EXERCISE 11.1

Choose the correct answer from the given four options:

Q1. If the sum of the areas of two circles with radii R_1 and R_2 in equal to the area of a circle of radius R, then

(*a*) $R_1 + R_2 = R$ (*b*) $R_1^2 + R_2^2 = R^2$

(*c*) $R_1 + R_2 < R$ (*d*) $R_1^2 + R_2^2 < R^2$

Sol. (*b*): Area of circle with radius R

= Area of circle with radius R_1 + Area of circle with radius R_2

$\Rightarrow \quad \pi R^2 = \pi R_1^2 + \pi R_2^2$

$\Rightarrow \quad \pi R^2 = \pi(R_1^2 + R_2^2)$

$\Rightarrow \quad R^2 = R_1^2 + R_2^2$

Hence, verifies the option (*b*).

Q2. If the sum of circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(*a*) $R_1 + R_2 = R$ (*b*) $R_1 + R_2 > R$

(*c*) $R_1 + R_2 < R$

(*d*) Nothing definite can be said about the relation among R_1, R_2 and R.

Sol. (*a*): According to the given condition,

$2\pi R = 2\pi R_1 + 2\pi R_2$

$\Rightarrow \quad 2\pi R = 2\pi(R_1 + R_2)$

$\Rightarrow \quad R = R_1 + R_2$

Hence, verifies the option (*a*).

Q3. If the circumference of a circle and the perimeter of a square are equal, then

(*a*) Area of circle = Area of the square

(*b*) Area of circle > Area of the square

(*c*) Area of circle < Area of the square

(*d*) Nothing definite can be said about the relation between the areas of the circle and square.

Sol. (*b*): According to the given condition,

Circumference of circle = Perimeter of square

$\Rightarrow \quad 2\pi r = 4a$ (where a = side of the square)

$\Rightarrow \quad \pi r = 2a$

$\Rightarrow \quad \frac{22}{7} r = 2a \Rightarrow r = \frac{7}{22} \times 2a$

$$\Rightarrow \quad r = \frac{7}{11}a$$

Now, area of circle = πr^2

$$= \frac{22}{7} \times \frac{7}{11}a \times \frac{7}{11}a = \frac{14}{11}a^2 = \frac{14}{11} \text{ area of square}$$

$\Rightarrow$ Area of circle = 1.2 area of square

$\therefore$ Area of circle > Area of the square

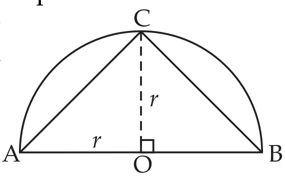
Hence, verifies the option (*b*).

Q4. Area of the largest triangle that can be inscribed in a semicircle of radius r units is

(*a*) r^2 square units (*b*) $\frac{1}{2}r^2$ square units

(*c*) $2r^2$ square units (*d*) $\sqrt{2}r^2$ square units

Sol. (*a*): Base AB of triangle ABC in semicircle is constant, i.e., equal to $2r$, and maximum altitude may be equal to r.



$$\therefore \quad \text{Area of triangle} = \frac{1}{2}\text{base} \times \text{altitude}$$

$$= \frac{1}{2}\text{AB} \times \text{OC} = \frac{1}{2}(2r) \times r = r^2$$

$\therefore$ Area of triangle in semicircle = r^2 square units.

Hence, verifies the option (*a*).

Q5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(*a*) 22 : 7 (*b*) 14 : 11 (*c*) 7 : 22 (*d*) 11 : 14

Sol. (*b*): Let r be the radius of a circle and side of a square is 'a', then according to the given condition,

$$2\pi r = 4a$$

$$\Rightarrow \quad 2 \times \frac{22}{7}r = 4a$$

$$\Rightarrow \quad r = \frac{4a \times 7}{2 \times 22} = \frac{7}{11}a$$

$$\therefore \quad \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{a^2} = \frac{22}{7} \times \frac{7}{11} \times \frac{7}{11} \times \frac{a^2}{a^2}$$

$$\Rightarrow \quad \frac{\text{Area of circle}}{\text{Area of square}} = \frac{14}{11}$$

Hence, the required ratio is 14 : 11.

Verifies the option (*b*).

Q6. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(*a*) 10 m (*b*) 15 m (*c*) 20 m (*d*) 24 m

Sol. (*a*): Let the radius of the new park be R.

∴ Area of new park = Area of old park I + Area of park II

$$\Rightarrow \pi R^2 = \pi r_1^2 + \pi r_2^2$$

$$\Rightarrow \pi R^2 = \pi[r_1^2 + r_2^2]$$

$$\Rightarrow \pi R^2 = \pi[8^2 + 6^2]$$

$$\Rightarrow R^2 = 64 + 36$$

$$\Rightarrow R = \sqrt{100} = 10 \text{ m}$$

$$\left[r_1 = \frac{16}{2} = 8 \text{ m} \quad r_2 = \frac{12}{2} = 6 \text{ m} \right]$$

Hence, verifies the option (*a*).

Q7. Area of the circle that can be inscribed in a square of side 6 cm is

(*a*) $36\pi \text{ cm}^2$ (*b*) $18\pi \text{ cm}^2$ (*c*) $12\pi \text{ cm}^2$ (*d*) $9\pi \text{ cm}^2$

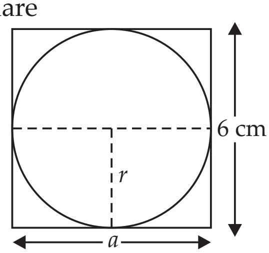
Sol. (*d*): Diameter of the circle inscribed in a square

= Side of square

$$\therefore 2r = a$$

$$r = \frac{a}{2} = \frac{6}{2} = 3 \text{ cm}$$

$$\therefore \text{Area of circle} = \pi r^2 = \pi(3)^2 = 9\pi = 9\pi \text{ cm}^2$$



Hence, verifies the option (*d*).

Q8. The area of the square that can be inscribed in a circle of radius 8 cm is

(*a*) 256 cm^2 (*b*) 128 cm^2 (*c*) $64\sqrt{2} \text{ cm}^2$ (*d*) 64 cm^2

Sol. (*b*): Let the side of square be *a* cm.

Radius of the circle = 8 cm

$$\therefore \text{Diameter} = 2r = 2 \times 8 = 16 \text{ cm}$$

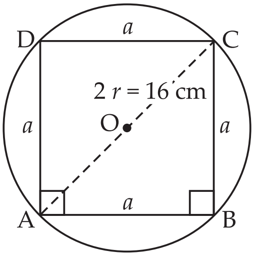
By Pythagoras theorem in right ΔABC,

$$a^2 + a^2 = (16)^2$$

$$\Rightarrow 2a^2 = 256$$

$$\Rightarrow a^2 = \frac{256}{2} = 128$$

$$\text{Area of square} = a^2 = 128 \text{ cm}^2$$



Hence, verifies the option (*b*).

Q9. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(*a*) 56 cm (*b*) 42 cm (*c*) 28 cm (*d*) 16 cm

Sol. (*c*): According to the given condition,

Circumference of circle = Sum of circumferences of two circles

$$\left[r_1 = \frac{36}{2} = 18 \text{ cm} \quad r_2 = \frac{20}{2} = 10 \text{ cm} \right]$$

$$\Rightarrow 2\pi R = 2\pi r_1 + 2\pi r_2$$

$$\Rightarrow 2\pi R = 2\pi(r_1 + r_2)$$

$$\Rightarrow R = r_1 + r_2 = 18 + 10$$

$\Rightarrow$ R = 28 cm, which verifies the option (*c*).

Q10. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(*a*) 31 cm (*b*) 25 cm (*c*) 62 cm (*d*) 50 cm

Sol. (*d*): According to the question,

$$\pi R^2 = \pi r_1^2 + \pi r_2^2 \qquad \begin{bmatrix} r_1 = 24 \text{ cm} \\ r_2 = 7 \text{ cm} \end{bmatrix}$$

$$\Rightarrow \quad \pi(R^2) = \pi(r_1^2 + r_2^2)$$

$$\Rightarrow \quad R^2 = r_1^2 + r_2^2 = (24)^2 + (7)^2 = 576 + 49 = 625$$

$$\Rightarrow \quad R = \sqrt{625}$$

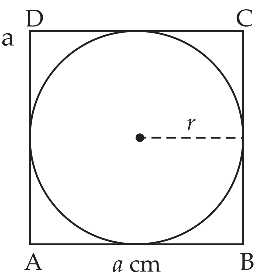
$$\Rightarrow \quad R = 25$$

$$\therefore \quad \text{Diameter} = 2R = 2 \times 25 = 50 \text{ cm.}$$

Hence, verifies the option (*d*).

EXERCISE 11.2

Q1. Is the area of circle inscribed in a square of side a cm, πa^2 cm^2? Give reasons for your answer.



Sol. False: The radius of the circle inscribed in a square of side a cm is $r = \frac{a}{2}$

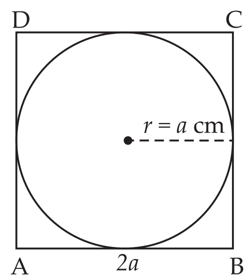
$$\therefore \quad \text{Area of circle} = \pi r^2$$

$$= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4} \text{ cm}^2$$

$$\neq \pi a^2 \text{ cm}^2$$

Hence, the given statement is false.

Q2. Will it be true to say that the perimeter of the square circumscribing a circle of radius a cm is $8a$ cm? Give reasons for your answer.



Sol. True: Side of square = Diameter of circle

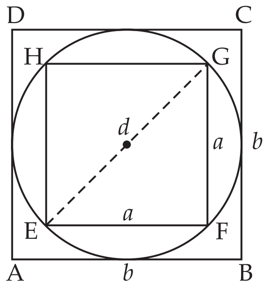
$$\therefore \quad AB = 2a$$

So, the perimeter of square

$$= 4 \times AB$$
$$= 4 \times 2a$$
$$= 8a \text{ cm}$$

Hence, the given statement is true.

Q3. In the given figure, a square is inscribed in a circle of diameter d and another square is circumscribing the circle. Is the area the outer square four times the area of inner square? Give reasons for your answer.



Sol. False: Let the side of the smaller square is a units and that of bigger square is b units.

Diameter of circle = d

So, diagonal of square EFGH = d

Then, by Pythagoras theorem,

$$a^2 + a^2 = d^2$$

$$\Rightarrow \quad 2a^2 = d^2$$

$$\Rightarrow \quad a^2 = \frac{d^2}{2}$$

$\therefore$ Area of small square = $a^2 = \dfrac{d^2}{2}$

Side of outer square = b = Diameter of circle

$\therefore$ Area of outer square = $b^2 = d^2$

$$= \frac{2}{2}d^2 = 2 \times \frac{1}{2}d^2$$

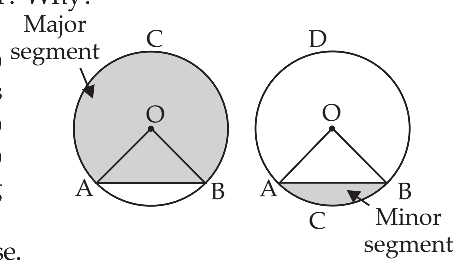
$\Rightarrow$ Area of larger square = 2 Area of smaller square

So, the given statement is false.

Q4. Is it true to say that area of a segment of a circle is less than the area of its corresponding sector? Why?

Sol. False:

Area of major segment (ACB) is always greater than its corresponding sector (OACB) and area of minor segment (ACB) is smaller than its corresponding minor sector (OACB).



Hence, the given statement is false.

Q5. Is it true that the distance travelled by circular wheel of diameter d cm in one revolution is $2\pi d$ cm? Why?

Sol. False: Distance travelled by wheel in one revolution is equal to the circumference of wheel = $2\pi r = \pi(2r) = \pi d$.

Hence, the given statement is false.

Q6. In covering a distance s metres, a circular wheel of radius r m makes $\dfrac{s}{2\pi r}$ revolutions. Is the statement true? Why?

Sol. True: Distance covered by a circular wheel in n revolutions = $2\pi r n$ where n = number of revolutions

$$\therefore \quad s = 2\pi r n \quad \text{or} \quad n = \frac{s}{2\pi r}.$$

Hence, verifies the given statement true.

Q7. The numerical values of the area of a circle is greater than the numerical value of its circumference. Is this statement true? Why?

Sol. False. Let the radius of circle is r $(0 < r < 2)$. Then, the area of circle $A = \pi r^2$ for $r = 1.5$, $A = \pi \times (1.5)^2$

$$\therefore \quad A = 2.25\pi$$

Circumference (C) = $2\pi r = 2 \times \pi \times 1.5$

$\Rightarrow \quad C = 3.0\pi$

$\therefore \quad C > A$

So, the area of a circle is not always greater than its circumference.
Hence, the given statement is false.

Q8. If the length of an arc of a circle of radius r is equal to that of an arc of a circle of radius $2r$, then the angle of the corresponding sector of the first circle is double the angle of the corresponding sector of other circle. Is this statement false? Why?

Sol. False: Consider two circles C_1 and C_2 of radii r and $2r$ respectively.
Let the lengths of two arcs be l_1 and l_2.

$$l_1 = \overparen{AB} \text{ of } C_1 = \frac{2\pi r\theta_1}{360^\circ}$$

$$l_2 = \overparen{CD} \text{ of } C_2 = \frac{2\pi r'\theta_2}{360^\circ} = \frac{2\pi . 2r\theta_2}{360^\circ}$$

According to question,

$$l_1 = l_2$$

$$\Rightarrow \quad \frac{2\pi r\theta_1}{360^\circ} = \frac{2\pi . 2r\theta_2}{360^\circ}$$

$$\Rightarrow \quad \theta_1 = 2\theta_2$$

i.e., Angle of sector of the Ist circle is twice the angle of the sector of the other circle.
Hence, the given statement is true.

Q9. The areas of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true? Why?

Sol. False. Consider two circles of radii, r_1, r_2 of arc length, l_1 and l_2, and their corresponding angles of sectors θ_1, θ_2 respectively.

$$l_1 = \frac{2\pi r_1\theta_1}{360^\circ}, \quad l_2 = \frac{2\pi r_2\theta_2}{360^\circ}$$

Now, $\quad l_1 = l_2 \quad$ [Given]

$$\Rightarrow \quad \frac{2\pi r_1\theta_1}{360^\circ} = \frac{2\pi r_2\theta_2}{360^\circ}$$

$$\Rightarrow \quad r_1\theta_1 = r_2\theta_2 = x \quad \text{[say]}$$

Area of sectors A_1 and A_2 are given by

$$A_1 = \frac{\pi r_1^2\theta_1}{360^\circ}, \quad A_2 = \frac{\pi r_2^2\theta_2}{360^\circ}$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{\frac{\pi r_1 \theta_1 r_1}{360^\circ}}{\frac{\pi r_2 \theta_2 r_2}{360^\circ}} = \frac{xr_1}{xr_2} = \frac{r_1}{r_2}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Area of sector can be equal when $\frac{r_1}{r_2} = 1$ i.e., equal radii. So, the areas of sectors of two circles of same arcs length are not equal.
Hence, the given statement is false.

Q10. The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal? Why?

Sol. False: Using usual abbreviations for sectors

$$A_1 = A_2 \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{\pi r_1^2 \theta_1}{360°} = \frac{\pi r_2^2 \theta_2}{360°}$$

$$\Rightarrow \quad r_1^2\,\theta_1 = r_2^2\,\theta_2$$

$$\Rightarrow \quad \frac{\theta_1}{\theta_2} = \frac{r_2^2}{r_1^2}$$

Now, $$\frac{l_1}{l_2} = \frac{2\pi r_1 \theta_1}{2\pi r_2 \theta_2} = \frac{r_1}{r_2} \times \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1} \Rightarrow \frac{l_1}{l_2} = \frac{r_2}{r_1}$$

Hence, arcs length can be equal if $\frac{r_2}{r_1} = 1$ i.e., $r_1 = r_2 = r$.

Hence, the given statement is false.

Q11. Is the area of the largest circle that can be drawn inside a rectangle of length a cm and breadth b cm ($a > b$) is πb^2 cm^2? Why?

Sol. False: The diameter of circle that can be drawn inside the rectangle is equal to the breadth of rectangle.
The length of the rectangle = a cm
The breadth of the rectangle = b cm
∴ Diameter of circle = b cm

$$\Rightarrow \quad r = \frac{b}{2} \text{ cm}$$

$$\therefore \quad \text{Area of circle } A = \pi r^2 = \pi\left(\frac{b}{2}\right)^2 = \frac{1}{4}\pi b^2 \text{ cm}^2$$

Hence, the given statement is false.

Q12. Circumferences of two circles are equal. Is it necessary that their areas be equal? Why?

Sol. True. ∵ $$2\pi r_1 = 2\pi r_2 \quad \text{[Given]}$$

$$\Rightarrow \quad r_1 = r_2 = r \quad \text{[say]}$$

Now, $$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r^2}{r^2} = 1$$

∴ $A_1 = A_2$ *i.e.*, their areas are equal.

Hence, the given statement is true.

Q13. Areas of the two circles are equal. Is it necessary that their circumferences are equal? Why?

Sol. True: $\because$ $A_1 = A_2$ [Given]

$\Rightarrow$ $\pi r_1^2 = \pi r_2^2$

$\Rightarrow$ $r_1^2 = r_2^2$

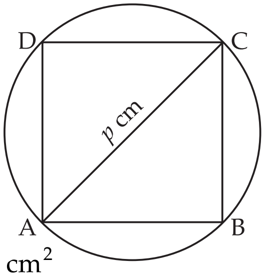
$\Rightarrow$ $r_1 = r_2 = r$ (say) [taking square root]

Now, $\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{r}{r} = \frac{1}{1}$

Hence, $C_1 = C_2$

Hence, the given statement is true.

Q14. Is it true to say that area of a square inscribed in a circle of diameter p cm is p^2 cm^2? Why?



Sol. False: The diameter of the circle is p cm.

So, the diagonal of the square will be p cm.

Now, $AB^2 + BC^2 = AC^2$

$\Rightarrow$ $AB^2 + AB^2 = p^2$

$\Rightarrow$ $2AB^2 = p^2 \Rightarrow AB = \frac{p}{\sqrt{2}}$

Now, Area of square $= \frac{p}{\sqrt{2}} \times \frac{p}{\sqrt{2}} = \frac{p^2}{2}$ cm^2

Hence, the given statement is false.

EXERCISE 11.3

Q1. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.

Sol.

Circle I (C_1)	Circle II (C_2)	Circle III (C)
$r_1 = 15$ cm	$r_2 = 18$ cm	$r = ?$

According to the question,

$$C_1 + C_2 = C$$

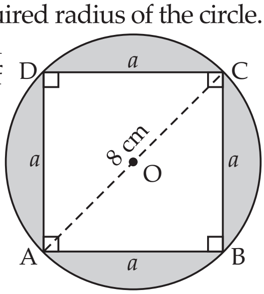
$\Rightarrow$ $$2\pi r_1 + 2\pi r_2 = 2\pi r$$

$\Rightarrow$ $$2\pi [r_1 + r_2] = 2\pi r$$

$\Rightarrow$ $$r_1 + r_2 = r \Rightarrow 15 + 18 = r$$

$\Rightarrow$ $r = 33$ cm is the required radius of the circle.

Q2. In the given figure, a square of diagonal 8 cm is inscribed in a circle. Find the area of shaded region.



Sol. Let the side of the square be a cm.

So, radius of the circle, $r = OA = \frac{AC}{2}$

$\Rightarrow$ $$r = \frac{8}{2} = 4 \text{ cm}$$

So, in right angled ΔABC,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad a^2 + a^2 = 8^2$$

$$\Rightarrow \quad 2a^2 = 64 \Rightarrow a^2 = \frac{64}{2} \Rightarrow a^2 = 32$$

Area of shaded part = Area of circle – Area of square

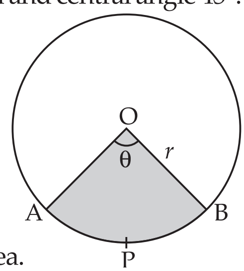
$$= \pi r^2 - a^2 = \frac{22}{7} \times 4 \times 4 - 32$$

$$= 16\left[\frac{22}{7} - \frac{2}{1}\right] = 16\left[\frac{22-14}{7}\right] = \frac{16 \times 8}{7} = \frac{128}{7}$$

Area of shaded region = 18— cm

Q3. Find the area of a sector of a circle of radius 28 cm and central angle 45°.

Sol. Sector (OAPBO) of a circle is given whose



radius (r) = 28 cm

and central angle(θ) = 45°

$$\therefore \quad \text{Area of sector (A)} = \frac{\pi r^2 \theta}{360°}$$

$$= \frac{22}{7} \times \frac{28 \times 28 \times 45°}{360°}$$

$\Rightarrow$ Area of sector = 308 cm^2 is the required area.

Q4. The wheel of a motor cycle is of radius 35 cm. How many revolution per minute must the wheel make so as to keep a speed of 66 km/hr?

Sol. Speed of the wheel (v) = 66 km/hr

$$v = 66 \times \frac{5}{18} = \frac{55}{3} \text{ m/s}$$

$$r = 35 \text{ cm} = 0.35 \text{ m}$$

$$n = ?$$

$$t = 1 \text{ min} = 60 \text{ sec}$$

Now,

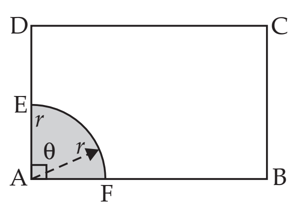
$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r n}{t} \Rightarrow v = \frac{2\pi r n}{t}$$

$$\Rightarrow \quad n = \frac{v.t}{2\pi r} = \frac{\frac{55}{3} \times 60}{2 \times \frac{22}{7} \times 0.35} = \frac{55 \times 60 \times 7 \times 100}{3 \times 2 \times 22 \times 35}$$

$\Rightarrow$ n = 500 revolutions per min.

Q5. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20 m × 16 m. Find the area of the field in which the cow can graze.

Sol. Field is rectangular. So if cow is tied at its vertex, it will graze the field in the shape of sector. The length of rope is less than length and breadth of rectangle. So, the required area is of sector.



Area of field grazed by cow

$= \text{Area of sector}$

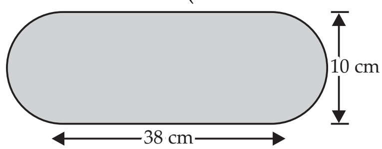
$\Rightarrow \quad A = \dfrac{\pi r^2 \theta}{360°}$

where $\quad r = 14 \text{ m}, \quad \theta = 90°$

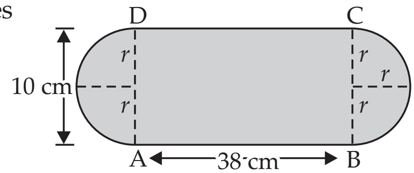
$\therefore \quad A = \dfrac{22}{7} \times \dfrac{14 \times 14 \times 90°}{360°} = 11 \times 14 \Rightarrow A = 154 \text{ m}^2.$

So, the required area grazed by cow is 154 m^2.

Q6. Find the area of flower bed (with semi-circular ends) shown in figure.



Sol. The figure has two semi-circles and one rectangle.



The radius of the semi-circles $= \dfrac{10}{2}$ cm

$\Rightarrow \quad r = 5 \text{ cm}$

The length of the rectangle

$= \quad l = 38 \text{ cm}$

$\text{Breadth} = b = 10 \text{ cm}$

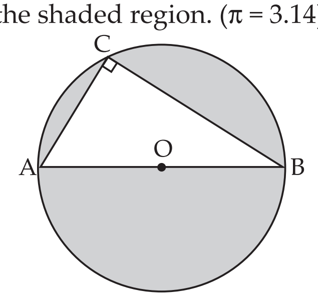
Area of the flower bed = Area of rectangle + Area of two semi-circles

$= l \times b + 2 \times \dfrac{\pi r^2}{2} = l \times b + \pi r^2$

$= 38 \times 10 + \dfrac{22}{7} \times 5 \times 5 = (380 + 25\pi) \text{ cm}^2$

Hence, the area of flower bed is $(380 + 25\pi) \text{ cm}^2$.

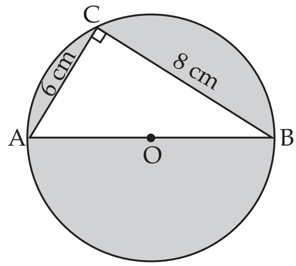
Q7. In the given figure, AB is diameter of circle, AC = 6 cm and BC = 8 cm. Find the area of the shaded region. ($\pi = 3.14$).



Sol. Identify the figure as a circle, and a right angled triangle (and semicircle, segments also) because AOB is diameter and angle in semicircle is 90°. So, $\angle C = 90°$

In right angled ΔABC,

$b = \text{base} = BC = 8 \text{ cm}$



a = altitude = AC = 6 cm

By Pythagoras theorem in right ΔABC,

$$AB^2 = BC^2 + AC^2$$
$$= 8^2 + 6^2 = 64 + 36$$
$$\Rightarrow \quad AB^2 = 100 \text{ cm}^2$$
$$\Rightarrow \quad AB = 10 \text{ cm}$$

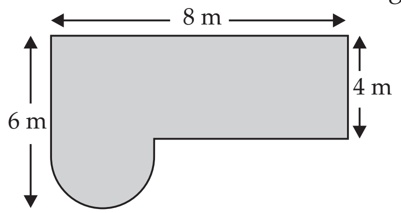
Hence, $r = \dfrac{10}{2} = 5$ cm

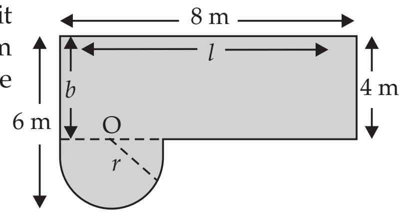
∴ Area of shaded region = Area of circle – Area of right ΔABC

$$= \pi r^2 - \frac{1}{2}\text{Base} \times \text{Alt.}$$
$$= 3.14 \times 5 \times 5 - \frac{1}{2} \times 8 \times 6$$
$$= 3.14 \times 25 - 8 \times 3 = (78.50 - 24) \text{ cm}^2 = 54.50 \text{ cm}^2$$

∴ Area of shaded region = 54.50 cm^2

Q8. Find the area of the shaded field shown in the given figure.





Sol. Redraw the figure and divide it into well known shapes. It is clear from the figure that there is one semicircle and one rectangle.

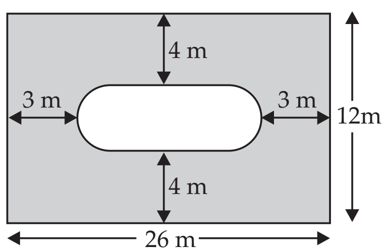
Rectangle	Circle
$l = 8$ m	$r = 6 - 4 = 2$ m
$b = 4$ m	

Area of shaded region = Area of rectangle + Area of semicircle

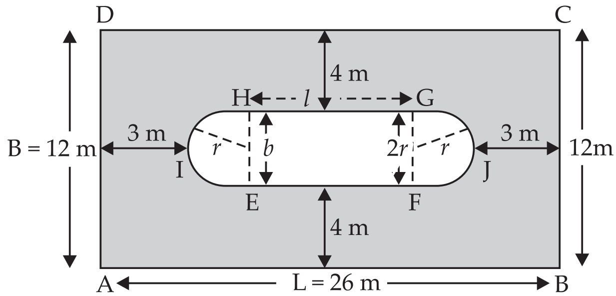
$$= l \times b + \frac{\pi r^2}{2} = 8 \times 4 + \pi \times \frac{2 \times 2}{2} = (32 + 2\pi) \text{ m}^2$$

Hence, the required area of shaded region = $(32 + 2\pi)$ m^2.

Q9. Find the area of shaded region in the given figure.



Sol. Redraw the given figure and identify the shapes (well known) from figure.



There are two rectangles ABCD and EFGH and two semicircles EHI and GFJ. For dimensions of shapes,

Rectangle ABCD	**Rectangle EFGH**	**Semicircles HEI and GFJ**
$L = 26$ m	$l = 26 - 3 - 3 - 2r$	$r = \dfrac{b}{2}$
$B = 12$ m	$\Rightarrow \quad l = 26 - 6 - 2r$	$\Rightarrow \quad r = \dfrac{4}{2}$ m
	$\Rightarrow \quad l = 20 - 2r$	$\Rightarrow \quad r = 2$ m
	$\Rightarrow \quad l = 20 - 2r = 20 - 2 \times 2 = 20 - 4$	
	$\Rightarrow \quad l = 16$ m	
	$b = 12 - 4 - 4$	
	$\Rightarrow \quad b = 12 - 8 = 4$ m	

Area of required shaded region

= Area of rectangle ABCD – [Area of 2 semicircles + Area of rectangle EFGH ...(*i*)

$$\Rightarrow \text{ Area of shaded region } = L \times B - \left[2.\frac{\pi r^2}{2} + l \times b\right]$$

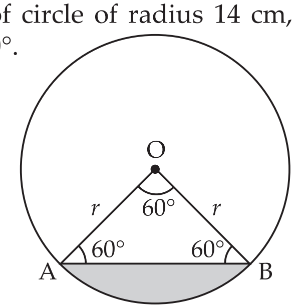
$$= 26 \times 12 - [\pi r^2 + l \times b]$$

$$= 26 \times 12 - [\pi \times 2 \times 2 + 16 \times 4] = 312 - 4\pi - 64 = (248 - 4\pi) \text{ m}^2$$

Hence, the area of shaded region = $(248 - 4\pi)$ m^2.

Q10. Find the area of the minor segment of circle of radius 14 cm, when the angle of corresponding sector is 60°.

Sol. Shaded region is a minor segment.



In ΔOAB,

$\theta = 60°$, $\quad$ OA = OB = r = 14 cm

$\therefore$ Area of minor segment

= Area of sector – Area of ΔOAB

In ΔOAB,

OA = OB $\quad$ [Radii of same circle]

Let $\angle A = \angle B = x°$ [($\because$ Angles opposite to equal sides are equal]

$$\angle O + \angle A + \angle B = 180° \quad \text{[Angle sum property of a triangle]}$$

$$\Rightarrow \quad 60° + x + x = 180°$$

$$\Rightarrow \quad 2x = 180° - 60°$$

$$\Rightarrow \quad x = \frac{120°}{2} \Rightarrow x = 60°$$

So, ΔOAB is an equilateral Δ with side 14 cm.

$$\text{Area of minor segment} = \frac{\pi r^2 \theta}{360°} - \text{Area of } \Delta OAB$$

$$\text{So, Area of minor segment} = \frac{\pi r^2 \theta}{360} - \frac{\sqrt{3}}{4} r^2$$

$$= \frac{22 \times 14 \times 14 \times 60°}{7 \times 360°} - \frac{\sqrt{3}}{4} \times 14 \times 14$$

$$= \frac{22 \times 14}{3} - 49\sqrt{3} = \left(\frac{308}{3} - 49\sqrt{3}\right) \text{cm}^2$$

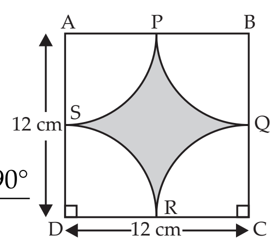
$$= (102.666 - 84.870) \text{ cm}^2 = 17.796 \text{ cm}^2$$

Q11. Find the area of the shaded region in figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD. (Use $\pi = 3.14$)

Sol. From figure,

Area of shaded part = Area of square ABCD – Area of 4 sectors

Side of the square ABCD = 12 cm, $\theta = 90°$



Radii of the sectors $r = \frac{12}{2} = 6$ cm.

$\therefore$ Area of shaded region

= Area of square ABCD – Area of 4 sectors

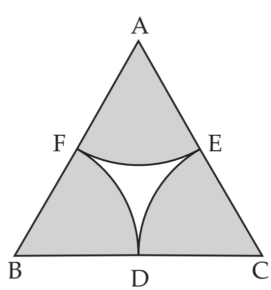
$$= (12)^2 - 4.\frac{\pi r^2 \theta}{360} = 12 \times 12 - \frac{4 \times 3.14 \times 6 \times 6 \times 90°}{360°}$$

$$= 6 \times 6\,[2 \times 2 - 3.14 \times 1 \times 1] = 36\,[4 - 3.14]$$

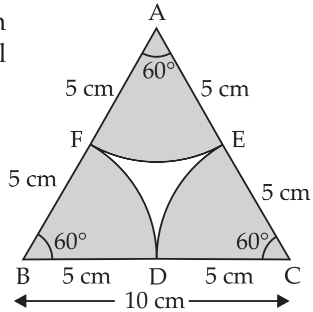
$\Rightarrow$ Area of shaded region = $36 \times 0.86 \text{ cm}^2 = 30.96 \text{ cm}^2$

Hence, the required area is 30.96 cm^2.

Q12. In the given figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm to intersect the sides BC, CA and AB at their respective mid points D, E and F respectively. Find the are of the shaded region. (Use $\pi = 3.14$)



Sol. From the given figure, area of the shaded part is equal to the sum of areas of three sectors at points A, B and C.



As ΔABC is equilateral triangle of side 10 cm and radius of the sector is half of the side. All the three sectors are identical.

$$\theta = 60°$$

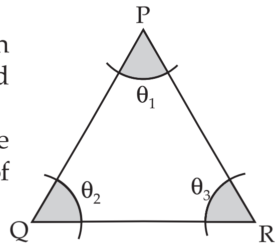
Radius of each sector (r) = $\frac{10}{2} = 5$ cm

$\therefore$ Area of shaded part = 3.(Area of sector)

$= \frac{3 \times \pi r^2 \theta}{360°} = \frac{3 \times 3.14 \times 5 \times 5 \times 60°}{360°}$

$= 1.57 \times 25 = 39.25$ cm^2

Hence, the required area is 39.25 cm^2.



Q13. In the given figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.

Sol. The area of the shaded region is equal to the sum of areas of three sectors of same radius but of different angles θ_1, θ_2 and θ_3.

$$\angle\theta_1 + \angle\theta_2 + \angle\theta_3 = 180° \quad \text{[Int. } \angle\text{s of } \Delta]$$

$\therefore$ Area of shaded region = $\frac{\pi r_1^2 \theta_1}{360°} + \frac{\pi r_2^2 \theta_2}{360°} + \frac{\pi r_3^2 \theta_3}{360°}$

where $r_1 = r_2 = r_3 = r = 14$ cm

$$= \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3)$$

$$= \frac{22}{7} \times \frac{14 \times 14}{360°} \times 180° = 22 \times 14 = 308 \text{ cm}^2$$

$\therefore$ Area of shaded region = 308 cm^2

Hence, required area = 308 cm^2.

Q14. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, then find the area of the road.

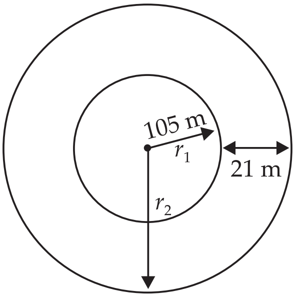
Sol. Circular road and park are concentric circles.

Radius of the park = r_1 = 105 m

Width of road = 21 m

Radius of circular road and park = r_2

= 105 m + 21 m = 126 m



So, Area of road = Area of park and road – Area of park

$$= \pi r_2^2 - \pi r_1^2$$

$$= \pi[r_2^2 - r_1^2]$$

$$= \frac{22}{7}[(126)^2 - (105)^2]$$

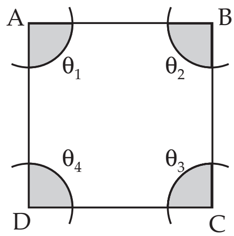
$$= \frac{22}{7}[126 - 105][126 + 105]$$

$$= \frac{22}{7} \times 21 \times 231$$

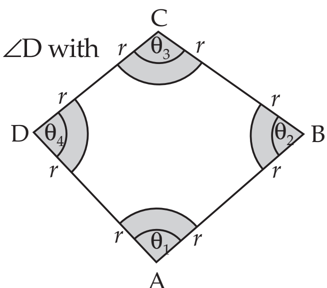
$$= 22 \times 3 \times 231 = 66 \times 231 = 15246 \text{ m}^2$$

$\therefore$ Area of road $= 15246 \text{ m}^2$

Q15. In the given figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of shaded region.



Sol. Specification of quadrilateral are not given, so quadrilateral may be of any shape.
As the radius of all 4 arcs are same equal to $r = 21$ cm but of different angles ∠A, ∠B, ∠C and ∠D.
So, there are four sectors of ∠A, ∠B, ∠C and ∠D with $r = 21$ cm.



$\therefore$ Area of shaded region

$$= \frac{\pi r_1^2(\theta_1)}{360°} + \frac{\pi r_2^2(\theta_2)}{360°} + \frac{\pi r_3^2(\theta_3)}{360°} + \frac{\pi r_4^2(\theta_4)}{360°}$$

$\because r_1 = r_2 = r_3 = r_4 = r$ and

$$\angle\theta_1 + \angle\theta_2 + \angle\theta_3 + \angle\theta_4 = 360°$$

[Interior ∠s of a quad.]

So, Area of shaded region $= \frac{\pi r^2(\theta_1)}{360°} + \frac{\pi r^2(\theta_2)}{360°} + \frac{\pi r^2(\theta_3)}{360°} + \frac{\pi r^2(\theta_4)}{360°}$

$$= \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3 + \theta_4)$$

$$= \frac{\pi r^2}{360°}(360°)$$

$$= \pi r^2 = \frac{22}{7} \times 21 \times 21 = 22 \times 63 = 1386 \text{ cm}^2$$

Hence, the area of shaded region = 1386 cm^2.

Q16. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

Sol. Arc is a part of circle that makes 60° between radii at end points A and B of wire.
So, it forms the shape of a sector.

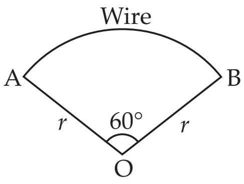
$r = ?$ $\quad l = 20$ cm

$\therefore$ Length of arc $l = \frac{2\pi r\theta}{360°}$

$$\Rightarrow \quad 20 \text{ cm} = \frac{2 \times \pi \times r \times 60^\circ}{360^\circ}$$

$$\Rightarrow \quad 2\pi r = 20 \times 6$$

$$\Rightarrow \quad r = \frac{120}{2\pi} = \frac{60}{\pi} \text{ cm}$$



Hence, radius $(r) = \frac{60}{\pi}$ cm.

EXERCISE 11.4

Q1. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹ 50 per m.

Sol. Fencing is made on circumference ($2\pi r$) of circular field. So, we require radius for it.

Area of the circular playground = 22176 m^2

$\Rightarrow \quad \pi r^2 = 22176$

$\Rightarrow \quad \frac{22}{7} r^2 = 22176$

$\Rightarrow \quad r^2 = \frac{7 \times 22176}{22} \Rightarrow r^2 = \sqrt{7 \times 1008}$

$\Rightarrow \quad r = \sqrt{7 \times 7 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$

$\Rightarrow \quad r = 7 \times 3 \times 2 \times 2$

$\Rightarrow \quad r = 84$ m

$\therefore$ Length of fencing = Circumference of circle

$= 2\pi r = 2 \times \frac{22}{7} \times 84 = 24 \times 22$ m

So, Cost of fencing = ₹ $50 \times 24 \times 22$ = ₹ 26400

Hence, cost of fencing = ₹ 26400.

Q2. The diameters of front and rear wheels of a tractor are 80 cm and 2 m, respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

Sol. $\begin{bmatrix}\text{Distance travelled by} \\ \text{rear wheel}\end{bmatrix} = \begin{bmatrix}\text{Distance travelled by} \\ \text{front wheel}\end{bmatrix}$

$\Rightarrow \quad 2\pi r_1 n_1 = 2\pi r_2 n_2$

$\Rightarrow \quad r_1 n_1 = r_2 n_2 \qquad$ (I)

Front wheel	**Rear wheel**
$r_2 = 80$ cm $= 0.8$ m	$r_1 = 2$ m
$n_2 = 1400$ revolutions	$n_1 = ?$

$\therefore$ From (I), we get

$2 \times n_1 = 0.8 \times 1400$

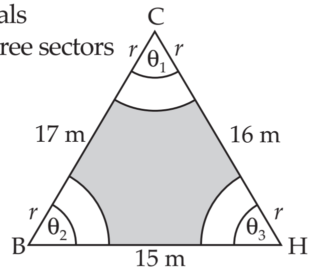
$\Rightarrow \quad n_1 = \frac{0.8 \times 1400}{2} = 0.4 \times 1400 \Rightarrow n_1 = 560$

Hence, the number of revolutions made by rear wheel = 560.

Q3. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field.
Find the area of the field which cannot be grazed by three animals.

Sol. Since with the three corners of the field a cow, a buffalo and a horse and tied separately with ropes of length 7 m each to graze in the field.
Area of field which cannot be grazed by animals

= Area of ΔBCH – Area of three sectors



Here, $a = 15$ m, $b = 16$ m, $c = 17$ m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{15+16+17}{2}$$

$$\Rightarrow \quad s = \frac{48}{2} = 24 \text{ m}$$

$$\text{Area of } \Delta BCH = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-15)(24-16)(24-17)}$$

$$= \sqrt{24 \times 9 \times 8 \times 7}$$

$$= \sqrt{\underline{2\times 2}\times \underline{2}\times 3\times \underline{3\times 3}\times \underline{2}\times \underline{2\times 2}\times 7}$$

$$\Rightarrow \quad ar(\Delta BCH) = 24\sqrt{21} \text{ m}^2$$

$$\text{Area of 3 sectors} = \frac{\pi r^2\theta_1}{360°} + \frac{\pi r^2\theta_2}{360°} + \frac{\pi r^2\theta_3}{360°}$$

$$= \frac{\pi r^2}{360°}[\theta_1 + \theta_2 + \theta_3]$$

$$= \frac{22}{7} \times \frac{7\times 7}{360°} \times 180° \qquad (\because \theta_1 + \theta_2 + \theta_3 = 180°)$$

$$= 77 \text{ m}^2$$

$\therefore$ Area of 3 sectors grazed by animals = 77 m^2.

Hence, the area which cannot be grazed by 3 animals is equal to $\left(24\sqrt{21} - 77\right)$ m^2.

Q4. Find the area of the segment of a circle of radius 12 cm whose corresponding sector has central angle 60°. (Use $\pi = 3.14$).

Sol. Area of minor segment = Area of sector – Area of ΔOAB

In ΔOAB,

$$\theta = 60°$$

$$OA = OB = r = 12 \text{ cm}$$

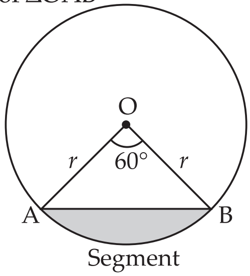
$$\angle B = \angle A = x$$

[$\angle$s opp. to equal sides are equal]

$$\Rightarrow \quad \angle A + \angle B + \angle O = 180°$$

$$\Rightarrow \quad x + x + 60° = 180°$$

$$\Rightarrow \quad 2x = 180° - 60°$$



$$\Rightarrow \quad x = \frac{120°}{2} = 60°$$

$\therefore$ ΔOAB is equilateral Δ with each side $(a) = 12$ cm

$$\text{Area of the equilateral } \Delta = \frac{\sqrt{3}}{4}a^2$$

$$\text{Area of minor segment} = \text{Area of the sector} - \text{Area of } \Delta OAB$$

$$= \frac{\pi r^2 \theta}{360°} - \frac{\sqrt{3}}{4}a^2$$

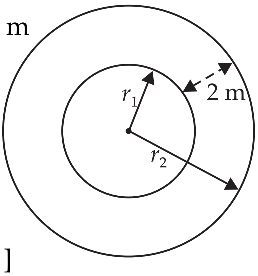
$$= \frac{3.14 \times 12 \times 12 \times 60°}{360°} - \frac{\sqrt{3}}{4} \times 12 \times 12$$

$$= 6.28 \times 12 - 36\sqrt{3}$$

$\therefore$ Area of minor segment = $\left(75.36 - 36\sqrt{3}\right)$ cm^2.

Q5. A circular pond is 17.5 m in diameter. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of ₹ 25 per m^2.

Sol. Radius of the circular pond $r_1 = \frac{17.5}{2}$ m = 8.75 m



Width of path = 2 m

$\therefore$ Radius of the path including pond

$$r_2 = 8.75 + 2 = 10.75 \text{ m}$$

$$\text{Area of path} = \pi r_2^2 - \pi r_1^2 = \pi[r_2^2 - r_1^2]$$

Cost of constructing the path = ₹ 25 $\pi(r_2^2 - r_1^2)$

$$\therefore \quad \text{Required cost} = ₹\, 25 \times \frac{22}{7}[(10.75)^2 - (8.75)^2]$$

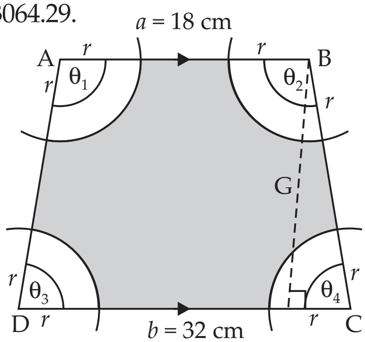
$$= 25 \times \frac{22}{7}[10.75 - 8.75][10.75 + 8.75]$$

$$= 25 \times \frac{22}{7} \times 2 \times 19.5 = \frac{50 \times 22 \times 19.5}{7}$$

$$= \frac{1100 \times 19.5}{7} = \frac{21450}{7} = ₹\, 3064.29$$

Hence, the cost of constructing path is ₹ 3064.29.

Q6. In the given figure, ABCD is a trapezium with AB || CD. AB = 18 cm, DC = 32 cm and distance between AB and DC is 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure.



Sol. In the given figure, there are 4 sectors and one trapezium.

4 sectors	**Trapezium**
$r = 7$ cm	$a = 18$ cm
	$b = 32$ cm
	$h = 14$ cm

Area of shaded part = Area of trapezium – Area of 4 sectors

$$= \frac{(a+b)h}{2} - \left[\frac{\pi r^2\theta_1}{360°} + \frac{\pi r^2\theta_2}{360°} + \frac{\pi r^2\theta_3}{360°} + \frac{\pi r^2\theta_4}{360°}\right]$$

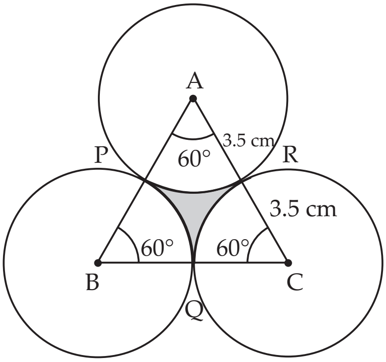
$$= \frac{(18+32)\times 14}{2} - \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3 + \theta_4)$$

$$= \frac{50 \times 14}{2} - \frac{22}{7} \times \frac{7 \times 7}{360°} \times 360°$$

$\Rightarrow$ Area of shaded region = 350 – 22 × 7 = 350 – 154 = 196 cm^2

Hence, the area of shaded region is 196 cm^2.

Q7. Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles.



Sol. Area of equilateral triangle with side 7 cm

$$= \frac{\sqrt{3}}{4} \times (7)^2 \text{ cm}^2$$

$$= \left(\frac{\sqrt{3} \times 49}{4}\right) \text{cm}^2$$

$$= 21.2176 \text{ cm}^2$$

Area of one sector with central angle 60° and radius 3.5 cm

$$= \frac{60°}{360°} \times \pi \times (3.5)^2$$

$$= \frac{\pi}{6}(12.25) \text{ cm}^2$$

$\therefore$ Area of three such sectors $= 3 \times \frac{\pi}{6}(12.25) \text{ cm}^2$

$$= \frac{\pi}{2}(12.25) \text{ cm}^2$$

$$= \frac{22}{14}(12.25) \text{ cm}^2 = 19.25 \text{ cm}^2$$

Now, area enclosed between these circles = Area of Δ – Area of three sectors

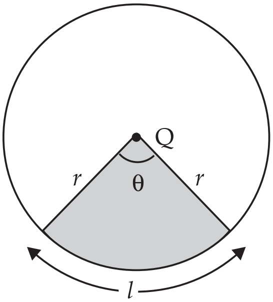
$$= 21.2176 \text{ cm}^2 - 19.25 \text{ cm}^2$$

$$= 1.9676 \text{ cm}^2$$

Q8. Find the area of sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Sol. Here, $l = 3.5$ cm

$r = 5$ cm



$$\text{Length of arc } l = \frac{2\pi r\theta}{360°}$$

$$\Rightarrow \quad 3.5 = \frac{2\times\pi\times5\times\theta}{360°}$$

$$\Rightarrow \quad \frac{\pi\theta}{36} = 3.5$$

$$\Rightarrow \quad \theta = \frac{3.5\times36}{\pi}$$

$$\text{Now,} \quad \text{Area of sector} = \frac{\pi r^2\theta}{360°} = \frac{\pi\times5\times5\times35\times36}{360°\times\pi\times10}$$

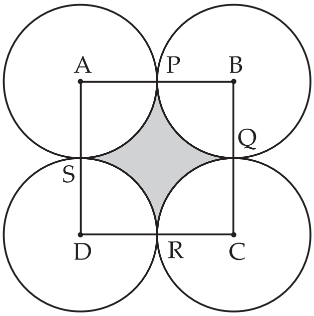
$$= \frac{25\times35}{100} = \frac{875}{100} = 8.75 \text{ cm}^2$$

$$\therefore \quad \text{Area of sector} = 8.75 \text{ cm}^2$$

Q9. Four circular cardboard pieces of radii 7 cm are placed on a paper in such a way that each piece touches other two pieces. Find the area of the portion enclosed between these pieces.

Sol. As we know that point of contact of two circles lies on the line joining their centres.

So, the line segments AB, BC, CD and AD will pass through the corresponding point of contact P, Q, R, S respectively.



As SD and AS are radius at contact point S. So, AD will be perpendicular to the tangent through S. It implies that interior angles of quadrilateral are 90° each.

As radius of each circle is equal.

So quadrilateral ABCD will be square with side $2r = 2 \times 7 = 14$ cm.

In the given figure, there is a square and 4 sectors.

4 sectors		**square**
$r = 7$ cm		$a = 2r = 2 \times 7 = 14$
$\theta = 90°$	$\Rightarrow$	$a = 14$ cm

Area enclosed between circles = Area of square – Area of 4 sectors

$$= a^2 - 4.\frac{\pi r^2\theta}{360°}$$

$$= 14\times14 - \frac{4\times\pi\times7\times7\times90°}{360°}$$

$\therefore$ Required Area $= (196 - 49\pi)\text{ cm}^2$

$= \left(196 - 49 \times \frac{22}{7}\right)\text{cm}^2$

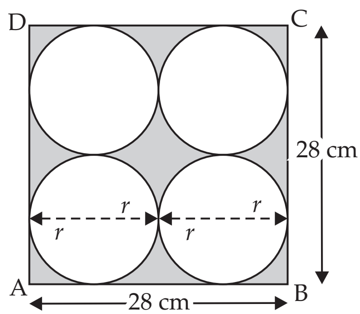
$= (196 - 7 \times 22)\text{ cm}^2 = (196 - 154)\text{ cm}^2$

$= 42\text{ cm}^2$

Hence, the area enclosed between circles = 42 cm^2.

Q10. On a square cardboard sheet of area 784 cm^2, four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.

Sol. Let a be the side of square ABCD.



Area of square ABCD = 784 cm^2

$\Rightarrow \quad a^2 = 784 \Rightarrow a = \sqrt{784}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 7 \times 7}$

$= 2 \times 2 \times 7 \Rightarrow a = 28\text{ cm}$

Now, in four circles,

$4r = \text{AB}$

$\Rightarrow \quad 4r = 28\text{ cm}$

$\Rightarrow \quad r = 7\text{ cm}$

Area enclosed between circles and square

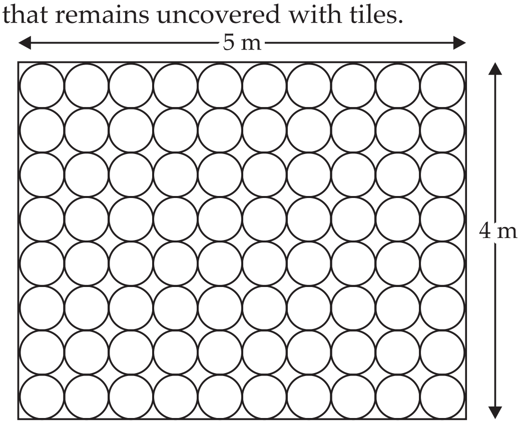
= Area of square – Area of 4 circles

$= 784 - 4\pi r^2$

$= 784 - 4 \times \frac{22}{7} \times 7 \times 7 = 784 - 616 = 168\text{ cm}^2$

Hence, the area of square sheet not covered by circular plates is 168 cm^2.

Q11. Floor of a room is of dimensions 5 m × 4 m and it is covered with circular tiles of diameters 50 cm each as shown in figure. Find the area of the floor that remains uncovered with tiles. (Use $\pi = 3.14$)



Sol. As the diameter of circular tile is 50 cm each, then $r = \frac{0.5}{2} = 0.25$ m

Number of tiles length wise $= \frac{5 \text{ m}}{0.5 \text{ m}} = 10$ tiles

Number of tiles width wise $= \frac{4 \text{ m}}{0.5 \text{ m}} = 8$ tiles

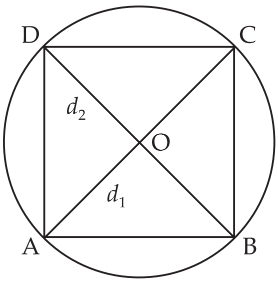
So, 10 tiles are length wise and 8 tiles are width wise.
So, total number of tiles = $10 \times 8 = 80$.
∴ Area of floor not covered by tiles

$$= \text{Area of rectangular floor} - \text{Area of 80 tiles}$$
$$= 5 \times 4 - 80\pi r^2 = 20 - 80 \times \pi \times 0.25 \times 0.25$$
$$= 20 - \frac{80 \times 314 \times 25 \times 25}{100 \times 100 \times 100} = 20 - \frac{157}{10} = 20 - 15.7 = 4.3 \text{ m}^2$$

Hence, the area of floor not covered by tiles = 4.3 m^2.

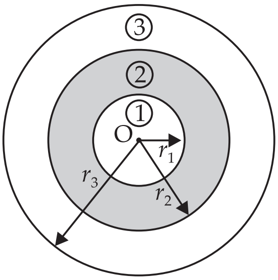
Q12. All the vertices of a rhombus lie on a circle. Find the area of the rhombus if area of the circle is 1256 cm^2. (Use $\pi = 3.14$)

Sol. All the vertices of a rhombus lie on a circle so rhombus is a square and its diagonals are of length $2r$ cm.



$$\text{Area of circle} = 1256 \text{ cm}^2$$
$$\Rightarrow \quad \pi r^2 = 1256 \text{ cm}^2$$
$$\Rightarrow \quad r^2 = \frac{1256}{\pi}$$
$$\Rightarrow \quad r^2 = \frac{1256 \times 100}{314} = 400$$
$$\Rightarrow \quad r = \sqrt{400} = 20 \text{ cm}$$
$$\therefore \quad \text{Area of rhombus} = \frac{1}{2}d_1 d_2 = \frac{1}{2} \times 2r \times 2r$$
$$= 2r^2 = 2 \times 20 \times 20$$
$$\Rightarrow \quad \text{Area of rhombus} = 800 \text{ cm}^2.$$

Q13. An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1 : 2 : 3, then find the ratio of the areas of three regions.



Sol. $d_1 : d_2 : d_3 = 1 : 2 : 3$ [given]

$= 1d : 2d : 3d$ [× by d]

$\Rightarrow \quad d_1 : d_2 : d_3 = \frac{d}{2} : \frac{2d}{2} : \frac{3d}{2}$ $\left[\times \text{ by } \frac{1}{2}\right]$

$\Rightarrow \quad r_1 : r_2 : r_3 = r, 2r, 3r$

$\therefore \quad r_1 = r, \quad r_2 = 2r, \quad r_3 = 3r$

Now, $A_1 = \pi r^2$

$$A_2 = \pi(2r)^2 = 4\pi r^2$$
$$A_3 = \pi(3r)^2 = 9\pi r^2$$

Area of innermost circle = $\pi r_1^2 = \pi r^2$

Area of region between first and second circles

$$= A_2 - A_1 = 4\pi r^2 - \pi r^2 = 3\pi r^2$$

Area of region between second and third circles

$$= A_3 - A_2 = 9\pi r^2 - 4\pi r^2 = 5\pi r^2$$

$\therefore$ Required ratio $= \pi r^2 : 3\pi r^2 : 5\pi r^2$

On dividing all the three ratios by πr^2, we get the required ratio of areas of three regions as 1 : 3 : 5.

Q14. The length of minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time 6 : 05 am and 6 : 40 am.

Sol. Time difference = (6 : 40 am – 6 : 05 a.m) = 35 min.

Time swept by min hand is 35 min.

Length of min. hand will be radius of circle swept.

$\therefore$ $r = 5$ cm

In 60 minutes time, area swept by min. hand = πr^2

In 1 minute time, area swept by min. hand = $\dfrac{\pi r^2}{60}$

In 35 minutes time, area swept by min. hand = $\dfrac{\pi r^2}{60} \times 35$

$\therefore$ Required area swept by min. hand = $\dfrac{22}{7} \times \dfrac{5 \times 5 \times 35}{60}$

$$= \frac{11 \times 25}{6} = \frac{275}{6} = 45\frac{5}{6}\text{ cm}^2$$

Hence, the required area swept by the min. hand is $45\frac{5}{6}\text{ cm}^2$.

Q15. Area of sector of central angle 200° of a circle is 770 cm². Find the length of the corresponding arc of this sector.

Sol. In the given sector,

$\theta = 200°$, $A = 770\text{ cm}^2$ and $l = ?$

$$A = \frac{\pi r^2 \theta}{360°}$$

$$\Rightarrow \quad r^2 = \frac{A \times 360°}{\pi\theta} = \frac{770 \times 360° \times 7}{22 \times 200°} = 21\text{ cm} \Rightarrow r = 21\text{ cm}$$

Now, length of arc $l = \dfrac{2\pi r\theta}{360°}$

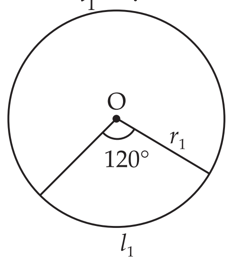
$$= 2 \times \frac{22}{7} \times \frac{21 \times 200°}{360°} = \frac{220}{3}\text{ cm} = 73\frac{1}{3}\text{ cm}$$

Hence, the required length of arc = $73\frac{1}{3}$ cm.

Q16. The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the lengths of the corresponding arcs. What do you observe?

Sol. For the first circle, we have

$r_1 = 7$ cm
$\theta_1 = 120°$
$A_1 = ?$
$l_1 = ?$

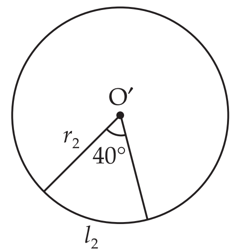


$$A_1 = \frac{\pi r_1^2 \theta_1}{360°} = \frac{22}{7} \times \frac{7 \times 7 \times 120°}{360°}$$
$$= \frac{154}{3} \text{ cm}^2$$

Now, $l_1 = \dfrac{2\pi r_1 \theta_1}{360°} = 2 \times \dfrac{22}{7} \times \dfrac{7 \times 120°}{360°}$
$$= \frac{2 \times 22}{3} = \frac{44}{3} \text{ cm}$$

For the second circle, we have

$r_2 = 21$ cm
$\theta_2 = 40°$
$A_2 = ?$
$l_2 = ?$



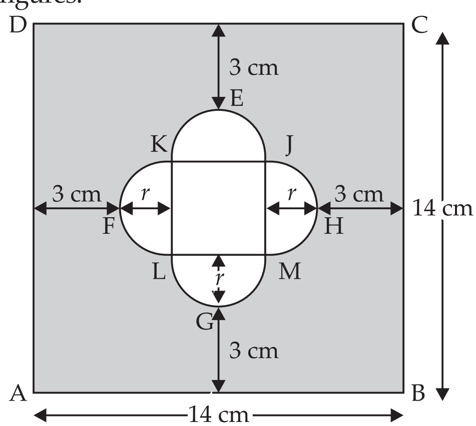
$$A_2 = \frac{\pi r_2^2 \theta_2}{360°} = \frac{22}{7} \times \frac{21 \times 21 \times 40°}{360°}$$
$$= 22 \times 7 = 154 \text{ cm}^2$$
$$l_2 = \frac{2\pi r_2 \theta_2}{360°} = 2 \times \frac{22}{7} \times \frac{21 \times 40°}{360°}$$
$$= \frac{44}{3} \text{ cm}$$

Hence, the length of arcs of two given circles are equal but area of IInd circle is three times that of Ist i.e., unequal.

Q17. Find the area of the shaded region given in figure here.

Sol. Identification of shapes of figures:

(*i*) 4 semi circles of radius r
(*ii*) square ABCD of side 14 cm
(*iii*) square JKLM of side $2r$



From figure,

$$AB = 3 + 3 + r + 2r + r$$
$$\Rightarrow \quad 14 = 6 + 4r$$
$$\Rightarrow \quad 4r = 14 - 6$$
$$\Rightarrow \quad 4r = 8$$
$$\Rightarrow \quad r = \frac{8}{4} = 2 \text{ cm}$$

So, the area of shaded region

= Area of square – Area of 4 semi circles – Area of square (JKLM)

One square ABCD	One square JKLM	Four semi-circles
$a_1 = 14$ cm	$a_2 = 2r$ $\Rightarrow a_2 = 2 \times 2$ $\Rightarrow a_2 = 4$ cm	$r = 2$ cm

$$\therefore \quad \text{Required area} = a_1^2 - 4 \times \frac{\pi r^2}{2} - a_2^2$$

$$= 14 \times 14 - \frac{4 \times \pi \times 2 \times 2}{2} - 4 \times 4$$

$$= 196 - 16 - 8\pi = (180 - 8\pi) \text{ cm}^2$$

Hence, the shaded area = $(180 - 8\pi)$ cm^2.

Q18. Find the number of revolutions made by circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

Sol. Distance covered by wheel in n revolutions with radius $r = 2\pi r n$.

$$\therefore \quad 2\pi r n = 176 \text{ m} \quad ...(i)$$

Area of wheel (circular) = 1.54 m^2

$$\Rightarrow \quad \pi r^2 = 1.54$$

$$\Rightarrow \quad r^2 = \frac{1.54}{\pi} = \frac{154 \times 7}{22 \times 100} = \frac{7 \times 7}{10 \times 10}$$

$$\Rightarrow \quad r = 0.7 \text{ m}$$

Now, $\quad 2\pi r n = 176$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times 0.7 \times n = 176 \quad \text{[from } (i)\text{]}$$

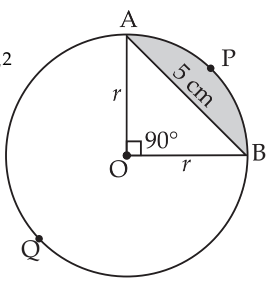
$$\Rightarrow \quad n = \frac{176 \times 7 \times 10}{2 \times 22 \times 7} = 40$$

Hence, $n = 40$ revolutions.

Q19. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre.

Sol. Chord AB = 5 cm divides the circle in two segments minor segment APB, and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, $\theta = 90°$.



Area of ΔOAB = $\frac{1}{2}$ Base × Altitude = $\frac{1}{2} r \times r = \frac{1}{2} r^2$

Area of minor segment APB

$$= \frac{\pi r^2 \theta}{360°} - \text{Area of } \Delta AOB$$

$$= \frac{\pi r^2 90°}{360°} - \frac{1}{2} r^2$$

$\Rightarrow$ Area of minor segment $= \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ ...(*i*)

Area of major segment AQB = Area of circle – Area of minor segment

$$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ ...(*ii*)

Difference between areas of major and minor segment

$$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$$

$$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right angle ΔOAB,

$$r^2 + r^2 = AB^2 \Rightarrow 2r^2 = 5^2 \Rightarrow r^2 = \frac{25}{2}$$

So, required area $= \left[\frac{1}{2}\pi\frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25\pi}{4} + \frac{25}{2}\right] \text{cm}^2$

Q20. Find the difference of the areas of a sector of angle 120° and its corresponding major sector of a circle of radius 21 cm.

Sol.

For minor sector	For major sector
$r_1 = 21$ cm	$r_2 = 21$ cm
$\theta_1 = 120°$	$\theta_2 = 360° - 120° = 240°$

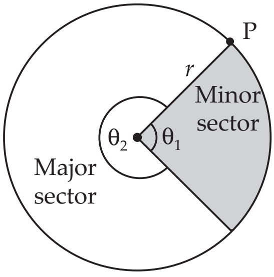
Difference in areas of major and minor sectors

$$= \frac{\pi r^2}{360°}(\theta_2 - \theta_1) \quad [\because r_1 = r_2 = r]$$

$$= \frac{22 \times 21 \times 21}{7 \times 360°}[240° - 120°]$$

$$= \frac{22 \times 21 \times 21 \times 120°}{7 \times 360°}$$

$$= 462 \text{ cm}^2$$



Hence, the difference in areas of major and minor sectors of given circle is 462 cm^2.