** $\sqrt{5}$　**30.** $\sqrt{6}$　**31.** $\sqrt{n}$　**32.** $3\sqrt{7}$

33. $-2\sqrt{8}$　**34.** $a\sqrt{b}$　**35.** $3\sqrt{5}$　**36.** $4-\sqrt{3}$　**37.** $-2+\sqrt{5}$

38. $\sqrt{5}+\sqrt{6}$　**39.** $\sqrt{a}+\sqrt{b}$　**40.** $2+3\sqrt{2}$　**41.** $(3+2\sqrt{5})^2$　**42.** $12\sqrt{3}-41$　**43.** $15+17\sqrt{3}$

TYPE V: DETERMINATION OF DECIMAL NUMBERS

Without actual division, find which of the following rationals are terminating decimals. Also write after how many places it will terminate.

44. 11/24　**45.** 9/35　**46.** 17/320　**47.** 24/125　**48.** 171/800

49. 19/3125　**50.** 32/455　**51.** 341/14000　**52.** $\frac{136}{2^3 \times 3^2 \times 7^2}$　**53.** 77/210

TYPE VI: COVERSION INTO P/Q FORM

Express each as a rational in simplest form :

54. $0.\overline{8}$　**55.** $2.\overline{4}$　**56.** $0.\overline{24}$　**57.** $0.\overline{12}$　**58.** $2.\overline{24}$　**59.** $0.\overline{365}$

Express each of the following p/q in the simplest form and write the prime factors of q.

60. 2.54　**61.** 321.064　**62.** 10.8713　**63.** 21.123456789　**64.** $2.123\overline{45}$　**65.** $0.\overline{1234}$

66. Write the denominator of 91/1250 in the form of $2^m.5^n$,where m and n are non - negative integers. Also write the decimal expansion without actual division.

67. Express each number as a product of its prime factors using factor tree method.

a. 140　**b.** 56　**c.** 3825　**d.** 5005　**e.** 7429

EXERCISE – B

TYPE I : APPLICATIONS OF EUCLID'S DIV. LEMMA

1. Show that every positive even integer is of the form 2q, & that every positive odd integer is of the form 2q + 1, where q is some integer.

2. Show that every positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

3. Show that every positive odd integer is of the form 6q + 1 or 6q + 3 or 6q + 5 for some integer q.

4. Show that one and only one out of n, n + 2, n+ 4 is divisible by 3, where n is any positive integer.

5. Show that $n^2 - 1$ is divisible by 8, if n is an odd positive integer.

6. Prove that x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

7. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

8. Show that square of any positive integer is of form 3m or 3m + 1 for some integer m.

9. Show that the cube of any positive integer is either of the form 9q, 9q +1 or, 9q + 8 for some integer q.

TYPE II: APPLICATIONS OF HCF AND LCM

10. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.

11. Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

12. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

13. A sweet seller has 420 Kaju burfis and 130 Badam burfis she wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of burfis that can be placed in each stack for this purpose?

14. Two tankers contain 850 litres and 680 litres of petrol respectively. Find their maximum capacity of a container which can measure the petrol of either tanker is exact number of times.

15. In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

16. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic-wise and the height of each stack is the same. The number of English books is 96, the number of Hnidi books is 240 and the number of Mathematics books is 336. Assuming that the books are of the same same thickness, determine the number of stacks of English, Hindi and Mathematics books.

17. A book seller purchased 117 books out of which 45 books are of Maths and the remaining are of Physics. Each book has same size. These all books are to be packed in separate bundles and each bundle must contain same number of books. Find the least number of bundles which can be made for these books.

18. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

19. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

20. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size in incyhes of the tile required that has to be cut and how many such tiles are required?

21. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

22. During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

23. On a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

24. Find the least number that is divisible by all the numbers between 1 & 10 (both inclusive).

25. On GT road, three consecutive traffic lights change after 36, 42 and 72 sec. If the lights are first switched on at 9:00 am, then at what time will they change again together.

26. A forester wants to plant 66 mango trees, 88 orange trees and 110 apple trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). Find the number of minimum rows.

27. On a morning walk, three persons steps off together & their steps measures 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? What are the benefits of morning walk?

28. A bookseller has 420 Science stream books and 130 Arts stream books. He wants to stack them in such a way that each stack has the same number and they take up the least area of the surface.

(i) What is the maximum number of Science stream books that can be placed in each stack for this purpose?

(ii) What mathematical concept is used to solve the above problem?

(iii) If the bookseller make a stack, then which kinds of quality are shown by the bookseller?

29. There is a rectangular park around a public school. Ram takes 24 min to walk around the school while Geeta takes 18 min to walk around the school. Suppose they both start at the same point and at the same time and go in the same direction.

(i) After how many minutes will they meet again at the starting point?

(ii) What mathematical concept is used to solve the above problem? **(iii)** What are the advantage of walking?

30. A trader was moving along a road selling eggs. An idler who did not have much work to do, started to get the trader into a words duel. They grew a fight, he pulled the basket with eggs and dashed it on the floor. The eggs broke. The trader requests the panchayat to ask the idler to pay for broken eggs. The panchayat asked the trader how many eggs were broken?

He gave the following response:

If counted in pairs, one will remain. If counted in 3, two willl remain. If counted in 4, three will remain. If counted in 5, four will remain. If counted in 6, five will remain.If counted in 7, nothing will remain and my basket cannot accomodate more than 150 eggs.

(i) How many eggs were there? **(ii)** What mathematical concept is used to solve the above problem?

(iii) What is the value shown by trader?

TYPE III: MISCELLENOUS ON REAL NUMBERS

31. Show that any number of the form 4^n, $n \in N$ can never end with the digit 0.

32. Show that any number of the form 6^n, $n \in N$ can never end with digit 0.

33. Is $7^5 \times 3^2 \times 5 + 3$ a composite number? Justify your answer.

34. Explain why $5 \times 4 \times 3 \times 2 \times 1 + 3$ is a composite number.

35. Explain why $11 \times 13 \times 15 \times 17 + 17$ is a composite number.

ANSWERS

EXERCISE – A

1. 2 **2.** 6 **3.** 10 **4.** 5 **5.** 3 **6.** 13 **7.** 5

8. 24 **9.** 195 **10.** 2880 **11.** 17 **12.** 6 **13.** HCF = 13, LCM=182

14. HCF = 2, LCM = 23460 **15.** HCF = 6, LCM = 3024 **16.** HCF = 1, LCM = 1139

17. HCF = 1, LCM = 1800 **18.** HCF = 2, LCM = 2520 **22.** 435 **23.** 207 **24.** 36

25. 192 **26.** 14976 **27.** HCF = 4 **44.** No **45.** No **46.** Yes and 6 places

47. Yes and 3 places **48.** Yes and 5 places **49.** Yes and 5 places **50.** No

51. No **52.** No **53.** No **54.** 8/9 **55.** 22/9 **56.** 8/33

57. 11/90 **58.** 101/45 **59.** 181/450 **60.** $\frac{127}{50}$ &(2,5) **61.** $\frac{40133}{125}$ &(5) **62.** $\frac{10817}{10^4}$ &(2,5)

63. $\frac{21123456789}{10^9}$ **64.** $\frac{11679}{5500}$ &(2,5,11) **65.** $\frac{1234}{9999}$ & (3,11,101) **66.** $2\times5^4, 0.0728$

EXERCISE – B

10. 16 **11.** 64 **12.** 17 **13.** 10 **14.** 170 L **15.** 21

16. 2, 5 & 7 **17.** 13 **18.** 8 **19.** 75 cm **20.** 24 inches, 20 litres **21.** 18

22. 4 packets of pencils, 3 packets of crayons **23.** 122m 40 cm **24.** 2520 **25.** 9:08:24 AM

26. 12 rows

27. 2520 cm. Benefits of morning walk is that it reduces stress and also increases the level of thinking by increasing the supply of fresh oxygen to the brain and lungs.

28. 10 books, Concept of HCF, art of management of books and also space management.

29. 72 min, concept of LCM.

30. 119, Concept of LCM, Value of the trader is honesty and a law abiding citizen