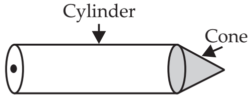
EXERCISE 12.1

Choose the correct answer from the given four options:

Q1. A cylindrical pencil sharpened at one edge is the combination of

(*a*) a cone and a cylinder
(*b*) frustrum of a cone and a cylinder
(*c*) a hemisphere and a cylinder
(*d*) two cylinders

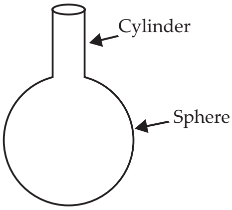


Sol. (*a*): The sharpened part of the pencil is cone and unsharpened part is cylinder.

Q2. A surahi is the combination of

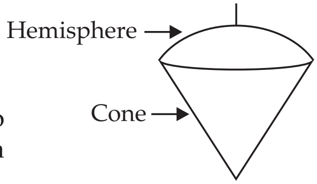
(*a*) a sphere and a cylinder (*b*) a hemisphere and a cylinder
(*c*) two hemispheres (*d*) a cylinder and a cone

Sol. (*a*): A surahi is the combination of a sphere and a cylinder.



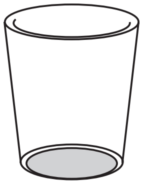
Q3. A plumbline (Sahul) is the combination of

(*a*) a cone and a cylinder
(*b*) a hemisphere and a cone
(*c*) frustrum of a cone and a cylinder
(*d*) sphere and cylinder



Sol. (*b*): Plumbline is an instrument used to check the verticality of an object. It is a combination of a hemisphere and a cone.

Q4. The shape of a glass (Tumbler) (see figure) is usually in the form of a



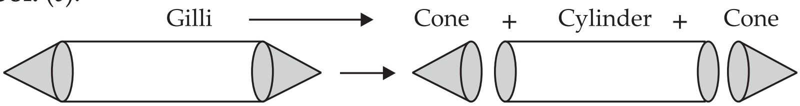
(*a*) cone (*b*) frustrum of a cone
(*c*) cylinder (*d*) sphere

Sol. (*b*): The radius of the lower circular part is smaller than the upper part. So, it is frustrum of a cone.

Q5. The shape of a gilli, in the gilli-danda game (see in figure) is the combination of

(*a*) two cylinders (*b*) a cone and a cylinder
(*c*) two cones and a cylinder (*d*) two cylinders and a cone

Sol. (*c*):

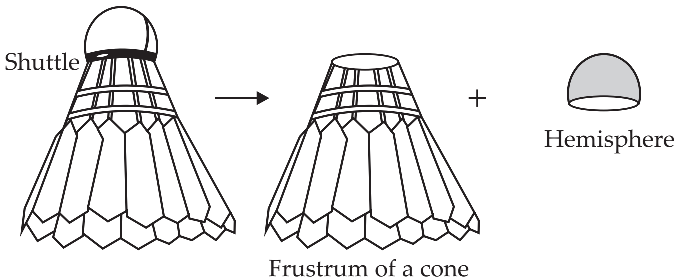


The shape of a gilli, in the gilli-danda game is a combination of two cones and a cylinder.

Q6. A shuttle cock used for playing badminton has the shape of the combination of

(*a*) a cylinder and a sphere (*b*) a cylinder and a hemisphere
(*c*) a sphere and a cone (*d*) frustrum of a cone and hemisphere

Sol. (*d*):

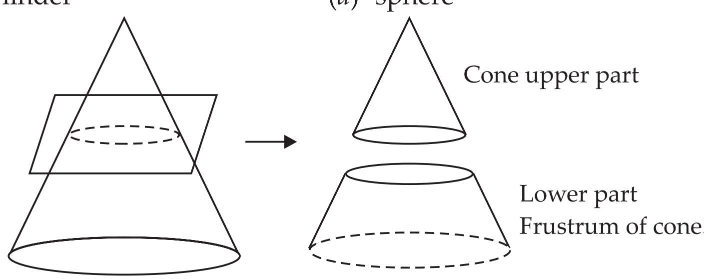


A shuttle cock used for playing badminton has the shape of the combination of frustrum of a cone and a hemisphere.

Q7. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called

(*a*) a frustrum of a cone (*b*) cone
(*c*) cylinder (*d*) sphere

Sol. (*a*):

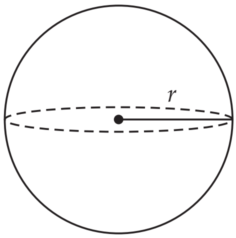


The new part that is left over on the other side of the plane is called a frustrum of a cone.

Q8. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is

(*a*) 142296 (*b*) 142396 (*c*) 142496 (*d*) 142596

Sol. (*a*): Let the spherical marble has radius r.

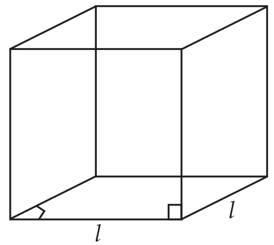


Diameter of the marble = 0.5 cm

$$\Rightarrow r = \frac{0.5}{2} \text{ cm}$$

$$\Rightarrow r = 0.25 \text{ cm}$$

Cube l = 22 cm



Let n marbles can fill the cube.

$$\therefore \quad \text{Volume of } n \text{ marbles} = \left(1 - \frac{1}{8}\right) \text{ part of volume of cube}$$

$$\Rightarrow \quad n.\frac{4}{3}\pi r^3 = \frac{7}{8} \times l^3$$

$$\Rightarrow \quad n = \frac{7l^3}{8} \times \frac{3}{4\pi r^3} = \frac{7 \times 3 \times 22 \times 22 \times 22 \times 7}{8 \times 4 \times 22 \times 0.25 \times 0.25 \times 0.25}$$

$$\Rightarrow \quad n = \frac{7 \times 3 \times 22 \times 22 \times 22 \times 100 \times 100 \times 100 \times 7}{8 \times 4 \times 22 \times 25 \times 25 \times 25}$$

$$= 7 \times 3 \times 22 \times 22 \times 2 = 42 \times 487 \times 7$$

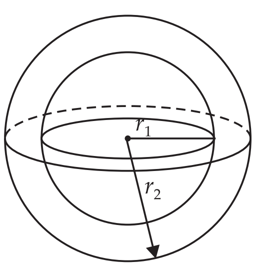
$$n = 142296$$

So, cube can accommodate upto 142296 marbles so right option is 142296, i.e., (*a*) other options are more than 142296. So, cannot accommodate.

Q9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recasted into the form of a cone of base diameter 8 cm. The height of cone is

(*a*) 12 cm (*b*) 14 cm (*c*) 15 cm (*d*) 18 cm

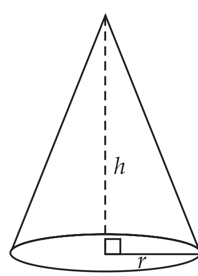
Sol. (*b*): **Main concept:** During recasting a shape into another it's volume does not change.



Spherical shell

$$r_1 = \frac{4}{2} = 2 \text{ cm}$$

$$r_2 = \frac{8}{2} = 4 \text{ cm}$$



Cone

$$r = \frac{8}{2} = 4 \text{ cm}$$

h = ?

During recasting volume remains same so

Volume of cone = Volume of hollow spherical shell

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$$

$$\Rightarrow \quad \frac{\pi}{3} r^2 h = \frac{\pi}{3} \times 4\left[r_2^3 - r_1^3\right]$$

$$\Rightarrow \quad r^2 h = 4\,[r_2^3 - r_1^3]$$

$$\Rightarrow \quad 4 \times 4\, h = 4[(4)^3 - (2)^3]$$

$$\Rightarrow \quad 4h = 64 - 8$$

$$\Rightarrow \quad h = \frac{56}{4}$$

$$\Rightarrow \quad h = 14 \text{ cm}$$

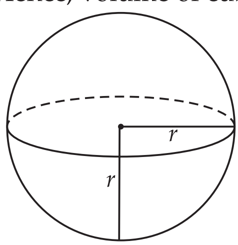
Hence, right option is (*b*).

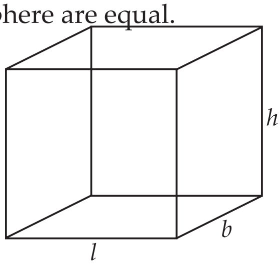
Q10. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is moulded to form a solid sphere. The radius of the sphere is

(*a*) 21 cm (*b*) 23 cm (*c*) 25 cm (*d*) 19 cm

Sol. (*a*): Solid cuboid of iron is moulded into solid sphere.

Hence, volume of cuboid and sphere are equal.





∴ Volume of sphere (solid) = Volume of cuboid

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = l \times b \times h$$

$$\Rightarrow \quad r^3 = \frac{l \times b \times h \times 3}{4 \times \pi} = \frac{49 \times 33 \times 24 \times 3 \times 7}{4 \times 22}$$

$$\Rightarrow \quad r^3 = 7 \times 7 \times 7 \times 3 \times 3 \times 3$$

$$\Rightarrow \quad r = 21 \text{ cm.}$$

Hence, right option is 21 cm *i.e.*, option (*a*).

Q11. A mason constructs a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is

(*a*) 11100 (*b*) 11200 (*c*) 11000 (*d*) 11300

Sol. (*b*): The volume of the wall covered by mortar = $\frac{1}{8}$ part

So, the volume covered by bricks of wall $= \left(1 - \frac{1}{8}\right)$ volume of wall

$= \frac{7}{8}$ volume of wall

Bricks (Cuboid)	Wall (Cuboid)
$l_1 = 22.5$ cm	$l = 270$ cm
$b_1 = 11.25$ cm	$b = 300$ cm
$h_1 = 8.75$ cm	$h = 350$ cm

Let n be the number of bricks.

According to the question, we have

$$\text{Volume of bricks} = \frac{7}{8} \text{ Volume of wall (cuboid)}$$

$$\Rightarrow \quad n \times l_1 \times b_1 \times h_1 = \frac{7}{8} l \times b \times h$$

$$\Rightarrow \quad n = \frac{7 \times l \times b \times h}{8 \times l_1 \times b_1 \times h_1} = \frac{7 \times 270 \times 300 \times 350}{8 \times 22.5 \times 11.25 \times 8.75}$$

$$\Rightarrow \quad n = \frac{7 \times 270 \times 300 \times 350 \times 10 \times 100 \times 100}{8 \times 225 \times 1125 \times 875}$$

$$\Rightarrow \quad n = 2 \times 4 \times 350 \times 4 = 32 \times 350 = 11200 \text{ bricks}$$

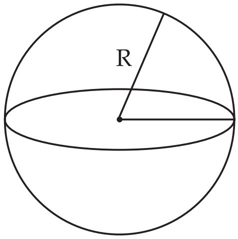
Hence, right option is (*b*).

Q12. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(*a*) 4 cm (*b*) 3 cm (*c*) 2 cm (*d*) 6 cm

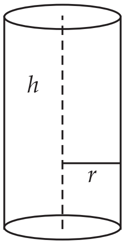
Sol. (*c*): Solid cylinder is recasted into 12 spheres.

So, the volume of 12 spheres will be equal to cylinder.



12 Spheres

R = ?



Cylinder

$r = \frac{2}{2} = 1$ cm

$h = 16$ cm

$\therefore$ Volume of 12 spheres = Volume of cylinder

$$\Rightarrow \quad \frac{4}{3}\pi R^3 \times 12 = \pi r^2 h$$

$$\Rightarrow \quad 12 \times \frac{4}{3} R^3 = r^2 h$$

$$\Rightarrow \quad R^3 = \frac{3r^2 h}{4 \times 12} = \frac{3 \times 1 \times 1 \times 16}{4 \times 12} = 1$$

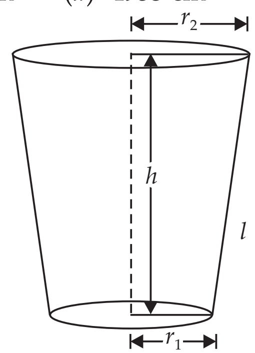
$\Rightarrow \qquad R = 1 \text{ cm}$

Hence, diameter (2R) is 2 cm. So, right option is (*c*).

Q13. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm respectively. The curved surface area of the bucket is

(*a*) 4950 cm^2 (*b*) 4951 cm^2 (*c*) 4952 cm^2 (*d*) 4953 cm^2

Sol. (*a*): Here, $r_1 = 7$ cm, $r_2 = 28$ cm, $l = 45$ cm



Curved surface area of bucket $= \pi l(r_1 + r_2)$

$$= \frac{22}{7} \times 45[7 + 28]$$

$$= \frac{22}{7} \times 45 \times 35$$

$\Rightarrow$ Curved surface area of bucket

$$= 22 \times 45 \times 5 \text{ cm}^2$$
$$= 4950 \text{ cm}^2$$

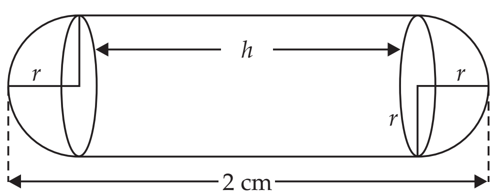
Hence, right option is (*a*).

Q14. A medicine capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

(*a*) 0.36 cm^3 (*b*) 0.35 cm^3 (*c*) 0.34 cm^3 (*d*) 0.33 cm^3

Sol. (*a*): Capsule consists of

2 Hemispheres	Cylinder
$r = 0.25$ cm	$r = \frac{0.5}{2}$ cm
	$\Rightarrow r = 0.25$ cm



$$\text{Total length of capsule} = r + h + r$$
$$\Rightarrow \quad 2 \text{ cm} = 2r + h$$
$$\Rightarrow \quad 2 = 2 \times 0.25 + h \Rightarrow h = 2 - 0.5 = 1.5 \text{ cm}$$
$$\text{Volume of capsule} = \text{Vol. of two hemispheres + Vol. of cylinder}$$
$$= 2 \times \left(\frac{4}{3}\pi r^3 \times \frac{1}{2}\right) + \pi r^2 h = \frac{4}{3}\pi r^3 + \pi r^2 h$$
$$= \pi r^2 \left[\frac{4}{3} r + h\right] = \frac{22}{7} \times 0.25 \times 0.25 \left[\frac{4}{3} \times 0.25 + \frac{15}{10}\right]$$
$$= \frac{22}{7} \times 0.25 \times 0.25 \left[\frac{4}{3} \times \frac{25}{100} + \frac{3}{2}\right]$$

$$= \frac{22}{7} \times 0.25 \times 0.25 \left[\frac{1}{3} + \frac{3}{2}\right]$$

$$= \frac{22}{7} \times 0.25 \times 0.25 \left[\frac{2+9}{6}\right]$$

$$= \frac{22 \times 25 \times 25 \times 11}{7 \times 6 \times 100 \times 100} = \frac{121}{42 \times 8} = \frac{121}{336}$$

$\therefore$ Volume of capsule $= 0.36 \text{ cm}^3$

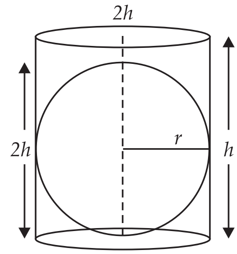
Hence, verifies the option (*a*).

Q15. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(*a*) $4\pi r^2$ (*b*) $6\pi r^2$ (*c*) $3\pi r^2$ (*d*) $8\pi r^2$

Sol. (*a*): When two hemispheres of equal radii are joined base to base new solid becomes sphere and curved surface area of sphere is $4\pi r^2$. So, the right option is (*a*).

Q16. A right circular cylinder of radius r cm and height h cm (where $h > 2r$) just encloses a sphere of diameter



(*a*) r cm (*b*) $2r$ cm

(*c*) h cm (*d*) $2h$ cm

Sol. (*b*): As the cylinder just encloses the sphere so the radius or diameter of cylinder and sphere are equal *i.e.*, $2r$ and height $h > 2r$.

Hence, verifies the option (*b*).

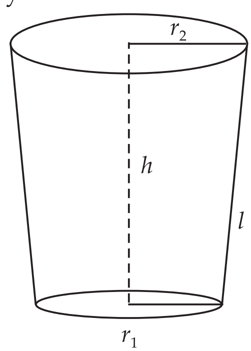
Q17. During conversion of a solid from one shape to another, the volume of new sphere will

(*a*) increase (*b*) decrease

(*c*) remains unaltered (*d*) be doubled

Sol. (*c*): During reshaping a solid, the volume of new solid will be equal to old one or remains unaltered.

Q18. The diameters of two circular ends of the bucket are 44 cm and 24 cm. The height of bucket is 35 cm. The capacity of bucket is



(*a*) 32.7 L (*b*) 33.7 L

(*c*) 34.7 L (*d*) 31.7 L

Sol. (*a*): Bucket is frustrum of a cone.

Here, $r_1 = \frac{24}{2} = 12 \text{ cm}$, $r_2 = \frac{44}{2} = 22 \text{ cm}$, $h = 35 \text{ cm}$

The volume of the bucket is given by

$$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 35[12^2 + 22^2 + 12 \times 22]$$

$$= \frac{1 \times 22 \times 35}{3 \times 7} [144 + 484 + 264]$$

$$= \frac{22 \times 35 \times 892}{3 \times 7} = \frac{22 \times 5 \times 892}{3} = \frac{110 \times 892}{3} \text{ cm}^3$$

$$= \frac{110 \times 892}{3 \times 1000} = \frac{9812}{300} = 32.706 \text{ litre.}$$

It is close to option (*a*).

Q19. In a right circular cone, the cross-section made by a plane parallel to the base is a

(*a*) circle (*b*) frustrum of a cone

(*c*) sphere (*d*) hemisphere

Sol. (*a*): In a right circular cone, if any cut is made parallel to its base, we get a circle. Hence, verifies option (*a*).

Q20. Volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(*a*) 3 : 4 (*b*) 4 : 3 (*c*) 9 : 16 (*d*) 16 : 9

Sol. (*d*): $\frac{V_1}{V_2} = \frac{64}{27}$ [V_1, V_2 are the volumes of two spheres]

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$ [r_1, r_2 are the radii of spheres]

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

Now, the ratio of their surface areas is given by

$$\frac{\text{T.S.A}_1}{\text{T.S.A}_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the required ratio 16 : 9 verifies option (*d*).

EXERCISE 12.2

Write True or False and justify your answer in the following:

Q1. Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

Sol. False: When two hemispheres of equal bases are stuck base to base it forms a sphere and total surface area of resulting sphere is $4\pi r^2$. Hence, the given statement is false.

Q2. A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $4\pi rh + 4\pi r^2$.

Sol. False: When two identical cylinders of same radius 'r' and height 'h' are stuck base (circular) to base, then the resulting cylinder will have $h' = 2h, \quad r' = r$

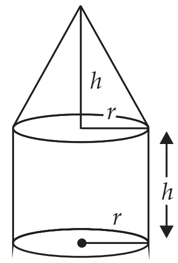
$$\therefore \quad \text{T.S.A} = 2\pi r'(r' + h) = 2\pi r(r + 2h) = 2\pi r^2 + 2\pi r.2h$$
$$= 4\pi rh + 2\pi r^2$$

Hence, the given statement is false.

Q3. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r\left[\sqrt{r^2 + h^2} + 3r + 2h\right]$.

Sol. False:

	Cone	**Cylinder**
	Radius = r	Radius = r
	Height = h	Height = h



Total surface area of the combined solid

= Curved surface area of cone + Curved surface area of cylinder + Area of the base of cylinder

$$= \pi rl + 2\pi rh + \pi r^2 = \pi r[l + 2h + r]$$

$$\because \quad l = \sqrt{r^2 + h^2}$$

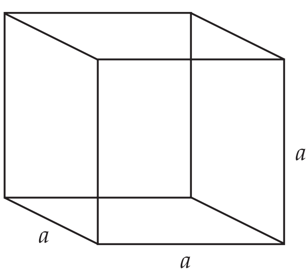
$\therefore$ Total surface area of the combined solid

$= \pi r\left[\sqrt{r^2 + h^2} + 2h + r\right]$ which is not according to the given statement.

Hence, the given statement is false.

Q4. A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is $\frac{4}{3}\pi a^3$.

Sol. False: Clearly from figure when a ball (spherical) is exactly fitted inside the cubical box then diameter of the ball becomes equal to side of cube so



$$\text{Diameter} = d = a$$

$$\Rightarrow \quad \text{Radius} = r = \frac{a}{2}$$

$$\therefore \quad \text{Volume of spherical ball} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3}\pi\frac{a^3}{8} = \frac{1}{6}\pi a^3 \neq \frac{4}{3}\pi a^3$$

Hence, the given statement is false.

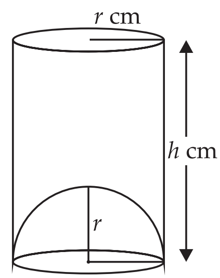
Q5. The volume of the frustrum of a cone is $\frac{1}{3}\pi h[r_1^2 + r_2^2 - r_1 r_2]$, where h is the vertical height of the frustrum and r_1, r_2 are the radii of the ends.

Sol. False: As we know that the volume of the frustrum

$$V = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2] \neq \frac{1}{3}\pi h[r_1^2 + r_2^2 - r_1 r_2]$$

Hence, the given statement is false.

Q6. The capacity of a cylindrical vessel with a hemispherical portion raised upward, at the bottom as shown in figure is $\frac{\pi r^2}{3}(3h - 2r)$.



Sol. True:

	Cylinder	**Hemisphere**
	Radius = r	Radius = r
	Height = h	

Capacity of vessel = Volume of cylinder – Volume of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad \text{Volume of vessel} = \frac{\pi r^2}{3}[3h - 2r]$$

which is equal to the volume given in the statement.

Hence, the given statement is true.

Q7. The curved surface area of a frustrum of a cone is $\pi l(r_1 + r_2)$, where $l = \sqrt{h^2 + (r_1 + r_2)^2}$, r_1, r_2 are the radii of the two ends of frustrum and h is vertical height.

Sol. False: We know that the curved surface area of frustrum $= \pi l[r_1 + r_2]$

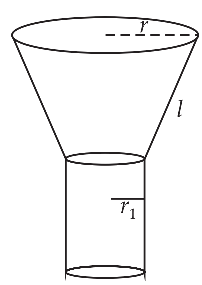
where $\quad l = \sqrt{h^2 + (r_1 - r_2)^2}$

But, $\quad l = \sqrt{h^2 + (r_1 + r_2)^2}$ in the given statement.

So, the given statement is false.

Q8. An open metallic bucket is in the shape of a frustrum of a cone, mounted on a hollow cylindrical base made of same metallic sheet. The surface area of the metallic sheet used is equal to the curved surface area of frustrum of a cone + area of circular base + curved surface area of cylinder.

Sol. True: The surface area of the sheet used for vessel will be equal to the total surface area of cylinder excluding the top and only curved surface area of frustrum of a cone.



So, total surface area of vessel

= Curved surface area of frustrum + Curved surface area of cylinder + Area of base of cylinder

It is equal to the surface area of the metallic sheet given in the statement.

Hence, the given statement is true.

EXERCISE 12.3

Q1. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edges of the cube so formed.

Sol. Volume of new cube will be equal to the sum of volumes of three cubes in recasting process.

Cube (New)	**Cube I**	**Cube II**	**Cube III**
$a = ?$	$a_1 = 3$ cm	$a_2 = 4$ cm	$a_3 = 5$ cm

$$V = V_1 + V_2 + V_3$$

$$\Rightarrow \quad a^3 = a_1^3 + a_2^3 + a_3^3$$

$$\Rightarrow \quad a^3 = (3)^3 + (4)^3 + (5)^3 = 27 + 64 + 125$$

$$\Rightarrow \quad a^3 = 216 = (6)^3$$

$$\Rightarrow \quad a = 6 \text{ cm.}$$

Hence, the edge of new recasted cube is 6 cm.

Q2. How many shots each having diameter 3 cm, can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm?

Sol.

Cuboid	***n* spherical shots**
$l = 12$ cm	$r = \frac{3}{2} = 1.5$ cm
$b = 11$ cm	
$h = 9$ cm	

Lead cuboid is recasted in lead shots (spherical) so

Volume of n spherical shots = Vol. of cuboid.

$$\Rightarrow \quad n.\frac{4}{3}\pi r^3 = l \times b \times h$$

$$\Rightarrow \quad n \times \frac{4}{3} \times \frac{22}{7} \times 1.5 \times 1.5 \times 1.5 = 9 \times 11 \times 12$$

$$\Rightarrow \quad n = \frac{9 \times 11 \times 12 \times 3 \times 7 \times 1000}{4 \times 22 \times 15 \times 15 \times 15}$$

$$\Rightarrow \quad n = 3 \times 7 \times 4 = 84$$

Hence, 84 lead shots can be made.

Q3. A bucket is in the shape of a frustrum of a cone and holds 28.490 liters of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

Sol. Here, $r_1 = 21$ cm, $r_2 = 28$ cm, $h = ?$

$$V = 28.490 \text{ L} = 28.490 \times 1000 \text{ cm}^3$$

$$\Rightarrow \quad V = 28490 \text{ cm}^3 \qquad \text{[given]}$$

Now, $$V = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2] = 28490$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times h[(21)^2 + (28)^2 + (21)(28)] = 28490$$

$$\Rightarrow \quad \frac{22}{3 \times 7} \times 7^2 h\,[3^2 + 4^2 + 3 \times 4] = 28490$$

$$\Rightarrow \quad \frac{22 \times 7 \times 7}{3 \times 7} h[9 + 16 + 12] = 28490$$

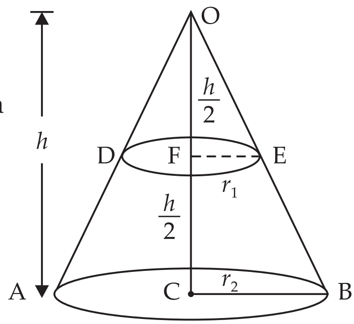
$$\Rightarrow \quad \frac{22 \times 7 \times 7 \times 37h}{3 \times 7} = 28490$$

$$\Rightarrow \quad h = \frac{28490 \times 3 \times 7}{22 \times 7 \times 7 \times 37} = 15$$

Hence, the height of frustrum = 15 cm.

Q4. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

Sol.



Cone I	**Cone II**	**Frustrum**
(ABO)	(ODE)	(DEBA)
$r = 8$ cm	$r_1 = 4$ cm	$r_1 = 4$ cm
$h = 12$ cm	$h_1 = \frac{h}{2} = \frac{12}{2}$	$r_2 = 8$ cm
	$\Rightarrow h_1 = 6$ cm	$h_2 = 6$ cm

$\Delta OBC \sim \Delta OEF$ [By AA criterion of similarity]

$$\therefore \quad \frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow \frac{r_1}{8} = \frac{h/2}{h} = \frac{1}{2}$$

$$\Rightarrow \quad r_1 = - \quad 4 \text{ cm}$$

$$\frac{\text{Vol. of frustrum (DEBA)}}{\text{Vol. of cone (ODE)}} = \frac{\frac{1}{3}\pi h_2[r_1^2 + r_2^2 + r_1 r_2]}{\frac{1}{3}\pi r_1^2 h_1}$$

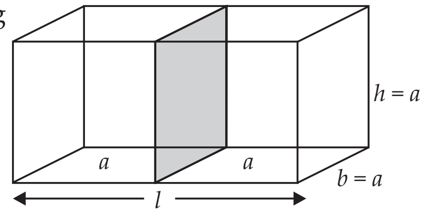
$$= \frac{6[4^2 + 8^2 + 4 \times 8]}{4 \times 4 \times 6} = \frac{(16 + 64 + 4 \times 8)}{4 \times 4} = \frac{112}{4 \times 4} = \frac{7}{1}$$

$\therefore$ Volume of frustrum : Volume of smaller cone = 7 : 1.

Q5. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of resulting cuboid?

Sol. Two identical cubes of side a are joined end to end to form a cuboid then

2 Cubes	**Cuboid**
Let length = a units	$l = 2a$ units
and breadth = a units	$b = a$ units
	$h = a$ units



So, the surface area of the resulting cuboid

$= 2[lb + lh + bh]$

$= 2[2a.a + 2a.a + a.a] = 2[2a^2 + 2a^2 + a^2]$

$= 10a^2$ ...(*i*)

Volume of the cube = 64 cm^3

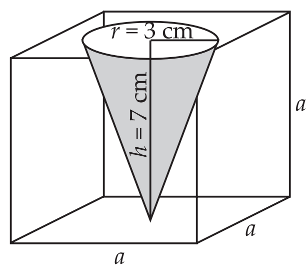
$\Rightarrow \quad a^3 = (4)^3$

$\Rightarrow \quad a = 4$ cm

$\therefore$ Total surface area of cuboid $= 10 \times 4 \times 4$ [From (*i*)]

Hence, the required surface area = 160 cm^2.

Q6. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of remaining solid.



Sol. Cone (cavity) **Cube**

$r = 3$ cm side (a) = 7 cm

$h = 7$ cm

Vol. of remaining solid

= Vol. of cube – Vol. of cone

$= a^3 - \frac{1}{3}\pi r^2 h$

$= 7 \times 7 \times 7 - \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$

$= 343 - 66 = 277$ cm^3

Hence, the volume of remaining solid = 277 cm^3.

Q7. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Sol. When two identical cones are joined base to base, the total surface area of new solid becomes equal to the sum of curved surface areas of both the cones.

So, total surface area of solid $= \pi rl + \pi rl = 2\pi rl$

In two cones, $r = 8$ cm, $h = 15$ cm

Now, $l^2 = r^2 + h^2 = 8^2 + 15^2 = 64 + 225 = 289$

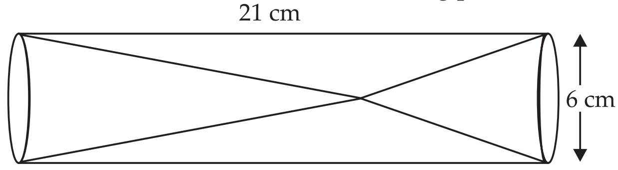
$\Rightarrow \quad l^2 = (17)^2 \Rightarrow l = 17$ cm

$\therefore$ Total surface area of solid $= 2\pi rl$

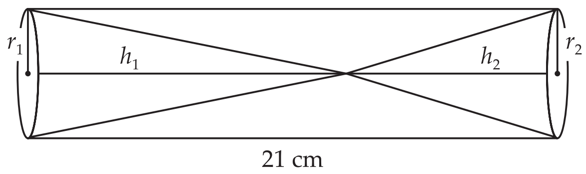
$= 2 \times \pi \times 8 \times 17 = 272\pi$ cm^2 = 854.857 cm^3

Hence, the surface area of new solid = 854.857 cm^3.

Q8. Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of their capacities is 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



Sol. As the ratio of volumes of cone c_1 and c_2 is 2 : 1, their radii are same equal to $r = \frac{6}{2} = 3$ cm .



$$\therefore \qquad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\Rightarrow \qquad \frac{2}{1} = \frac{(3)^2 h_1}{(3)^2 h_2}$$

$$\Rightarrow \qquad h_1 = 2h_2 \qquad ...(i)$$

Also, $\quad h_1 + h_2 = 21$ cm

$\Rightarrow \quad 2h_2 + h_2 = 21$

$\Rightarrow \quad 3h_2 = 21$ [Using (*i*)]

$\Rightarrow \quad h_2 = 7$ cm

Now, $\quad h_1 = 21 \text{ cm} - 7 \text{ cm} = 14$ cm ...(*ii*)

Hence, height of cone I = 14 cm and height of cone II = 7 cm.

Cone I	**Cone II**	**Cylinder**
$r_1 = \frac{6}{3} = 3$ cm	$r_2 = 3$ cm	$r = 3$ cm
$h_1 = 14$ cm	$h_2 = 7$ cm	$h = 21$ cm

$$\text{Volume of cone I} = \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14$$

$$= 132 \text{ cm}^3$$

$$\text{Volume of cone II} = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$$

$$= 22 \times 3 = 66 \text{ cm}^3$$

Volume of remaining portion of tube

$$= \text{Vol. of cylinder} - \text{Vol. of cone I} - \text{Vol. of cone II}$$

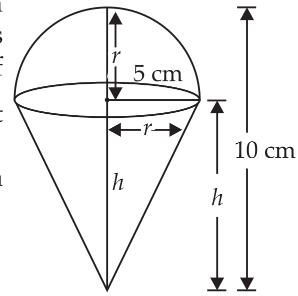
$$= \pi r^2 h - 66 - 132$$

$$= \frac{22}{7} \times 3 \times 3 \times 21 - 198$$

$$= 22 \times 27 - 198 = 594 - 198 = 396 \text{ cm}^3$$

Hence, the required volume is 396 cm^3.

Q9. An ice-cream cone full of ice-cream having radius 5 cm, and height 10 cm, as shown in figure. Calculate the volume of ice-cream, provided that its $\frac{1}{6}$ part is left unfilled with ice cream.



Sol. Ice-cream cone can be considered as a hemisphere on a cone.

Hemisphere	**Cone**
$r = 5$ cm	$r = 5$ cm
	$h = 10 - 5 = 5$ cm

$\frac{1}{6}$ part of ice-cream is left unfilled.

So, Vol. of ice-cream $= \left(1 - \frac{1}{6}\right)$ [Volume of cone and hemisphere]

$$= \frac{5}{6}\left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right] = \frac{5}{6} \times \frac{1}{3}\pi r^2 [h + 2r]$$

$$= \frac{5 \times 22 \times 5 \times 5}{6 \times 3 \times 7}[5 + 2 \times 5] = \frac{5 \times 22 \times 5 \times 5 \times 15}{6 \times 3 \times 7}$$

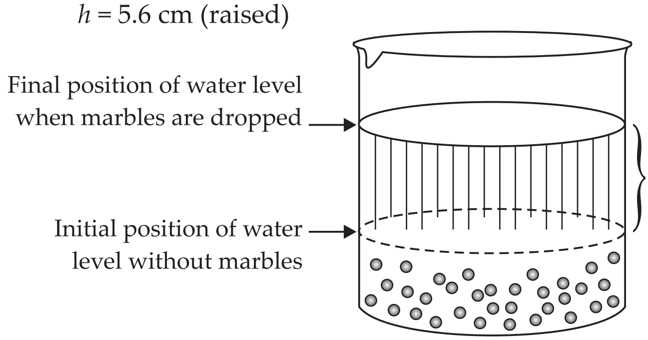
$$= \frac{55 \times 125}{21} = \frac{6875}{21} \cong 327.4 \text{ cm}^3$$

Hence, the volume of ice-cream in cone is 327.4 cm^3.

Q10. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm, containing some water. Find the number of marbles that should be dropped into the beaker so that water level rises by 5.6 cm.

Sol. When marbles are dropped in beaker filled partially with water, the volume of water raised in beaker will be equal to the volume of n marbles. The shape of water raised in beaker is cylindrical.

Cylindrical beaker	**Spherical marbles**
$r = \frac{7}{2} = 3.5$ cm	$R = \frac{1.4}{2} = 0.7$ cm
$h = 5.6$ cm (raised)	



Beaker

$\therefore$ Vol. of n spherical balls = Vol. of water raised in cylinders

$$\Rightarrow \quad n \times \frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\Rightarrow \quad n \times \frac{4}{3} \times 0.7 \times 0.7 \times 0.7 = 3.5 \times 3.5 \times 5.6$$

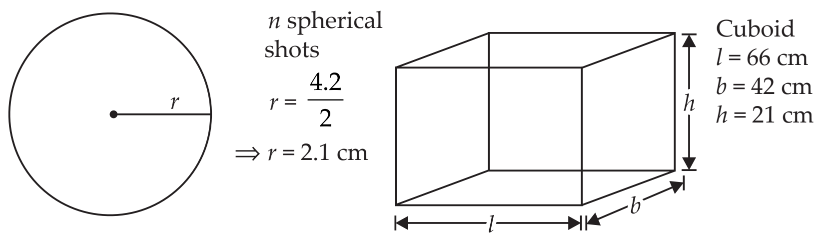
$$\Rightarrow \quad n = \frac{35 \times 35 \times 56 \times 3}{4 \times 7 \times 7 \times 7}$$

$$\Rightarrow \quad n = 50 \times 3 = 150$$

Hence, required number of marbles = 150.

Q11. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm?

Sol.



Spherical lead shots are recasted from cuboid of lead. So, volume of n spherical lead shots is equal to the volume of cuboid.

$\therefore$ Volume of n spherical lead shots = Vol. of lead cuboid

$$\Rightarrow \quad n \times \frac{4}{3}\pi r^3 = l \times b \times h$$

$$\Rightarrow \quad n \times \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 = 66 \times 42 \times 21$$

$$\Rightarrow \quad n = \frac{66 \times 42 \times 21 \times 3 \times 7 \times 1000}{4 \times 22 \times 21 \times 21 \times 21}$$

$$\Rightarrow \quad n = 3 \times 500 = 1500$$

Hence, the number of shots are 1500.

Q12. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm?

Sol. Solid cube is recasted into spherical lead shots.

Cube	**Spherical lead shots**
$a = 44$ cm	$r = \frac{4}{2} = 2$ cm

$\therefore$ Vol. of n spherical lead shots = Vol. of cube

$$\Rightarrow \quad n . \frac{4}{3}\pi r^3 = a^3$$

$$\Rightarrow \quad n \times \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 = 44 \times 44 \times 44$$

$$\Rightarrow \qquad n = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 2 \times 2 \times 2} = 121 \times 21 \Rightarrow n = 2541$$

Hence, the number of lead shots are 2541.

Q13. A wall 24 m long, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}$th of the volume of the wall, then find the number of bricks used in constructing the wall.

Sol. Wall is 24 m long, 0.4 m thick and 6 m high.

So, volume of wall = 24 m × 0.4 m × 6 m = 57.6 m^3

Since $\frac{}{10}$th of the volume of the wall is occupied by mortar, so the volume of bricks in the wall

$$= \left(1 - \frac{1}{10}\right) \text{ part of the wall.}$$

$$= \frac{9}{10} \text{th part of the wall}$$

$$= \frac{9}{10} \times 57.6 \text{ m}^3 = 51.84 \text{ m}^3$$

$$\text{Volume of one brick} = 25 \text{ cm} \times 16 \text{ cm} \times 10 \text{ cm}$$

$$= \frac{25}{100} \times \frac{16}{100} \times \frac{10}{100} \text{ m}^3 = 0.004 \text{ m}^3$$

$$\therefore \text{ Required number of bricks} = \frac{\text{Volume of bricks in the wall}}{\text{Volume of one brick}}$$

$$= \frac{51.84}{0.004} = 12960$$

So, 12960 bricks are used in constructing the wall.

Q14. Find the number of metallic circular discs with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Sol. Required number of metallic discs

$$= \frac{\text{Volume of right circular cylinder}}{\text{Volume of one metallic circular disc}}$$

$$= \frac{\pi\left(\frac{4.5}{2}\right)^2 \times 10}{\pi\left(\frac{15}{20}\right)^2 \times 0.2}$$

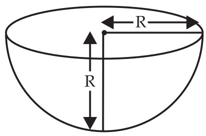
$$= (2.25)^2 \times \frac{400}{225} \times 50$$

$$= \frac{225}{100} \times \frac{225}{100} \times \frac{400}{225} \times 50 = 450$$

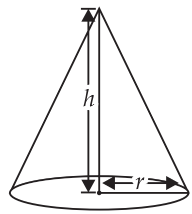
EXERCISE 12.4

Q1. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone.

Sol.



Hemisphere
R = 8 cm



Cone
$r = 6$ cm
$h = ?$

As the hemisphere is recasted into a cone. So,

$$\text{Volume of cone} = \text{Volume of hemisphere}$$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi R^3$$

$$\Rightarrow \quad r^2 h = 2R^3$$

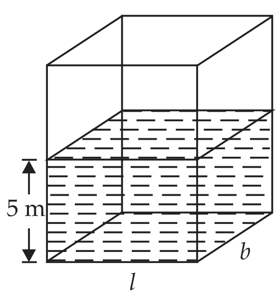
$$\Rightarrow \quad h = \frac{2R^3}{r^2} = \frac{2 \times 8 \times 8 \times 8}{6 \times 6} = \frac{32 \times 8}{9}$$

$$= \frac{256}{9} = 28.44 \text{ cm} \Rightarrow h = 28.44 \text{ cm.}$$

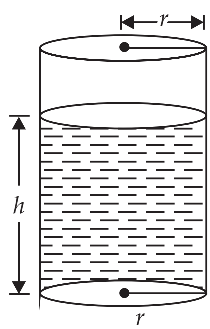
Hence, the height of the cone is 28.44 cm.

Q2. A rectangular water tank of base 11 m $\times$ 6 m contains water upto a height of 5 m. If the water in tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

Sol.



Rectangular water tank
$l = 11$ m
$b = 6$ m
H = 5 m



Cylindrical tank
$r = 3.5$ m
$h = ?$

Water is transferred from cuboid to cylinder, so, the volume of water in both the vessels will be same.

$$\therefore \quad \pi r^2 h = l \times b \times H$$

$$\Rightarrow \quad \frac{22}{7} \times 3.5 \times 3.5 \times h = 11 \times 6 \times 5$$

$\Rightarrow \qquad h = \dfrac{11 \times 6 \times 5 \times 7 \times 100}{22 \times 35 \times 35} = \dfrac{60}{7}$

$\Rightarrow \qquad h \cong 8.6$ m (approx.)

Hence, the height of water level in cylindrical tank is 8.6 m.

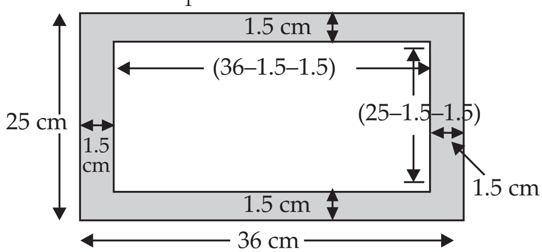
Q3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic centimetre of iron weighs 7.5 g, then find the weight of the box.

Sol. External dimensions **Internal dimensions**

$l_2 = 36$ cm $\qquad l_1 = 36 - 1.5 - 1.5 = 36 - 3 = 33$ cm

$b_2 = 25$ cm $\qquad b_1 = 25 - 3 = 22$ cm

$h_2 = 16.5$ cm $\qquad h_1 = 16.5 - 1.5 = 15$ cm



Volume of iron in the open box

$$= l_2b_2h_2 - l_1b_1h_1$$

$$= (36 \times 25 \times 16.5) - (33 \times 22 \times 15)$$

$$= 9 \times 5 \times 11 \left[\frac{36 \times 25 \times 165}{10 \times 9 \times 5 \times 11} - \frac{33 \times 22 \times 15}{9 \times 5 \times 11} \right]$$

$\Rightarrow$ Volume of iron in the open box

$$= 9 \times 5 \times 11 \left[\frac{4 \times 5 \times 15}{10} - 22 \right]$$

$$= 45 \times 11\ [30 - 22] = 495 \times 8 = 3960 \text{ cm}^3$$

Volume of iron is 3960 cm^3.

1 cm^3 of iron weighs = 7.5 gm

So, 3960 cm^3 of iron will weigh $= \dfrac{3960 \times 75}{10} = 396 \times 75$ gm

$$= \frac{396 \times 75}{1000} \text{ kg} = \frac{297}{10} \text{ kg}$$

Hence, the weight of the box = 29.7 kg

Q4. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one-fifth of a litre?

Sol. Let n times the barrel of pen is filled.

Radius of barrel, $r = \frac{5 \text{ mm}}{2} = \frac{5}{20} \text{ cm} = \frac{1}{4} \text{ cm}$

Height of barrel, $h = 7$ cm

$\therefore \quad n \times \text{Volume of barrel} = \text{Volume of Ink}$

$\Rightarrow \quad n \times \pi r^2 h = \frac{1}{5} \times \text{one litre}$

$\Rightarrow \quad n\, \pi r^2 h = \frac{1}{5} \times 1000 \text{ cm}^3$

$\Rightarrow \quad n \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 7 = 200 \text{ cm}^3$

$\Rightarrow \quad n = \frac{200 \times 7 \times 4 \times 4}{22 \times 7} = \frac{1600}{11}$

Ink in one full barrel can write words = 3300

So, n barrels can write words = $3300n$

$= 3300 \times \frac{1600}{11} = 4{,}80{,}000$

Hence, the required number of words = 4,80,000 words.

Q5. Water flows at the rate of 10 m per min. through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

Sol. When a fluid (water) flows through a pipe of area of cross-section A with velocity v, then volume of water coming from pipe in time t

= Area of cross-section × Length

$= \text{A} \times v.t \qquad [\because \text{V} = \text{Area of base} \times \text{Height}]$

Cylinder

$\text{A} = \pi r^2$

$r = \frac{5 \text{ mm}}{2} = \frac{5}{2000} \text{ m}$

$r = \frac{1}{400} \text{ m}$

$v = 10 \text{ m/min} \Rightarrow v = \frac{10}{60} \text{ m/s} = \frac{1}{6} \text{ m/s}$

Cone

$\text{R} = \frac{40}{2} \text{ cm} = 0.2 \text{ m}$

$\text{H} = 24 \text{ cm} = 0.24 \text{ m}$

$\therefore \quad \text{Volume of flowing water} = \text{Volume of cone}$

$\Rightarrow \text{Area of base} \times \text{height (dist.)} = \frac{1}{3}\pi \text{R}^2\text{H}$

$\Rightarrow \quad \text{A} \times v.t = \frac{1}{3}\pi \text{R}^2\text{H}$

$\Rightarrow \quad \pi r^2.v.t = \frac{1}{3}\pi \text{R}^2\text{H}$

$\Rightarrow \quad r^2.v.t = \frac{1}{3}\text{R}^2\text{H}$

$$\Rightarrow \quad \frac{1}{400} \times \frac{1}{400} \times \frac{1}{6} t = \frac{1}{3} \times 0.2 \times 0.2 \times 0.24$$

$$\Rightarrow \quad t = \frac{2 \times 2 \times 24 \times 400 \times 400 \times 6}{3 \times 10000}$$

$$\Rightarrow \quad t = 4 \times 24 \times 4 \times 4 \times 2 \text{ sec}$$

$$= \frac{4 \times 24 \times 4 \times 4 \times 2}{60} \text{min} = \frac{512}{10} = 51.2 \text{ min}$$

$$= 51 \text{ min} + 0.2 \text{ min} = 51 \text{ min} + 0.2 \times 60 \text{ sec}$$

$$\Rightarrow \quad t = 51 \text{ min and } 12 \text{ sec.}$$

Hence, conical tank will fill in 51 min and 12 sec.

Q6. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of rice. How much canvas cloth is required to just cover the heap?

Sol. Heap of rice is in shape of cone, so

$$r = \frac{9}{2} \text{ m} = 4.5 \text{ m}$$

$$h = 3.5 \text{ m}$$

$$\therefore \quad V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 3.5$$

$$\Rightarrow \quad V = \frac{22 \times 9 \times 9 \times 35}{3 \times 7 \times 2 \times 2 \times 10} = \frac{33 \times 9}{4} = \frac{297}{4}$$

$$\Rightarrow \quad V = 74.25 \text{ m}^3$$

Hence, volume of rice = 74.25 m^3.

For Canvas:

Area of canvas = Curved surface area of cone

$= \pi r l$

But, $l^2 = r^2 + h^2 = (4.5)^2 + (3.5)^2 = 20.25 + 12.25$

$\Rightarrow \quad l^2 = 32.50$

$\Rightarrow \quad l = \sqrt{32.5} = 5.7 \text{ m}$

$\therefore$ Area of canvas = $\frac{22}{7} \times 4.5 \times 5.7 = 80.614$

$\Rightarrow$ Area of canvas = 80.61 m^2

Q7. A factory manufactures 1,20,000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹ 0.05 per dm^2.

Sol. Shape of the pencil is cylindrical.

Here, $h = 25$ cm, $2\pi r = 1.5$ cm

Curved surface area of one pencil = $2\pi rh$

$\therefore$ Curved surface area of 1,20,000 pencils

$$= 1{,}20{,}000 \times 2\pi rh = 1{,}20{,}000 \times 1.5 \times 25 \text{ cm}^2$$

$$= \frac{1{,}20{,}000 \times 15 \times 25}{10 \times 100} \text{ dm}^2 = 600 \times 75 \text{ dm}^2$$

$\therefore$ Cost of colouring = ₹ $600 \times 75 \times 0.05$ = ₹ 2250

Hence, cost of colouring is ₹ 2250.

Q8. Water is flowing at the rate of 15 km/hr through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

Sol. Main concept: Volume of flowing water = A.v.t

Here, A = Area of cross-section of pipe or flowing water

v = Speed of water

t = Time

Here, A is equivalent to area of base and height equal to distance ($v.t$) and we know that V = area of base $\times$ height.

Flowing water	**Pond**
$A = \pi r^2$ (circular pipe)	$l = 50$ m
$r = \frac{14}{2}$ cm = 7 cm = 0.07 m	$b = 44$ m
v = 15 km/hr = 15000 m/hr	h = 21 cm = 0.21 m

$\therefore$ Volume of flowing water = Volume of same water in pond

$$\Rightarrow \quad A.v.t = l \times b \times h$$

$$\Rightarrow \quad \pi r^2.v.t = l \times b \times h$$

$$\Rightarrow \quad \frac{22}{7} \times 0.07 \times 0.07 \times 15000t \text{ (hrs.)} = 50 \times 44 \times 0.21$$

$$\Rightarrow \quad t = \frac{50 \times 44 \times 0.21 \times 7}{22 \times 0.07 \times 0.07 \times 15000}$$

$$= \frac{50 \times 44 \times 21 \times 7 \times 100}{22 \times 7 \times 7 \times 15000} \Rightarrow t = 2 \text{ hours}$$

Hence, time required is 2 hours.

Q9. A solid iron cuboidal block of dimensions 4.4 m $\times$ 2.6 m $\times$ 1 m is recasted into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

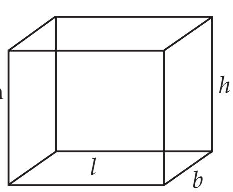
Sol.

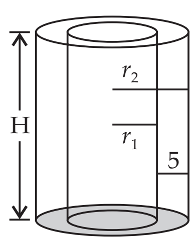
Cuboid

l = 4.4 m = 440 cm

b = 2.6 m = 260 cm

h = 1 m = 100 cm





Cylindrical pipe

Thickness = 5 cm

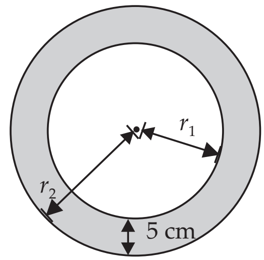
r_1 = 30 cm

r_2 = 30 + 5 = 35 cm

H = ?

Cuboid is recasted into hollow cylindrical pipe.

$\therefore$ Volume of cuboid = Volume of cylindrical pipe (hollow)



$$\Rightarrow \quad l\,bh = \pi r_2^2 H - \pi r_1^2 H$$

$$\Rightarrow \quad l\,bh = \pi H[r_2^2 - r_1^2]$$

$$\Rightarrow \quad 440 \times 260 \times 100 = \frac{22}{7} \times H\,[35^2 - 30^2]$$

$$\Rightarrow \quad 440 \times 260 \times 100 = \frac{22}{7} \times H\,[1225 - 900]$$

$$\Rightarrow \quad 440 \times 260 \times 100 = \frac{22}{7} \times H\,[325]$$

$$\Rightarrow \quad H = \frac{100 \times 440 \times 260 \times 7}{22 \times 325}\text{ m}$$

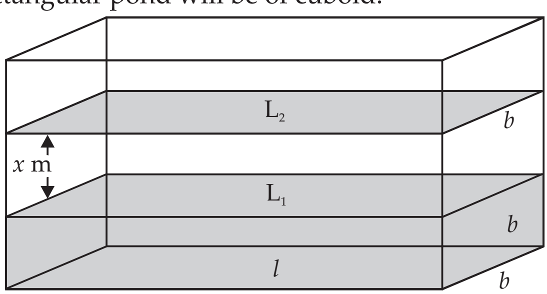
$$= \frac{1600 \times 7}{100}\text{ m}$$

$$\Rightarrow \quad H = 112\text{ m}$$

Hence, the length of pipe is 112 m.

Q10. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3?

Sol. Let the rise of water level in the pond be x m. The shape of water rise in rectangular pond will be of cuboid.



Here, $l = 80$ m, $b = 50$ m, $h = x$ m

Number of persons = 500

Let the water level before the persons took a dip was at L_1. Now, when 500 persons dipped into the pond, water level rises from L_1 to L_2 of height x m. The volume of water between two levels will be equal to the water displaced by 500 persons.

$\therefore$ Volume of water raised = Volume of cuboid

$$\Rightarrow \quad l \times b \times h = 500 \times 0.04$$

$$\Rightarrow \quad 80 \times 50 \times x = 500 \times 0.04$$

$$\Rightarrow \quad x = \frac{500 \times 0.04}{80 \times 50} = \frac{1}{200}\text{ m} = \frac{100}{200}\text{ cm} = 0.5\text{ cm}$$

$\Rightarrow \qquad x = 0.5$ cm

Hence, the rise of water level in the pond is 0.5 cm.

Q11. 16 glass spheres each of radius 2 cm are packed into cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box.

Sol. **Cuboidal box** — $l = 16$ cm, $b = 8$ cm, $h = 8$ cm

16 spheres — $r = 2$ cm

Volume of water filled in the box = Volume of box – Volume of 16 glass spheres

$\therefore$ Vol. of water filled in the box $= lbh - 16.\frac{4}{3}\pi r^3$

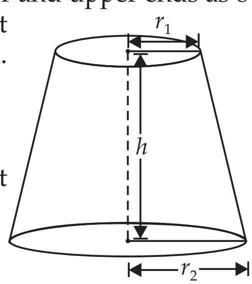
$$= 16 \times 8 \times 8 - \frac{16 \times 4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2$$

$$= 16 \times 8\left[\frac{16 \times 8 \times 8}{16 \times 8} - \frac{4 \times 22 \times 2 \times 2 \times 2}{3 \times 7 \times 1 \times 8}\right]$$

$$= 16 \times 8\left[8 - \frac{88}{21}\right] = 16 \times 8\left[\frac{168 - 88}{21}\right]$$

$$= \frac{16 \times 8 \times 80}{21} = \frac{10240}{21} = 487.6 \text{ cm}^3$$

Q12. A milk container of height 16 cm is made of metal sheet in the form of a frustrum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of ₹ 22 per L, which the container can hold.



Sol. Here, $r_1 = 8$ cm

$r_2 = 20$ cm

$h = 16$ cm

$\therefore$ Volume of milk = Volume of frustrum as it is filled completely

$$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 \times r_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 16\,[8^2 + 20^2 + 8 \times 20]$$

$$= \frac{22 \times 16}{21}[64 + 400 + 160]$$

$$= \frac{352}{21} \times 624 = \frac{352 \times 208}{7} = \frac{73216}{7}$$

$= 10459.428 \text{ cm}^3 = 10.459$ litre

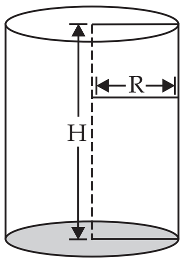
Volume of milk = 10.459 litre

$\therefore$ Cost of milk = ₹ 22 × 10.459 = ₹ 230.107

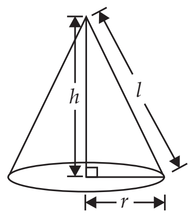
Hence, the cost of milk = ₹ 230.107.

Q13. A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground, and a conical heap of sand is formed. If the height of conical heap is 24 cm, find the radius and slant height of the heap.

Sol. By identifying the shapes, we have cone and cylinder. On reshaping from cylindrical to conical, the volume of sand emptied out remains same.



Cylinder
R = 18 cm
H = 32cm



Cone (heap)
r = ?
h = 24 cm
l = ?

$\therefore$ Volume of conical heap = Volume of cylinder

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \pi R^2 H \quad \Rightarrow \quad \frac{1}{3} r^2 h = R^2 H$$

$$\Rightarrow \quad r^2 = \frac{3R^2H}{h} = \frac{3 \times 18 \times 18 \times 32}{24}$$

$$\Rightarrow \quad r^2 = 18 \times 18 \times 2 \times 2$$

$$\Rightarrow \quad r = 18 \times 2 \text{ cm} = 36 \text{ cm}$$

Radius of conical heap is 36 cm.

Now, $l^2 = r^2 + h^2 = 36 \times 36 + 24 \times 24$

$$\Rightarrow \quad l^2 = 4 \times 4\,[9 \times 9 + 6 \times 6] = 4 \times 4\,[81 + 36]$$

$$= 4 \times 4 \times 117$$

$$\Rightarrow \quad l = \sqrt{4 \times 4 \times 3 \times 3 \times 13} = 4 \times 3\sqrt{13}$$

$$\Rightarrow \quad l = 12\sqrt{13} \cong 12 \times 3.60555$$

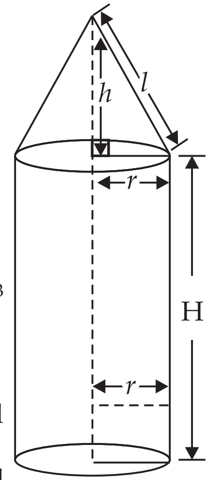
$$\Rightarrow \quad l = 43.2666 \text{ cm}$$

Hence, radius and slant height are 36 cm and 43.2666 cm respectively.

Q14. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of cylinder. The diameter and height of cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of rocket. (Use $\pi = 3.14$)

Sol.	**Cylinder**	**Cone**
	$r = \frac{6}{2} = 3$ cm	$r = 3$ cm
	H = 12 cm	$l = 5$ cm

$\therefore \quad l^2 = r^2 + h^2$ or $h^2 = l^2 - r^2$



$$= 5^2 - 3^2 = 25 - 9$$

$$\Rightarrow \quad h = \sqrt{16} = 4 \text{ cm}$$

Now, Volume of rocket

$$= \text{Volume of cylinder} + \text{Volume of cone}$$

$$= \pi r^2 H + \frac{1}{3}\pi r^2 h = \pi r^2 \left[H + \frac{1}{3}h\right]$$

$$= 3.14 \times 3 \times 3 \left[12 + \frac{1}{3} \times 4\right]$$

$$= 3.14 \times 9 \left[\frac{40}{3}\right] = 3.14 \times 3 \times 40 = 376.8 \text{ cm}^3$$

$\therefore$ Volume of Rocket = 376.8 cm^3

Total surface area of rocket

= Curved surface area of cylinder + Curved surface area of cone + Area of base of cylinder

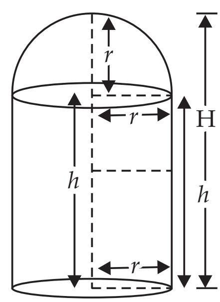
[As it is closed (Given)]

$= 2\pi rH + \pi rl + \pi r^2 = \pi r\,[2H + l + r]$

$= 3.14 \times 3[2 \times 12 + 5 + 3] = 3.14 \times 3 \times 32 = 301.44 \text{ cm}^2$

Hence, the surface area of rocket is 301.44 cm^2.

Q15. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building.



Sol. **Dome (hemisphere)** **Cylinder**

Radius = r Radius = r

$$H = 2r \quad \text{[given]}$$

$$\Rightarrow \quad h + r = 2r$$

$$\Rightarrow \quad h = 2r - r = r$$

$$\text{Volume of building} = 41\frac{19}{21} = \frac{880}{21} \text{ m}^3$$

$$\Rightarrow \text{Vol. of cylinder} + \text{Vol. of hemisphere} = \frac{880}{21} \text{ m}^3$$

$$\Rightarrow \quad \pi r^2 h + \frac{2}{3}\pi r^3 = \frac{880}{21}$$

$$\Rightarrow \quad \pi r^2 r + \frac{2}{3}\pi r^3 = \frac{880}{21} \qquad [\because h = r]$$

$$\Rightarrow \quad \frac{5}{3}\pi r^3 = \frac{880}{21} \Rightarrow \frac{5}{3} \times \frac{22}{7} \times r^3 = \frac{880}{21}$$

$$\Rightarrow \quad r^3 = \frac{880 \times 3 \times 7}{21 \times 5 \times 22} \Rightarrow r^3 = 8$$

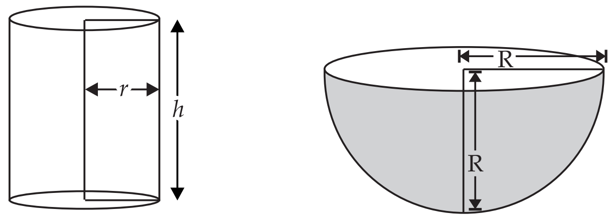
$$\Rightarrow \quad r = 2 \text{ m}$$

Hence, height of the building is $2 \times 2 = 4$ m. $[\because H = 2r \text{ (Given)}]$

Q16. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

Sol.

n cylindrical bottles	**Hemisphere**
$r = 1.5$ cm	$R = 9$ cm
$h = 4$ cm	



Cylindrical bottle Hemispherical bowl

As the volume of liquid does not change

So, Volume of n bottles = Volume of hemisphere

$$\Rightarrow \quad n\pi r^2 h = \frac{2}{3}\pi R^3 \Rightarrow nr^2h = \frac{2}{3}R^3$$

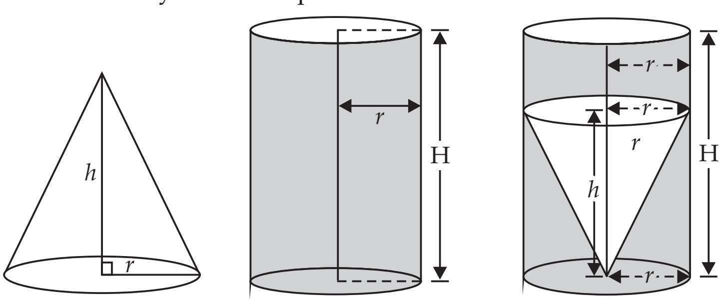
$$\Rightarrow \quad n \times 1.5 \times 1.5 \times 4 = \frac{2}{3} \times 9 \times 9 \times 9$$

$$\Rightarrow \quad n = \frac{2 \times 9 \times 9 \times 9 \times 100}{3 \times 15 \times 15 \times 4} = 54$$

Hence, 54 bottles are needed.

Q17. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in cylinder, if the radius of the cylinder is equal to the radius to the cone.

Sol.



Cone Cylinder Cone & Cylinder

Cone	**Cylinder**
$r = 60$ cm	$R = r = 60$ cm
$h = 120$ cm	$H = 180$ cm

Cone is placed inside the cylindrical vessel full of water. So, the volume of water from cylinder will over flow equal to the volume of cone.

Hence, the water left in cylinder = Vol. of cylinder – Vol. of cone (*i*)

Volume of water left after immersing the cone into cylinder full of water

$$= \text{Volume of cylinder} - \text{Volume of cone}$$

$$= \pi R^2 H - \frac{1}{3}\pi r^2 h$$

$\therefore$ Required volume of water in cylinder

$$= \pi r^2 H - \frac{1}{3}\pi r^2 h \qquad [\because R = r]$$

$$= \pi r^2\left[H - \frac{1}{3}h\right] = \frac{22}{7} \times 60 \times 60\left[180 - \frac{120}{3}\right]$$

$$= \frac{22}{7} \times 60 \times 60 \times 140 \text{ cm}^3$$

$$= \frac{22 \times 60 \times 60 \times 140}{7 \times 100 \times 100 \times 100} = \frac{22 \times 72}{1000} = \frac{1584}{1000}$$

$\therefore$ Vol. of water in cylinder = 1.584 m^3

Hence, required volume of water left = 1.584 m^3.

Q18. Water flows through a cylindrical pipe, whose inner radius is 1 cm at the rate of 80 cm per second in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?

Sol. Main concept: Volume of flowing water = $A.v.t$

Area of base = A = Area of cross-section of flowing water

height = distance = speed × time = $v.t$

Flowing water is filled in cylindrical tank. Hence, the volume of flowing water is equal to volume of water in cylindrical tank.

Flowing water	**Cylinder**
$A = \pi r^2$ (cylinder)	$R = 40$ cm
$v = 80$ cm/s	$H = x$

$r = 1$ cm and $t = \frac{1}{2}$ hr $= \frac{1}{2} \times 3600$ sec = 1800 sec

$\therefore$ Volume of water in cylindrical tank = Volume of flowing water

$$\Rightarrow \quad \pi R^2 H = A.v.t$$

$$\Rightarrow \quad \pi R^2 H = \pi r^2\, v.t$$

$$\Rightarrow \quad 40 \times 40 \times x = 1 \times 1 \times 80 \times 1800$$

$$\Rightarrow \quad x = \frac{80 \times 1800}{40 \times 40} = 5 \times 18 = 90 \text{ cm}$$

Hence, the rise of water level in cylindrical tank is 90 cm.

Q19. The rain water from a roof of dimensions 22 m × 20 cm drains into a cylindrical vessel having diameter of the base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm.

Sol.	**Cuboid**	**Cylinder**
	$l = 22\text{ m} = 2200\text{ cm}$	$r = \frac{2}{2} = 1\text{ m} = 100\text{ cm}$
	$b = 20\text{ m} = 2000\text{ cm}$	$h = 3.5\text{ m} = 350\text{ cm}$
	$H = x\text{ cm}$	

If water from roof is not allowed to flow, then water level on roof rises upto x cm (let) then volume of cuboidal shape of water will be equal to the volume of cylinder.

$\therefore$ Volume of rain water = Volume of cylinder

$$\Rightarrow \quad l \times b \times h = \pi r^2 h$$

$$\Rightarrow \quad 2200 \times 2000 \times x = \frac{22}{7} \times 100 \times 100 \times 350$$

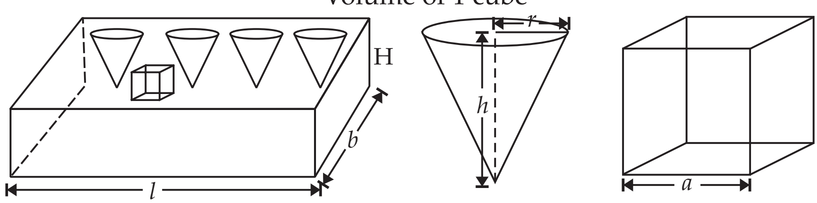
$$\Rightarrow \quad x = \frac{22 \times 100 \times 100 \times 350}{7 \times 2200 \times 2000} = \frac{5}{2} = 2.5\text{ cm}$$

Hence, the rainfall is 2.5 cm.

Q20. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins respectively. The dimensions of cuboid are 10 cm, 5 cm, 4 cm. The radius of each of the conical depressions is 0.5 cm and depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.

Sol. From a cuboidal piece of wood, depressions (4 cones and 1 cube) are made.

So, the volume of wood = Volume of cuboid – Volume of 4 cones – Volume of 1 cube



$l = 10$ cm	$r = 0.5$ cm	$a = 3$ cm
$b = 5$ cm	$h = 2.1$ cm	
$H = 4$ cm		

Hence, the volume of the wood in the entire pen stand

$$= l \times b \times h - \left(\frac{1}{3}\pi r^2 h\right) \times 4 - a^3$$

$$= 10 \times 5 \times 4 - \frac{4}{3} \times \frac{22}{7} \times \frac{5 \times 5 \times 21}{1000} - (3)^3$$

$$= 200 - \frac{11 \times 5 \times 4}{100} - 27 = 200 - 2.20 - 27 = 200 - 29.2 = 170.8\text{ cm}^3$$

So, the volume of the wood in the pen stand after making depressions is 170.8 cm^3.