EXERCISE 13.1

Choose the correct answer from the given four options in the following questions:

Q1. In the formula $\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are the deviations from a of

(*a*) lower limits of the classes
(*b*) upper limits of the classes
(*c*) mid points of the classes
(*d*) frequencies of the class marks

Sol. (*c*): In the given formula, a is assumed mean from class marks (x_i) and $d_i = x_i - a$
Hence, d_i is the deviation of class mark (mid-value) from the assumed mean 'a'. Hence, verifies the option (*c*).

Q2. While computing mean of grouped data, we assume that the frequencies are

(*a*) evenly distributed over all the classes
(*b*) centred at the class marks of the classes
(*c*) centred at the upper limits of the classes
(*d*) centred at the lower limits of the classes.

Sol. (*b*): In grouping the data from ungrouped data all the observations between lower and upper limits of class marks are taken in one group then mid value or class mark is taken for further calculation.

Hence frequencies or observations must be centred at the class marks of the classes.

Hence, verifies the option (*b*).

Q3. If x_i's are the mid points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\overline{x}$ is the mean, then $\sum (f_i x_i - \overline{x})$ is equal to

(*a*) 0 (*b*) – 1 (*c*) 1 (*d*) 2

Sol. (*a*): $\because$ $$\overline{x} = \sum_{i=1}^{n} \frac{f_i x_i}{n}$$

$\therefore$ $$\sum_{i=1}^{n} f_i x_i = n\overline{x} \quad \text{(I)}$$

$$\sum_{i=1}^{n} \overline{x} = \overline{x} + \overline{x} + \overline{x} + \cdots n \text{ times}$$

$$\Rightarrow \quad \sum_{i=1}^{n} \overline{x} = n\overline{x} \qquad \text{(II)}$$

From (I) and (II), we have

$$\sum_{i=1}^{n} f_i x_i = \sum_{i=1}^{n} \overline{x}$$

$$\Rightarrow \quad \sum_{i=1}^{n} f_i x_i - \sum_{i=1}^{n} \overline{x} = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} (f_i x_i - \overline{x}) = 0 \quad \text{or} \quad \sum f_i x_i - \overline{x} = 0$$

Hence, verifies the option (*a*).

Q4. In the formula $\overline{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped frequency distribution, μ_i =

(*a*) $\frac{x_i + a}{h}$ (*b*) $h(x_i - a)$ (*c*) $\frac{x_i - a}{h}$ (*d*) $\frac{a - x_i}{h}$

Sol. (*c*): $\overline{x} = a + h\left[\frac{\sum f_i u_i}{\sum f_i}\right]$

$u_i = \frac{(x_i - a)}{h}$ verifies the option (*c*).

Q5. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its

(*a*) mean (*b*) median (*c*) mode (*d*) all of these

Sol. (*b*): The point of intersection of the less than type and of the more than type cumulative frequency curves give the median on abscissa as on X-axis we take the upper or lower limits respectively and on Y-axis we take cumulative frequency.

Hence verifies the option (*b*).

Q6. For the following distribution:

Class	0–5	5–10	10–15	15–20	20–25
Frequency	10	15	12	20	9

the sum of lower limits of median class and modal class is

(*a*) 15 (*b*) 25 (*c*) 30 (*d*) 35

Sol. (*b*):

Class	Frequency	Cumulative frequency
0–5	10	10
5–10	15	25
10–15	12	37
15–20	20	57
20–25	9	66

The modal class is the class having the maximum frequency.
The maximum frequency 20 belongs to class (15–20).
Here, $n = 66$

So, $\frac{n}{2} = \frac{66}{2} = 33$

33 lies in the class 10 – 50.
Therefore, 10 – 15 is the median class.
So, sum of lower limits of (15–20) and (10–15) is (15 + 10) = 25 verifies the option (*b*).

Q7. Consider the following frequency distribution:

Class	0–5	6–11	12–17	18–23	24–29
Frequency	13	10	15	8	11

the upper limit of the median class is

(*a*) 7 (*b*) 17.5 (*c*) 18 (*d*) 18.5

Sol. (*b*):

Class	Frequency	Cumulative frequency
0.5–5.5	13	13
5.5–11.5	10	23
11.5–17.5	15	38
17.5–23.5	8	46
23.5–29.5	11	57

The median of 57 (odd) observations = $\frac{57+1}{2} = \frac{58}{2}$ = 29th term

29th term lies in class 11.5–17.5.
So, upper limit is 17.5 verifies option (*b*).

Q8. For the following distribution the modal class is

(*a*) 10–20 (*b*) 20–30 (*c*) 30–40 (*d*) 50–60

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

Sol. (*c*):

Marks	Number of students or Frequency	f_i
0–10	3 – 0 = 3	3
10–20	12–3 = 9	9
20–30	27 – 12 = 15	15
30–40	57 – 27 = 30	30
40 – 50	75 – 57 = 18	18
50 – 60	80 – 75 = 5	5

Modal class has maximum frequency (30) in class 30–40.
Hence, verifies the option (*c*).

Q9. Consider the data

Class	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is

(*a*) 0 (*b*) 19 (*c*) 20 (*d*) 38

Sol. (*c*):

Class	Frequency	Cumulative frequency
65–85	4	4
85–105	5	9
105–125	13	22
125–145	20	42
145–165	14	56
165–165	7	63
185–205	4	67

Hence, $n = 67$ (odd)

So, Median = $\frac{67+1}{2} = 34$

34 lies in class 125 – 145.
So, median class is 125 – 145 and upper limit is 145.
Now, the maximum frequency is 20 and it lies in class 125 – 145 (Modal class).
Lower limit of modal class = 125.

Hence, the required difference 145 – 125 = 20, verifies the option (*c*).

Q10. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below–

Class	13.8–14	14–14.2	14.2–14.4	14.4–14.6	14.6–14.8	14.8–15
Frequency	2	4	5	71	48	20

The number of athletes who completed the race in less than 14.6 seconds is:

(*a*) 11 (*b*) 71 (*c*) 82 (*d*) 130

Sol. (*c*): The number of athletes who completed the race in less than 14.6 sec = 2 + 4 + 5 + 71 = 82.

Hence verifies the option (*c*).

Q11. Consider the following distribution:

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

The frequency of the class 30–40 is

(*a*) 3 (*b*) 4 (*c*) 48 (*d*) 51

Sol. (*a*):

Class	Number of Students	*f*
0–10	63 – 58 = 5	5
10–20	58 – 55 = 3	3
20–30	55 – 51 = 4	4
30–40	51 – 48 = 3	3
40–50	48 – 42 = 6	6
50–60	42 – 0 = 42	42

Hence the frequency of 30–40 class interval is 3 which verifies the option (*a*).

Q12. If an event cannot occur, then its probability is

(*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) 0

Sol. (*d*): An event that cannot occur has 0 probability, such an event is called impossible event. Hence, (*d*) is the correct answer.

Q13. Which of the following cannot be the probability of an event?

(*a*) $\frac{1}{3}$ (*b*) 0.1 (*c*) 3% (*d*) $\frac{17}{16}$

Sol. (*d*): Probability of any event cannot be more than one or negative as $\frac{17}{16} > 1$.

Hence, verifies the option (*d*).

Q14. An event is very unlikely to happen. Its probability is closest to

(*a*) 0.0001 (*b*) 0.001 (*c*) 0.01 (*d*) 0.1

Sol. (*a*): The probability of the event which is very unlikely to happen will be very close to zero. So it's probability is 0.0001 which is minimum among the given values.

Hence, verifies the option (*a*).

Q15. If the probability of an event is p, then the probability of its complementary event will be

(*a*) $p - 1$ (*b*) p (*c*) $1 - p$ (*d*) $1 - \frac{1}{p}$

Sol. (*c*): Probability of an event + Probability of its complementary event = 1

$\therefore$ p + Probability of complement = 1

or Probability of complement = $1 - p$

Hence, verifies the option (*c*).

Q16. The probability expressed as a percentage of a particular occurrence can never be

(*a*) less than 100 (*b*) less than 0

(*c*) greater than 1 (*d*) anything but a whole number

Sol. (*b*): $\because$ Probability lies between 0 and 1 and when it is converted into percentage it will be between 0 and 100. So, cannot be negative. So, verifies the option (*b*).

Q17. If P(A) denotes the probability of an event A, then

(*a*) $P(A) < 0$ (*b*) $P(A) > 1$

(*c*) $0 \leq P(A) \leq 1$ (*d*) $-1 \leq P(A) \leq 1$

Sol. (*c*): As the probability of an event can be between 0 and 1. Hence, verifies the option (*c*).

Q18. If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

(*a*) $\frac{3}{26}$ (*b*) $\frac{3}{13}$ (*c*) $\frac{2}{13}$ (*d*) $\frac{1}{2}$

Sol. (*a*): In a deck of 52 cards, there are 26 red cards.

Number of red face cards = 3 of hearts + 3 of diamonds
= 6

So, probability of having a red face card = $\frac{6}{52} = \frac{3}{26}$

Hence, verifies the option (*a*).

Q19. The probability that a non leap year selected at random will contains 53 sundays is

(*a*) $\frac{1}{7}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{5}{7}$

Sol. (*a*): Number of days in non leap year = 365

Number of weeks $= \frac{365}{7} = 52\frac{1}{7} = 52$ weeks

Number of days left = 1

i.e., may be any of 7 days which from Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday so T(E) = 7

$$F(E) = 1 \text{ (Sunday)}$$

$$P(F) = \frac{F(E)}{T(E)} = \frac{1}{7}$$

Hence, verifies option (*a*).

Q20. When a die is thrown, the probability of getting an odd number less than 3 is

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) 0

Sol. (*a*): Total number of outcomes favourable for event E are (1, 2, 3, 4, 5, 6) *i.e.*, T(E) = 6

A number which is odd and less than 3 is 1 so, F(E) = 1

So, probability $P(E) = \frac{F(E)}{T(E)} = \frac{1}{6}$

Hence, verifies option (*a*).

Q21. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is

(*a*) 4 (*b*) 13 (*c*) 48 (*d*) 51

Sol. (*d*): Favourable event E is all cards except the ace of heart and ace of heart is only one. Hence, the number of outcomes favourable for event E are 52 – 1 = 51, verifies the option (*d*).

Q22. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(*a*) 7 (*b*) 14 (*c*) 21 (*d*) 28

Sol. (*b*): T(E) = 400

Number of outcomes favourable for event E, *i.e.*, F(E) = ?

$$P(F) = 0.035$$

$$\therefore \quad P(F) = \frac{F(E)}{T(E)} \Rightarrow 0.035 = \frac{F(E)}{400}$$

So, F(E) = 0.035 × 400 = 14 eggs. So, the number of bad eggs are 14.

Hence, verifies the option (*b*).

Q23. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

(*a*) 40 (*b*) 240 (*c*) 480 (*d*) 750

Sol. (*c*): T(E) = 6000

$$F(E) = ?$$
$$P(F) = 0.08$$
$$\therefore \quad P(F) = \frac{F(E)}{T(E)} \Rightarrow 0.08 = \frac{F(E)}{6000}$$
$$\therefore \quad F(E) = 6000 \times 0.08 = 480$$

Hence, verifies the option (*c*).

Q24. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(*a*) $\frac{1}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{4}{5}$ (*d*) $\frac{1}{3}$

Sol. (*a*): T(E) = 40

Number of outcomes favourable for event E are 5, 10, 15, 20, 25, 30, 35, 40 *i.e.*, F(E) = 8

$$P(F) = \frac{F(E)}{T(E)} = \frac{8}{40} = \frac{1}{5}$$

Hence, verifies option (*a*).

Q25. Someone is asked to take a number from 1 to 100. The probability that it is a prime is

(*a*) $\frac{1}{5}$ (*b*) $\frac{6}{25}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{13}{50}$

Sol. (*c*): T(E) = 100

F(E) prime numbers (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97)

$$F(E) = 25$$
$$P(F) = \frac{F(E)}{T(E)} = \frac{25}{100} = \frac{1}{4}$$ Hence, verifies option (*c*).

Q26. A school has five houses, A B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

(*a*) $\frac{4}{23}$ (*b*) $\frac{6}{23}$ (*c*) $\frac{8}{23}$ (*d*) $\frac{17}{23}$

Sol. (*b*): T(E) = 23

F(E) = not from A, B, C i.e. = 23 – (4 + 8 + 5)

$$F(E) = 23 - 17 = 6$$
$$\therefore \quad P(F) = \frac{6}{23}$$ verifies the option (*b*).

EXERCISE 13.2

Q1. The median of an ungrouped data and the median calculated, when the same data is grouped are always the same. Do you think that this is a correct statement? Give reason.

Sol. The median of an ungrouped data and the median calculated when the same data is grouped are not always the same because the median for ungrouped data is calculated by arranging the data in increasing or decreasing order. But for calculating the median of a grouped data, the formula used is based on the assumption that the observations are uniformly distributed in the classes.

Q2. In calculating the mean of grouped data, grouped in classes of equal width, we may use the formula

$$\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

where a = assumed mean.

a must be one of the mid-points of the classes. Is the last statement correct? Justify your answer.

Sol. Not always. Assumed mean can be considered any convenient number which makes calculation easy.

Q3. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

Sol. Not always. The median, mean and mode can be the same.

They may be equal if number of observations are odd and are equispaced.

Q4. Will the median class and modal class of grouped data always be different? Justify your answer.

Sol. The median and modal class may be same if modal class is median class which is not always possible as the number of frequencies may be maximum in any class.

So given statement is not true.

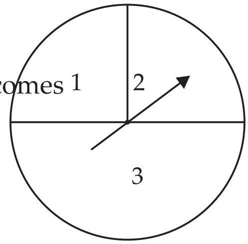
Q5. In a family having three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is 1/4. Is this correct? Justify your answer.

Sol. False: In a family of three children events are (b, b, b,) (g, b, b), (g, g, b), (g, g, g)

T(E) = 4

The probability of each is not 1/4, because the outcomes are not equally likely.

Q6. A game consists of spinning an arrow which comes to rest pointing at one of the regions (1, 2, or 3) see figure. Are the outcomes 1, 2 and 3 equally likely to occur? Give reason.



Sol. The area of region 3 is double either of 1 or 2 and area of 1 and 2 are equal so no. of outcomes (or probability) of region 3 is double of either 1 or 2.

So the outcomes of 1, 2, 3 are not equally likely to occur.

Q7. Apoorv throws two dice once and compules the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has better chance of getting the number 36? Why?

Sol. For Apoorv T(E) = 36

Favourable is only (6, 6) *i.e.*, F(E) = 1

then P(F) by Apoorv = $\frac{F(E)}{T(E)} = \frac{1}{36}$

Now for Peehu T′(E) = 6

$$F'(E) = 1$$

$$P'(A) = \frac{F'(E)}{T'(E)} = \frac{1}{6}$$

$$\frac{1}{6} > \frac{1}{36}$$

$$\therefore \quad P'(A) > P(A)$$

Hence, Peehu has the better chance.

Q8. When we toss a coin, there are two possible outcomes—head or tail. Therefore, the probability of each outcome is 1/2. Justify your answer.

Sol. There are two outcomes of equally in all manner. So probability of both head and tail are equal to 1/2 each.

Hence, the given statement is true.

Q9. A student says that if you throw a die, it will show up 1 or not 1. Therefore, the probability of getting 1 and the probability of getting not 1 each is equal to 1/2. Is this correct? Give reason.

Sol. A dice can be thrown in 6 different equally likely ways. Possible outcomes are given by S = {1, 2, 3, 4, 5, 6}.

P(getting 1) = $\frac{1}{6}$ and P(not getting 1) = $\frac{5}{6}$.

Hence, the given statement is not correct.

Q10. I toss three coins together. The possible outcomes are no heads, 1 head, 2 heads and 3 heads. So, I say that probability of no heads is 1/4. What is wrong with this conclusion?

Sol. Three coins are tossed together.

Total outcomes T(E) = 2^3 = 8

(T T H), (T H H), (H T H), (H H T), (H T T), (T H T) and (H H H), (T T T), so, the number of favourable outcomes for event (getting no head) = 1

$\therefore$ Probability (getting no head) = $\frac{1}{8}$

Hence the given statement is wrong $\left(\because \quad \frac{1}{8} \neq \frac{1}{4}\right)$.

Q11. If you toss a coin 6 times and it comes down head on each occasion. Can you say that the probability of getting a head is 1? Given reasons.

Sol. A coin is tossed 6 times so

$$T(E) = 6$$

In total six events, number of outcomes for getting head are 3 so F(E) = 3 again

P(F) getting head = $\frac{3}{6} = \frac{1}{2}$

Hence, the given statement is false.

Q12. Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be tail? Give reasons?

Sol. As the coin is tossed 3 times and gets tail each time but it is not necessary that 4th time will be tail it may be either tail or head in any further toss.

Hence, the given statement is false.

Q13. If I toss a coin 3 times and get head each time, should I except a tail to have a higher chance in the 4th toss? Give reason in support of your answer.

Sol. As we know that a coin has two equal chances always either head or tail. So next time on tossing he can get either tail or head.

So, the given statement is false.

Q14. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since this situation has only two possible outcomes, so, the probability of each is 1/2. Justify.

Sol. From 1 to 100 numbers, there are 50 even and 50 odd numbers.

Total number of outcomes T(E) = 100

Number of outcomes favourable for event E = F(E) = 50

So, $P(F) = \frac{50}{100} = \frac{1}{2}$

Similarly, the probability of getting odd numbers is $\frac{1}{2}$. Hence the probability of getting odd and even each is $-$. Hence, the given statement is true.

EXERCISE 13.3

Q1. Find the mean of the distribution:

Class	1–3	3–5	5–7	7–10
Frequency	9	22	27	17

Sol.

Class	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
1–3	2	9	18
3–5	4	22	88

5–7	6	27	162
7–10	8.5	17	144.5
		$\Sigma f_i = 75$	$\Sigma f_i x_i = 412.5$

$$\text{Mean } \overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{412.5}{75} = 5.5$$

Hence, the mean of the given distribution is 5.5.

Q2. Calculate the mean of the scores of 20 students in a mathematics test:

Marks	10–20	20–30	30–40	40–50	50–60
No. of students	2	4	7	6	1

Sol.

Marks	**Class Mark (x_i)**	**No. of students (f_i)**	$f_i x_i$
10–20	15	2	30
20–30	25	4	100
30–40	35	7	245
40–50	45	6	270
50–60	55	1	55
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 700$

$$\therefore \quad \text{Means } \overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{700}{20} = 35$$

Q3. Calculate the mean of the following data:

Class	4–7	8–11	12–15	16–19
Frequency	5	4	9	10

Sol. Class marks of these classes are same, so no need to convert given data to continuous.

Class	**Class marks (x_i)**	$d_i = x_i - a$	**Frequency (f_i)**	$f_i d_i$
4–7	5.5	−4	5	−20
8–11	$\boxed{9.5} = a$	0	4	0
12–15	13.5	+4	9	36
16–19	17.5	+8	10	80
			$\Sigma f_i = 28$	$\Sigma f_i d_i = 96$

a = assumed mean, d_i = deviation from mean

$$\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 9.5 + \frac{96}{28} = 9.5 + 3.43$$

$$\therefore \quad \overline{x} = 12.93$$

Hence, the mean = 12.93.

Q4. The following table gives the number of pages written by Sarika for completing her own book for 30 days:

No. of pages written per day	16–18	19–21	22–24	25–27	28–30
No. of days	1	3	4	9	13

Find the mean number of pages written per day.

Sol. No need to change the class-intervals into continuous intervals as Class marks of continuous and discontinuous classes are same. d_i is deviation from assumed mean.

Class interval	**Mid Value (x_i)**	$d_i = (x_i - a)$	**No. of days (f_i)**	$f_i d_i$
16–18	17	– 6	1	– 6
19–21	20	– 3	3	–9
22–24	$a = 23$	0	4	0
25–27	26	3	9	27
28–30	29	6	13	78
			$\Sigma f_i = 30$	$\Sigma f_i d_i = 90$

a = assumed mean, $a = 23$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 23 + \frac{90}{30} = 23 + 3 = 26$$

$\therefore \quad \bar{x} = 26$

Hence, the mean of pages written per day is 26.

Q5. The daily income of a sample of 50 employees are tabulated as follows:

Income (in ₹)	1–200	201–400	401–600	601–800
No. of employees	14	15	14	7

Find the mean daily income of employees.

Sol. No need to convert discontinuous classes into continuous for class mark because class mark of both C.I. are same and gives same result of $\bar{x}$.

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
1–200	100.5	– 200	14	– 2800
201–400	$\boxed{300.5} = a$	0	15	0
401–600	500.5	+ 200	14	2800
601–800	700.5	+ 400	7	2800
			$\Sigma f_i = 50$	$\Sigma f_i d_i = 2800$

a = assumed mean $\qquad d_i = x_i - a$

Let $a = 300.5$

$$\therefore \quad \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 300.5 + \frac{2800}{50} = ₹\ 356.5$$

$\bar{x} = ₹\ 356.5$

Hence, the average daily income of employees is ₹ 356.5.

Q6. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table:

No. of seats	100–104	104–108	108–112	112–116	116–120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied over the flights.

Sol. Let a = assumed mean

d_i = deviation of x_i from assumed mean = $x_i - \bar{a}$

f_i = frequencies (No. of passengers)

C.I. = Number of seats occupied in that flight

x_i = Class mark of ith C.I.

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
100–104	102	– 8	15	– 120
104–108	106	– 4	20	– 80
108–112	110 = a	0	32	0
112–116	114	4	18	72
116–120	118	8	15	120
			$\Sigma f_i = 100$	$\Sigma f_i d_i = -8$

Here, $a = 110$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 110 + \frac{-8}{100} = 110 - 0.08$$

$\bar{x} = 109.92$, but, seat cannot be in decimal, so,

$\Rightarrow \quad \bar{x} = 109$

Hence, the mean number of seats occupied over the flights is 109.

Q7. The weights (in kg) of 50 wrestlers are recorded in the following table:

Weight in Kg	100–110	110–120	120–130	130–140	140–150
No. of Wrestlers	4	14	21	8	3

Find the mean weight of wrestlers.

Sol. a = assumed mean from x_i (weight in kg) = 125

x_i = class mark of classes (in kg)

d_i = deviation of x_i from $a = (x_i - a)$ (kg)

f_i = frequency (no. of wrestlers)

(C.I.) class interval = Number of wrestlers

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
100–110	105	–20	4	–80
110–120	115	–10	14	–140
120–130	125 = a	0	21	0
130–140	135	10	8	80
140–150	145	20	3	60
			$\Sigma f_i = 50$	$\Sigma f_i d_i = -80$

a = 125 kg

$$\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$\overline{x} = 125 + \frac{(-80)}{50} = 125 - \frac{8}{5} = 125 - 1.6$$

$\Rightarrow$ $\overline{x}$ = 123.4 kg

Hence, the mean weight of wrestlers = 123.4 kg

Q8. The mileage (km/litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below:

Mileage (km/l)	10–12	12–14	14–16	16–18
Number of cars (f_i)	7	12	18	13

Find the mean mileage.

The manufacturer claimed that the mileage of the model was 16 km L^{-1}. Do you agree with this claim?

Sol. $d_i = x_i - a$

x_i = class mark and a = assumed mean.

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
10–12	11	–2	7	–14
12–14	13 = a	0	12	0
14–16	15	2	18	36
16–18	17	4	13	52
			$\Sigma f_i = 50$	$\Sigma f_i d_i = 74$

a = 13

$$\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 13 + \frac{74}{50} = 13 + 1.48 = 14.48 \text{ km } L^{-1}$$

Hence, mean mileage of car is 14.48 km/litre.

So, the manufacturer's is statement is wrong that mileage is 16 km L^{-1}.

Q9. The following is the distribution of weights (in kg) of 40 persons.

Weight (in kg)	40–45	45–50	50–55	55–60	60–65	65–70	70–75	75–80
No. of person	4	4	13	5	6	5	2	1

Construct a cumulative frequency distribution (of the less than type) table for the above data.

Sol.

C.I.	f_i	Weight (in kg)	Cumulative frequency
40–45	4	less than 45	4 + 0 = 4
45–50	4	less than 50	4 + 4 = 8
50–55	13	less than 55	13 + 8 = 21
55–60	5	less than 60	5 + 21 = 26
60–65	6	less than 65	6 + 26 = 32
65–70	5	less than 70	5 + 32 = 37
70–75	2	less than 75	2 + 37 = 39
75–80	1	less than 80	1 + 39 = 40

Q10. The following table show the cumulative frequency distribution of marks of 800 students in an examination:

Marks	Number of Students	Marks	Number of Students
Below 10	10	Below 60	570
Below 20	50	Below 70	670
Below 30	130	Below 80	740
Below 40	270	Below 90	780
Below 50	440	Below 100	800

Construct a frequency distribution table for the data above.

Sol.

Marks	Number of students	C.I. (Marks)	Frequency
Below 10	10	0–10	10 – 0 = 10
Below 20	50	10–20	50–10 = 40
Below 30	130	20–30	130–50 = 80
Below 40	270	30–40	270–130 = 140
Below 50	440	40–50	440 – 270 = 170
Below 60	570	50–60	570 – 440 = 130
Below 70	670	60–70	670–570 = 100
Below 80	740	70–80	740 – 670 = 70
Below 90	780	80–90	780 – 740 = 40
Below 100	800	90–100	800–780 = 20

Q11. Form the frequency distribution table from the following data:

Marks (out of 90)	Number of students (c.f)
More than or equal to 80	4
More than or equal to 70	6
More than or equal to 60	11

More than or equal to 50	17
More than or equal to 40	23
More than or equal to 30	27
More than or equal to 20	30
More than or equal to 10	32
More than or equal to 0	34

Sol.

Marks (out of 90)	No. of Students	C.I.	No. of Students (f_i)
More than or equal to 0	34	0–10	34 – 32 = 2
More than or equal to 10	32	10–20	32 – 30 = 2
More than or equal to 20	30	20–30	30 – 27 = 3
More than or equal to 30	27	30–40	27 – 23 = 4
More than or equal to 40	23	40–50	23 – 17 = 6
More than or equal to 50	17	50–60	17 – 11 = 6
More than or equal to 60	11	60–70	11 – 6 = 5
More than or equal to 70	6	70–80	6 – 4 = 2
More than or equal to 80	4	80–90	4 – 0 = 4

Q12. Find the unknown entries a, b, c, d e and f in the following distribution of heights of students in a class.

Height (in cm)	Frequency	Cumulative frequency
150–155	12	a
155–160	b	25
160–165	10	c
165–170	d	43
170–175	e	48
175–180	2	f
Total	50	

Sol.

Height (in cm)	Frequency (f_i)	*c.f.* (given)	*c.f.* calculated from (f_i)
150–155	12	a	12
155–160	b	25	$12 + b$
160–165	10	c	$12 + b + 10$
165–170	d	43	$22 + b + d$
170–175	e	48	$22 + b + d + e$
175–180	2	f	$22 + 2 + b + d + e$

Comparing the two *c.f.* (cumulative frequency) calculated one and given one $a = 12$

$$12 + b = 25 \Rightarrow b = 25 - 12 = 13 \qquad \Rightarrow b = 13$$
$$c = 12 + b + 10 = 12 + 13 + 10 = 35 \qquad \Rightarrow c = 35$$
$$22 + b + d = 43 \Rightarrow d = 43 - 22 - b = 21 - 13 = 8 \qquad \Rightarrow d = 8$$
$$22 + b + d + e = 48 \Rightarrow e = 48 - 22 - 13 - 8 = 48 - 43 = 5 \qquad \Rightarrow e = 5$$
$$f = 24 + b + d + e = 24 + 13 + 8 + 5 = 24 + 26 = 50 \Rightarrow f = 50$$

Q13. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
No. of patients	60	42	55	70	53	20

Form

(*i*) less than type cumulative frequency distribution.

(*ii*) more than type cumulative frequency distribution.

Sol. (*i*) For less than type cumulative frequency (*c.f.*), it is clear from the table that patients less than 10 years of age are zero and less than 20 years are 60, and for less than 30 it will include from 10 – 30 (i.e., 60 + 42) i.e. less than 30 are 102 and so on.

(*ii*) For more than type *c.f.* the Ist C.I. is 10–20 so more than 10 will include all 300 patients or from the last C.I. (60–70) we observe that patients more than or equal to 70 are zero, more than 60 or equal to 60 patient are 20, and more than or equal to 50 are 20 + 53 = 73 and so on.

Less than type		More than type	
Age of Patients (in years)	Number of Patients	Age of Patients (in years)	
Less than 10	0	More than or equal to 10	60 + 240 = 300
Less than 20	60 + 0 = 60	More than or equal to 20	42 + 198 = 240
Less than 30	42 + 60 = 102	More than or equal to 30	55 + 143 = 198
Less than 40	55 + 102 = 157	More than or equal to 40	70 + 73 = 143
Less than 50	70 + 157 = 227	More than or equal to 50	53 + 20 = 73
Less than 60	53 + 227 = 280	More than or equal to 60	20 + 0 = 20
Less than 70	20 + 280 = 300	More than or equal to 70	0 = 0

Q14. Given below is a cumulative frequency distribution showing the marks secured by 50 students of a class:

Marks	Below 20	Below 40	Below 60	Below 80	Below 100
No. of students	17	22	29	37	50

Form the frequency distribution table for the data.

Sol. Class size is 40 – 20 = 20

So below 20 means C.I. is 0 – 20 and frequency is 17.

Frequency 22 includes 0 – 20 and 20 – 40 both class intervals.

Hence, the frequency between 20–40 is (22 – 17) = 5

Frequency 29 includes all 0–10, 10–20 and 20–60 class intervals.

So, 40–60 = 29 – 22 = 7

Marks (C.I.)	Number of students (f_i)
0–20	17 – 0 = 17
20–40	22 – 17 = 5
40–60	29 – 22 = 7
60–80	37 – 29 = 8
80–100	50 – 37 = 13

Q15. Weekly income of 600 families is tabulated below:

Weekly income (in ₹)	Number of families
0–1000	250
1000–2000	190
2000–3000	100
3000–4000	40
4000–5000	15
5000–6000	5
Total	600

Compute the median income.

Sol. For calculating the median of grouped data, we first form *c.f.* table.

Weekly income (₹)	No. of families f_i	*c.f.*
0–1000	250	250
1000–2000	190	440
2000–3000	100	540
3000–4000	40	580
4000–5000	15	595
5000–6000	5	600

The median of 600 (even) obser. = mean of 300 and 301 obs.

= Median lies in range (1000–2000)

So Median class = 1000 – 2000

$$\text{Median} = \frac{l + \left(\frac{n}{2} - c.f.\right) \times h}{f},$$

where,

l = lower limit of median class = 1000

n = Total no. of observations = 600

c.f. = *c.f.* preceding the median class = 250

h = the class size = 2000 – 1000 = 1000

f = frequency of median class = 190

$$\therefore \text{ Median} = 1000 + \frac{\left(\frac{600}{2} - 250\right) \times 1000}{190}$$

$$= 1000\left[1 + \frac{50}{190}\right] = 1000\,[1 + 0.26315]$$

$$= 1000\,[1.26315] = 1263.15$$

Hence, the median income of family is ₹ 1263.15 per week.

Q16. The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows.

Speed (in km/h)	85–100	100–115	115–130	130–145
Number of players	11	9	8	5

Calculate the median bowling speed.

Sol. To calculate median we form *c.f.* table.

Speed (in km/h) (C.I.)	**No. of players (f_i)**	***c.f.***
85–100	11	11
100–115	9	20
115–130	8	28
130–145	5	33

N = No. of observations = 33.

Median obs. of 33 odd observations = $\frac{33+1}{2} = \frac{34}{2} = 17\text{th obs}$.

17th obs. lies in class 100–115

$\therefore \quad l = 100, \quad f = 9, \quad c.f. = 11, \quad h = 100 - 85 = 15$

$$\therefore \qquad \text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

$$= 100 + \frac{\left(\frac{33}{2} - 11\right)15}{9} = 100 + \frac{(16.5 - 11)\,15}{9}$$

$$= 100 + \frac{5.5 \times 15}{9} = 100 + \frac{82.5}{9}$$

$$= 100 + 9.166 = 109.17 \text{ km/h}$$

Hence, the median bowling speed is 109.17 km/h.

Q17. The monthly income of 100 families are given below:

Income (in ₹)	**Number of families**	**Income (in ₹)**	**Number of families**
0–5000	8	20000–25000	3
5000–10000	26	25000–30000	3
10000–15000	41	30000–350000	2
15000–20000	16	35000–40000	1

Calculate the modal income.

Sol. For modal income, we have to calculate mode.

The mode of grouped data = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] h$

Modal class = class having maximum frequency *i.e.*, 41 is class (10000–15000)

f_0 = frequency preceding the modal class = 26
f_1 = frequency of modal class = 41
f_2 = frequency of class succeeding the modal class = 16
l = lower limit of modal class = 10000
h = 5000

$$\therefore \quad \text{Mode} = 10000 + \frac{(41 - 26)\,5000}{2 \times 41 - 26 - 16}$$

$$= 5000\left[2 + \frac{15}{82 - 42}\right] = 5000\left[2 + \frac{15}{40}\right]$$

$$= 5000\,[2 + 0.375] = 5000 \times 2.375 = 11875$$

Hence, the modal income is ₹ 11,875 per month.

Q18. The weight of coffee in 70 packets are shown in the following table.

Weight (in g)	200–201	201–202	202–203	203–204	204–205	205–206
No. of packets	12	26	20	9	2	1

Determine the modal weight.

Sol.

C.I.	**(f_i)**	**C.I.**	**(f_i)**
200–201	12	203–204	9
201–202	26	204–205	2
202–203	20	205–206	1

Modal class = (201–202) [$\because$ maximum frequency is 26]

f_0 = 12
f_1 = 26
f_2 = 20
h = 201 – 200 = 1
l = 201

$$\therefore \quad \text{Mode} = l + \frac{(f_1 - f_0)\,h}{(2f_1 - f_0 - f_2)}$$

$$\text{Mode} = 201 + \frac{(26 - 12) \times 1}{(2 \times 26 - 12 - 20)} = 201 + \frac{14}{(52 - 32)}$$

$$= 201 + \frac{14}{20} = 201 + 0.7$$

Mode = 201.7

Hence, the modal weight is 201.7 g.

Q19. Two dice are thrown at the same time. Find the probability of getting

(*i*) same number on both dice.

(*ii*) different numbers on both dice.

Sol. (*i*) Let E be the event of getting same number on both dice.

Total number of all possible outcomes T(E) = 36

No. of outcomes favourable to E, F(E) = 6

F(E) are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*ii*) Probability of getting different number on both the dice

= 1 – Probability of getting same number on both the dice

$$\therefore \quad = 1 - \frac{1}{6} = \frac{5}{6}$$

Q20. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is

(*i*) 7? (*ii*) a prime number? (*iii*) 1?

Sol. Total number of all positive outcomes when two dice are thrown simultaneously T(E) = 36

(*i*) The sum of the numbers appearing on both dice is 7. So, combinations are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1)

$$\Rightarrow \quad F(E) = 6$$

$$\therefore \quad \text{Required probability} = P(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*ii*) Sum of numbers on both dice is a prime number, *i.e.*, (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)

Hence, number of outcomes favourable to E = F(E) = 15

$$\Rightarrow \quad P(F) = \frac{F(E)}{T(E)} = \frac{15}{36} = \frac{5}{12}$$

(*iii*) Sum of two numbers on both dice 1, F(E) = 0

$$\therefore \quad P(F) = 0$$

Q21. Two dice are thrown together. Find the probability that the product of the numbers on the top of dice is

(*i*) 6 (*ii*) 12 (*iii*) 7

Sol. Main concept: The two dice are not identical, so, (4, 3) and (3, 4) will be different outcomes.

To get favourable outcomes: Choose I entry from 1 to 6, then place 1 to 6 at IInd place if given condition satisfies.

Total number of all possible outcomes if two dice are thrown together = T(E) = 36

(*i*) Let E be the event of getting the product on 6.
Number of outcomes favourable to event E are (1, 6), (2, 3), (3, 2), (6, 1), F(E) = 4

$$\Rightarrow P(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(*ii*) Number of outcomes favourable when product of numbers on both dice is 12, (2, 6), (3, 4), (4, 3), (6, 2)

$$\therefore \quad F(E) = 4$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(*iii*) Let E be the event of getting the product of numbers on both dice is 7

$$\therefore \quad F(E) = 0$$

$$\Rightarrow \quad P(E) = 0$$

Q22. Two dice are thrown at the same time and the product of the numbers appearing on them is noted. Find the probability that the product is less than 9.

Sol. Total number of all possible outcomes when two dice thrown together T(E) = 36

Product of the numbers on both dice is less than 9 so favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (6, 1)

$$\therefore \quad F(E) = 16$$

$$\Rightarrow \quad P(E) = \frac{F(E)}{T(E)} = \frac{16}{36} = \frac{4}{9}$$

Q23. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Sol. Total number of all possible outcomes, *i.e.*, T(E) = 36.

(*i*) Number of favourable outcomes when
Sum of numbers on two dice is 2 are (1, 1), (1, 1) *i.e.*, F(E) = 2

$$P_1(E) = \frac{F(E)}{T(E)} = \frac{2}{36} = \frac{1}{18}$$

(*ii*) Number of favourable outcomes when
Sum of numbers on two dice is 3 are (1, 2), (1, 2), (2, 1), (2, 1)

$$\therefore \quad F(E) = 4$$

$$\Rightarrow \quad P_2(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(*iii*) Number of favourable outcomes when
Sum of numbers on two dice is 4 are (1, 3), (1, 3), (2, 2), (2, 2), (3, 1), (3, 1)

$$F(E) = 6.$$

$$\therefore \quad P_3(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*iv*) Number of favourable outcomes when
Sum of numbers on two dice is 5 are (2, 3), (2, 3), (3, 2), (3, 2), (4, 1), (4, 1)

$$F(E) = 6$$

$$\therefore \quad P_4(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*v*) Number of favourable outcomes when
Sum of numbers on two dice is 6 *i.e.*, (3, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1)

$$F(E) = 6$$

$$\therefore \quad P_5(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*vi*) Number of favourable outcomes when
Sum of numbers on both dice is 7 are (4, 3), (4, 3) (6, 1), (6, 1), (5, 2), (5, 2)

$$F(E) = 6$$

$$P_6(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*vii*) Number of favourable outcomes when
Sum of numbers on both dice is 8 are (5, 3), (5, 3), (6, 2), (6, 2)

$$F(E) = 4$$

$$\therefore \quad P_7(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(*viii*) Number of favourable outcomes when
Sum of numbers on both dice is 9 are (6, 3), (6, 3) *i.e.*, F(E) = 2

$$\therefore \quad P_8(E) = \frac{F(E)}{T(E)} = \frac{2}{36} = \frac{1}{18}$$

Q24. A coin is tossed two times. Find the probability of getting almost one head.

Sol. Total number of possible outcomes if a coin is tossed 2 times (HH), (HT), (TH), (TT) *i.e.*, $T(E) = 2^2 = 4$

No. of favourable outcomes of getting almost one head *i.e.*,

$$F(E) = 3$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{3}{4}$$

Q25. A coin is tossed 3 times. List the possible outcomes, find the probability of getting

(*i*) all heads (*ii*) at least two heads

Sol. Total number of possible outcomes when if a coin is tossed
Number of favourable outcomes of getting (HHH), (HHT), (HTH), (THH) (TTT) (TTH) (THT) (HTT) so T(E) = 8

(*i*) All head *i.e.*, F(E) = 1

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$$

(*ii*) Number of favourable outcomes of getting at least 2 heads

$$F(E) = 4$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{4}{8} = \frac{1}{2}$$

Q26. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

Sol. Total number of possible outcomes when 2 dice (option 6 each) are tossed together = 6^2 = 36

$$\therefore \quad T(E) = 36$$

Number of favourable outcomes of getting the difference of the numbers as (1, 3), (2, 4), (3, 5), (4, 6), (3, 1), (4, 2), (5, 3), (6, 4)

$$\therefore \quad F(E) = 8$$

$$\Rightarrow \quad P(E) = \frac{F(E)}{T(E)} = \frac{8}{36} = \frac{2}{9}$$

Q27. A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random, Find the probability of this ball being a

(*i*) red ball (*ii*) green ball (*iii*) not a blue ball

Sol. No. of red balls = 10
Number of blue balls = 5
Number of green balls = 7
Total number of balls T(E) = (10 + 5 + 7) = 22

(*i*) Number of favourable outcomes of getting a red ball = F(E) = 10

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{10}{22} = \frac{5}{11}$$

(*ii*) Number of favourable outcomes of getting a green ball = F(E) = 7

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{7}{22}$$

(*iii*) Number of favourable outcomes of not getting a blue ball = F(E) = 22– 5 = 17

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{17}{22}$$

Q28. The King, Queen, and Jack of clubs are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is

(*i*) a heart (*ii*) a king

Sol. Total number of cards after removing King, Queen and Jack of club
T(E) = 52 – 3 = 49

(*i*) Number of favourable outcomes of getting a card of heart (any) = F(E) = 13

$$\therefore \quad P_1(E) = \frac{F(E)}{T(E)} = \frac{13}{49}$$

(*ii*) Number of favourable outcomes of getting a card of King *i.e.*, F(E) = (4 –1) = 3

$$\therefore \quad P_2(E) = \frac{F(E)}{T(E)} = \frac{3}{49}$$

Q29. Refer to Q. 28. What is the probability that the card is

(*i*) a club? (*ii*) 10 of heart?

Sol. Total number of cards = T(E) = 52 – 3 = 49

(*i*) Number of favourable outcomes of getting a club
F(E) = 13 – 3 = 10

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{10}{49}$$

(*ii*) Number of favourable outcomes of getting 10 of heart F(E) = 1

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{1}{49}$$

Q30. All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1, similar value for other cards, find the probability that the card has a value

(*i*) 7 (*ii*) greater than 7 (*iii*) less than 7

Sol. Out of 52 playing cards, 4 Jacks, 4 queens and 4 kings are removed.
$\therefore$ Total number of cards removed = 3×4 =12
Total number of cards remained = 52 – 12 = 40

$$\therefore \quad T(E) = 40$$

As ace has been given value 1, and similar value for other cards.
So, all the four aces are numbered by 1 and so on.

(*i*) Number of favourable outcomes of getting a card that has a value 7 = 4

$$\therefore \quad F(E) = 4$$

$$\therefore \quad P_1(E) = \frac{F(E)}{T(E)} = \frac{4}{40} = \frac{1}{10}$$

(*ii*) The numbers greater than 7 are 8, 9, and 10
So, number of favourable outcomes of getting a card that has a value of greater than 7 = 3×4 = 12

$$\therefore \quad F_2(E) = 12$$

$$\therefore \quad P_2(E) = \frac{F_2(E)}{T(E)} = \frac{12}{40} = \frac{3}{10}$$

(*iii*) The numbers less than 7 are = 1, 2, 3, 4, 5, 6

So, number of favourable outcomes of getting a card that was a value less than 7 = $F_3(E) = 6 \times 4 = 24$

$$\therefore \quad P_3(E) = \frac{F_3(E)}{T(E)} = \frac{24}{40} = \frac{3}{5}$$

Q31. An integer is chosen between 0 and 100. What is the probability that it is

(*i*) divisible by 7? (*ii*) not divisible by 7?

Sol. (*i*) Numbers between 0 and 100 divisible by 7 are 7, 14, 21, …., 98 = (AP)

Here, $a_n = 98$, $a = 7$, $d = 7$

$$a_n = a + (n-1)\,d$$
$$\Rightarrow \quad 98 = 7 + (n-1)\,7$$
$$\Rightarrow \quad 98 - 7 = (n-1)\,7$$
$$\Rightarrow \quad \frac{91}{7} = (n-1)$$
$$\Rightarrow \quad (n-1) = 13$$
$$\Rightarrow \quad n = 13 + 1 = 14$$
$$\therefore \quad F(E) = 14 \quad \text{and} \quad T(E) = 99$$
$$\therefore \quad P(E) = \frac{14}{99}$$

(*ii*) Number of favourable outcomes of getting a number which is not divisible by 7 = 99 – 14 = 85

$$\therefore \quad F(E) = 85$$
$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{85}{99}$$

Q32. Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has

(*i*) an even number (*ii*) a square number

Sol. (*i*) Total number of the cards (101 – 1) = 100

$$\therefore \quad T(E) = 100$$

Out of 100 cards, even number cards are 50

$$\therefore \quad F(E) = 50$$
$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{50}{100} = \frac{1}{2}$$

(*ii*) Square numbers from 2 to 101 are 4, 9, 16, 25, 36, 49, 64, 81, 100

$$\therefore \quad F(E) = 9$$
$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{9}{100}$$

Q33. A letter of English alphabets is chosen at random. Determine the probability that the letter is a consonant.

Sol. In 26 English alphabets there are 5 vowels and 21 consonants.
So,number of favourable outcomes of getting a consonant *i.e.*, F(E) = 21

Total alphabets = T(E) = 26

$$\therefore \quad \text{Probability (getting a consonant)} = \frac{F(E)}{T(E)} = \frac{21}{26}$$

Q34. There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize?

Sol. Total number of envelopes, T(E) = 1000
Number of envelopes containing cash prizes = 200 + 100 + 10 = 310
So, number of envelopes containing no cash prize = 1000 – 310 = 690
i.e., F(E) = 690

∴ Probability of getting an envelope of no cash prize = P(E)

$$= \frac{F(E)}{T(E)} = \frac{690}{1000} = \frac{69}{100}$$

Q35. Box 'A' contains 25 slips of which 19 are marked ₹ 1 and other are marked ₹ 5 each. Box B contains 50 slips of which 45 are marked ₹ 1 each and others are marked ₹ 13 each. Slip of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than ₹ 1.

Sol. Total number of slips poured in third box = 25 + 50 = 75

∴ T(E) = 75

Number of slips in third box marked ₹ 1 = 19 + 45 = 64
Hence, the number of favourable outcomes of drawing a slip from IIIrd box other than ₹ 1

= 75 – 64 = 11

∴ F(E) = 11

$$\therefore \quad \text{Required probability } P(E) = \frac{F(E)}{T(E)} = \frac{11}{75}$$

Q36. A carton of 24 bulbs contain 6 defective bulbs one bulb is drawn at random. What is the probability that the bulb is not defective? If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest, what is the probability that the second bulb is defective?

Sol. Total bulbs in carton = 24 ⇒ T(E) = 24
Defective bulbs = 6
Number of favourable outcomes of drawing a bulb which is not defective = 24 – 6 = 18 ⇒ F(E) = 18

$\therefore$ Probability that bulb is not defective = $P(E) = \dfrac{F(E)}{T(E)} = \dfrac{18}{24} = \dfrac{3}{4}$

According to the question selected bulb is defective and not replaced.

So, the total remaining bulb = 23 $\Rightarrow$ $T'(E) = 23$

Number of favourable outcomes of drawing a second defective bulb *i.e.*, $F'(E) = 6 - 1 = 5$

$$\therefore \quad P'(E) = \frac{F'(E)}{T'(E)} = \frac{5}{23}$$

Q37. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

(*i*) triangle (*ii*) square

(*iii*) square of blue colour (*iv*) triangle of red colour

Sol. Total no. of triangles = 8

Triangles with blue colour = 3

Triangles with red colour = 8 – 3 = 5

Total no. of squares = 10

Squares with blue colour = 6

Squares with red colour = 10 – 6 = 4

(*i*) Number of favourable outcomes for the event that lost figure is triangle

i.e., $F(E) = 8$

Total figures (square and triangle) = 8 + 10 = 18

i.e., $T(E) = 18$

$\therefore$ Probability (getting a triangle) $P(E) = \dfrac{F(E)}{T(E)} = \dfrac{8}{18} = \dfrac{4}{9}$

(*ii*) Number of favourable outcomes for the events that squares is lost

i.e., $F(E) = 10$

$T(E) = 8 + 10 = 18$

$\therefore$ P (getting a square) = $P(E) = \dfrac{10}{18} = \dfrac{5}{9}$

(*iii*) Number of favourable outcomes for the events that lost figure is square of blue colour

i.e., $F(E) = 6$

$T(E) = 18$

P(E) (getting a blue square) = $\dfrac{F(E)}{T(E)}$

$$\therefore \quad P(E) = \frac{6}{18} = \frac{1}{3}$$

(*iv*) Number of favourable outcomes for the event that lost figure is triangle of red colour = 5 i.e., $F(E) = 5$

$T(E) = 18$

$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{5}{18}$

Q38. In a game, the entry fee is ₹ 5. The game consists of a tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she

(*i*) loses the entry fee (*ii*) gets double entry fee

(*iii*) just gets her entry fee

Sol. One coin is tossed 3 times so total number of favourable outcomes $= 2^3 = 8$, which are (HHH), (HHT), (HTH), (THH) and (replacing $H \to T$ and $T \to H$) (TTT), (TTH), (THT), (HTT)

(*i*) Losing the game means getting no head

Number of favourable outcomes of getting no head = F(E) = 1

So, P (Losing the entry fee) *i.e.*, $P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$

(*ii*) Gets double entry fee back by getting 3 heads

Number of favourable outcomes of getting 3 heads i.e., F(E) = 1

$\therefore$ (getting double entry fee) $P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$

(*iii*) Just gets her entry fees back by getting either one or two heads.

Number of favourable outcomes of getting either one or two heads *i.e.*, F(E) = 6

(just getting entry fee) *i.e.*, $P(E) = \frac{F(E)}{T(E)} = \frac{6}{8}$

$\therefore \quad P(E) = \frac{3}{4}$

Q39. A die has six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and total scores are recorded.

(*i*) How many different scores are possible?

(*ii*) What is the probability of getting a total of 7?

Sol. Total number of possible outcomes $= 6^2 = 36$

(*i*) Number of favourable outcomes are (0, 0), (0, 1), (0, 6), (1, 0), (1, 1), (1, 6), (6, 0), (6, 1), (6, 6) *i.e.*, 9.

Total of both numbers are 0, 1, 6, 2, 7, 12

So, 6 differentiate scores are possible.

(*ii*) Number of favourable outcomes for getting a total of 7 are 2

$\Rightarrow \quad F(E) = 2$

Probability of getting sum 7 = $\frac{F(E)}{T(E)}$

Total no. of all possible outcomes of getting sum either (0, 1, 2, 6, 7 and 12) = 6

$\therefore$ (sum of numbers on both dice is 7) = $\frac{2}{6} = \frac{1}{3}$

Hence, the probability of getting sum on both dice 7 is = $\frac{1}{3}$

Q40. A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Varnika will buy a phone, if it is good, but the trader will only buy a mobile, if it has no major defect. One phone is selected at random from the lot. What is probability that it is

(*i*) Acceptable to Varnika?

(*ii*) Acceptable to trader?

Sol. Total number of mobile phones = T(E) = 48

(*i*) Let E be the event that Varnika's selected mobile should be good.

$\therefore$ Number of favourable outcomes for event E = F(E) = 42

$\therefore$ P (for good mobile) = $\frac{F(E)}{T(E)} = \frac{42}{48} = \frac{7}{8}$

(*ii*) Trader buys a phone which has no major defect.

No. of phones with major defect = 3

$\therefore$ Phones which do not have major defects = 48 – 3 = 45

$\therefore$ F(E) = 45

$\Rightarrow$ P(E) = $\frac{F(E)}{T(E)} = \frac{45}{48} = \frac{15}{16}$

Q41. A bag contains 24 balls of which x are red, $2x$ are white and $3x$ are blue. A ball is selected at random. What is the probability that it is

(*i*) not red (*ii*) white

Sol. Total number of balls = 24

Number of red balls = x

Number of white balls = $2x$

Number of blue balls = $3x$

Total balls = 24

$\therefore$ $1x + 2x + 3x = 24$

$\Rightarrow$ $6x = 24$

$\Rightarrow$ $x = 4$

So, Number of red balls = $x = 1 \times 4 = 4$

Number of white balls = $2x = 2 \times 4 = 8$

Number of blue balls = $3x = 3 \times 4 = 12$

(*i*) Randomly selected ball is not red

$\therefore$ Number of favourable outcomes for the event that ball is not red = 24 – 4 = 20

$\therefore$ F(E) = 20

and T(E) = 24

$$\therefore \quad P\text{ (not red)} = \frac{F(E)}{T(E)} = \frac{20}{24} = \frac{5}{6}$$

$$\therefore \quad P(E) = \frac{5}{6} \text{ is the required probability}$$

(*ii*) Number of favourable outcomes for the event that the selected ball is white, F(E) = 8

$$\therefore \quad P\text{ (ball is white)} = \frac{8}{24} = \frac{1}{3}$$

Q42. At a fete, cards bearing numbers 1 to 1000, one number on one card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that

(*i*) the first player wins the prize?

(*ii*) the second player wins a prize, if the first has won?

Sol. (*i*) First player can select a card from a box in 1000 ways.

Perfect square greater than 500 are 529, 576, 625, 676, 729, 784, 841, 900, 961

$= (23)^2, (24)^2, (25)^2, (26)^2, (27)^2, (28)^2, (29)^2, (30)^2, (31)^2$

$\therefore$ F(E) = 9

So, the probability P(E) that the first player wins the prize will be

$$P(E) = \frac{F(E)}{T(E)} = \frac{9}{1000} = 0.009$$

(*ii*) For IInd player, the card selected by earlier player is not replaced.

$\therefore$ Total number of cards for IInd player = 1000 – 1 = 999

$\therefore$ T′(E) = 999

As the first player wins the prize. So, cards having perfect square greater than 500 become one less.

So, number of favourable outcomes for IInd player to win a prize = 9 – 1

i.e., F′(E) = 8

$$\therefore \quad \text{Probability} = P'(E) = \frac{F'(E)}{T'(E)} = \frac{8}{999}$$

(winning second player a prize)

EXERCISE 13.4

Q1. Find the mean marks of the students for the following distribution:

Marks	Number of Students	Marks	Number of Students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

Sol.

Marks	*c.f.*	Marks C.I.	x_i	$d_i = x_i - a$	f_i	$f_i d_i$
0 and above	80	0–10	5	– 50	80 – 77 = 3	– 150
10 and above	77	10–20	15	– 40	77 – 72 = 5	– 200
20 and above	72	20–30	25	– 30	72 – 65 = 7	– 210
30 and above	65	30–40	35	– 20	65 – 55 = 10	– 200
40 and above	55	40–50	45	– 10	55 – 43 = 12	– 120
50 and above	43	50–60	55	0	43 – 28 = 15	0
60 and above	28	60–70	65	10	28 – 16 = 12	120
70 and above	16	70–80	75	20	16 – 10 = 6	120
80 and above	10	80–90	85	30	10 – 8 = 2	60
90 and above	8	90–100	95	40	8 – 0 = 8	320
100 and above	0	100–110	105	50	0	0
					$\Sigma f_i = 80$	$\Sigma f_i d_i = -260$

a = assumed mean = 55

$\Sigma f_i d_i = -260$

$\Sigma f_i = 80$

$$\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 55 - \frac{260}{80} = 55 - \frac{13}{4} = 55 - 3.25$$

$\Rightarrow \quad \overline{x} = 51.75$

This method is called deviation method.

Hence, the mean marks of students = 51.75.

Q2. Determine the mean of the following distribution:

Marks	Number of Students	Marks	Number of Students
Below 10	5	Below 60	60
Below 20	9	Below 70	70
Below 30	17	Below 80	78
Below 40	29	Below 90	83
Below 50	45	Below 100	85

Sol. From the given table, we observe that the students getting marks below 10 are 5, and marks cannot be negative. So, 5 students lie in (0–10) class interval.

Similarly, no. of students getting marks below 20 are 9 and below 10 are 5 so number of students getting marks 10–20 are (9 – 5) = 4. So,

Marks	*c.f.*	C.I.	x_i	$u_i = \frac{(x_i - a)}{h}$	f_i	$f_i u_i$
Below 10	5	0–10	5	– 4	5 – 0 = 5	– 20
Below 20	9	10–20	15	– 3	9 – 5 = 4	– 12
Below 30	17	20–30	25	– 2	17 – 9 = 8	– 16
Below 40	29	30–40	35	– 1	29 – 17 = 12	– 12
Below 50	45	40–50	45	0	45 – 29 = 16	0
Below 60	60	50–60	55	1	60 – 45 = 15	15
Below 70	70	60–70	65	2	70 – 60 = 10	20
Below 80	78	70–80	75	3	78 – 70 = 8	24
Below 90	83	80–90	85	4	83 – 78 = 5	20
Below 100	85	90–100	95	5	85 – 83 = 2	10
					$\Sigma f_i = 85$	$\Sigma f_i u_i = 29$

a = assumed mean = 45

$\Sigma f_i = 85$

$\Sigma f_i u_i = 29$

$$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h \quad \text{[Step deviation method]}$$

$$\Rightarrow \quad \bar{x} = 45 + \frac{29}{85} \times 10 = 45 + \frac{58}{17} = 45 + 3.41 = 48.41$$

$$\Rightarrow \quad \bar{x} = 48.41 \text{ marks}$$

Hence, the average marks of students are 48.41.

Q3. Find the mean age of 100 residents of a town from the following data:

Age	Number of persons	Age	Number of persons
equal and above 0	100	equal and above 40	25
equal and above 10	90	equal and above 50	15
equal and above 20	75	equal and above 60	5
equal and above 30	50	equal and above 70	0

Sol. Age above 70 years is zero. So, last C.I. is 60–70. Age above zero is 100 and above 10 is 90 so age of persons in class interval (0–10) is (100 – 90) and so on.

Age	*c.f.*	C.I.	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
equal and above 0	100	0–10	5	– 3	100 – 90 = 10	– 30
equal and above 10	90	10–20	15	– 2	90 – 75 = 15	– 30
equal and above 20	75	20–30	25	– 1	75 – 50 = 25	– 25
equal and above 30	50	30–40	35	0	50 – 25 = 25	0
equal and above 40	25	40–50	45	1	25 – 15 = 10	10
equal and above 50	15	50–60	55	2	15 – 5 = 10	20
equal and above 60	5	60–70	65	3	5 – 0 = 5	15
equal and above 70	0					
					$\Sigma f_i = 100$	$\Sigma f_i u_i = -40$

a = assumed mean = 35

$\Sigma f_i = 100$

$\Sigma f_i u_i = -40$

$h = 10$

$$\therefore \quad \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h \quad \text{[Step deviation method]}$$

$$\Rightarrow \quad \bar{x} = 35 + \frac{(-40)(10)}{100}$$

$$= 35 - \frac{400}{100} = 31$$

$$\Rightarrow \quad \bar{x} = 31 \text{ years}$$

Hence, the mean age of 100 persons = 31 years

Q4. The weight of tea in 70 packets are shown in the following table:

Weight (in g)	200–201	201–202	202–203	203–204	204–205	205–206
Number of packets	13	27	18	10	1	1

Find the mean weight of packets.

Sol.

C.I.	x_i	d_i	f_i	$f_i d_i$
200–201	200.5	– 2	13	– 26
201–202	201.5	– 1	27	– 27
202–203	$\boxed{202.5}$	0	18	0
203–204	203.5	1	10	10
204–205	204.5	2	1	2
205–206	205.5	3	1	3
			$\Sigma f_i = 70$	$\Sigma f_i d_i = -38$

$a = 202.5$

$$\therefore \quad \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \quad \text{(By deviation method)}$$

$$= 202.5 - \frac{38}{70} = 202.5 - 0.5428 = 201.9572$$

$$\Rightarrow \quad \bar{x} = 201.96 \text{ (approx.)}$$

Hence, the mean weight of packets is 201.96 g.

Q5. Refer to Q4 above. Draw the less than type ogive for the data, and use it to find the mean weight and median weight.

Sol.

C.I.	f_i	**Weight**	*c.f.*
		less than 200	0 = 0
200–201	13	less than 201	13 + 0 = 13
201–202	$\boxed{27} = f$	less than 202	27 + 13 = 40
202–203	18	less than 203	18 + 40 = 58
203–204	10	less than 204	10 + 58 = 68
204–205	1	less than 205	1 + 68 = 69
205–206	1	less than 206	1 + 69 = 70

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

The median class of 70 even obs. = $\frac{70}{2}$ = 35th obs.
35th obs. lies in 201–202 class

$\therefore \quad l = 201, \quad h = 1$

$N = 70, \quad f = 27$

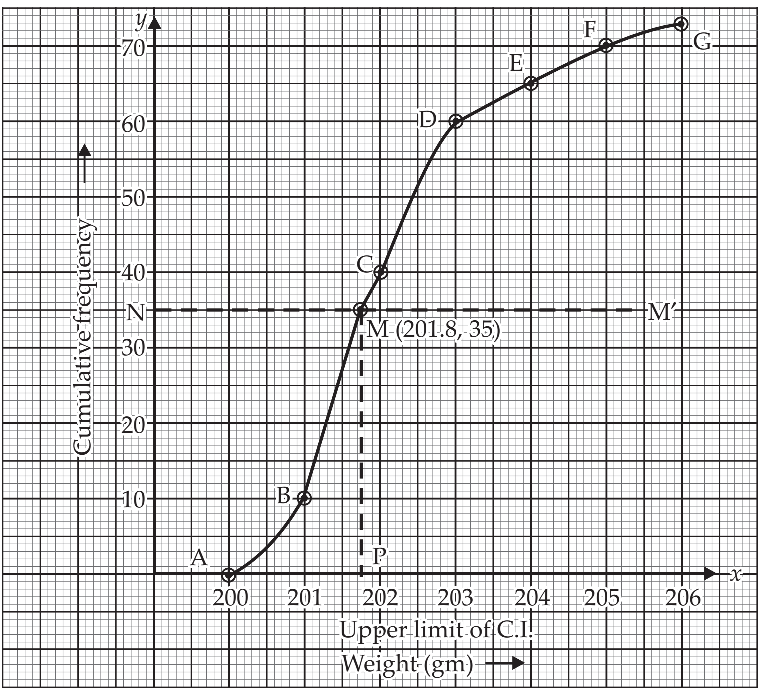
$c.f. = 13$

$$\therefore \quad \text{Median} = 201 + \frac{\left(\frac{70}{2} - 13\right)1}{27} = 201 + \frac{(35 - 13)}{27}$$

$$= 201 + \frac{22}{27} = 201 + 0.8148 = 201.8148$$

Hence, the median weight is 201.8148 g.

Points for less than type ogive are A(200, 0), B(201, 13), C(202, 40), D(203, 58), E(204, 68), F(205, 69), G(206, 70).



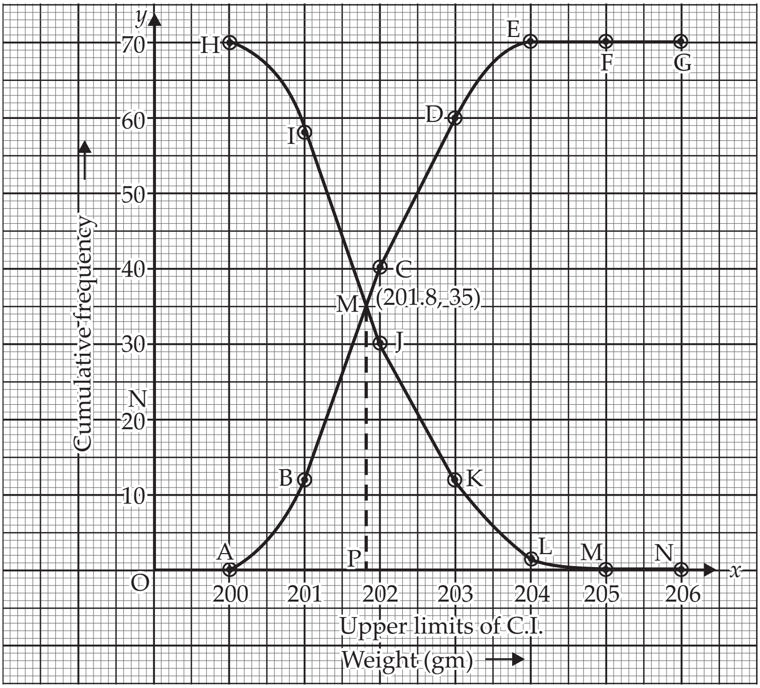
To find out median from graph take $\frac{N}{2} = \frac{70}{2} = 35$ at Y-axis and draw a line NM′ parallel to X-axis. Which meet the plotted graph at M; Draw MP perpendicular to X-axis. It meets on X-axis at 201.8 which is the median of data.

Q6. Refer to Q.5 above. Draw less than type and more than type ogives for the data and use them to find the median weight.

Sol.

C.I.	f_i wt (in gm)	*c.f.* less than	weight (in g)	*c.f.* more than	Points for more than ogive
200	0 (less than 200)	0	more than or equal 200	70	H(200, 70)
200–201	13 (less than 201)	13	more than or equal 201	57	I(201, 57)
201–202	27 (less than 202)	40	more than or equal 202	30	J(202, 30)
202–203	18 (less than 203)	58	more than or equal 203	12	K(203, 12)
203–204	10 (less than 204)	68	more than or equal 204	2	L(204, 2)
204–205	1 (less than 205)	69	more than or equal 205	1	M(205, 1)
205–206	1 (less than 206)	70	more than or equal 206	0	N(206, 0)

Graph (ogive) must be smooth having no edge. In both the graphs, upper limits of class intervals are taken on X-axis and cumulative frequency is taken on Y-axis. The intersection point of less than and more than ogive gives N/2 on Y-axis and median on X-axis.



Hence, the median weight of packets is 201.8 g.

Q7. The table below shows the salaries of 280 persons. Calculate the median and mode of the data.

Salary (in thous. ₹)	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50
No. of Persons	49	133	63	15	6	7	4	2	1

Sol.

Salary (₹ 1000) (C.I.)	No. of persons (f_i)	*c.f.*
5–10	49	49 + 0 = 49
10–15	133	133 + 49 = 182
15–20	63	63 + 182 = 245
20–25	15	15 + 245 = 260
25–30	6	6 + 260 = 266
30–35	7	7 + 266 = 273
35–40	4	4 + 273 = 277
40–45	2	2 + 277 = 279
45–50	1	1 + 279 = 280

(*i*) **Median:** Median class = $\frac{280}{2}$ = 140 (*c.f.*) obs.

Median class is 10–15.

$\therefore \quad l = 10, \quad N = 280, \quad h = 5, \quad f = 133, \quad c.f. = 49$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

$$\therefore \quad \text{Median} = 10 + \frac{\left(\frac{280}{2} - 49\right)5}{133} = 10 + \frac{(140 - 49) \times 5}{133}$$

$$= 10 + \frac{91 \times 5}{133} = 10 + \frac{455}{133} = 10 + 3.4210 = 13.421$$

$$= 13.421 \text{ (₹ in 1000)}$$

$\therefore$ Median = 13.421 × 1000 = ₹ 13, 421

(*ii*) **Mode:** Modal class (of maximum frequency) is (10–15)

$$\text{Mode} = l + \frac{(f_1 - f_0)h}{(2f_1 - f_0 - f_2)}$$

$$= 10 + \frac{(133 - 49) \times 5}{(2 \times 133 - 49 - 63)}$$

$(f_0 = 49, \quad f_1 = 133, \quad f_2 = 63, \quad h = 5)$

$$= 10 + \frac{84 \times 5}{266 - 112} = 10 + \frac{84 \times 5}{154} = 10 + \frac{30}{11}$$

$= 10 + 2.727 = 12.727$ (₹ in 1000)

$\Rightarrow$ Mode $= 12.727 \times 1000 =$ ₹ 12,727

Hence, the median and mode of the salaries are ₹ 13,421 and ₹ 12,727 respectively.

Q8. The mean of following frequency distribution is 50, but the frequencies f_1 and f_2 in classes (20–40) and (60–80) respectively are not known. Find these frequencies, if the sum of all the frequencies is 120.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	17	f_1	32	f_2	19

Sol. Mean of observations is 50.

C.I.	x_i	f_i	$d_i = (x_i - a)$	$f_i d_i$
0–20	10	17	– 20	– 340
20–40	30	f_1	0	0
40–60	50	32	20	+ 640
60–80	70	f_2	40	$+ 40 f_2$
80–100	90	19	60	+ 1140
		$\Sigma f_i = 120$		$\Sigma f_i d_i = 1440 + 40 f_2$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$a =$ Assumed mean $= 30$, $\bar{x} = 50$ (Given)

$$\Rightarrow \quad 50 = 30 + \frac{1440 + 40 f_2}{120}$$

$$\Rightarrow \quad 50 - 30 = \frac{1440}{120} + \frac{40 f_2}{120}$$

$$\Rightarrow \quad 20 = 12 + \frac{f_2}{3}$$

$$\Rightarrow \quad 20 - 12 = \frac{f_2}{3}$$

$$\Rightarrow \quad 8 \times 3 = f_2$$

$$\Rightarrow \quad f_2 = 24$$

From frequencies, we have $= 17 + f_1 + 32 + f_2 + 19 = 120$ (Given)

$$\Rightarrow \quad 68 + f_1 + f_2 = 120 \qquad (f_2 = 24)$$

$$\Rightarrow \quad 68 + f_1 + 24 = 120$$

$$\Rightarrow \quad 92 + f_1 = 120$$

$$\Rightarrow \quad f_1 = 120 - 92$$

$\Rightarrow \qquad f_1 = 28$

and $\qquad f_2 = 24$

Q9. The median of the following data is 50. Find the values of p and q, if sum of all the frequencies is 90.

Marks	f
20–30	p
30–40	15
40–50	25
50–60	20
60–70	q
70–80	8
80–90	10

Sol. Here, median of observations is 50 so, we have to calculate the values of p and q.

Marks (C.I.)	f_i	*c.f.*
20–30	p	p
30–40	15	$p + 15$
40–50	25	$p + 15 + 25 = p + 40$
50–60	20	$p + 40 + 20 = p + 60$
60–70	q	$p + q + 60$
70–80	8	$p + q + 60 + 8 = p + q + 68$
80–90	10	$p + q + 68 + 10 = p + q + 78$
	$\Sigma f_i = 90$	

Now, $p + 15 + 25 + 20 + q + 8 + 10 = 90$ (Given)

$\Rightarrow \qquad 78 + p + q = 90$

$\Rightarrow \qquad p + q = 90 - 78 = 12$ (I)

The median is 50. (Given)

$\therefore$ Median class is (50–60)

$\therefore$ $l = 50, \quad c.f. = (p + 40), \quad f = 20, \quad h = 10$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f\right)h}{f}$$

$$\therefore \qquad \text{Median} = 50 + \frac{[45 - (40 + p)] \times 10}{20} \qquad \left(\because \frac{N}{2} = \frac{90}{2} = 45\right)$$

$$\Rightarrow \qquad 50 = 50 + \frac{(45 - 40 - p)}{2}$$

$\Rightarrow \quad 50 - 50 = \frac{5-p}{2}$

$\Rightarrow \quad 5 - p = 0$

$\Rightarrow \quad p = 5$

But, $\quad p + q = 12$ [From (I)]

$\Rightarrow \quad q = 12 - 5 = 7$

$\therefore \quad p = 5$ and $q = 7$.

Q10. The distribution of heights (in cm) of 96 children is given below:

Height (in cm)	Number of children	Height (in cm)	Number of children
124–128	5	144–148	12
128–132	8	148–152	6
132–136	17	152–156	4
136–140	24	156–160	3
140–144	16	160–164	1

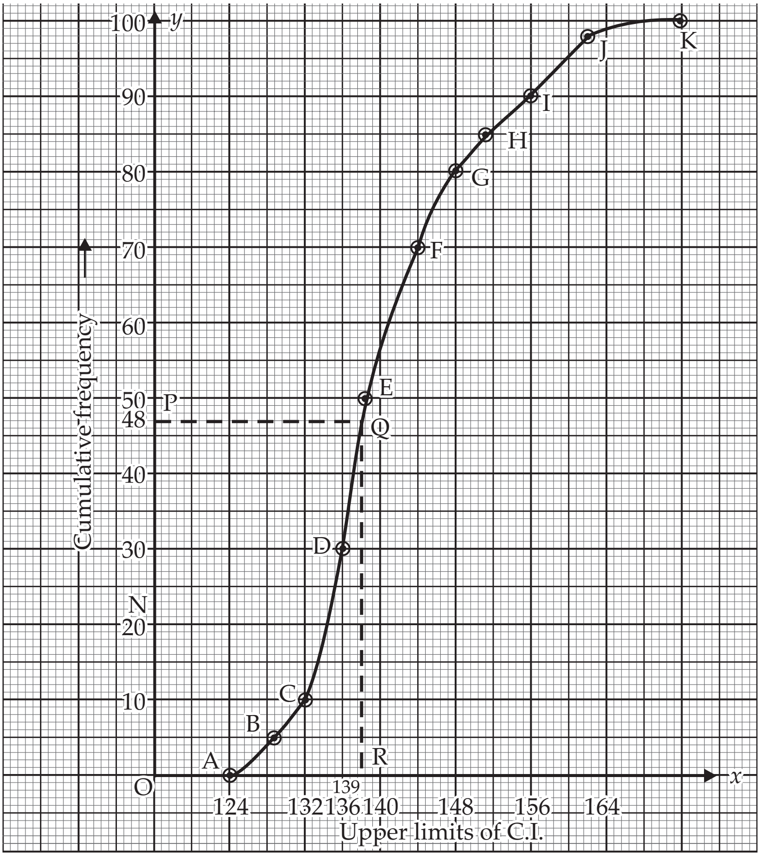
Draw a less than type cumulative frequency curve for this data and use it to compute median height of the children.

Sol. From given table, we have,

Height (in cm)	No. of children	Points for less than type ogive
less than 124	0 + 0 = 0	A(124, 0)
less than 128	5 + 0 = 5	B(128, 5)
less than 132	8 + 5 = 13	C(132, 13)
less than 136	17 + 13 = 30	D(136, 30)
less than 140	24 + 30 = 54	E(140, 54)
less than 144	16 + 54 = 70	F(144, 70)
less than 148	12 + 70 = 82	G(148, 82)
less than 152	6 + 82 = 88	H(152, 88)
less than 156	4 + 88 = 92	I(156, 92)
less than 160	3 + 92 = 95	J(160, 95)
less than 164	1 + 95 = 96	K(164, 96)

By plotting the graph with the above points, we get less than type ogive. Taking $y = \frac{N}{2} = \frac{96}{2} = 48$ at point P draw a line PQ parallel to x-axis and draw QR $\perp$ on x-axis. Point R on x-axis gives the value of median of the given observations.

Hence, the median height of observations is 139.2 cm.



Q11. The size of agricultural holdings in a survey of 200 families is given in the following table:

Size of agricultural holdings (in Hectare)	Number of families
0–5	10
5–10	15
10–15	30
15–20	80
20–25	40
25–30	20
30–35	5

Compute median and mode size of the holdings.

Sol.

C.I. (in hectare)	f_i (No. of families)	*c.f.*
0–5	10	10
5–10	15	25

C.I. (in hectare)	f_i (No. of families)	*c.f.*
10–15	30	55
15–20	80	135 → Median class
20–25	40	175
25–30	20	195
30–35	5	200

Sol. (*i*) Median class = $\frac{200}{2}$th observation = 100th observation *i.e.*, (15–20)

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}, \text{ where}$$

l = lower limit of median class = 15
N = Total number of observations = 200
c.f. = *c.f.* preceding the median class = 55
f = frequency of median class = 80
h = 5

$$\therefore \quad \text{Median} = 15 + \frac{\left(\frac{200}{2} - 55\right)5}{80} = 15 + \frac{(100-55)5}{80}$$

$$= 15 + \frac{45 \times 5}{80} = 15 + \frac{45}{16} = 15 + 2.8125$$

$\therefore$ Median = 17.8125 hectare

(*ii*) **Mode:** Maximum frequency in the given table is 80. So, modal class is (15–20)

$$\text{Mode} = l + \frac{(f_1 - f_0)h}{(2f_1 - f_0 - f_2)}$$

Here, $l = 15$, N/2 = 100, $f_0 = 30$, $f_1 = 80$, $f_2 = 40$

$$\therefore \quad \text{Mode} = 15 + \frac{(80-30)\times 5}{2\times 80 - 30 - 40} = 15 + \frac{50\times 5}{160 - 70}$$

$$= 15 + \frac{50 \times 5}{90} = 15 + \frac{25}{9} = 15 + 2.77 = 17.77$$

$\therefore$ Mode = 17.77 hectare.

Q12. The annual rainfall record of a city for 66 days is given in the following table:

Rainfall (cm)	0–10	10–20	20–30	30–40	40–50	50–60
No. of days	22	10	8	15	5	6

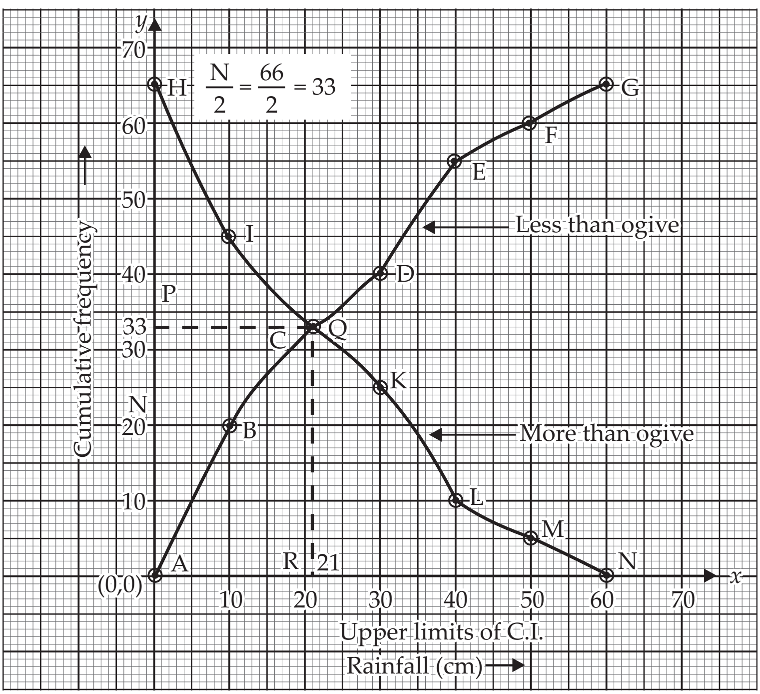
Calculate the median rainfall using ogives (more than type and less than type)

Sol. From the given table, we observe that the lowest limit is 0 so less than 0 rainfall is zero. The highest limit is 60 so more than 60 or 60 rainfall is zero.

Rainfall (cm)	No. of days (*c.f.*)	Rainfall (cm)	No. of days (*c.f.*)
less than 0	0	more than or equal 0	66
less than 10	22 + 0 = 22	more than or equal 10	66 – 22 = 44
less than 20	10 + 22 = 32	more than or equal 20	44 – 10 = 34
less than 30	8 + 32 = 40	more than or equal 30	34 – 8 = 26
less than 40	15 + 40 = 55	more than or equal 40	26 – 15 = 11
less than 50	5 + 55 = 60	more than or equal 50	11 – 5 = 6
less than 60	6 + 60 = 66	more than or equal 60	6 – 6 = 0

Co-ordinates on graph for less than type ogive are A(0, 0), B(10, 22), C(20, 32), D(30, 40), E(40, 55), F(50, 60) and G(60, 66).

Co-ordinates for more than type ogive are H(0, 66), I(10, 44), J(20, 34), K(30, 26), L(40, 11), M(50, 6), N(60, 0).



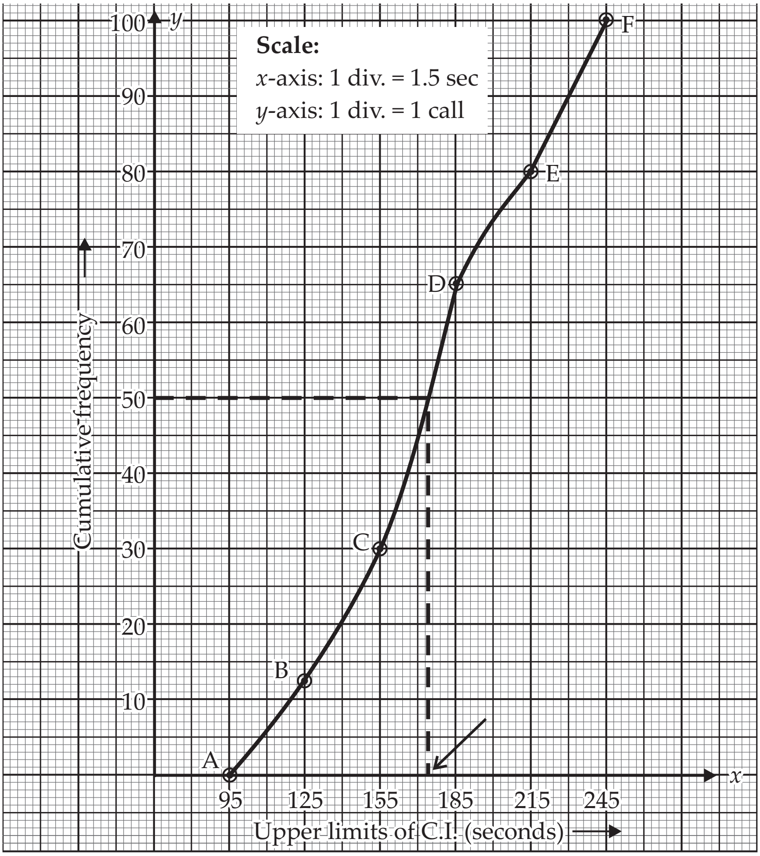
Both more than and less that type ogives intersect at point Q(21, 33). Hence, the median is 21 cm rainfall.

Q13. The following is the frequency distribution of duration for 100 calls made on a mobile phone.

Duration (in seconds)	Number of calls
95–125	14
125–155	22
155–185	28
185–215	21
215–245	15

Calculate the average duration (in sec) of a call and also find the median from the cumulative frequency curve.

Sol.



Duration (in sec) C.I.	No. of calls (f_i)	c. f.	x_i	$d_i = (x_i - a)$	$u_i = \frac{d_i}{h}$	f_iu_i	Points for ogive
Less than 95	0	0	0				A(95, 0)
95–125	14	14	110	– 60	– 2	– 28	B(125, 14)
125–155	22	36	140	– 30	– 1	– 22	C(155, 36)
155–185	28	64	170	0	0	0	D(185, 64)
185–215	21	85	200	30	1	21	E(215, 85)
215–245	15	100	230	60	2	30	F(245, 100)
	$\Sigma f_i = 100$					$\Sigma f_iu_i = 1$	

Here, $a = 170$, $h = 30$, $\Sigma f_iu_i = 1$, $\Sigma f_i = N = 100$

(*i*) Mean $\overline{x} = a + \frac{(\sum f_iu_i)h}{\sum f_i}$

$$= 170 + \frac{1 \times 30}{100} = 170 + 0.3 = 170.3$$

$\therefore$ $\overline{x} = 170.3$ seconds

Hence, the average duration for a call is 170.3 seconds.

(*ii*) **Median:** Median class $= \left(\frac{N}{2}\right)$th observation $= \frac{100}{2}$th observation = 50th observation

After plotting the ogive, median can be find out by taking *y* axis at $\frac{N}{2} = \frac{100}{2} = 50$ calls. Note the call time on *x*-axis corresponding to 50 calls which is shown by arrows *i.e.*, 170.

Hence, the median time is 170 seconds.

Q14. 50 students enter for a school Javelin throw competition. The distance (in metre) thrown are recorded below.

Distance (m)	0–20	20–40	40–60	60–80	80–100
No. of students	6	11	17	12	4

(*i*) Construct a cumulative frequency table.

(*ii*) Draw cumulative frequency curve (less than type) and calculate the median distance thrown by using this curve.

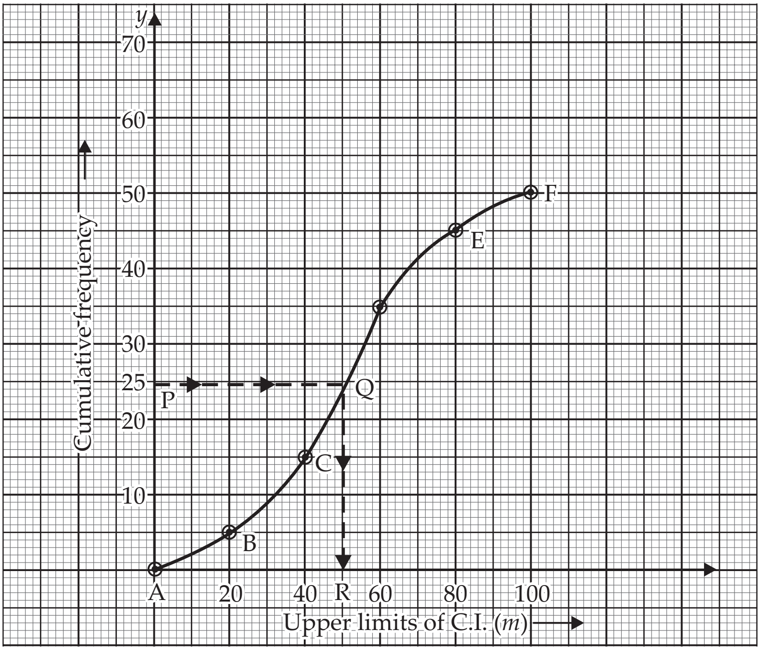
(*iii*) Calculate the median distance by using the formula for median.

(*iv*) Are the median distance calculated in (*ii*) and (*iii*) same ?

Sol. (*i*) Cumulative frequency table

Distance (m) C.I.	**No. of students (f_i)**	***c.f.***	**Points of less than ogive**
less than 0	0	0	A(0, 0)
0–20	6	6	B(20, 6)
20–40	11	17	C(40, 17)
40–60	17	34	D(60, 34)
60–80	12	46	E(80, 46)
80–100	4	50	F(100, 50)

(*ii*) Cumulative frequency (less than type) curve



To obtain median distance from less than or cumulative frequency ogive, we have to find out the distance of $\frac{50}{2} = 25$ observations from Y-axis and its corresponding distance on X-axis. On x-axis R (50 m) is the median distance.

(*iii*) Median by formula

The median class is 25th obs. that lies in 40–60 class

$\therefore \quad l = 40$

$$N = 50$$
$$c.f. = 17 \text{ (Preceding the median class)}$$
$$h = 20$$
$$f = 17 \text{ (median class)}$$

$$\therefore \text{ Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

$$= 40 + \frac{\left(\frac{50}{2} - 17\right)20}{17} = 40 + \frac{(25-17)\,20}{17}$$

$$= 40 + \frac{8 \times 20}{17} = 40 + \frac{160}{17} = 40 + 9.41176 = 49.41176$$

$\therefore$ Median distance = 49.41176 metre

(*iv*) Median distance calculated by formula and graph are almost equal *i.e.*, differ only by 0.588 m. So, we can say that the median distance calculated in part (*ii*) and (*iii*) are same.