1. REAL NUMBER (2024-25)

• FUNDAMENTAL THEOREM OF ARITHMETIC •

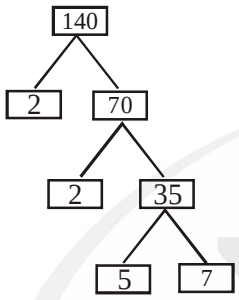
Every composite number can be expressed (factorise) as a product of primes and this factorisation is unique. (neglecting the order in which the prime factors occur).

EXAMPLE : factorizing 90, we get $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

ILLUSTRATION

Q.1 Express 140 as a product of prime factors using factor tree

Sol. Factors of 140 = 2 x 2 x 5 x 7



Q.2 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. (i) $7 \times 11 \times 13 + 13 = 13 \times \{7 \times 11 + 1\} = 13 \times 78$ which is a composite number.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times \{7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1\} = 5 \times 1009$ which is a composite number.

Q.3 Check whether 6^n can end with the digit 0 for any natural number n.

Sol. Let (if possible) 6^n ends with digit 0 $\Rightarrow 6^n = 10 \times q \Rightarrow 2^n \times 3^n = 2 \times 5 \times q \Rightarrow$ 5 is a prime factor of $2^n \times 3^n$ which is not possible because $2^n \times 3^n$ can have only 2 and 3 prime factors. Hence, 6^n cannot end with the digit 0 for any natural number n.

PRACTICE PROBLEMS

1. Express the following as the product of prime number using factor tree: (i) 3825 (ii) 468
2. Prove that there is no natural number for which 4^n ends with the digit zero.
3. Explain why $5 \times 17 \times 23 + 5$ is a composite number.

• APPLICATIONS OF FUNDAMENTAL THEOREM OF A ARITHMETIC •

FINDING HCF AND LCM OF POSITIVE INTEGERS

Fundamental theorem of arithmetic can be used to find the HCF and LCM of two or more positive integers. This method is also called prime factorisation method. In this method, first express the given two or more numbers into the product of prime numbers seperately. Then,

HCF of two or more numbers = Product of smallest power of each common prime factor involved in the numbers.

LCM of two or more numbers = Product of greatest power of each prime factor involved in the numbers.

• RELATION BETWEEN HCF AND LCM OF TWO NUMBERS •

HCF × LCM = product of the two numbers

For any two positive integers a and b, we have

HCF(a, b) × LCM (a, b) = a × b

For any three positive integers a, b and c, we have

$$\text{HCF (a, b, c)} = \frac{a \times b \times c \times \text{LCM}(a,b,c)}{\text{LCM}(a,b) \times \text{LCM}(b,c) \times \text{LCM}(a,c)}; \quad \text{LCM(a,b, c)} = \frac{a \times b \times c \times \text{HCF}(a,b,c)}{\text{HCF}(a,b) \times \text{HCF}(b,c) \times \text{HCF}(a,c)}$$

ILLUSTRATION

Q.4 Find the L.C.M and H.C.F. of 60 and 45 by the prime factorisation method

Sol. We have 60 = 2 × 2 × 3 × 5 45 = 3 × 3 × 5

HCF (60, 45) = 3 × 5 = 15

LCM (60, 45) = 2 × 2 × 3 × 3 × 5 = 180

LCM can also be found by using the below given formula.

$$\text{LCM (a, b)} = \frac{a \times b}{\text{HCF}(a, b)} \qquad \text{LCM (60, 45)} = \frac{60 \times 45}{\text{HCF}(60, 45)} = \frac{60 \times 45}{15} = 180$$

PRACTICE PROBLEMS

4. Find LCM and HCF of the following integers by using prime factorisation method.

(i) 24, 30, 54 (ii) 120, 144, 336 (iii) 28, 49, 84

5. Find the LCM of 66 & 486 by the Prime factorisation method. Hence find their HCF.

6. Find the HCF of 145 and 382 by the Prime factorisation method. Hence find their L.C.M.

7. HCF of two numbers is 113 and their LCM is 56952. If one number is 904, then find the other number.

8. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

9. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

• IRRATIONAL NUMBER •

Any number, which cannot be written in the form p/q (where p and q are integers and $q \neq 0$) is called irrational.

EXAMPLE : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, \frac{1}{\sqrt{7}}$ etc are irrational numbers.

• PROVING IRRATIONAL NUMBERS •

POINTS TO NOTE :

(i) Sum or difference of a rational and an irrational number is irrational.

(ii) The product and quotient of a non-zero rational and irrational number is irrational.

(iii) If p is a prime and p divides a^2, then p divides 'a' where a is a positive integer.

ILLUSTRATION

Q.5 Prove $\sqrt{2}$ is irrational.

Sol. Let us assume that $\sqrt{2}$ is a rational.

So $\sqrt{2} = \frac{r}{s}$ where r, s are integers and $s \neq 0$. r and s are co-prime (does not have any common factors)

On squaring, we get $2 = \frac{r^2}{s^2} \Rightarrow s^2 = \frac{r^2}{2} \Rightarrow$ 2 divides $r^2 \Rightarrow$ 2 divides r.

Now, let r = 2a for some integer a. Putting r = 2a in $2s^2 = r^2$, we get $2s^2 = (2a)^2$

$\Rightarrow s^2 = 2a^2 \Rightarrow \frac{s^2}{2} = a^2$ i.e., 2 divides $s^2 \Rightarrow$ 2 divides s.

$\Rightarrow$ 2 is divisor of both r and s that contradicts our assumption that r and s are co-prime.

So, our assumption of taking $\sqrt{2}$ as rational is incorrect. So, $\sqrt{2}$ is irrational.

Q.6 Show that $\sqrt{2}+5$ is irrational.

Sol. Let us assume $\sqrt{2}+5$ be rational. So, $\sqrt{2}+5 = \frac{r}{s}$ where r, s are co-prime integers and $s \neq 0$

$\Rightarrow \sqrt{2} = \frac{r}{s} - 5$ or $\sqrt{2} = \frac{r-5s}{s}$

since r, s are integers, So, $\frac{(r-5s)}{2}$ is a rational numbers. So, $\sqrt{2}$ should be rational number.

But we know the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}+5$ is rational.

So, $\sqrt{2}+5$ is irrational.

PRACTICE PROBLEMS

10. Prove that $\sqrt{3}$ is irrational.

11. Prove that $7+4\sqrt{5}$ is irrational.

PRACTICE PROBLEMS ANSWERS

1. 3825 = 3 × 3 × 5 × 5 × 17, 468 = 2 × 2 × 3 × 3 × 13

4. (i) 1080, 6 (ii) 5040, 24 (iii) 588, 7

5. LCM (66, 486) = 5346; HCF (66, 486) = 6

6. HCF(145, 382) = 1; LCM (145, 382) = 55390

7. 7119

8. 8 column

9. 36 minutes

EXERCISE

I. MULTIPLE TYPE QUESTIONS [MCQ'S 1 MARK EACH]

EASY LEVEL

1. The HCF and LCM of two numbers are 4 and 9696, then the product of the two numbers is :

   a) 9696 b) 24242 c) 38784 d) 4848

2. If p and q are two distinct prime numbers , then LCM (p ,q) is :

   a) 1 b) p c) q d) p q

3. The number of prime factors of $3 \times 5 \times 7 + 7$ is :

   a) 1 b) 3 c) 4 d) 5

4. The sum of the HCF and LCM of 12,21,15 is:

   a) 140 b) 417 c) 423 d) 420

5. If HCF(n,8)=4, LCM(n,8)=24, then n is:

   a) 8 b)12 c) 10 d) 14

II. ASSERTION AND REASON TYPE QUESTIONS [1 MARK EACH]

EASY LEVEL

Each of the following examples contains STATEMENT -1 (ASSERTION) and STATEMENT-2 (REASON) has following four choices (a), (b), (c), and (d), only one of which is the correct answer.

a) Statement -1 is true, Statement- 2 is true; Statement-2 is a correct explanation for Statement-1.

b) Statement -1 is true, Statement- 2 is true; Statement-2 is not a correct explanation for Statement-1.

c) Statement -1 is true, Statement- 2 is false.

d) Statement -1 is false, Statement- 2 is true.

1. Statement-1(Assertion): If HCF(90,144) =18, then LCM (90,144) = 720

Statement- 2(Reason): If HCF (a, b) $\times$ LCM (a, b) = a $\times$ b

III. CASE BASED QUESTIONS

EASY LEVEL

1. To enhance the reading skills in your school of grade 10 students, the school nominates you and two of your friends to setup a class library. There are two Section A and Section B of class 10. There are 32 students in Section A and 36 students in Section B.

Based on the above information answer the following questions:

a) Expressed 36 as a product of primes**.**

b) 2×17+2 is a composite number or not.

c) Find the minimum number of books required for their class-library so that they can be distributed equally among students of Section A or Section B.

OR

What is the HCF of 32 and 36.

IV. SHORT ANSWER TYPE QUESTIONS (SA I) [2 MARKS EACH]

EASY LEVEL

1. Is 7×5×3×2+3 a composite number? Justify your answer.

2. Find the HCF and LCM of 17, 23 and 37 by using prime factorization method.

3. Find the HCF and LCM of 336 and 54 and verify that LCM × HCF = Product of the two numbers.

4. **Using prime factorisation method find H.C.F. and L.C.M. of :**

i. 26 and 91 **ii.** 510 and 92 **iii.** 336 and 54

5. Prove that $5 + \sqrt{2}$ is irrational.

V. SHORT ANSWER TYPE QUESTIONS (SA II) [3 MARKS EACH]

EASY LEVEL

1. A sweet seller has 420 Kaju burfis and 130 Badam burfis she wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of burfis that can be placed in each stack for this purpose?

2. In a morning walk, three persons step off together. Their steps measure 80 cm , 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

3. A merchant has 120 litres of oil of one kind, 180 litres of another kind and 240 litres of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

4. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

5. Prove that $\sqrt{2}$ is irrational.

VI. LONG ANSWER TYPE QUESTIONS (LA) [5 MARKS EACH]

EASY LEVEL

1. In a seminar, the number of participants in physics, mathematics and chemistry are 84, 108 and 144 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

2. During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

3. Prove that $\sqrt{3}$ is irrational. Hence prove that $7 + \sqrt{3}$ is also irrational.

I. MULTIPLE TYPE QUESTIONS [MCQ'S 1 MARK EACH]

MEDIUM LEVEL

1. If p is a prime number and p divides k^2, then p does not divides :

   a) $2k^2$ b) k c) 3k d) 2k + 1

2. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF(a, b) is :

   a) xy b) xy^2 c) x^3y^3 d) x^2y^2

3. The HCF of the smallest prime number and the smallest composite number is:

   a) 2 b) 4 c) 6 d) 8

4. The HCF of two consecutive positive integers is :

   a) 0 b) 1 c) 4 d) 2

5. If HCF (26, 169) = 13 then LCM (26, 169) is

   a) 26 b) 2 c) 338 d) 13

II. ASSERTION AND REASON TYPE QUESTIONS [1 MARK EACH]

MEDIUM LEVEL

Each of the following examples contains STATEMENT -1 (ASSERTION) and STATEMENT-2 (REASON) has following four choices (a), (b), (c), and (d), only one of which is the correct answer.

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c) Statement -1 is true, Statement- 2 is false.

d) Statement -1 is false, Statement- 2 is true.

1. Statement-1(Assertion): If HCF(60, 72) = 12, then LCM (60, 72) = 360

   Statement- 2(Reason): If HCF (a, b) $\times$ LCM (a, b) = a + b

III. CASE BASED QUESTIONS

MEDIUM LEVEL

1. A seminar is being conducted by an Educational Organization, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

Based on the above information answer the following questions:

a) Find the LCM of 84p and 108q, where p and q are co-prime.

b) Find the HCF of 60p and 108q, where p and q are co-prime.

c) Find the maximum number of participants that can be accommodated in each room, so that in each room the same number of participants are to be seated and all of them being in the same subject.

OR

Find the LCM of 60, 84 and 108.

IV. SHORT ANSWER TYPE QUESTIONS (SA I) [2 MARKS EACH]

MEDIUM LEVEL

1. If HCF (6, a) =2 and LCM (6, a) = 60 then find the value of a.

2. Prove that $15 + 17\sqrt{3}$ is an irrational number.

3. Show that 12^n cannot end with digit 0 or 5 for any natural number n.

4. Can two numbers have 16 as their HCF and 380 as their LCM? Give reason

5. If $A= p^3q^4r^5$, $B = p^5q^2r^3$, Find HCF (A, B)

6. If $p = a^2b^4c^2d^5$, $q = a^5cd^2$, Find HCF (p, q) and LCM (p q)

V. SHORT ANSWER TYPE QUESTIONS (SA II) [3 MARKS EACH]

MEDIUM LEVEL

1. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.
2. The length, breadth and height of a room are 8m25cm, 6m75cm and 4m50cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.
3. Find the HCF of the smallest prime number and the smallest composite number.
4. Find the lcm of smallest two digit composite number and smallest composite number.
5. What is the LCM of the least composite and the least prime number?
6. What is the ratio of HCF and LCM of the smallest prime and composite number?
7. Find the HCF of the smallest odd prime number and the smallest odd composite number.
8. Determine the number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

VI. LONG ANSWER TYPE QUESTIONS (LA) [5 MARKS EACH]

MEDIUM LEVEL

1. Three sets of physics, chemistry and mathematics books have to be stacked in such a way that all the books are stored topic-wise and height of each stack is the same. The number of physics books is 260, the number of chemistry books is 364 and the number of mathematics books is 416. Assuming that the books are of same thickness, determine the number of stacks of physics, chemistry and mathematics books.
2. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic-wise and the height of each stack is the same. The number of English books is 96, the number of Hnidi books is 240 and the number of Mathematics books is 336. Assuming that the books are of the same same thickness, determine the number of stacks of English, Hindi and Mathematics books.

I. MULTIPLE TYPE QUESTIONS [MCQ'S 1 MARK EACH]

DIFFICULT LEVEL

1. Let p be a prime number, the sum of its factors is :

   a) p b) p+1 c) 1 d) None of these
2. The LCM of the smallest two digit composite number and the smallest composite number is:

   a) 12 b) 20 c) 4 d) 44
3. The smallest number divisible by all natural numbers between 1 and 10, including both is:

   a) 2020 b) 2520 c) 1010 d) 5040
4. If 3 is the least prime factors of m and 5 is the least prime factor of n, then the least prime factor of m + n is:

   a) 2 b) 1 c) 8 d) none of these
5. The smallest number which when divided by 17, 23 and 29 leaves a remainder 11 in each case :

   a) 667 b) 11339 c) 11350 d) 493

II. ASSERTION AND REASON TYPE QUESTIONS [1 MARK EACH]

DIFFICULT LEVEL

Each of the following examples contains STATEMENT -1 (ASSERTION) and STATEMENT-2 (REASON) has following four choices (a), (b), (c), and (d), only one of which is the correct answer.

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c) Statement -1 is true, Statement- 2 is false.

d) Statement -1 is false, Statement- 2 is true.

1. Statement-1(Assertion): If product of two numbers is 5780 and HCF is 17, then their LCM is 340.

Statement- 2 (Reason): HCF is always a factor of LCM.

III. CASE BASED QUESTIONS

DIFFICULT LEVEL

1. Mohak wants to host a party on his 16th birthday in a large banquet hall having a certain number of chairs. He wants that guests should sit in different groups like in pairs, triplets, quadruplets, fives and sixes etc. When the banquet hall manager arrange chairs in such pattern like in 2's, 3's, 4's and 6's then 1, 2, 3, 4 & 5 chairs are left respectively. But when he arrange in groups of 11's no chairs is left.

Based on the above information answer the following questions:

a) How many chairs are in the banquet hall

i) 407 ii) 209 iii) 149 iv) 539

b) If one chair is added, then the total number of chairs can be arranged in what group?

c) If one chair is added, how many chairs will be left when arranged in groups of 11's.

OR

How many chairs will be left in original arrangement if number of chairs is arranged in Groups of 9's.

IV. SHORT ANSWER TYPE QUESTIONS (SQ I) [2 MARKS EACH]

DIFFICULT LEVEL

1. If p, q are prime positive integers, prove that $\sqrt{p}+\sqrt{q}$ is an irrational number.

2. Show that there are infinitely many positive primes.

3. If $\sqrt{pq}$ is an irrational number, prove that $\sqrt{p}+\sqrt{q}$ is also irrational.

V. SHORT ANSWER TYPE QUESTIONS (SA II) [3 MARKS EACH]

DIFFICULT LEVEL

1. A mason has to fit the floor of a hall with square marble tiles of the largest possible size. The size of the hall is 30 ft by 14ft. What would be the size of the tiles required that has to be cut and how many such tiles are required?
2. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.
3. Find the least number that is divisible by all the numbers between 1 & 10 (both inclusive).
4. On GT road, three consecutive traffic lights change after 36, 42 and 72 sec. If the lights are first switched on at 9:00 am, then at what time will they change again together.
5. Find the smallest number which when increased by 17 is exactly divisible by 520 and 468?
6. Find the smallest numbet which when increased by 20 is exactly divisible by 90 and 144?
7. Find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

ANSWERS

EASY LEVEL

I. MCQ	**1.** c,	**2.** d,	**3.** d,	**4.** c,	**5.** b
II. Assertion Reason	**1.** a				
III. Case Study	**a.** 36 = 2^2 x 3^2	**b.** Yes,	**c**. 288	**c.OR** 4	
IV. SA I	**1.** Yes,	**2.** HCF = 1, LCM = 14467,	**3.** HCF = 6, LCM = 3024,		
	4. i. HCF = 13, LCM=182	**ii.** HCF = 2, LCM = 23460	**iii.** HCF = 6, LCM = 3024		
V. SA II	**1.** 10,	**2.** 12240 cm,	**3.** 60 L,	**4.** 18	
VI. LA	**1.** 21 rooms,	**2.** 4 packs of pencils and 3 packs of crayons.			

MEDIUM LEVEL

I. MCQ	**1.** d,	**2.** b,	**3.** a,	**4.** b,	**5.** c
II. Assertion Reason	**1.** b				
III. Case Study	**a.** 756pq,	**b.** 12,	**c.** 12	**c. OR** 3780	
IV. SA I	**1.** 20,	**4.** No	**5.** $p^3q^2r^3$	**6.** HCF = a^2cd^2 , LCM = $a^5b^4c^2d^5$	
V. SA II	**1.** 63,	**2.** 75 cm,	**3.** HCF = 2,	**4.** 20, **5.** 4, **6.** 1: 2, **7.** 3, **8.** 109200	
VI. LA	**1.** 52,	**2.** 2, 5, 7			

DIFFICULT LEVEL

I. MCQ	**1.** b,	**2.** b,	**3.** b,	**4.** a,	**5.** c
II. Assertion Reason	**1.** b				
III. Case Study	**a. iv.** 539,	**b.** 2's,	**c.** 1	**c. OR** 8 chairs left	
V. SA II	**1.** 24 tiles,	**2.** 999720,	**3.** 2520,	**4.** 9:08:24, **5.** 4663, **6.** 700, **7.** 204	