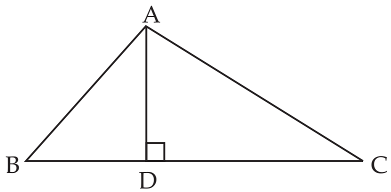
EXERCISE 6.1

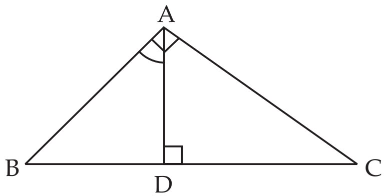
Choose the correct answer from the given four options:

Q1. In the given figure, if $\angle BAC = 90°$ and $AD \perp BC$. Then,



(*a*) $BD \cdot DC = BC^2$
(*b*) $AB \cdot AC = BC^2$
(*c*) $BD \cdot CD = AD^2$
(*d*) $AB \cdot AC = AD^2$

Sol. (*c*): In ΔADC and ΔADB,



$\angle BDA = \angle ADC = 90°$ [Given]

$\angle B = \angle DAC = (90° - C)$

$\therefore \quad \Delta ADB \sim \Delta CDA$

[By AA similarity critierion]

$$\Rightarrow \quad \frac{AD}{CD} = \frac{AB}{CA} = \frac{DB}{DA}$$

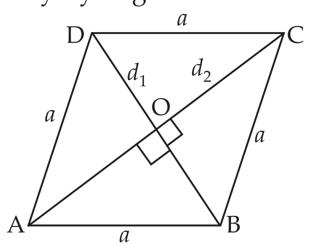
$\therefore \quad AD^2 = BD \cdot DC$

Q2. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is

(*a*) 9 cm (*b*) 10 cm (*c*) 8 cm (*d*) 20 cm

Sol. (*b*): Let the length of the side of the rhombus is *a* cm.

As the diagonals of rhombus bisect at 90° so by Pythagoras theorem in right angled ΔOAB,



$$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$

$$= \left(\frac{12}{2}\right)^2 + \left(\frac{16}{2}\right)^2$$

$$= (6)^2 + (8)^2 = 36 + 64$$

$\Rightarrow \quad a^2 = 100$

$\Rightarrow \quad a = 10$ cm

Q3. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(*a*) $BC \cdot EF = AC \cdot FD$ (*b*) $AB \cdot EF = AC \cdot DE$
(*c*) $BC \cdot DE = AB \cdot EF$ (*d*) $BC \cdot DE = AB \cdot FD$

Sol. (*c*): $\Delta ABC \sim \Delta EDF$ [Given]

$$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF} \quad ...(i)$$

So, every statement will be true if it satisfies the above relation, *i.e.*, LHS from option and RHS from (*i*).

(*a*) BC · EF = AC · DF True
(*b*) AB · EF = AC · DE True
(*c*) BC · DE = AB · EF False
(*d*) BC · DE = AB · DF True

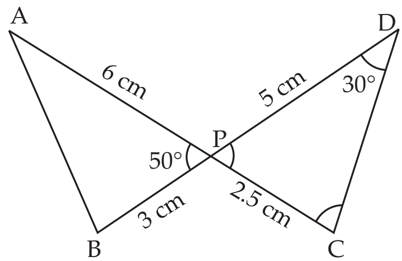
Q4. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(*a*) ΔPQR ~ ΔCAB (*b*) ΔPQR ~ ΔABC
(*c*) ΔCBA ~ ΔPQR (*d*) ΔBCA ~ ΔPQR

Sol. (*a*): Here, vertex P corresponds to vertex C, vertex Q corresponds to vertex A and vertex R corresponds to vertex B. Symbolically, we write the similarity of these two triangles as ΔPQR ~ ΔCAB.

Hence, (*a*) is the correct answer.

Q5. In the given figure, two line segments AC and BD intersect each other at P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°, then ∠PBA is equal to



(*a*) 50° (*b*) 30°
(*c*) 60° (*d*) 100°

Sol. (*d*): Considering ΔAPB and ΔDPC

$$\frac{PA}{PC} = \frac{6.0}{2.5} = \frac{12}{5}$$

$$\frac{PB}{PD} = \frac{3}{5} \neq \frac{PA}{PC}$$

So, the above solution is rejected.

Now, $$\frac{PA}{PD} = \frac{6}{5}$$

$$\frac{PB}{PC} = \frac{3.0}{2.5} = \frac{6}{5}$$

$$\Rightarrow \quad \frac{PA}{PD} = \frac{PB}{PC}$$

$\angle APB = \angle CPD = 50°$ [Vertically opp ∠s]

∴ $\Delta APB \sim \Delta DPC$ [By SAS similarity criterion]

$\angle PBA = \angle PCD$ [∵ Corresponding ∠s of similar Δs are equal]

In ΔDPC, $\angle DPC = \angle APB = 50°$ [Vertically opp. ∠s]

$\angle D = 30°$

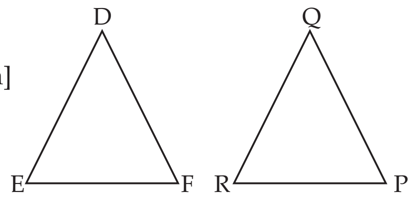
$\therefore \quad \angle PCD = \angle C = 180° - 50° - 30° = 180 - 80° = 100°$

$\Rightarrow \quad \angle PBA = 100°$ verifies the option (*d*).

Q6. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(*a*) $\frac{EF}{PR} = \frac{DF}{PQ}$ (*b*) $\frac{DE}{PQ} = \frac{EF}{RP}$ (*c*) $\frac{DE}{QR} = \frac{DF}{PQ}$ (*d*) $\frac{EF}{RP} = \frac{DE}{QR}$

Sol. (*b*): In ΔDEF and ΔPQR,



$$\left.\begin{aligned}\angle D &= \angle Q\\ \angle E &= \angle R\end{aligned}\right\} \text{[Given]}$$

$\therefore \quad \angle F = \angle P$

$\therefore \quad \Delta DEF \sim \Delta QRP$

$$\therefore \quad \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{RP}$$

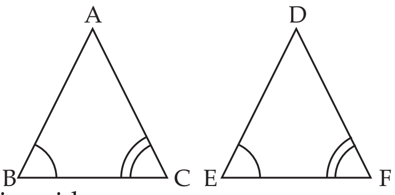
Hence, (*b*) is not true.

Q7. In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are

(*a*) congruent but not similar (*b*) similar but not congruent

(*c*) neither congruent nor similar (*d*) congruent as well as similar

Sol. (*b*): In ΔABC and ΔDEF,



$$\left.\begin{aligned}\angle B &= \angle E\\ \angle C &= \angle F\end{aligned}\right\} \text{[Given]}$$

$\therefore \quad \Delta ABC \sim \Delta DEF$

[By AA similarity criterion]

So, AB and DE sides are corresponding sides.

But, $AB = 3DE$ [Given]

So, ΔABC cannot be congruent to ΔDEF.

So, Δs are similar but not congruent.

Q8. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{3}$. Then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to

(*a*) 9 (*b*) 3 (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{9}$

Sol. (*a*): $\Delta ABC \sim \Delta PQR$ [Given]

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9} \quad \text{[By area theorem]}$$

$$\text{or} \quad = \frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{9}{1}$$

Hence, verifies option (*a*).

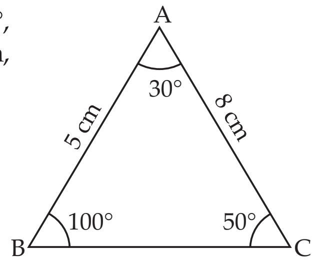
Q9. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, AB = 5 cm, AC = 8 cm and DF = 7.5 cm, then which of the following is true?

(*a*) DE = 12 cm, $\angle F = 50°$

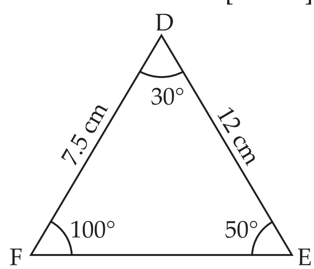
(*b*) DE = 12 cm, $\angle F = 100°$

(*c*) EF = 12 cm, $\angle D = 100°$

(*d*) EF = 12 cm, $\angle D = 30°$



Sol. (*b*): $\Delta ABC \sim \Delta DEF$ [Given]



$$\therefore \quad \frac{AB}{DF} = \frac{AC}{DE} = \frac{BC}{FE}$$

$$\Rightarrow \quad \frac{5}{7.5} = \frac{8}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \quad DE = \frac{8 \times 7.5}{5} = 12 \text{ cm}$$

Now, $\angle A = \angle D = 30°$

$\angle B = \angle F = 180° - 30° - 50° = 100°$

$\angle C = \angle E = 50°$

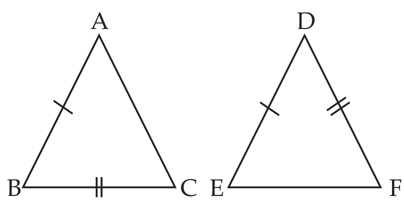
$\therefore$ Verifies the option (*b*) i.e., DE = 12 cm, $\angle F = 100°$.

Q10. If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(*a*) $\angle B = \angle E$ (*b*) $\angle A = \angle D$ (*c*) $\angle B = \angle D$ (*d*) $\angle A = \angle F$

Sol. (*c*): In ΔABC and ΔDEF,

$$\frac{AB}{DE} = \frac{BC}{FD}$$



Angle formed by AB and BC is $\angle B$.

Angle formed by DE and FD is $\angle D$.

So, $\angle B = \angle D$

$\therefore \quad \Delta ABC \sim \Delta EDF$ [By SAS similarity criterion]

Hence, (*c*) is the correct answer.

Q11. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then PR is equal to

(*a*) 10 cm (*b*) 12 cm (*c*) $\frac{20}{3}$ cm (*d*) 8 cm

Sol. (*a*): $\because \quad \Delta ABC \sim \Delta QRP$ [Given]

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{BC^2}{RP^2} = \frac{AB^2}{QR^2} \quad \text{[By area theorem]}$$

$$\Rightarrow \quad \frac{9}{4} = \frac{15^2}{RP^2} = \frac{18^2}{QR^2}$$

$\Rightarrow \quad RP^2 = \dfrac{15 \times 15 \times 4}{9}$

$\Rightarrow \quad RP^2 = 100$

$\Rightarrow \quad RP = 10$ cm

Hence, verifies the option (*a*).

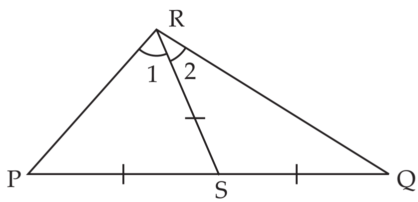
Q12. If S is a point on side PQ, of a ΔPQR such that PS = SQ = RS, then

(*a*) $PR \cdot QR = RS^2$

(*b*) $QS^2 + RS^2 = QR^2$

(*c*) $PR^2 + QR^2 = PQ^2$

(*d*) $PS^2 + RS^2 = PR^2$



Sol. (*c*): In ΔPQR,

$$PS = SQ = RS$$

Now, in ΔPSR,

$$PS = SR$$

$\therefore \quad \angle P = \angle 1$

[Angles opposite to equal sides in a triangle are equal]

Similarly, in $\angle SRQ$,

$$\angle Q = \angle 2$$

Now, in ΔPQR,

$\angle P + \angle Q + \angle R = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad \angle 1 + \angle 2 + (\angle 1 + \angle 2) = 180°$

$\Rightarrow \quad 2(\angle 1 + \angle 2) = 180°$

$\Rightarrow \quad \angle 1 + \angle 2 = 90°$

$\Rightarrow \quad \angle PRQ = 90°$

By Pythagoras theorem, we have

$$PQ^2 = PR^2 + RQ^2$$

Hence, verifies the option (*c*).

EXERCISE 6.2

Q1. Is the triangle with sides 25 cm, 5 cm, and 24 cm a right triangle? Give reasons for your answer.

Sol. False: By converse of Pythagoras theorem, this Δ will be right angle triangle if

$$(25)^2 = (5)^2 + (24)^2$$

$$\Rightarrow \quad 625 = 25 + 576$$

$$\Rightarrow \quad 625 \neq 601$$

So, the given triangle is not right angled triangle.

Q2. It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?

Sol. False: When $\Delta DEF \sim \Delta RPQ$, each angle of a triangle will be equal to the corresponding angle of similar triangle so

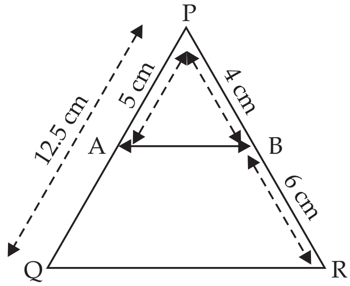
$$\angle D = \angle R$$

$$\angle E = \angle P$$
$$\angle F = \angle Q$$

So, $\angle D = \angle R$ is true but $\angle F \neq \angle P$.

Hence, it is not true that $\angle D = \angle R$ and $\angle F = \angle P$.

Q3. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reasons for your answer.



Sol. True: By converse of BPT, AB will be parallel to QR if AB, divides PQ and PR in the same ratio i.e.,

$$\frac{AP}{AQ} = \frac{PB}{BR}$$

$$\Rightarrow \quad \frac{5}{12.5 - 5} = \frac{4}{6}$$

$$\Rightarrow \quad \frac{5.0}{7.5} = \frac{2}{3} \quad \text{or} \quad \frac{2}{3} = \frac{2}{3}$$

So, AB is parallel to QR. Hence, the given statement AB || QR is true.

Q4. In the given figure, BD and CE intersect each other at P. Is $\Delta PBC \sim \Delta PDE$? Why?

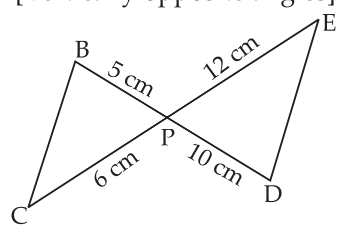
Sol. True: In ΔPBC and ΔPDE, we have

$$\angle BPC = \angle DPE \quad \text{[Vertically opposite angles]}$$

$$\frac{BP}{PD} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{PC}{PE} = \frac{6}{12} = \frac{1}{2}$$

$$\therefore \quad \frac{BP}{PD} = \frac{PC}{PE}$$



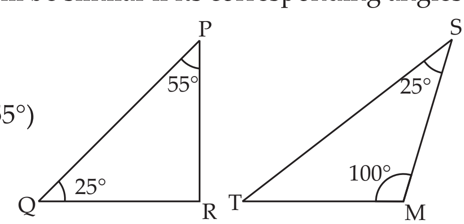
Hence, $\Delta BPC \sim \Delta DPE$ [By SAS similarity criterion]

Hence, the given statement is true.

Q5. In ΔPQR and ΔMST, $\angle P = 55°$, $\angle Q = 25°$, $\angle M = 100°$, $\angle S = 25°$. Is $\Delta QPR \sim \Delta TSM$? Why?

Sol. False: ΔQPR and ΔTSM will be similar if its corresponding angles are equal

$$\angle Q = 25°$$
$$\angle P = 55°$$
$$\Rightarrow \quad \angle R = 180° - (25° + 55°)$$
$$= 180° - 80°$$
$$\Rightarrow \quad \angle R = 100°$$
$$\angle S = 25°$$



$\angle M = 100°$

$\Rightarrow \quad \angle T = 180° - (100° + 25°) = 55°$

$\therefore \quad \angle Q \neq \angle T$

$\angle P \neq \angle S$

$\angle R \neq \angle M$

So, ΔQPR is not similar to ΔTSM. So, the given statement $\Delta QPR \sim \Delta TSM$ is false.

Q6. Is the following statement true? Why?

"Two quadrilaterals are similar if their corresponding angles are equal".

Sol. False: Two quadrilaterals will be similar if their corresponding angles as well as ratio of sides are also equal. So, the given statement is false.

Q7. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Sol. True: Let the two sides of ΔABC are AB = 3 cm, AC = 4 cm and perimeter AB + BC + AC = 13 cm, then BC = 13 – 7 = 6 cm.

According to the question, the sides of another ΔDEF are

$$DE = 3 \times 3 = 9,$$

$$DF = 3 \times 4 = 12,$$

and $\quad DE + DF + EF = 3 \times 13 = 39$

So, $\quad EF = 39 - 12 - 9 = 18$

$$\therefore \quad \frac{DE}{AB} = \frac{9}{3} = \frac{3}{1}$$

$$\frac{DF}{AC} = \frac{12}{4} = \frac{3}{1}$$

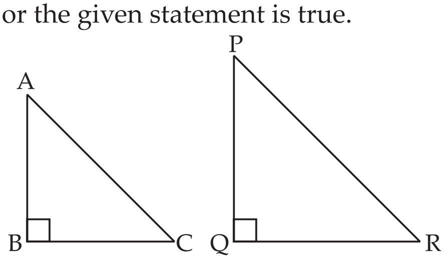
$$\frac{EF}{BC} = \frac{18}{6} = \frac{3}{1}$$

$$\therefore \quad \frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC} = \frac{3}{1}$$

As the ratio of corresponding sides in two Δs are same then $\Delta DEF \sim \Delta ABC$ by SSS similarity criterion.

Hence, the triangles are similar or the given statement is true.

Q8. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that two triangles will be similar? Why?



Sol. True: In ΔABC and ΔPQR,

$\angle B = \angle Q = 90°$ [Given]

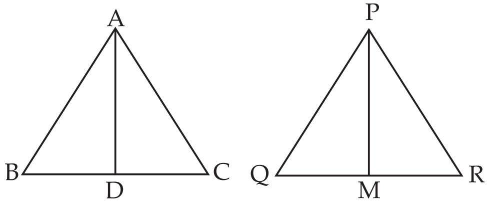
$\angle C = \angle R$ [Given]

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By AA similarity criterion]

Hence, the statement that two triangles are similar is true.

Q9. The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that ratio of their areas is $\frac{6}{5}$? Why?

Sol. False: If two triangles are similar, then the ratio of areas of two triangles will be equal to the square of the ratios of their corresponding sides or altitudes or angle bisectors,



If $\Delta ABC \sim \Delta PQR$, then

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PM}\right)^2$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25} \neq \frac{6}{5}$$

So, the given statement is false.

Q10. D is the point on side QR of ΔPQR such that $PD \perp QR$. Will it be correct to say that $\Delta PQD \sim \Delta RPD$? Why?

Sol. False: In ΔPDQ and ΔPDR,

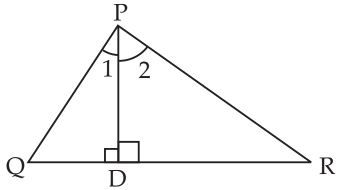
$PD \perp QR$ [Given]

$\therefore \quad \angle PDQ = \angle PDR = 90°$

PD does not bisect $\angle P$.

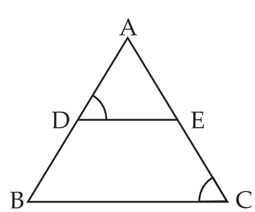
$\therefore \quad \angle 1 \neq \angle 2$

$\angle Q \neq \angle R \quad [\because PQ \neq QR]$



Any ratio of sides are also not equal. So, ΔPDQ is not similar to ΔPDR. Hence, the given statement is false.

Q11. In the given figure, $\angle D = \angle C$, then is it true that $\Delta ADE \sim \Delta ACB$? Why?



Sol. True: In ΔADE and ΔABC,

$\angle D = \angle C$ [Given]

$\angle A = \angle A$ [Common]

$\therefore \Delta ADE \sim \Delta ACB$ [By AA similarity criterion]

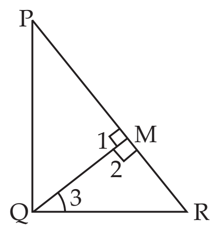
Q12. Is it true to say that if in two triangls, an angle of one triangle is equal to an angle of another triangle and, two sides of one triangle are proportional to the two sides of the other triangle, then triangles are similar? Give reasons for your answer.

Sol. False: Here, the ratio of two sides of a triangle is equal to the ratio of corresponding two sides of other triangle, although the one angle of one triangle is equal to one angle of other triangle but, not included angles of proportional sides are equal.

So, triangles are not similar. Hence, the given statement is false.

EXERCISE 6.3

Q1. In a ΔPQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that $QM \perp PR$. Prove that $QM^2 = PM \times MR$.



Sol. Given: In ΔPQR,

$$PR^2 - PQ^2 = QR^2$$

$\Rightarrow$ $PR^2 = PQ^2 + QR^2$

$\Rightarrow$ PR is hypotenuse.

Also, $QM \perp PR$

To Prove: $MQ^2 = MP \times MR$

Proof: In ΔPQR,

$PR^2 - PQ^2 = QR^2$ [Given]

$\Rightarrow$ $PR^2 = PQ^2 + QR^2$

$\therefore$ $\angle PQR = 90°$ [By conv. of Pythagoras theorem]

In ΔQMP and ΔQMR, [$\because$ Sides QM, MP and MR form these]

$QM \perp PR$

$\therefore$ $\angle 1 = \angle 2 = 90°$

$\angle 3 = 90° - \angle R$

$\angle P = 90° - \angle R$

$\Rightarrow$ $\angle 3 = \angle P$

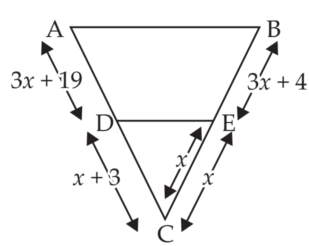
$\therefore$ $\Delta QMP \sim \Delta QMR$ [By AA similarity criterion]

$\Rightarrow$ $\dfrac{PQ}{QR} = \dfrac{PM}{QM} = \dfrac{QM}{RM}$

$\Rightarrow$ $QM^2 = PM \times RM$

Hence, proved.

Q2. Find the value of x for which DE || AB in the given figure.



Sol. In ΔABC, DE || AB.

$\Rightarrow$ $\dfrac{AD}{DC} = \dfrac{BE}{EC}$

$\Rightarrow$ $\dfrac{3x+19}{x+3} = \dfrac{3x+4}{x}$

$\Rightarrow$ $x(3x + 19) = (x + 3)(3x + 4)$

$\Rightarrow$ $3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$\Rightarrow$ $3x^2 - 3x^2 + 19x - 13x = 12$

$\Rightarrow \quad 6x = 12$

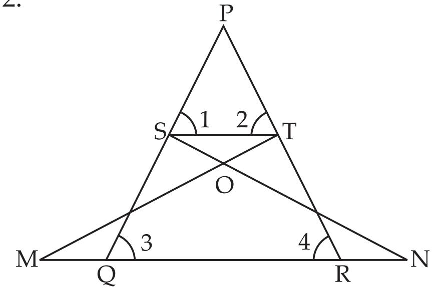
$\Rightarrow \quad x = \dfrac{12}{6}$

$\Rightarrow \quad x = 2$

Hence, the required value of x is 2.

Q3. In the given figure, $\angle 1 = \angle 2$ and $\Delta NQS \cong \Delta MTR$.

Prove that $\Delta PTS \sim \Delta PRQ$.



Sol. Given: In ΔPQR,
point S is on PQ and T is on PR
such that $\angle 1 = \angle 2$
and $\Delta NSQ \cong \Delta MTR$

To prove: $\Delta PTS \sim \Delta PRQ$

Proof: $\Delta NSQ \cong \Delta MTR$ [Given]

$\therefore \quad SQ = TR$ [CPCT] (I)

$\angle 1 = \angle 2$ [Given]

$\therefore \quad PT = PS$ [Sides opposite to equal angles in ΔPTS] (II)

$\Rightarrow \quad \dfrac{PT}{TR} = \dfrac{PS}{SQ}$ [From (I), (II)]

$\therefore \quad ST \parallel QR$ [By converse of BPT]

Now, in ΔPTS and ΔPRQ, we have

$ST \parallel QR$ [Proved above]

$\angle 1 = \angle 3$ [Corresponding $\angle$s]

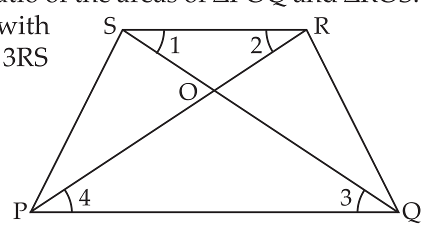
$\angle 2 = \angle 4$ [Corresponding $\angle$s]

$\therefore \quad \Delta PTS \sim \Delta PRQ$ [By AA similarity criterion]

Hence, proved.

Q4. Diagonals of a trapezium PQRS intersect each other at the point O, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of ΔPOQ and ΔROS.

Sol. Given: PQRS is a trapezium with
$PQ \parallel RS$ and $PQ = 3RS$



To find: $\dfrac{ar(\Delta POQ)}{ar(\Delta ROS)}$

Proof: In ΔPOQ and ΔROS,

$PQ \parallel RS$ [Given]

$\therefore \quad \angle 1 = \angle 3$ [Alt. int. $\angle$s]

$\angle 2 = \angle 4$ [Alt. int. $\angle$s]

$\therefore \quad \Delta POQ \sim \Delta ROS$ [By AA similarity criterion]

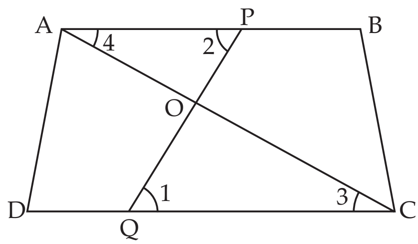
So, $\quad \dfrac{ar(\Delta POQ)}{ar(\Delta ROS)} = \left(\dfrac{PQ}{RS}\right)^2$ [By area theorem]

But, $PQ = 3RS$ [Given]

$$\Rightarrow \frac{ar(\Delta POQ)}{ar(\Delta ROS)} = \left(\frac{3RS}{RS}\right)^2 = \frac{9}{1}$$

Hence, the required ratio is 9:1.

Q5. In the given figure, if AB || DC, and AC and PQ intersect each other at O, prove that OA . CQ = OC · AP



Sol. Given: □ ABCD,

AB || DC

and PQ intersect AC at O (in figure)

To Prove: $OA \cdot CQ = OC \cdot AP$

Proof: In ΔOPA and ΔOQC,

$$\left.\begin{aligned}\angle 1 &= \angle 2\\ \angle 3 &= \angle 4\end{aligned}\right\}$$ [Alt. int. ∠s]

$\therefore$ $\Delta OPA \sim \Delta OQC$ [By AA similarity criterion]

$$\Rightarrow \frac{OQ}{OP} = \frac{OC}{OA} = \frac{QC}{PA}$$

$\Rightarrow$ $OA \cdot CQ = OC \cdot PA$

Hence, proved.

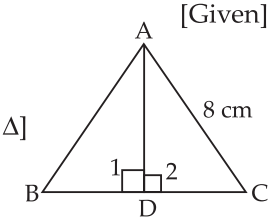
Q6. Find the altitude of an equilateral triangle of side 8 cm.

Sol. ΔABC is an equilateral triangle. [Given]

$AB = BC = AC = 8$ cm [Given]

$AD \perp BC$ [Given]

$\therefore$ $\angle 1 = \angle 2 = 90°$



In ΔADB and ΔADC,

$AB = AC$ [Sides of an equilateral Δ]

$\angle 1 = \angle 2 = 90°$

$AD = AD$ [Common]

$\therefore$ $\Delta ADB \cong \Delta ADC$ [By RHS congruence criterion]

$\Rightarrow$ $BD = DC$ [CPCT]

$$\therefore BD = DC = \frac{BC}{2} = \frac{AB}{2} = \frac{8}{2} = 4 \text{ cm}$$

$\therefore$ By Pythagoras theorem, we have

$$AD^2 + BD^2 = AB^2$$

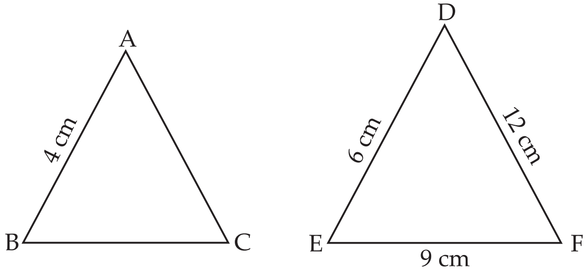
$$\Rightarrow AD^2 + (4)^2 = (8)^2$$

$$\Rightarrow AD^2 = 64 - 16$$

$$\Rightarrow AD^2 = 48$$

$$\Rightarrow AD = 4\sqrt{3} \text{ cm}$$

Q7. If $\Delta ABC \sim \Delta DEF$, AB = 4 cm, DE = 6 cm, EF = 9 cm, FD = 12 cm, then find the perimeter of ΔABC.



Sol. Given: In ΔABC and ΔDEF,

$$AB = 4 \text{ cm}, \quad DE = 6 \text{ cm}$$
$$EF = 9 \text{ cm}, \quad FD = 12 \text{ cm}$$

To find: Perimeter of ΔABC

Proof: $\Delta ABC \sim \Delta DEF$ [Given]

$$\therefore \quad \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$\Rightarrow \quad \frac{4}{6} = \frac{AC}{12} = \frac{BC}{9}$$

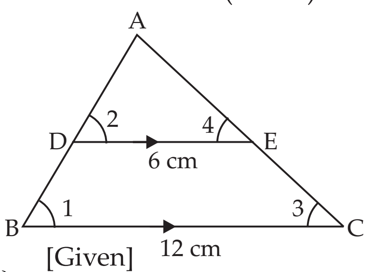
$$\Rightarrow \quad AC = \frac{4}{6} \times 12 = 8 \text{ cm}$$

and $$BC = \frac{4}{6} \times 9 = 6 \text{ cm}$$

$\therefore$ The perimeter of ΔABC = AB + BC + AC

= 4 cm + 6 cm + 8 cm = 18 cm

Q8. In the given figure, if DE || BC, then find the ratio of ar (ΔADE) and ar ($\square$DECB).



Sol. Given: In ΔABC, in which

DE || BC

and DE = 6 cm and BC = 12 cm

To find: $\dfrac{ar(\Delta ADE)}{ar(\square DECB)}$

In ΔADE and ΔABC,

DE || BC [Given]

$$\therefore \quad \left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 3 = \angle 4\end{array}\right\} \quad \text{[Corresponding angles]}$$

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity critrion]

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \left(\frac{BC}{DE}\right)^2$$

[$\because$ Ratio of the areas of two similar triangles is equal to the squares of the ratio of their corresponding sides]

$$\Rightarrow \quad \frac{ar(\square DECB) + ar(\Delta ADE)}{ar(\Delta ADE)} = \left(\frac{12}{6}\right)^2$$

$$\Rightarrow \quad \frac{ar(\square DECB)}{ar(\Delta ADE)} + \frac{ar(\Delta ADE)}{ar(\Delta ADE)} = (2)^2$$

$$\Rightarrow \quad \frac{ar(\square DECB)}{ar(\Delta ADE)} + 1 = 4$$

$$\Rightarrow \quad \frac{ar(\square DECB)}{ar(\Delta ADE)} = 4 - 1 = 3$$

$$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\square DECB)} = \frac{1}{3}$$

Hence, the required ratio is 1 : 3.

Q9. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively such that PQ|| DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.

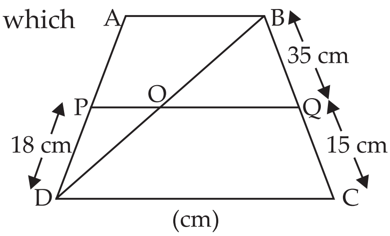
Sol. Given: ABCD is a trapezium in which

AB || CD and

PQ || DC (See figure)

Also, PD = 18 cm,

BQ = 35 cm and QC = 15 cm



To find: AD

Proof: In trapezium ABCD,

AB || CD

PQ || DC

$\therefore$ AB || CD || PQ (I)

In ΔBCD,

OQ || CD [From (I)]

$$\therefore \quad \frac{BO}{OD} = \frac{BQ}{QC} \quad \text{(II)} \quad \text{[By BPT]}$$

Similarly, in ΔDAB,

PO || AB [From (I)]

$$\therefore \quad \frac{BO}{OD} = \frac{AP}{PD} \quad \text{(III)} \quad \text{[By BPT]}$$

From (II) and (III)

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

$$\Rightarrow \quad \frac{AP}{18} = \frac{35}{15}$$

$$\Rightarrow \quad AP = \frac{35}{15} \times 18 = 7 \times 6$$

$\Rightarrow$ $\quad AP = 42\text{ cm}$

$\therefore$ AD = AP + PD = 42 cm + 18 cm = 60 cm

Q10. Corresponding sides of two similar triangles are in the ratio 2 : 3. If the area of the smaller triangle is 48 cm^2, then find the area of the larger triangle.

Sol. If $\Delta ABC \sim \Delta DEF$, then by area theorem,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$$

But, $\quad AB : DE = 2 : 3$

and $\quad ar(\Delta ABC)\text{ (smaller)} = 48\text{ cm}^2$

$$\therefore \quad \frac{48}{ar(\Delta DEF)} = \left(\frac{2}{3}\right)^2$$

$$\Rightarrow \quad ar(\Delta DEF) = \frac{48 \times 9}{4} = 108\text{ cm}^2$$

Q11. In a ΔPQR, N is the point on PR such that $QN \perp PR$. If $PN \times NR = QN^2$, then prove that $\angle PQR = 90°$.

Sol. Given: ΔPQR in which $QN \perp PR$ and $PN \times NR = QN^2$.

To Prove: $\angle PQR = 90°$

Proof: In ΔQNP and ΔQNR,

$QN \perp PR$ [Given]

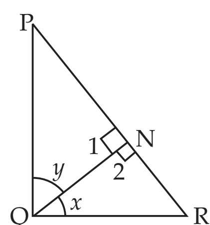
$\therefore \quad \angle 1 = \angle 2 = 90°$

$QN^2 = NR \times NP$ [Given]

$\Rightarrow \quad \frac{QN}{NR} = \frac{NP}{QN}$ or $\frac{QN}{NP} = \frac{NR}{QN}$

$\therefore \quad \Delta PNQ \sim \Delta QNR$

[By SAS similarity criterion]



$\angle P = \angle RQN = x$ (I)

$\angle 1 = \angle 2 = 90°$

$\angle PQN = \angle R = y$ (II)

In ΔPQR, we have

$\angle P + \angle PQR + \angle R = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad x + x + y + y = 180°$ [Using (I) and (II)]

$\Rightarrow \quad 2x + 2y = 180°$

$\Rightarrow \quad x + y = 90°$

$\Rightarrow \quad \angle PQR = 90°$

Hence, proved.

Q12. Areas of two similar triangles are 36 cm^2 and 100 cm^2. If the length of a side of the larger triangle is 20 cm, find the length of the corresponding side of the similar triangle.

Sol. Here, $ar(\Delta ABC) = 36\text{ cm}^2$, $ar(\Delta DEF) = 100\text{ cm}^2$, DE = 20 cm, AB = ?

If $\Delta ABC \sim \Delta DEF$, then by area theorem $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\dfrac{AB}{DE}\right)^2$

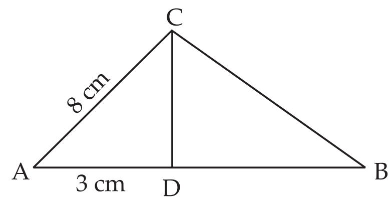
$$\Rightarrow \quad \frac{36}{100} = \left(\frac{AB}{DE}\right)^2$$

$$\Rightarrow \quad \frac{6}{10} = \left(\frac{AB}{DE}\right) \quad \text{[Taking square root]}$$

$$\text{or} \quad \frac{6}{10} = \frac{AB}{20} \Rightarrow AB = \frac{6 \times 20}{10} = 12 \text{ cm}$$

$\therefore$ AB = 12 cm. Hence, side of smaller Δ is 12 cm.

Q13. In the given figure, if $\angle ACB = \angle CDA$, AC = 8 cm, AD = 3 cm, then find BD.



Sol. In ΔACD and ΔACB, we have

$\angle CDA = \angle ACB$ [Given]

$\angle A = \angle A$ [Common]

$\therefore \quad \Delta ACD \sim \Delta ACB$ [By AA similarity criterion]

$$\text{So,} \quad \frac{AC}{AB} = \frac{DC}{BC} = \frac{AD}{AC} \Rightarrow \frac{8}{AB} = \frac{DC}{BC} = \frac{3}{8}$$

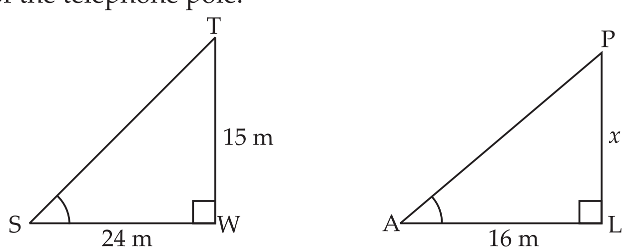
$$\text{Now,} \quad \frac{8}{AB} = \frac{3}{8} \Rightarrow AB = \frac{8 \times 8}{3} = \frac{64}{3}$$

$$BD = AB - AD = \frac{64}{3} - 3 = \frac{64 - 9}{3}$$

$$= \frac{55}{3} \text{ cm} = 18.33 \text{ cm}$$

Hence, BD = 18.33 cm.

Q14. A 15 m high tower casts a shadow 24 m long at a certain time and at the same time a telephone pole casts a shadow 16 m long. Find the height of the telephone pole.



Sol. Let TW = 15 m be the tower and SW = 24 m be its shadow. Also, let PL be the telephone pole and AL = 16 m be its shadow.

Let PL = x metres.

In ΔTWS and ΔPLA,

$$\angle W = \angle L = 90^\circ$$

$$\angle S = \angle A \qquad \text{[Each = Angular elevation of sun]}$$

$\therefore$ $$\Delta TWS \sim \Delta PLA$$

$\Rightarrow$ $$\frac{TW}{PL} = \frac{TS}{PA} = \frac{WS}{LA}$$

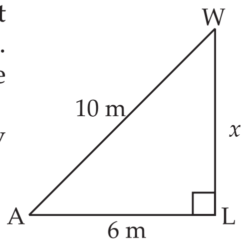
$\Rightarrow$ $$\frac{15}{x} = \frac{24}{16}$$

$\Rightarrow$ $$x = \frac{15 \times 16}{24} = 5 \times 2$$

$\Rightarrow$ $$x = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

Q15. Foot of a 10 m long ladder leaning against a vertical wall is 6 m away from the base of wall. Find the height of the point on the wall where the top of the ladder reaches.



Sol. As wall WL = x m is vertically up so by Pythagoras theorem,

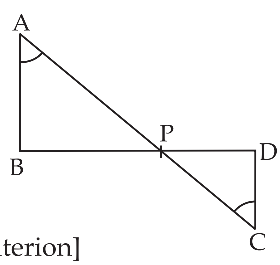
$$x^2 = 10^2 - 6^2 = 100 - 36$$

$\Rightarrow$ $$x^2 = 64$$

$\Rightarrow$ $$x = 8 \text{ m}$$

EXERCISE 6.4

Q1. In the given figure, if $\angle A = \angle C$, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.



Sol. In ΔABP and Δ CDP,

$\angle A = \angle C$ [Given]

$\angle 1 = \angle 2$

[Vertically opposite angles]

$\therefore$ $\Delta ABP \sim \Delta CDP$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{AB}{CD} = \frac{AP}{CP} = \frac{BP}{DP}$$

$$\Rightarrow \quad \frac{6}{y} = \frac{12}{4} = \frac{15}{x}$$

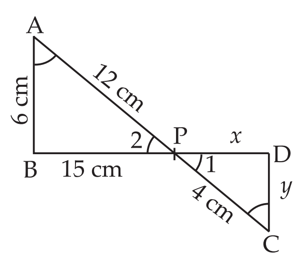
$$\Rightarrow \quad \frac{6}{y} = \frac{12}{4}$$

$$\Rightarrow \quad y = \frac{6}{3} = 2 \text{ cm}$$

$$\Rightarrow \quad \frac{15}{x} = \frac{12}{4}$$

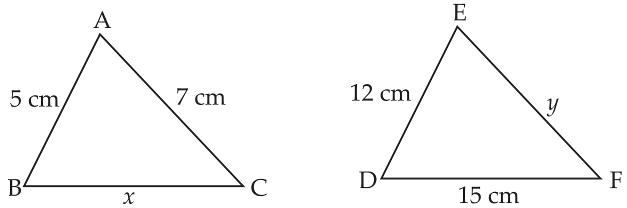
$$\Rightarrow \quad \frac{15}{3} = x$$

$$\Rightarrow \quad x = 5 \text{ cm}$$



$\therefore$ PD = 5 cm and DC = 2 cm

Q2. It is given that $\Delta ABC \sim \Delta EDF$ such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.



Sol. $\Delta ABC \sim \Delta EDF$ [Given]

$$\therefore \quad \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$$

$$\Rightarrow \quad \frac{5}{12} = \frac{7}{y} = \frac{x}{15}$$

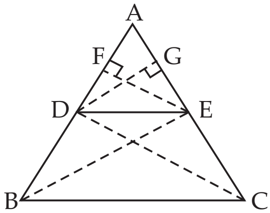
$$\Rightarrow \quad \frac{5}{12} = \frac{7}{y}$$

$$\Rightarrow \quad y = \frac{7 \times 12}{5} = \frac{84}{5} = 16.8 \text{ cm}$$

and $$x = \frac{5 \times 15}{12} = \frac{25}{4} = 6.25 \text{ cm}$$

Hence, the length of BC = 6.25 cm and EF = 16.8 cm.

Q3. Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.



Sol. Given: In ΔABC,

$$DE \parallel BC$$

To Prove: $$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction: Draw $EF \perp AB$ and $DG \perp AC$.

Join DC and BE.

Proof: $$\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{\frac{1}{2} AD \times EF}{\frac{1}{2} DB \times EF} = \frac{AD}{DB} \qquad \text{(I)}$$

and $$\frac{ar(\Delta AED)}{ar(\Delta ECD)} = \frac{\frac{1}{2} AE \times DG}{\frac{1}{2} EC \times DG} = \frac{AE}{EC} \qquad \text{(II)}$$

Note that ΔDBE and ΔECD are on same base DE and between same parallel lines DE and BC.

$\therefore$ $\quad$ ar (ΔDBE) = ar (ΔECD) $\quad$ (III)

From equations (II) and (III), we have

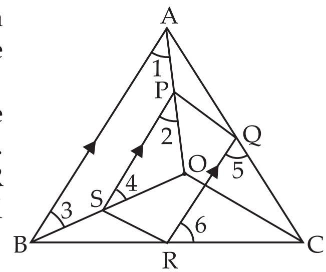
$$\frac{\text{ar}(\Delta AED)}{\text{ar}(\Delta DBE)} = \frac{AE}{EC} \quad \text{(IV)}$$

From equations (I) and (IV), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, proved.

Q4. In the given figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR.



Sol. Given: In ΔABC, O is any point in the interior of ΔABC. OA, OB, OC are joined. PQRS is a parallelogram such that P, Q, R and S lies on segments OA, AC, BC and OB and PS || AB.

To Prove: $\quad$ OC || SR

Proof: In ΔOAB and ΔOPS

PS || AB $\quad$ [Given]

$\therefore$ $\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 3 = \angle 4\end{array}\right\}$ $\quad$ [Corresponding angles]

$\therefore$ $\quad$ $\Delta OPS \sim \Delta OAB$ $\quad$ [By AA similarity criterion]

$\Rightarrow$ $\quad$ $\dfrac{OP}{OA} = \dfrac{OS}{OB} = \dfrac{PS}{AB}$ $\quad$ (I)

PQRS is a parallelogram so PS || QR. $\quad$ (II)

$\Rightarrow$ $\quad$ QR || AB $\quad$ (III) $\quad$ [From (I), (II]

In ΔCQR and ΔCAB,

QR || AB $\quad$ (III)

$\therefore$ $\left.\begin{array}{l}\angle CAB = \angle 5 \\ \angle CBA = \angle 6\end{array}\right\}$ $\quad$ [Corresponding angles]

$\therefore$ $\quad$ $\Delta CQR \sim \Delta CAB$ $\quad$ [By AA similarity criterion]

$\Rightarrow$ $\quad$ $\dfrac{CQ}{CA} = \dfrac{CR}{CB} = \dfrac{QR}{AB}$

PQRS is a parallelogram.

$\therefore$ $\quad$ PS || QR

$\therefore$ $\quad$ $\dfrac{PS}{AB} = \dfrac{CR}{CB} = \dfrac{CQ}{CA}$ $\quad$ (IV)

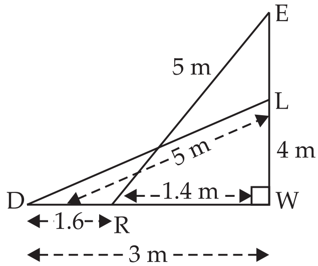
$\Rightarrow$ $\quad$ $\dfrac{CR}{CB} = \dfrac{OS}{OB}$ $\quad$ [From (I) and (IV)]

These are the ratios of two sides of ΔBOC and are equal so by converse of BPT, SR ||OC.

Hence, proved.

Q5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Sol. In figure ELW is a wall. DL and RE are two positions of ladder of length 5 cm.



Case I: In right angled ΔLWD,

$$DW^2 = DL^2 - LW^2$$

$$\Rightarrow \quad DW^2 = 5^2 - 4^2$$

$$= 25 - 16 = 9$$

$$\Rightarrow \quad DW = 3 \text{ m}$$

Case II: $RW = DW - DR$

$$= 3 - 1.6 = 1.4 \text{ m}$$

In right angled triangle RWE,

$$EW^2 = RE^2 - RW^2$$

$$= 5^2 - 1.4^2 = 25 - 1.96$$

$$= 23.04$$

$$EW = \sqrt{23.04} = 4.8 \text{ m.}$$

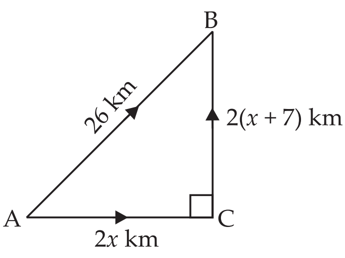
$\therefore$ The distance by which the ladder shifted upward = EL = 4.8 m – 4 m = 0.8 m

Hence, the ladder would slide upward on wall by 0.8 m.

Q6. For going to a city B from city A, there is route via city C, such that $AC \perp CB$, $AC = 2x$ km, and $CB = 2(x + 7)$ km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A, after the construction of the highway.

Sol. Distance saved by direct highway = (AC + BC) – AB

$\because$ $AC \perp BC$ so by Pythagoras theorem



$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow \quad (2x)^2 + [2(x+7)]^2 = 26^2$$

$$\Rightarrow \quad 2^2x^2 + 2^2(x+7)^2 = 676$$

$$\Rightarrow \quad 4x^2 + 4(x^2 + 49 + 14x) = 676$$

$$\Rightarrow \quad 4[x^2 + x^2 + 49 + 14x] = 676$$

$$\Rightarrow \quad 2x^2 + 14x + 49 = \frac{676}{4}$$

$$\Rightarrow \quad 2x^2 + 14x + 49 = 169$$

$$\Rightarrow \quad 2x^2 + 14x + 49 - 169 = 0$$

$$\Rightarrow \quad 2x^2 + 14x - 120 = 0$$

$$\Rightarrow \quad x^2 + 7x - 60 = 0$$

$$\Rightarrow \quad x^2 + 12x - 5x - 60 = 0$$

$$\Rightarrow \quad x(x+12) - 5(x+12) = 0$$

$$\Rightarrow \quad (x + 12)(x - 5) = 0$$
$$\Rightarrow \quad x + 12 = 0 \quad \text{or} \quad x - 5 = 0$$
$$\Rightarrow \quad x = -12 \quad \text{or} \quad x = 5$$
(rejected)

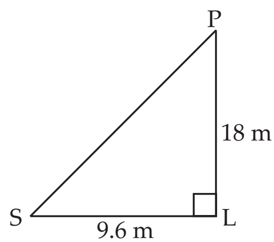
$\therefore$ The required distance $= AC + BC - AB$
$= 2x + 2x + 14 - 26$
$= 4x - 12$
$= 4 \times 5 - 12 = 20 - 12$ $[\because x = 5]$
$= 8$ km

Hence, the distance saved by highway is 8 km.

Q7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Sol. Pole PL = 18 m casts shadow LS = 9.6 m

The required distance between top of pole and far end of shadow is equal to PS as pole is vertical so $\angle L = 90°$.



$\therefore$ By Pythagoras theorem,

$$PS^2 = 18^2 + 9.6^2$$
$$\Rightarrow \quad PS^2 = 324 + 92.16 = 416.16$$
$$\Rightarrow \quad PS = \sqrt{416.16}$$
$$\Rightarrow \quad PS = 20.4 \text{ m}$$

Hence, the required distance = 20.4 m

Q8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, then find how far she is away from the base of the pole.

Sol. In ΔLPS and ΔNWS,

Bulb L is fixed at a height of 6 m above the road SP.

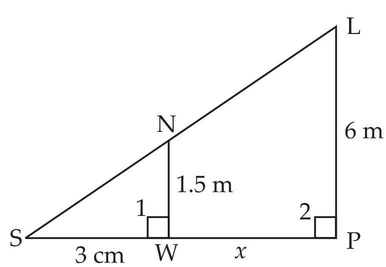
Woman and pole are vertical.

$\therefore$ $\angle 1 = \angle 2 = 90°$

$\angle S = \angle S$ [Common]

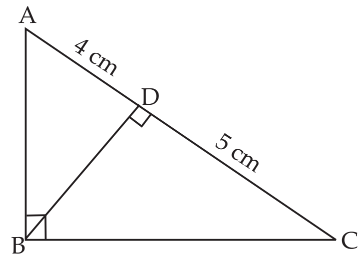
$\therefore$ $\Delta LPS \sim \Delta NWS$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{LP}{NW} = \frac{LS}{NS} = \frac{PS}{WS}$$
$$\Rightarrow \quad \frac{6 \text{ m}}{1.5 \text{ m}} = \frac{LS}{NS} \quad \frac{}{3}$$
$$\Rightarrow \quad \frac{6}{1.5} = \frac{3 + x}{3}$$
$$\Rightarrow \quad 4.5 + 1.5x = 18$$
$$\Rightarrow \quad 1.5x = 18 - 4.5$$
$$\Rightarrow \quad x = \frac{13.5}{1.5} = 9 \text{ m}$$



Hence, the woman is 9 m away from the pole.

Q9. In the given figure, ABC, is a triangle right angled at B and $BD \perp AC$. If AD = 4 cm, and CD = 5 cm then find BD and AB.



Sol. In ΔABC,

$\angle ABC = 90°$ [Given]

$BD \perp AC$ [Hypotenuse]

$\therefore \quad BD^2 = DA \times DC$

$\Rightarrow \quad BD^2 = 4 \times 5$

$\Rightarrow \quad BD = 2\sqrt{5}$ cm

In right angled ΔBDA,

$BD \perp AC$ [Given]

$\therefore \quad \angle BDA = 90°$

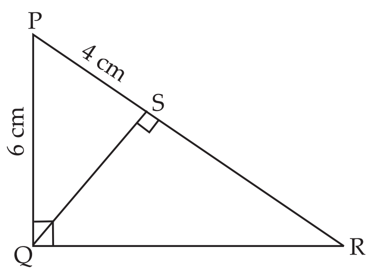
$\Rightarrow \quad AB^2 = AD^2 + BD^2$ [By Pythagoras theorem]

$= 4^2 + (2\sqrt{5})^2$

$= 16 + 20 = 36$

$\Rightarrow \quad AB = 6$ cm

Q10. In the given figure, PQR is a right triangle right angled at Q and $QS \perp PR$. If PQ = 6 cm and PS = 4 cm, then find QS, RS and QR.



Sol. In ΔPQR,

$\angle PQR = 90°$ [Given]

$QS \perp PR$

[From vertex Q to hypotenuse PR]

$\therefore \quad QS^2 = PS \times SR \quad$ (I) [By theorem]

Now, in ΔPSQ, we have

$QS^2 = PQ^2 - PS^2$ [By Pythagoraus theorem]

$= 6^2 - 4^2$

$= 36 - 16$

$\Rightarrow \quad QS^2 = 20$

$\Rightarrow \quad QS = 2\sqrt{5}$

$QS^2 = PS \times SR \quad$ (I)

$\Rightarrow \quad (2\sqrt{5})^2 = 4 \times SR$

$\Rightarrow \quad \dfrac{20}{4} = SR$

$\Rightarrow \quad SR = 5$ cm

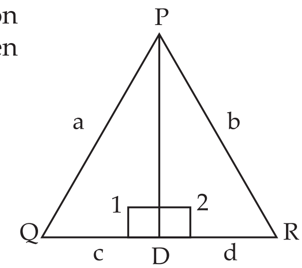
Now, $\quad QS \perp PR$

$\therefore \quad \angle QSR = 90°$

$$\Rightarrow \quad QR^2 = QS^2 + SR^2 \quad \text{[By Pythagoras theorem]}$$
$$= (2\sqrt{5})^2 + 5^2$$
$$= 20 + 25$$
$$\Rightarrow \quad QR^2 = 45$$
$$\Rightarrow \quad QR = 3\sqrt{5} \text{ cm}$$

Hence, $QS = 2\sqrt{5}$, $RS = 5$ cm and $QR = 3\sqrt{5}$ cm.

Q11. In ΔPQR, $PD \perp QR$ such that D lies on QR, if $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, then prove that $(a + b)(a - b) = (c + d)(c - d)$



Sol. Given: In ΔPQR, $PD \perp QR$ so $\angle 1 = \angle 2$.
$PQ = a$, $PR = b$, $QD = c$ and $DR = d$.

To Prove: $(a + b)(a - b) = (c + d)(c - d)$

Proof: In right angle ΔPDQ,

$$PD^2 = PQ^2 - QD^2$$

[By Pythagoras theorem]

$$\Rightarrow \quad PD^2 = a^2 - c^2 \quad \text{(I)}$$

Similarly, in right angled ΔPDR,

$$PD^2 = PR^2 - DR^2 \quad \text{[By Pythagoras theorem]}$$
$$\Rightarrow \quad PD^2 = b^2 - d^2 \quad \text{(II)}$$

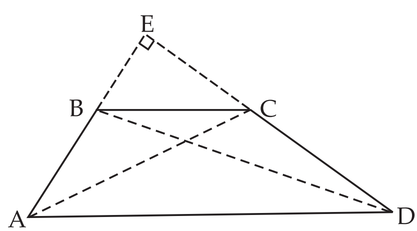
From (I) and (II), we have

$$a^2 - c^2 = b^2 - d^2$$
$$\Rightarrow \quad a^2 - b^2 = c^2 - d^2$$
$$\Rightarrow \quad (a - b)(a + b) = (c - d)(c + d)$$

Hence, proved.

Q12. In a quadrilateral ABCD, $\angle A + \angle D = 90°$. Prove that $AC^2 + BD^2 = AD^2 + BC^2$

[Hint: Produce AB and DC to meet at E.]



Sol. Given: A quadrilateral ABCD in which $\angle A + \angle D = 90°$.

To Prove: $AC^2 + BD^2 = AD^2 + BC^2$

Construction: Join AC and BD. Produce AB and DC to meet at E.

Proof: In ΔADE,

$$\angle BAD + \angle CDA = 90° \quad \text{[Given]}$$
$$\therefore \quad \angle E = 90° \quad \text{[Int. angles of a } \Delta\text{]}$$

By Pythagoras theorem in ΔADE and ΔBCE,

$$AD^2 = AE^2 + DE^2 \quad \text{(I)}$$
$$BC^2 = BE^2 + EC^2 \quad \text{(II)}$$

Adding (I) and (II), we get

$$AD^2 + BC^2 = AE^2 + EC^2 + DE^2 + BE^2 \quad \text{(III)}$$

By Pythagoras theorem in ΔECA and ΔEBD,

$$AC^2 = AE^2 + CE^2 \quad \text{(IV)}$$

$$BD^2 = BE^2 + DE^2 \quad \text{(V)}$$

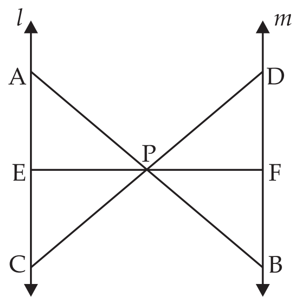
$$\Rightarrow \quad AC^2 + BD^2 = AE^2 + BE^2 + CE^2 + DE^2 \quad \text{(VI)[Adding (IV) and (V)]}$$

$$\Rightarrow \quad AC^2 + BD^2 = AD^2 + BC^2 \quad \text{[Using (III)]}$$

Hence, proved.

Q13. In the given figure, $l \parallel m$ and line segments AB, CD, and EF are concurrent at point P.

Prove that: $\dfrac{AE}{BF} = \dfrac{AC}{BD} = \dfrac{CE}{FD}$

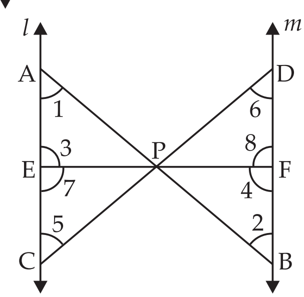


Sol. Given: $l \parallel m$

Line segments AB, CD and EF intersect at P.

Points A, E and C are on line l.

Points D, F and B are on line m.



To Prove: $\dfrac{AE}{BF} = \dfrac{AC}{BD} = \dfrac{CE}{FD}$

Proof: In ΔAEP and ΔBFP,

$l \parallel m$ [Given]

$\angle 1 = \angle 2$ [Alternate interior angles]

$\angle 3 = \angle 4$ [Same reason]

$\therefore$ ΔAEP ~ ΔBFP [By AA similarity criterion]

$$\Rightarrow \quad \frac{AE}{BF} = \frac{AP}{BP} = \frac{EP}{FP} \quad \text{(I)}$$

In ΔCEP and ΔDFP,

$l \parallel m$ [Given]

$\left.\begin{aligned}\angle 7 &= \angle 8\\ \angle 5 &= \angle 6\end{aligned}\right\}$ [Alternate interior angles]

$\therefore$ ΔCEP ~ ΔDFP [By AA similarity criterion]

$\Rightarrow \qquad \dfrac{CE}{DF} = \dfrac{CP}{DP} = \dfrac{EP}{FP} \qquad$ (II)

In ΔACP and ΔBDP,

$l \parallel m$ [Given]

$\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 5 = \angle 6\end{array}\right\}$ [Alternate interior angles]

$\therefore \qquad \Delta ACP \sim \Delta BDP$ [By AA similarity criterion]

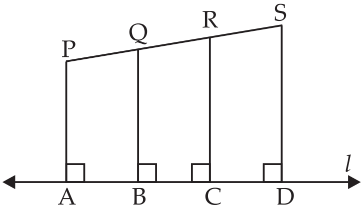
$\Rightarrow \qquad \dfrac{AC}{BD} = \dfrac{AP}{BP} = \dfrac{CP}{DP} \qquad$ (III)

$\Rightarrow \qquad \dfrac{AP}{PB} = \dfrac{AC}{BD} = \dfrac{CP}{DP} = \dfrac{CE}{DF} = \dfrac{EP}{FP} = \dfrac{AE}{BF}$

$\Rightarrow \qquad \dfrac{AC}{BD} = \dfrac{AE}{BF} = \dfrac{CE}{DF}$

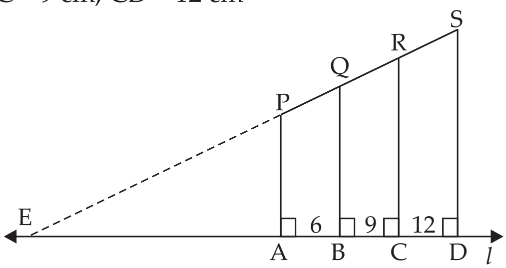
Hence, proved.

Q14. In the given figure, PA, QB, RC, and SD are all perpendiculars to line 'l', AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.



Sol. Given: PA, QB, RC and SD are perpendiculars on line l.

AB = 6 cm, BC = 9 cm, CD = 12 cm



To find: PQ, QR and RS

Construction: Produce SP and l to meet each other at E.

Proof: In ΔEDS,

$AP \parallel BQ \parallel DS \parallel CR$ [Given]

$\therefore \qquad PQ : QR : RS = AB : BC : CD$

$PQ : QR : RS = 6 : 9 : 12$

Let PQ = $6x$

then QR = $9x$

and RS = $12x$

$\therefore \qquad PQ + QR + RS = 36$ cm

$\Rightarrow \qquad 6x + 9x + 12x = 36$

$$\Rightarrow \quad 27x = 36$$

$$\Rightarrow \quad x = \frac{36}{27} = \frac{4}{3}$$

$$\therefore \quad PQ = 6 \times \frac{4}{3} = 8 \text{ cm}$$

$$QR = 9 \times \frac{4}{3} = 12 \text{ cm}$$

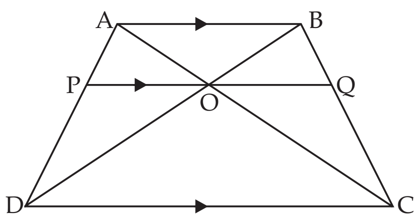
$$RS = 12 \times \frac{4}{3} = 16 \text{ cm}$$

Q15. 'O' is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || CD. Through 'O', a line PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO.

Sol. Given: In trapezium ABCD, AB || DC.

Diagonals BD and AC intersect at O and POQ || DC || AB

To Prove: PO = QO



Proof: In ΔABD,

$$PO \parallel AB \quad \text{[Given]}$$

$$\therefore \quad \frac{AP}{PD} = \frac{BO}{OD} \quad \text{(I)}$$

Similarly, in ΔBDC,

$$OQ \parallel DC$$

$$\therefore \quad \frac{BO}{OD} = \frac{BQ}{QC} \quad \text{(II)}$$

From (I) and (II), we have

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

$$\Rightarrow \quad \frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1 \quad \text{[Adding 1 on both sides]}$$

$$\Rightarrow \quad \frac{AP + PD}{PD} = \frac{BQ + QC}{QC}$$

$$\Rightarrow \quad \frac{AD}{PD} = \frac{BC}{QC} \text{ or } \frac{PD}{AD} = \frac{QC}{BC} \quad \text{(III)}$$

In ΔDOP and ΔDBA,

$$AB \parallel PO \quad \text{[Given]}$$

$$\therefore \quad \left.\begin{aligned} \angle DPO &= \angle DAB \\ \angle DOP &= \angle DBA \end{aligned}\right\} \quad \text{[Corresponding angles]}$$

$$\therefore \quad \Delta DOP \sim \Delta DBA \quad \text{[By AA similarity criterion]}$$

$$\Rightarrow \quad \frac{PO}{AB} = \frac{DP}{DA} \quad \text{(IV)}$$

Similarly, ΔCOQ ~ ΔCAB [By AA similarity criterion]

$$\therefore \qquad \frac{OQ}{AB} = \frac{QC}{BC} \qquad \text{(V)}$$

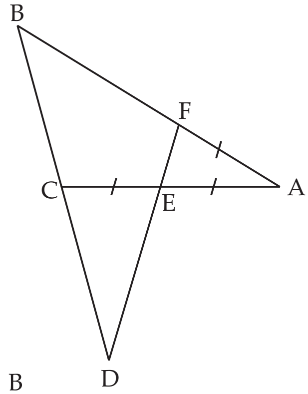
From (III), (IV) and (V), we have

$$\frac{PO}{AB} = \frac{OQ}{AB}$$

$$\Rightarrow \qquad PO = OQ$$

Hence, proved.

Q16. In the given figure, the line segment DF intersect the side AC of ΔABC at the point E such that E is mid point of AC and $\angle AFE = \angle AEF$.



Prove that: $\frac{BD}{CD} = \frac{BF}{CE}$.

[Hint: Take point G on AB such that CG || DF.]

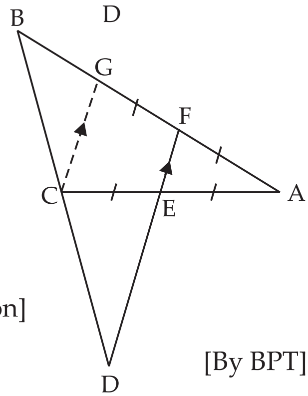
Sol. In the given figure of ΔABC,

$$EA = AF = EC$$

EF and BC meets at D.

To Prove: $\frac{BD}{CD} = \frac{BF}{CE}$

Construction: Draw $CG \parallel EF$.



Proof: In ΔACG, $CG \parallel EF$.

$\because$ E is mid-point of AC

$\therefore$ F will be the mid point of AG.

$\Rightarrow \qquad FG = FA$

But, $\qquad EC = EA = AF \qquad$ [Given]

$\therefore \qquad FG = FA = EA = EC \qquad$ (I)

In ΔBCG and BDF,

$CG \parallel EF \qquad$ [By construction]

$$\therefore \qquad \frac{BC}{CD} = \frac{BG}{GF} \qquad \text{[By BPT]}$$

$$\Rightarrow \qquad \frac{BC}{CD} + 1 = \frac{BG}{GF} + 1 \Rightarrow \frac{BC + CD}{CD} = \frac{BG + GF}{GF}$$

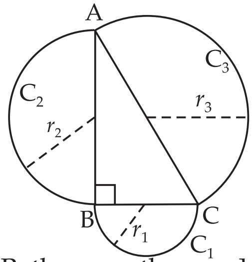
$$\Rightarrow \qquad \frac{BD}{CD} = \frac{BF}{GF}$$

But, $\qquad FG = CE \qquad$ [From (I)]

$$\Rightarrow \qquad \frac{BD}{CD} = \frac{BF}{CE}$$

Hence, proved.

Q17. Prove that the area of the semi-circle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of semi-circles drawn on the other two sides of the triangle.



Sol. Given: In figure, ΔABC is right angled at B. Three semi-circles taking as the sides BC, AB and AC of triangle ABC as diameter C_1, C_2 and C_3 are drawn.

To Prove: Area of semicircles $(C_1 + C_2)$ = Area of semi-circle C_3

Proof: In ΔABC,

$$\angle B = 90°$$

$\therefore \quad BC^2 + AB^2 = AC^2$ [By Pythagoras theorem]

$\Rightarrow \quad (2r_1)^2 + (2r_2)^2 = (2r_3)^2$

[From figure as BC, AB and AC are diameters]

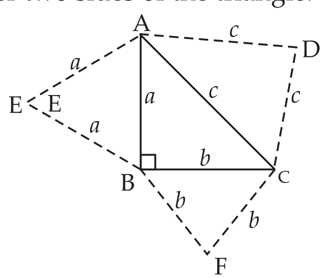
$\Rightarrow \quad 4(r_1^2 + r_2^2) = 4r_3^2 \quad \Rightarrow \quad r_1^2 + r_2^2 = r_3^2$

$\Rightarrow \quad \dfrac{1}{2}\pi r_1^2 + \dfrac{1}{2}\pi r_2^2 = \dfrac{1}{2}\pi r_3^2$

ar (semi-circle C_1) + ar (semi-circle C_2) = ar (semi-circle C_3)

Hence, proved.

Q18. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles drawn on the other two sides of the triangle.



Sol. Given: A right triangle ABC.

Let AB = a, BC = b, AC = c and B = $\angle 90°$.

Equilateral triangles with sides AB = a, BC = b and AC = c are drawn respectively.

To Prove: Area of equilateral triangle with side hypotenuse (c) is equal to the area of equilateral triangles with side a and b.

or $\dfrac{\sqrt{3}}{4}c^2 = \dfrac{\sqrt{3}}{4}a^2 + \dfrac{\sqrt{3}}{4}b^2$

Proof: In ΔABC,

$\angle ABC = 90°$ [Given]

$\therefore \quad AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow \quad c^2 = a^2 + b^2$

$\Rightarrow \quad \dfrac{\sqrt{3}}{4}c^2 = \dfrac{\sqrt{3}}{4}a^2 + \dfrac{\sqrt{3}}{4}b^2$ [Multiplying by $\dfrac{\sqrt{3}}{4}$ to both sides]

$$\Rightarrow \begin{pmatrix}\text{Area of equilateral}\\ \Delta \text{ with side } c\end{pmatrix} = \begin{pmatrix}\text{Area of equilateral}\\ \Delta \text{ with side } a\end{pmatrix} + \begin{pmatrix}\text{Area of equilateral}\\ \Delta \text{ with side } b\end{pmatrix}$$

Hence, the area of equilateral Δ with hypotenuse is equal to the sum of areas of equilateral triangles on other two sides.

Hence, proved.