## EXERCISE 2.1

**Choose the correct answer from the given four options in the following questions:**

**Q1.** If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is $-3$, then the value of $k$ is

(*a*) 4/3 (*b*) –4/3 (*c*) 2/3 (*d*) –2/3

**Sol.** (*a*): **Main concept:** If $a$ is root of a polynomial $f(x)$, then $f(a) = 0$.

Let $f(x) = (k-1)x^2 + kx + 1$

As $-3$ is a zero of $f(x)$, then

$$f(-3) = 0$$

$$\Rightarrow \quad (k-1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow \quad 9k - 9 - 3k + 1 = 0$$

$$\Rightarrow \quad 9k - 3k = +9 - 1$$

$$\Rightarrow \quad 6k = 8$$

$$\Rightarrow \quad k = 4/3$$

**Q2.** A quadratic polynomial, whose zeroes are $-3$ and 4, is

(*a*) $x^2 - x - 12$ (*b*) $x^2 + x + 12$ (*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (*d*) $2x^2 + 2x - 24$

**Sol.** (*c*): **Main concept:** Required quadratic polynomial

$$= x^2 - (\alpha + \beta) + \alpha\beta$$

Here, $\alpha = -3$ and $\beta = 4$

$$\therefore \quad \alpha + \beta = -3 + 4 = 1$$

and $\quad \alpha \cdot \beta = -3 \times 4 = -12$

$\therefore$ The quadratic polynomial is

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - 1x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - \frac{12}{2}$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

**Q3.** If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and $-3$, then

(*a*) $a = -7, b = -1$ (*b*) $a = 5, b = -1$

(*c*) $a = 2, b = -6$ (*d*) $a = 0, b = -6$

**Sol.** (*d*): **Main concept:** If $a$ is zero of a polynomial $f(x)$, then $f(a) = 0$.

Let $f(x) = x^2 + (a+1)x + b$

As 2, and $(-3)$ are zeroes of polynomial $f(x) = x^2 + (a+1)x + b$, then

$$f(2) = 0$$
$$\Rightarrow (2)^2 + (a+1)(2) + b = 0$$
$$\Rightarrow 4 + 2a + 2 + b = 0$$
$$\Rightarrow 2a + b = -6 \quad ...(i)$$

and
$$f(-3) = 0$$
$$\Rightarrow (-3)^2 + (a+1)(-3) + b = 0$$
$$\Rightarrow 9 - 3a - 3 + b = 0$$
$$\Rightarrow -3a + b = -6$$
$$\Rightarrow 3a - b = 6 \quad ...(ii)$$

$$5a = 0 \qquad \text{[Adding } (i) \text{ and } (ii)\text{]}$$
$$\Rightarrow a = 0$$

But, $2a + b = -6$ [From $(i)$]
$$\Rightarrow 2(0) + b = -6$$
$$\Rightarrow b = -6$$

Hence, $a = 0$ and $b = -6$ verifies option ($d$).

**Q4.** The number of polynomials having zeroes as –2 and 5 is

($a$) 1 ($b$) 2 ($c$) 3 ($d$) more than 3

**Sol.** ($d$): We know that if we divide or multiply a polynomial by any constant (real number), then the zeroes of polynomial remains same.

Here, $\alpha = -2$ and $\beta = +5$

$\therefore \quad \alpha + \beta = -2 + 5 = 3$ and $\alpha \cdot \beta = -2 \times 5 = -10$

So, required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$
$$= x^2 - 3x - 10$$

If we multiply this polynomial by any real number let 5 and 2, we get
$$5x^2 - 15x - 50$$
and
$$2x^2 - 6x - 20$$
which are different polynomials having same zeroes $-2$ and 5.

So, we can obtain so many (infinite polynomials) from two given zeroes.

**Q5.** Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

($a$) $\frac{-c}{a}$ ($b$) $\frac{c}{a}$ ($c$) 0 ($d$) $\frac{-b}{a}$

**Sol.** ($b$): Let $f(x) = ax^3 + bx^2 + cx + d$

If $\alpha, \beta, \gamma$ are the zeroes of $f(x)$, then
$$\boxed{\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}}$$

One root is zero (Given) so, $\alpha = 0$.
$$\Rightarrow \beta\gamma = \frac{c}{a}$$

**Q6.** If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is $-1$, then the product of other two zeroes is

($a$) $b - a + 1$ ($b$) $b - a - 1$ ($c$) $a - b + 1$ ($d$) $a - b - 1$

**Sol.** (*a*): Let $f(x) = x^3 + ax^2 + bx + c$

$\because$ Zero of $f(x)$ is $-1$ so

$$f(-1) = 0$$

$\Rightarrow \quad (-1)^3 + a(-1)^2 + b(-1) + c = 0$

$\Rightarrow \quad -1 + a - b + c = 0$

$\Rightarrow \quad a - b + c = 1$

$\Rightarrow \quad c = 1 + b - a$

Now, $\quad \alpha \cdot \beta \cdot \gamma = \dfrac{-d}{a} \qquad [\because c = b, \quad d = c]$

$\Rightarrow \quad -1\ \beta\gamma = \dfrac{-c}{1}$

$\Rightarrow \quad \beta\gamma = c$

$\Rightarrow \quad \beta\gamma = 1 + b - a$

**Q7.** The zeroes of quadratic polynomial $x^2 + 99x + 127$ are

(*a*) both positive (*b*) both negative

(*c*) one positive and one negative (*d*) both are equal

**Sol.** (*b*): Let $f(x) = x^2 + 99x + 127$

Now, $\quad b^2 - 4ac = (99)^2 - 4(1)\ 127 \qquad (a = 1, b = 99, c = 127)$

$\Rightarrow \quad b^2 - 4ac = 9801 - 508$

$\Rightarrow \quad \sqrt{b^2 - 4ac} = \sqrt{9293}$

$\Rightarrow \quad \sqrt{b^2 - 4ac} = 96.4$

So, zeroes of $f(x)$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-99 \pm 96.4}{2 \times 1}$

$\Rightarrow$ Both roots will be negative as $99 > 96.4$.

**Q8.** The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$

(*a*) cannot both be positive (*b*) cannot both be negative

(*c*) are always unequal (*d*) are always equal

**Sol.** (*a*): Let $f(x) = x^2 + kx + k$

For zeroes of $f(x)$, $f(x) = 0$

$\Rightarrow \quad x^2 + kx + k = 0$

But, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-k \pm \sqrt{k^2 - 4 \cdot k}}{2} = \dfrac{-k \pm \sqrt{k(k-4)}}{2}$

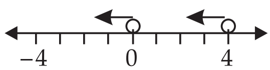
For real roots, $b^2 - 4ac > 0$

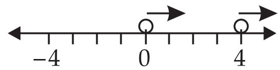
$\Rightarrow \quad k(k - 4) > 0 \qquad (k \neq 0)$

$\Rightarrow$ Either $\quad k < 0$ and $k - 4 < 0 \quad$ or $\quad k > 0$ and $(k - 4) > 0$

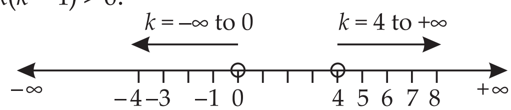
$k < 0$ and $k < 4 \qquad\qquad k > 0$ and $k > 4$

$k < 0, k < 4$




So, solution $k(k - 4) > 0$.




Let $k = -4$

(any point on number line)

$$x = \frac{-4 \pm \sqrt{-4(-4-4)}}{2} = \frac{-4 \pm \sqrt{32}}{2}$$

$$= \frac{-4 \pm 4\sqrt{2}}{2} = \frac{4[-1 \pm \sqrt{2}]}{2}$$

$$x = 2[-1 \pm \sqrt{2}]$$

$x_1 = 2[-1 + \sqrt{2}]$, which is positive

$x_2 = 2[-1 - \sqrt{2}]$, which is negative

Let $k = 8$ (any point on number line)

$$x = \frac{-8 \pm \sqrt{8(8-4)}}{2}$$

$$x = \frac{-8 \pm \sqrt{8 \times 4}}{2}$$

$$x = \frac{-8 \pm 4\sqrt{2}}{2}$$

$$x = \frac{+4[-2 \pm \sqrt{2}]}{2}$$

$$x = 2(-2 \pm \sqrt{2})$$

$x_1 = 2[-2 + \sqrt{2}]$, which is negative

$x_2 = 2[-2 - \sqrt{2}]$, which is negative

So, the roots cannot be both positive.

**Q9.** If the zeroes of the quadratic polynomial

$ax^2 + bx + c$, where, $c \neq 0$ are equal then

(*a*) $c$ and $a$ both have opposite signs
(*b*) $c$ and $b$ have opposite signs
(*c*) $c$ and $a$ have same sign
(*d*) $c$ and $b$ have the same sign

**Sol.** (*c*): For equal roots $b^2 - 4ac = 0$

or $\qquad b^2 = 4ac$

$b^2$ is always positive so $4ac$ must be positive or i.e., product of $a$ and $c$ must be positive i.e., $a$ and $c$ must have same sign either positive or negative.

**Q10.** If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other then it

(*a*) has no linear term and the constant term is negative
(*b*) has no linear term and the constant term is positive
(*c*) can have a linear term but the constant term is negative.
(*d*) can have a linear term but the constant term is positive.

**Sol.** (*a*): Let $f(x) = x^2 + ax + b$ and $\alpha$, $\beta$ are the roots of it.

Then, $\beta = -\alpha$ (Given)

$$\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \alpha - \alpha = \frac{-a}{1} \qquad \alpha(-\alpha) = \frac{b}{1}$$

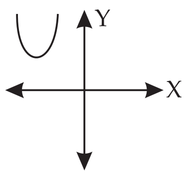
$$\Rightarrow \quad -a = 0 \qquad -\alpha^2 = b$$

$$\Rightarrow \quad a = 0 \qquad \Rightarrow \quad b < 0 \text{ or } b \text{ is negative}$$

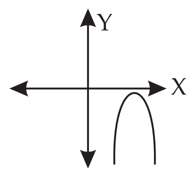
So, $f(x) = x^2 + b$ shows that it has no linear term.

**Q11.** Which of the following is not the graph of a quadratic polynomial?

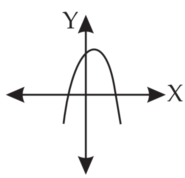
(*a*)



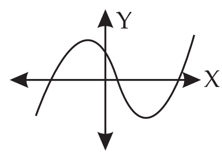

(*b*)




(*c*)




(*d*)




**Sol.** (*d*): Graph '*d*' intersect at three points on X-axis so the roots of polynomial of graph is three, so it is cubic polynomial. Other graphs are of quadratic polynomial. Graphs *a*, *b* have no real zeroes.

## EXERCISE 2.2

**Q1.** Answer the following and justify.

(*i*) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in $x$ of degree 5 ?

(*ii*) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s$, $p \neq 0$ ?

(*iii*) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero what is the relation between the degrees of $p(x)$ and $g(x)$?

(*iv*) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

(*v*) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

**Sol.** (*i*): Let the divisor of degree 5 is $g(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + 1$

Dividend $= p(x) = x^6 + 2x^3 + x - 1$,

$q(x) = x^2 - 1$ and let remainder be $r(x)$

So, by Euclid's division algorithm

$$p(x) = g(x)\, q(x) + r(x)$$

[deg $p(x)$ is 6] = [$g(x)$ of deg 5] [$q(x)$ degree 2] + $r(x)$ of degree less than 5

degree $p(x)$ = degree $g(x)$ + degree $q(x)$ + degree $r(x)$

$$6 = 5 + 2 + \text{any}$$

So, degree of $q(x)$ can never be 2 it may be only one.

So, $(x^2 - 1)$ can never be the quotient.

(*ii*) $p(x)$ (dividend) $= ax^2 + bx + c$

$g(x)$ (divisor) $= px^3 + qx^2 + r(x) + s$

As the degree of dividend is always greater than divisor but here degree $p(x) <$ degree $g(x)$.

When we divide $p(x)$ by $g(x)$, quotient will be zero and remainder will be $p(x)$.

(*iii*) The dividend = $p(x)$, divisor $g(x)$

quotient $q(x) = 0$

remainder = $r(x)$

Here, degree of divisor $g(x)$ is more than degree of dividend.

(*iv*) When $p(x)$ is divided by $g(x)$, the remainder is zero so the $g(x)$ is a factor of $p(x)$ and degree of $g(x)$ will be less than or equal to the degree of $p(x)$ or degree $g(x) \leq$ degree $p(x)$.

(*v*) Let $p(x) = x^2 + kx + k$

For equal zeroes, $b^2 - 4ac = 0$

$\Rightarrow$ $(k)^2 - 4(1)(k) = 0$

$\Rightarrow$ $k^2 - 4k = 0$

$\Rightarrow$ $k(k - 4) = 0$

$\Rightarrow$ $k = 0$ or $k = 4$

But $k > 1$ so $k = 4$

The given quadratic polynomial has equal zeroes at $k = 4$.

**Q2.** Are the following statements true or false? Justify your answers.

(*i*) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then $a$, $b$ and $c$ have the same sign.

(*ii*) If the graph of polynomial intersects the X-axis at only one points it cannot be a quadratic polynomial.

(*iii*) If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.

(*iv*) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(*v*) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of polynomial have the same sign.

(*vi*) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of $a$, $b$, and $c$ is non-negative.

(*vii*) The only value of $k$ for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is 1/2.

**Sol.** (*i*): **False:** Let α and β be the roots of the quadratic polynomial. If α and β are positive then $\alpha + \beta = \frac{-b}{a}$ it shows that $\frac{-b}{a}$ is negative but sum of two positive numbers (α, β) must be +ive i.e. either $b$ or $a$ must be negative. So $a$, $b$ and $c$ will have different signs.

(*ii*) **False:** The given statement is false, because when two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.

(*iii*) **True:** If a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, then graph of the polynomial will intersect at two points on x-axis.

(*iv*) **True:** Let $\beta = 0, \quad \gamma = 0$

$$f(x) = (x - \alpha)(x - \beta)(x - \gamma)$$
$$= (x - \alpha)\, x \cdot x$$
$$\Rightarrow \quad f(x) = x^3 - \alpha x^2$$

which has no linear (coefficient of $x$) and constant terms.

(*v*) **True:** α, β, and γ are all (–)ive for cubic polynomial $ax^3 + bx^2 + cx + d$.

$$\alpha + \beta + \gamma = \frac{-b}{a} \quad \text{...}(i)$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad \text{...}(ii)$$
$$\alpha\beta\gamma = \frac{-d}{a} \quad \text{...}(iii)$$

∵ α, β, γ are all negative so,

$$\alpha + \beta + \gamma = -x \quad \text{(Any negative number)}$$
$$\Rightarrow \quad \frac{-b}{a} = -x \quad \text{[From (i)]}$$
$$\Rightarrow \quad \frac{b}{a} = x$$

So, $a$, $b$, have same sign and product of any two zeroes will be positive.

$$\text{So,} \quad \alpha\beta + \beta\gamma + \gamma\alpha = +y \quad \text{(Any positive number)}$$
$$\Rightarrow \quad \frac{+c}{a} = +y \quad \text{[From (ii)]}$$

⇒ $c$ and $a$ have same sign

$$\alpha\beta\gamma = -z \quad \text{(Any negative number)}$$
$$\Rightarrow \quad \frac{-d}{a} = -z \quad \text{[From (ii)]}$$

$\Rightarrow \quad \dfrac{d}{a} = z$

So, $d$ and $a$ will have same sign.

Hence, signs of $b$, $c$, $d$ are same as of $a$.

So, signs of $a$, $b$, $c$, $d$ will be same either positive or negative.

(*vi*) **True:** As all zeroes of cubic polynomial are positive

Let $\quad f(x) = x^3 + ax^2 - bx + c$

$\therefore \quad \alpha + \beta + \gamma = +\text{ive say} + x$

$\Rightarrow \quad \dfrac{-b}{a} = x$

$\Rightarrow$ $a$ and $b$ has opposite signs ...(*i*)

$\alpha\beta + \beta\gamma + \gamma\alpha = +y$

$\Rightarrow \quad \dfrac{c}{a} = y$

So, signs of $a$ and $c$ are same. ...(*ii*)

Now, $\quad \alpha\beta\gamma = +\text{ive} = +z$

$\Rightarrow \quad \dfrac{-d}{a} = z$

$\Rightarrow$ $a$ and $d$ have opposite signs. [From (*i*)]

From (*i*), if $a$ is positive, then $b$ is negative.

From (*ii*) if $a$ is positive, then $c$ is also positive.

From (*iii*) if $a$ is positive, then $d$ is negative.

Hence, if zeroes $\alpha$, $\beta$, $\gamma$ of cubic polynomial are positive then out of $a$, $b$, $c$ at least one is positive.

(*vii*) **False:** $\quad f(x) = kx^2 + x + k \quad (a = k, b = 1, c = k)$

For equal roots

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad (1)^2 - 4(k)(k) = 0$$

$$\Rightarrow \quad 4k^2 = 1$$

$$\Rightarrow \quad k^2 = 1/4$$

$$\Rightarrow \quad k = \pm\frac{1}{2}$$

So, there are $\dfrac{1}{2}$ and $\dfrac{-1}{2}$ values of $k$ so that the given equation has equal roots.

## EXERCISE 2.3

**Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and coefficients of the polynomials.**

**Q1.** $4x^2 - 3x - 1$

**Sol.** Let $f(x) = 4x^2 - 3x - 1$

Splitting the middle term, we get

$$= 4x^2 - 4x + 1x - 1$$
$$= 4x(x-1) + 1(x-1)$$
$$= (x-1)(4x+1)$$

For $f(x) = 0$, we have

$$4x^2 - 3x - 1 = 0$$

or $(x-1)(4x+1) = 0$

Either $x - 1 = 0 \Rightarrow x = 1$

or $4x + 1 = 0 \Rightarrow 4x = -1 \Rightarrow x = \frac{-1}{4}$

$\therefore$ The zeroes of $f(x)$ are 1 and $\frac{-1}{4}$.

**Verification:** $\alpha = 1, \beta = \frac{-1}{4}$

$a = 4, b = -3$ and $c = -1$

| | |
|---|---|
| $\therefore \quad \alpha + \beta = \frac{-b}{a}$ | $\alpha \cdot \beta = \frac{c}{a}$ |
| $\Rightarrow \quad 1 - \frac{1}{4} = \frac{-(-3)}{4}$ | $\Rightarrow \quad 1 \times \left(\frac{-1}{4}\right) = \frac{-1}{4}$ |
| $\Rightarrow \quad \frac{3}{4} = \frac{3}{4}$ | $\Rightarrow \quad \frac{-1}{4} = \frac{-1}{4}$ |
| $\Rightarrow$ LHS = RHS | $\Rightarrow$ LHS = RHS |
| Hence, verified | Hence, verified |

**Q2.** $3x^2 + 4x - 4$

**Sol.** Let $f(x) = 3x^2 + 4x - 4$

For zeroes of $f(x)$, $f(x) = 0$

$$\therefore \quad 3x^2 + 4x - 4 = 0$$

Splitting the middle term, we get

$$3x^2 + 6x - 2x - 4 = 0$$
$$\Rightarrow \quad 3x(x+2) - 2(x+2) = 0$$
$$\Rightarrow \quad (x+2)(3x-2) = 0$$
$$\Rightarrow \quad x + 2 = 0 \quad \text{or} \quad 3x - 2 = 0$$
$$\Rightarrow \quad x = -2 \quad \text{or} \quad 3x = +2 \Rightarrow x = \frac{2}{3}$$

So, zeroes of $f(x)$ are –2, and 2/3.

$$\text{Sum of roots} = \frac{-b}{a} \qquad (a = 3, b = 4, c = -4)$$
$$\Rightarrow \quad -2 + \frac{2}{3} = \frac{-4}{3}$$
$$\Rightarrow \quad \frac{-6+2}{3} = \frac{-4}{3}$$

$\Rightarrow \quad \frac{-4}{3} = \frac{-4}{3}$

$\Rightarrow \quad \text{LHS} = \text{RHS}$

Hence, verified.

Product of roots = $\frac{c}{a}$

$\Rightarrow \quad -2 \times \frac{2}{3} = \frac{-4}{3}$

$\Rightarrow \quad \frac{-4}{3} = \frac{-4}{3}$

$\Rightarrow \quad \text{LHS} = \text{RHS}$

Hence, verified.

**Q3.** $5t^2 + 12t + 7$

**Sol.** Let $f(t) = 5t^2 + 12t + 7$

For zeroes of $f(t)$, $f(t) = 0$

$\Rightarrow \quad 5t^2 + 12t + 7 = 0$

$\Rightarrow \quad 5t^2 + 7t + 5t + 7 = 0$

$\Rightarrow \quad t(5t + 7) + 1(5t + 7) = 0$

$\Rightarrow \quad (5t + 7)(t + 1) = 0$

$\Rightarrow \quad 5t + 7 = 0 \quad \text{or} \quad (t + 1) = 0$

$\Rightarrow \quad t = \frac{-7}{5} \quad \text{or} \quad t = -1$

**Verification:** $\alpha = -\frac{7}{5} \qquad \beta = -1$

$a = 5, \quad b = 12, \quad c = +7$

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow \quad \frac{-7}{5} - 1 = \frac{-(+12)}{5}$

$\Rightarrow \quad \frac{-7-5}{5} = \frac{-12}{5}$

$\Rightarrow \quad \frac{-12}{5} = \frac{-12}{5}$

$\Rightarrow \quad \text{LHS} = \text{RHS}$

Hence, verified.

$\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow \quad \left(\frac{-7}{5}\right)(-1) = \frac{7}{5}$

$\Rightarrow \quad +\frac{7}{5} = \frac{7}{5}$

$\Rightarrow \quad \text{LHS} = \text{RHS}$

Hence, verified.

**Q4.** $t^3 - 2t^2 - 15t$

**Sol.** Let $f(t) = t^3 - 2t^2 - 15t$

For zeroes of $f(t)$, $f(t) = 0$

$\Rightarrow \quad t^3 - 2t^2 - 15t = 0$

$\Rightarrow \quad t[t^2 - 2 \cdot t - 15] = 0$

$\Rightarrow \quad t[t^2 - 5t + 3t - 15] = 0$

$\Rightarrow \quad t[t(t-5) + 3(t-5)] = 0$

$\Rightarrow \quad t(t-5)(t+3) = 0$

$\Rightarrow \quad t = 0$ or $t - 5 = 0$ or $t + 3 = 0$

$\Rightarrow \quad t = 0$ or $t = 5$ or $t = -3$

So, zeroes of cubic polynomial are $\alpha = 0$, $\beta = 5$, $\gamma = -3$

**Verification:** $\alpha = 0$, $\beta = 5$, $\gamma = -3$

Cubic polynomial,

$f(t) = t^3 - 2t^2 - 15\,t$, which is of the form $at^3 + bt^2 + ct + d$

where $a = 1$, $b = -2$, $c = -15$ and $d = 0$

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\Rightarrow \quad 0 + 5 - 3 = \frac{-(-2)}{1}$$

$$\Rightarrow \quad 2 = 2$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow (0)(5) + (5)(-3) + (-3)(0) = \frac{-15}{1}$$

$$\Rightarrow \quad 0 - 15 + 0 = -15$$

$$\Rightarrow \quad -15 = -15$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha\,\beta\,\gamma = \frac{-d}{a}$$

$$\Rightarrow \quad (0)(5)(-3) = \frac{-0}{1}$$

$$\Rightarrow \quad 0 = 0$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

**Q5.** $2x^2 + \frac{7}{2}x + \frac{3}{4}$

**Sol.** Let $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow \quad 2x^2 + \frac{7}{2}x + \frac{3}{4} = 0$$

$$\Rightarrow \quad 8x^2 + 14x + 3 = 0$$

[As $c$ is positive $(+3)$ so sum of $(8 \times 3)$ factors should be equal to 14]

$$\Rightarrow \quad 8x^2 + 12x + 2x + 3 = 0$$

$$\Rightarrow \quad 4x(2x+3) + 1(2x+3) = 0$$

$$\Rightarrow \quad (2x+3)(4x+1) = 0$$

$\Rightarrow \quad 2x + 3 = 0$ or $4x + 1 = 0$

$\Rightarrow \quad 2x = -3$ or $4x = -1$

$\Rightarrow \quad x = \frac{-3}{2} \quad$ or $\quad x = \frac{-1}{4}$

**Verification:** $\alpha = \frac{-3}{2}$ and $\beta = \frac{-1}{4}$

Quadratic polynomial $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$, which is of the form $ax^2 + bx + c$.

$\therefore a = 2, \quad b = \frac{7}{2}$ and $c = \frac{3}{4}$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \frac{-3}{2} - \frac{1}{4} = \frac{\frac{-7}{2}}{2}$$

$$\Rightarrow \frac{-7}{4} = \frac{-7}{2} \times \frac{1}{2}$$

$$\Rightarrow \frac{-7}{4} = \frac{-7}{4}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \left(\frac{-3}{2}\right)\left(\frac{-1}{4}\right) = \frac{\frac{3}{4}}{2}$$

$$\Rightarrow \frac{+3}{8} = \frac{3}{4} \times \frac{1}{2}$$

$$\Rightarrow \frac{3}{8} = \frac{3}{8}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

**Q6.** $4x^2 + 5\sqrt{2}x - 3$

**Sol.** Let $f(x) = 4x^2 + 5\sqrt{2}x - 3$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow 4x^2 + 5\sqrt{2}x - 3 = 0$$

$$\Rightarrow 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 = 0$$

$$\left[\begin{aligned} \because 4 \times 3 &= 2 \times 2 \times 3 \\ &= \sqrt{2} \times \sqrt{2} \times 2 \times 3 \\ &= 6\sqrt{2} \times \sqrt{2} \end{aligned}\right]$$

$$\Rightarrow 2x[2x + 3\sqrt{2}] - 1[\sqrt{2}x + 3] = 0$$

$$\Rightarrow 2\sqrt{2}x[\sqrt{2}x + 3] - 1[\sqrt{2}x + 3] = 0$$

$$\Rightarrow (\sqrt{2}x + 3)(2\sqrt{2}x - 1) = 0$$

$\Rightarrow \quad \sqrt{2}x + 3 = 0 \quad$ or $\quad 2\sqrt{2}x - 1 = 0$

$\Rightarrow \quad x = \frac{-3}{\sqrt{2}} \quad$ or $\quad x = \frac{1}{2\sqrt{2}}$

**Verification:**

$\alpha = \frac{-3}{\sqrt{2}}, \beta = \frac{1}{2\sqrt{2}}, a = 4, b = 5\sqrt{2}, c = -3$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \frac{-3}{\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{-5\sqrt{2}}{4}$$

$$\Rightarrow \quad \frac{(-6+1)}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{5\sqrt{2}}{4}$$

$$\Rightarrow \quad \frac{-5\sqrt{2}}{4} = -\frac{5\sqrt{2}}{4}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(-\frac{3}{\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}}\right) = \frac{-3}{4}$$

$$\Rightarrow \quad \frac{-3}{4} = \frac{-3}{4}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

**Q7.** $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

**Sol.** Let $f(s) = 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

For zeroes of $f(s)$, $f(s) = 0$

$$\Rightarrow \quad 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2} = 0 \qquad \left[\because \; a \times c = (2\sqrt{2})\right]$$

$$\Rightarrow \quad 2s^2 - 1s - 2\sqrt{2}s + \sqrt{2} = 0 \qquad \text{[Open the brackets]}$$

$$\Rightarrow \quad s(2s-1) - \sqrt{2}\,(2s-1) = 0$$

$$\Rightarrow \quad (2s-1)(s - \sqrt{2}) = 0$$

$$\Rightarrow \quad 2s - 1 = 0 \quad \text{or} \quad s - \sqrt{2} = 0$$

$$\Rightarrow \quad s = \frac{1}{2} \quad \text{or} \quad s = \sqrt{2}$$

**Verification of the relation between α, β, *a*, *b* and *c***

$$\alpha = \frac{1}{2}, \quad \beta = \sqrt{2}, \quad a = 2, \quad b = -(1 + 2\sqrt{2}), \quad c = \sqrt{2}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \frac{1}{2} + \sqrt{2} = \frac{+(1 + 2\sqrt{2})}{2}$$

$$\Rightarrow \quad \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \frac{2\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \sqrt{2}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

**Q8.** $v^2 + 4\sqrt{3}v - 15$

**Sol.** Let $f(v) = v^2 + 4\sqrt{3}v - 15$

For zeroes of $f(v)$, $f(v) = 0$

$$\Rightarrow \quad v^2 + 4\sqrt{3}v - 15 = 0$$

$$\Rightarrow \quad v^2 + 5\sqrt{3}v - 1\sqrt{3}v - 15 = 0 \qquad \begin{bmatrix} 15 = 5 \times 3 \\ \quad = 1 \times 5 \times \sqrt{3} \times \sqrt{3} \end{bmatrix}$$

$$\Rightarrow \quad v(v + 5\sqrt{3}) - \sqrt{3}\,(v + 5\sqrt{3}) = 0$$

$$\Rightarrow \quad (v + 5\sqrt{3})\,(v - \sqrt{3}) = 0$$

$$\Rightarrow \quad (v + 5\sqrt{3}) = 0 \quad \text{or} \quad (v - \sqrt{3}) = 0$$

$$\Rightarrow \quad v = -5\sqrt{3} \quad \text{or} \quad v = \sqrt{3}$$

**Verification of relations between α, β, *a*, *b*, *c***

$\alpha = -5\sqrt{3}, \quad \beta = \sqrt{3}, \quad a = 1, \quad b = 4\sqrt{3} \quad$ and $\quad c = -15$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad -5\sqrt{3} + \sqrt{3} = \frac{-4\sqrt{3}}{1}$$

$$\Rightarrow \quad -4\sqrt{3} = -4\sqrt{3}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad (-5\sqrt{3})\,(\sqrt{3}) = \frac{-15}{1}$$

$$\Rightarrow \quad -5 \times 3 = -15$$

$$\Rightarrow \quad -15 = -15$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

**Q9.** $y^2 + \frac{3}{2}\sqrt{5}y - 5$

**Sol.** Let $f(y) = y^2 + \frac{3}{2}\sqrt{5}y - 5$

For zeroes of $f(y)$, $f(y) = 0$

$$\Rightarrow \quad y^2 + \frac{3}{2}\sqrt{5}y - 5 = 0$$

$$\Rightarrow \quad 2y^2 + 3 \cdot \sqrt{5}y - 10 = 0$$

$$\Rightarrow \quad 2y^2 + 4\sqrt{5}y - 1\sqrt{5}y - 10 = 0 \qquad \begin{bmatrix} 2 \times 10 = 2 \times 2 \times 5 \\ \quad = 2 \times 2 \times \sqrt{5} \times \sqrt{5} \\ \quad = (4 \times 5) \end{bmatrix}$$

$$\Rightarrow \quad 2y(y + 2\sqrt{5}) - \sqrt{5}[y + 2\sqrt{5}] = 0$$

$$\Rightarrow \quad (y + 2\sqrt{5})\,(2y - \sqrt{5}) = 0$$

$$\Rightarrow \quad y + 2\sqrt{5} = 0 \quad \text{or} \quad 2y - \sqrt{5} = 0$$

$$\Rightarrow \quad y = -2\sqrt{5} \quad \text{or} \quad y = \frac{\sqrt{5}}{2}$$

**Verification of the relations between α, β, and *a*, *b*, *c***

$\alpha = -2\sqrt{5}, \quad \beta = \frac{\sqrt{5}}{2}, \quad a = 1, \quad b = \frac{3}{2}\sqrt{5} \quad$ and $\quad c = -5$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad -2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{\frac{-3}{2}\sqrt{5}}{1}$$

$$\Rightarrow \quad \frac{-4\sqrt{5} + \sqrt{5}}{2} = \frac{-3}{2}\sqrt{5}$$

$$\Rightarrow \quad \frac{-3\sqrt{5}}{2} = \frac{-3}{2}\sqrt{5}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad (-2\sqrt{5})\left(\frac{\sqrt{5}}{2}\right) = \frac{-5}{1}$$

$$\Rightarrow \quad -5 = -5$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

**Q10.** $7y^2 - \frac{11}{3}y - \frac{2}{3}$

**Sol.** Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of $f(y)$, $f(y) = 0$

$$\Rightarrow \quad 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

$$\Rightarrow \quad 21y^2 - 11y - 2 = 0$$

$$\Rightarrow \quad 21y^2 - 14y + 3y - 2 = 0$$

$$\Rightarrow \quad 7y(3y-2) + 1(3y-2) = 0$$

$$\Rightarrow \quad (3y-2)(7y+1) = 0$$

$$\Rightarrow \quad 3y - 2 = 0 \quad \text{or} \quad 7y + 1 = 0$$

$$\Rightarrow \quad y = \frac{2}{3} \quad \text{or} \quad y = \frac{-1}{7}$$

**Verification of the relations between $\alpha$, $\beta$, $a$, $b$ and $c$**

$$\alpha = \frac{2}{3}, \quad \beta = \frac{-1}{7}, \quad a = 7, \quad b = -\frac{11}{3}, \quad c = \frac{-2}{3}$$

$$\Rightarrow \quad \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right) - \frac{1}{7} = \frac{+\frac{11}{3}}{7}$$

$$\Rightarrow \quad \frac{14-3}{21} = \frac{11}{3} \times \frac{1}{7}$$

$$\Rightarrow \quad \frac{11}{21} = \frac{11}{21}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right) \times \left(\frac{-1}{7}\right) = \frac{\frac{-2}{3}}{7}$$

$$\Rightarrow \quad \frac{-2}{21} = \frac{-2}{3} \times \frac{1}{7}$$

$$\Rightarrow \quad \frac{-2}{21} = \frac{-2}{21}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

## EXERCISE 2.4

**Q1.** For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomial by factorisation.

(*i*) $\frac{-8}{3}, \frac{4}{3}$ (*ii*) $\frac{21}{8}, \frac{5}{16}$ (*iii*) $-2\sqrt{3}, -9$ (*iv*) $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

**Sol. Main concept:** (*a*) If α, β are the zeroes of $f(x)$, then

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

(*b*) The zeroes of $f(x)$ are given by $f(x) = 0$.

(*i*) $\alpha + \beta = \frac{-8}{3}$ and $\alpha \cdot \beta = \frac{4}{3}$ [Given]

$\therefore$ $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ [Formula]

$$= x^2 - \left(\frac{-8}{3}\right)x + \frac{4}{3}$$

Multiplying or dividing $f(x)$ by any real number does not affect the zeroes of polynomial.

So, $f(x) = 3x^2 + 8x + 4$ [Multiplying by LCM 3]

For zeroes of $f(x)$, $f(x) = 0$

$\Rightarrow$ $3x^2 + 8x + 4 = 0$

$\Rightarrow$ $3x^2 + 6x + 2x + 4 = 0$

$\Rightarrow$ $3x(x + 2) + 2(x + 2) = 0$

$\Rightarrow$ $(x + 2)(3x + 2) = 0$

$\Rightarrow$ $x + 2 = 0$ or $3x + 2 = 0$

$\Rightarrow$ $x = -2$ or $x = \frac{-2}{3}$

$\therefore$ $\alpha = -2$ and $\beta = \frac{-2}{3}$

(*ii*) $\alpha + \beta = \frac{21}{8}$ and $\alpha \cdot \beta = \frac{5}{16}$ [Given]

$f(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta$ [Formula]

$\Rightarrow$ $f(x) = x^2 - \left(\frac{21}{8}\right)x + \left(\frac{5}{16}\right)$

Multiplying (or dividing) $f(x)$ by any real number does not affect the zeroes of $f(x)$ so, multiplying $f(x)$ by 16 (LCM), we get

$$f(x) = 16x^2 - 42x + 5$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$\Rightarrow$ $16x^2 - 42x + 5 = 0$

$\Rightarrow$ $16x^2 - 40x - 2x + 5 = 0$

$\Rightarrow$ $8x(2x - 5) - 1(2x - 5) = 0$

$$\Rightarrow \quad (2x-5)(8x-1) = 0$$
$$\Rightarrow \quad 2x-5 = 0 \quad \text{or} \quad 8x-1 = 0$$
$$\Rightarrow \quad x = \frac{5}{2} \quad \text{or} \quad x = \frac{1}{8}$$
$$\therefore \alpha = \frac{5}{2} \text{ and } \beta = \frac{1}{8}$$

(*iii*) $\alpha + \beta = -2\sqrt{3}$ and $\alpha\beta = -9$ [Given]

$$f(x) = x^2 - (\alpha+\beta)x + \alpha\beta \qquad \text{[Formula]}$$
$$= x^2 - (-2\sqrt{3})x + (-9)$$
$$\Rightarrow \quad f(x) = x^2 + 2\sqrt{3}x - 9$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow \quad x^2 + 2\sqrt{3}x - 9 = 0$$
$$\Rightarrow \quad x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$$
$$\Rightarrow \quad x(x+3\sqrt{3}) - \sqrt{3}(x+3\sqrt{3}) = 0$$
$$\Rightarrow \quad (x+3\sqrt{3})(x-\sqrt{3}) = 0$$
$$\Rightarrow \quad x+3\sqrt{3} = 0 \quad \text{or} \quad (x-\sqrt{3}) = 0$$
$$\Rightarrow \quad x = -3\sqrt{3} \quad \text{or} \quad x = \sqrt{3}$$
$$\therefore \quad \alpha = -3\sqrt{3} \quad \text{and} \quad \beta = \sqrt{3}$$

(*iv*) $\alpha + \beta = \dfrac{-3}{2\sqrt{5}}$ and $\alpha \cdot \beta = -\dfrac{1}{2}$ [Given]

$$f(x) = x^2 - (\alpha+\beta)x + \alpha\beta \qquad \text{[Formula]}$$
$$= x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(-\frac{1}{2}\right)$$
$$\Rightarrow \quad f(x) = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$$

Multiplying or dividing $f(x)$ by any real number does not affect the zeroes of $f(x)$. On multiplying $f(x)$ by $2\sqrt{5}$ (LCM), we get

$$f(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow \quad 2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$$
$$\Rightarrow \quad 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$$
$$\Rightarrow \quad \sqrt{5}x(2x+\sqrt{5}) - 1(2x+\sqrt{5}) = 0$$
$$\Rightarrow \quad (2x+\sqrt{5})(\sqrt{5}x-1) = 0$$
$$\Rightarrow \quad (2x+\sqrt{5}) = 0 \quad \text{or} \quad \sqrt{5}x - 1 = 0$$

$\Rightarrow \quad x = \dfrac{-\sqrt{5}}{2} \quad$ or $\quad x = \dfrac{1}{\sqrt{5}}$

$\therefore \quad \alpha = \dfrac{-\sqrt{5}}{2} \quad$ and $\quad \beta = \dfrac{1}{\sqrt{5}}$

**Q2.** Given that the zeroes of cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a$, $(a + b)$, $(a + 2b)$ for some real numbers $a$ and $b$, find the values of $a$ and $b$ as well as the zeroes of the given polynomial.

**Sol. Main concept:** $\alpha + \beta + \gamma = \dfrac{-b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{+c}{a}$ and $\alpha\beta\gamma = \dfrac{-d}{a}$

Let $f(x) = x^3 - 6x^2 + 3x + 10$ [given] ...($i$)

$\alpha = a, \quad \beta = a + b \quad$ and $\quad \gamma = a + 2b$ [Given]

But, $f(x) = ax^3 + bx^2 + cx + d$ ...($ii$)

$\therefore\ a = 1, \quad b = -6, \quad c = 3 \quad$ and $\quad d = +10$ [Comparing ($i$) and ($ii$)]

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$\Rightarrow \quad a + a + b + a + 2b = \dfrac{+6}{1} \Rightarrow 3a + 3b = 6$

$\Rightarrow \quad a + b = 2$

$\Rightarrow \quad b = 2 - a$ ...($iii$)

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$\Rightarrow \quad a(a + b) + (a + b)(a + 2b) + (a + 2b)(a) = \dfrac{3}{1}$

$\Rightarrow \quad a^2 + ab + a^2 + 2ab + ab + 2b^2 + a^2 + 2ab = 3$

$\Rightarrow \quad 3a^2 + 6ab + 2b^2 = 3$

$\Rightarrow \quad 3a^2 + 6a(2 - a) + 2(2 - a)^2 = 3$ [Using ($iii$)]

$\Rightarrow \quad 3a^2 + 12a - 6a^2 + 2(4 + a^2 - 4a) = 3$

$\Rightarrow \quad -3a^2 + 12a + 8 + 2a^2 - 8a - 3 = 0$

$\Rightarrow \quad -a^2 + 4a + 5 = 0$

$\Rightarrow \quad a^2 - 4a - 5 = 0$

$\Rightarrow \quad a^2 - 5a + a - 5 = 0$

$\Rightarrow \quad a(a - 5) + 1(a - 5) = 0$

$\Rightarrow \quad (a + 1)(a - 5) = 0$

$\Rightarrow \quad (a + 1) = 0 \quad$ or $\quad (a - 5) = 0$

$\Rightarrow \quad a = -1 \quad$ or $\quad a = 5$

Now, $\quad b = 2 - a$ [From ($iii$)]

When $a = 5$, $b = 2 - 5 = -3$

When $a = -1$, $b = 2 - (-1) = 3$

If $a = -1$ and $b = 3$, then zeroes are, $a, (a + b), (a + 2b)$

$$= -1, (-1 + 3), [-1 + 2(3)]$$
$$= -1, 2, 5$$

If $a = 5$, and $b = -3$, then zeroes are $5, [5 + (-3)], [5 + 2(-3)] = 5, 2, -1$

So, zeroes in both cases are $\beta = 2$, $\gamma = -1$ and $\alpha = 5$.

**Q3.** Given that $\sqrt{2}$ is a zero of a cubic polynomial

$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

**Sol. Main concept:** Using Euclid's division algorithm here, remainder is zero. Then quotient will be quadratic whose zeroes can be find out by factorisation.

Let $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

If $\sqrt{2}$ is the zero of $f(x)$, then $(x - \sqrt{2})$ will be a factor of $f(x)$. So, by remainder theorem when $f(x)$ is divided by $(x - \sqrt{2})$, the quotient comes out to be quadratic.

$$\begin{array}{r|l}
 & 6x^2 + 7\sqrt{2}x + 4 \\
\hline
x - \sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & \underset{-}{6x^3} \underset{+}{-} 6\sqrt{2}x^2 \\
\hline
 & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & \underset{-}{7\sqrt{2}x^2} \underset{+}{-} 14x \\
\hline
 & 4x - 4\sqrt{2} \\
 & \underset{-}{4x} \underset{+}{-} 4\sqrt{2} \\
\hline
 & 0
\end{array}$$

$\therefore \quad f(x) = (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4)$ (By Euclid's division algorithm)

$$= (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)$$

For zeroes of $f(x)$, $f(x) = 0$

$$\therefore \quad (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) = 0$$

$$\Rightarrow \quad (x - \sqrt{2})[2x(3x + 2\sqrt{2}) + \sqrt{2}(3x + 2\sqrt{2})] = 0$$

$$\Rightarrow \quad (x - \sqrt{2})(3x + 2\sqrt{2})(2x + \sqrt{2}) = 0$$

$$\Rightarrow x - \sqrt{2} = 0 \quad \text{or} \quad 3x + 2\sqrt{2} = 0 \quad \text{or} \quad 2x + \sqrt{2} = 0$$

$$\Rightarrow \quad x = \sqrt{2} \quad \text{or} \quad x = \frac{-2\sqrt{2}}{3} \quad \text{or} \quad x = \frac{-\sqrt{2}}{2}$$

So, other two roots are = $\dfrac{-2\sqrt{2}}{3}$ and $\dfrac{-\sqrt{2}}{2}$.

**Q4.** Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of two polynomials.

**Sol. Main concept:** Factor theorem and Euclid's division algorithm.

By factor theorem and Euclid's division algorithm, we get

$$f(x) = g(x) \times q(x) + r(x)$$

Let $f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6$ ...(*i*)

and $g(x) = x^2 + 2x + k$

$$\therefore \quad \begin{array}{r|l} & 2x^2 - 3x - 8 - 2k \\ \hline x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\ & \underset{-}{2x^4} \underset{-}{+} 4x^3 \underset{-}{+} 2kx^2 \\ \hline & -3x^3 - 14x^2 - 2kx^2 + 5x + 6 \\ & \underset{+}{-3x^3} \underset{+}{-} 6x^2 \quad \underset{+}{-} 3kx \\ \hline & -8x^2 - 2kx^2 + 5x + 3kx + 6 \\ & \underset{+}{-8x^2} \quad \underset{+}{-} 16x \quad \underset{+}{-} 8k \\ \hline & -2kx^2 + 21x + 3kx + 8k + 6 \\ & \underset{+}{-2kx^2} \quad \underset{+}{-} 4kx \underset{+}{-} 2k^2 \\ \hline & 21x + 7kx + 2k^2 + 8k + 6 \end{array}$$

But, $r(x) = 0$

$\therefore$ $(21 + 7k)x + 2k^2 + 8k + 6 = 0x + 0$

$\Rightarrow \quad 21 + 7k = 0 \quad$ and $\quad 2k^2 + 8k + 6 = 0$

$2k^2 + 6k + 2k + 6 = 0$

$\Rightarrow \quad k = \dfrac{-21}{7}$

$\Rightarrow \quad k = -3$

$\Rightarrow \quad 2k(k + 3) + 2(k + 3) = 0$

$\Rightarrow \quad (k + 3)(2k + 2) = 0$

$\Rightarrow \quad k + 3 = 0$ or $2k + 2 = 0$

$\Rightarrow \quad k = -3$ or $k = -1$

$\therefore$ Common solution is $k = -3$

So, $\quad q(x) = 2x^2 - 3x - 8 - 2(-3)$

$= 2x^2 - 3x - 8 + 6$

$\Rightarrow \quad q(x) = 2x^2 - 3x - 2$

$\therefore \quad f(x) = g(x)\, q(x) + 0$

$= (x^2 + 2x - 3)(2x^2 - 3x - 2)$

$= (2x^2 - 4x + 1x - 2)(x^2 + 3x - 1x - 3)$

$$= [2x(x-2) + 1\,(x-2)]\,[x(x+3) - 1(x+3)]$$

$\Rightarrow \qquad f(x) = (x-2)\,(2x+1)\,(x+3)\,(x-1)$

For zeroes of $f(x)$, $f(x) = 0$

$\therefore \qquad (x-1)\,(x-2)\,(x+3)\,(2x+1) = 0$

$\Rightarrow (x-1) = 0, \quad (x-2) = 0, \quad (x+3) = 0 \quad \text{and} \quad 2x+1 = 0$

$\Rightarrow x = 1,\ x = 2,\ x = -3 \text{ and } x = \dfrac{-1}{2}$

So, zeroes of $f(x)$ are 1, 2, $-3$, and $\dfrac{-1}{2}$.

**Q5.** Given that $(x - \sqrt{5})$ is a factor of cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

**Sol. Main concept:** Factor theorem, Euclid's division algorithm.

Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$

and $\qquad g(x) = (x - \sqrt{5})$

$\because\ g(x)$ is a factor of $f(x)$ so $f(x) = q(x)\,(x - \sqrt{5})$

$$\begin{array}{r|l}
 & x^2 - 2\sqrt{5}x + 3 \\ \hline
x - \sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & \underset{-}{x^3}\ \underset{+}{-}\ \sqrt{5}x^2 \\ \hline
 & -2\sqrt{5}x^2 + 13x - 2\sqrt{5} \\
 & \underset{+}{-}\ 2\sqrt{5}x^2\ \underset{-}{+}\ 10x \\ \hline
 & +3x - 3\sqrt{5} \\
 & \underset{-}{+}\ 3x\ \underset{+}{-}\ 3\sqrt{5} \\ \hline
 & 0
\end{array}$$

But, $\qquad f(x) = q(x)\,g(x)$

$\therefore \qquad f(x) = (x^2 - 2\sqrt{5}x + 3)\,(x - \sqrt{5})$

$\Rightarrow\ f(x) = [x^2 - \{(\sqrt{5}+\sqrt{2}) + (\sqrt{5}-\sqrt{2})\}x + (\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})][(x) - \sqrt{5}]$

$\qquad = [x^2 - (\sqrt{5}+\sqrt{2})x - (\sqrt{5}-\sqrt{2})x + (\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})][x - \sqrt{5}]$

$\qquad = x[x - (\sqrt{5}+\sqrt{2})] - (\sqrt{5}-\sqrt{2})\,[x - (5+\sqrt{2})]\,[x - \sqrt{5}]$

$\Rightarrow\ f(x) = (x - \sqrt{5} - \sqrt{2})\,(x - \sqrt{5} + \sqrt{2})\,(x - \sqrt{5})$

For zeroes of $f(x)$, $f(x) = 0$

$\Rightarrow\ (x - \sqrt{5} - \sqrt{2})\,(x - \sqrt{5} + \sqrt{2})\,(x - \sqrt{5}) = 0$

$\Rightarrow (x - \sqrt{5} - \sqrt{2}) = 0$ or $(x - \sqrt{5} + \sqrt{2}) = 0$ or $(x - \sqrt{5}) = 0$

$\Rightarrow \qquad x = \sqrt{5} + \sqrt{2}$ or $x = \sqrt{5} - \sqrt{2}$ or $x = +\sqrt{5}$

$\therefore$ Zeroes are $(\sqrt{5} + \sqrt{2}), (\sqrt{5} - \sqrt{2})$ and $\sqrt{5}$.

**Q6.** For which values of $a$ and $b$ are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

**Sol. Main concept:** Factor theorem and Euclid's division algorithm.
By factor theorem if $q(x)$ is a factor of $p(x)$, then $r(x)$ must be zero.

$$p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$$

$$q(x) = x^3 + 2x^2 + a$$

$$\begin{array}{r|l}
 & x^2 - 3x + 2 \\ \hline
x^3 + 2x^2 + a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\
 & \underset{-}{x^5} \underset{-}{+} 2x^4 \qquad \underset{-}{+} ax^2 \\ \hline
 & -3x^4 - 4x^3 - ax^2 + 3x^2 + 3x + b \\
 & \underset{+}{-3x^4} \underset{+}{-} 6x^3 \qquad \underset{+}{-} 3ax \\ \hline
 & 2x^3 - ax^2 + 3x^2 + 3ax + 3x + b \\
 & \underset{-}{2x^3} \qquad \underset{-}{+} 4x^2 \qquad \underset{-}{+} 2a \\ \hline
 & -ax^2 - x^2 + 3ax + 3x - 2a + b
\end{array}$$

So, by factor theorem remainder must be zero i.e.,

$$r(x) = 0$$

$$\Rightarrow \qquad -(a+1)x^2 + (3a+3)x + (b-2a) = 0x^2 + 0x + 0$$

Comparing the coefficients of $x^2$, $x$ and constt. on both sides, we get

$$-(a+1) = 0 \quad \text{and} \quad 3a + 3 = 0 \quad \text{and} \quad b - 2a = 0$$

$$\Rightarrow \qquad a = -1 \quad \text{and} \quad a = -1 \quad \text{and} \quad b - 2(-1) = 0$$

$$\Rightarrow \qquad b = -2$$

For $a = -1$ and $b = -2$, zeroes of $q(x)$ will be zeroes of $p(x)$.

For zeroes of $p(x)$, $p(x) = 0$

$$\Rightarrow \qquad (x^3 + 2x^2 + a)(x^2 - 3x + 2) = 0 \qquad [\because \ a = -1]$$

$$\Rightarrow \qquad [x^3 + 2x^2 - 1][x^2 - 2x - 1x + 2] = 0$$

$$\Rightarrow \qquad (x^3 + 2x^2 - 1)[x(x-2) - 1(x-2)] = 0$$

$$\Rightarrow \qquad (x^3 + 2x^2 - 1)(x-2)(x-1) = 0$$

Hence, $x = 2$ and 1 are not the zeroes of $q(x)$.