3. LINEAR EQUATION (2024-25)

• PAIR OF LINEAR EQUATION IN TWO VARIABLES •

LINEAR EQUATION IN TWO VARIABLES:

An equation of the form ax + by + c = 0 is called a linear equation in two variables.
Where a is called coefficient of x, b is called coefficient of y and c is the constant term. a, b, c can be any real numbers and a, b $\neq$ 0

- If $a \neq 0, b \neq 0, c \neq 0$ equation will be of the form a x + by + c = 0.
- If $a \neq 0, b = 0$ equation will be of the form ax + c = 0. [Line || to Y-axis]
- If $a = 0, b \neq 0$, equation will be of the form by + c = 0. [Line || to X-axis]
- If $a \neq 0, b \neq 0, c = 0$ equation will be of the form ax + by = 0. [Line passing through origin]

NOTE: Since it involves two variables therefore a single equation will have infinite set of solution i.e. indeterminate solution. So we require a pair of equation i.e. simultaneous equations.

Standard form of system(pair) of linear equations: $a_1x + b_1y + c_1 = 0$ ....(i) ; $a_2x + b_2y + c_2 = 0$ ....(ii)

Solution of a linear equation: Those values of x and y which satisfy the equation constitute the solution of the linear equation. For eg: If x + y =5 then x = 4 and y =1 is one of the solutions.

Solution of System of Linear Equations: Those values of x and y which satisfy the system of linear equations simultaneously are known as solution of the system of linear equations.

EXAMPLE : x + y = 5 and 3x + 2y = 14 are satisfied by x = 4 and y =1.

• ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES •

For solving such equations we have three methods.

(i) Substitution Method (ii) Elimination Method

A. SUBSTITUTION METHOD:

ILLUSTRATION

Q.1 Solve x + 4y = 14 .....(i) ; 7x – 3y = 5 ....(ii)

Sol. From eq. (i) x = 14 - 4y ....(iii)

Substitute the value of x in eq. (ii)

$\Rightarrow$ 7 (14 – 4y) – 3y = 5 $\Rightarrow$ 98 – 28y – 3y = 5 $\Rightarrow$ 98 – 31y = 5 $\Rightarrow$ 93 = 31y $\Rightarrow y = \frac{93}{31} \Rightarrow y = 3$

Now substitute value of y in eq. (iii)

$\Rightarrow$ 7x – 3 (3) = 5 $\Rightarrow$ 7x – 3 (3) = 5 $\Rightarrow$ 7x = 14 $\Rightarrow x = \frac{14}{7} = 2$

So, solution is x = 2 and y = 3

B. ELIMINATION METHOD:

ILLUSTRATION

Q.2 Solve 9x – 4y = 8 .....(i) ; 13x + 7y = 101 ....(ii)

Sol. Multiply equation (i) by 7 and equation (ii) by 4, we get, 63x – 28y = 56 ; 52x + 28y= 404

Adding 115x = 460 $\Rightarrow x = \frac{460}{115} \Rightarrow x = 4$.

Substitute x = 4 in equation (i) 9 (4) – 4y = 8 $\Rightarrow$ 36 - 8 = 4y $\Rightarrow$ 28 = 4y $\Rightarrow y = \frac{28}{4} = 7$

So, solution is x = 4 and y = 7.

PRACTICE PROBLEMS

1. Solve by **substitution** method.

a. 2x + y = 7 and 4x + 3y = 1.

b. 4x + 3y = 11 and x –3y = 7.

c. x + 2y = –1 and 2x – 3y = 12

d. x + ay = 5 and 2x – by = 4

2. Solve by **elimination** method.

a. x + 2y = 140 and 3x + 4y = 360

b. 5x + 7y = 35 and 3x + 5y = 15.

c. 7y –2x = 14 and 4y + 3x = 2.

d. $\frac{1}{2}x - y = -1$ and $x + \frac{1}{2}y = 8$

e. ax + by = 4a and bx – ay = 4b

• CONDITIONS FOR SOLVABILITY (OR CONSISTENCY) OF SYSTEM OF EQUATIONS •

A. Unique Solution (Consistent) : Two equations $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$, have unique solution (i.e. only one solution) or consistent (solvable) solution or lines on the graph are intersecting if $\frac{a_1}{b_2} \neq \frac{b_1}{b_2}$

B. No Solution(Inconsistent) : Two equations $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$, have no solution or inconsistent solution or lines on the graph are parallel or solution is not possible , if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

C. Many Solution (Infinite Solutions) : Two equations $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$, has infinitely many solution or consistent but dependent solution or lines on the graph are coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

ILLUSTRATION

Q.3 Check whether the following equations have unique solution or no solution or infinite solution?

2x + 4y = 10 & 3x + 6y = 12

Sol. $a_1 = 2, b_1 = 4, c_1 = 10$; $a_2 = 3, b_2 = 6, c_2 = 12$

Conditions for unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, Infinite solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, No Solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Checking the equation we get, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. So No Solution is the answer

Q.4 Find the value of 'P' for which given system of equations has only 1 solution (i.e. unique sol.)

Px - y = 2 ....(i) ; 6x - 2y = 3 ....(ii)

Sol. $a_1 = P, b_1 = -1, c_1 = -2$; $a_2 = 6$ $b_2 = -2, c_2 = -3$

Conditions for unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{P}{6} \neq \frac{+1}{+2} \Rightarrow \quad P \neq \frac{6}{2} \Rightarrow P \neq 3 \Rightarrow$ P can have all real values except 3

Q.5 Find the value of k for which the system of linear equation kx + 4y = k - 4; 16x + ky = k has infinite solution.

Sol. $a_1 = k, b_1 = 4, c_1 = -(k - 4)$; $a_2 = 16, b_2 = k, c_2 = -k$

Here condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k}{16} = \frac{4}{k} = \frac{(k-4)}{(k)} \Rightarrow \frac{k}{16} = \frac{4}{k}$ also $\frac{4}{k} = \frac{k-4}{k}$

$\Rightarrow k^2 = 64$ and $4k = k^2 - 4k \Rightarrow k = \pm 8$ and $k(k - 8) = 0 \Rightarrow k = \pm 8$ and $k = 0$ or $k = 8$

but k = 0 is not possible. $\therefore$ **k = 8** is correct value

Q.6 Determine the value of k so that the linear equations has no solution.

$(3x + 1)\,x + 3y - 2 = 0$; $(k^2 + 1)\,x + (k - 2)\,y - 5 = 0$

Sol. Here $a_1 = 3k + 1, b_1 = 3$ and $c_1 = -2$; $a_2 = k^2 + 1, b_2 = k - 2$ and $c_2 = -5$

For no solution, condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$; $\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{-2}{-5}$

$\Rightarrow \frac{3k+1}{k^2+1} = \frac{3}{k-2}$ and $\frac{3}{k-2} \neq \frac{2}{5}$

Now, $\frac{3k+1}{k^2+1} = \frac{3}{k-2} \Rightarrow (3k + 1)(k - 2) = 3(k^2 + 1) \Rightarrow 3k^2 - 5k - 2 = 3k^2 + 3$

$\Rightarrow -5k - 2 = 31 \Rightarrow -5k = 5$ Þ $k = -1$

Clearly, $\frac{3}{k-2} \neq \frac{2}{5}$ for $k = -1$.

Hence, the equations will have no solution for **k = – 1**.

PRACTICE PROBLEMS

3. Determine whether the following system of linear equations has a unique solution or no solution or infinite solution:

   a. 4x + 5y = 7 and 3x – 7y = 5 **b.** 7x + 9y = 8 and 14x + 18y = 15 **c.** x + 2y + 7 = 0 and 2x + 4y + 14=0

4. Find the value of k for which the given system of equations has a Unique solution

   3x + 5y = 12 and 4x – ky = 7

5. Find the value of k for which the given system of equations becomes consistent

   3x – 7y = 6 and 21x – ky = 31

6. Find the value of k for which the given system of linear equation have infinitely many solution or represent the coincident lines.

   a. 6x + 3y = k – 3 & 2k x + 6y = 6 **b.** x + 2y + 7 = 0 & 2x + ky + 14 = 0

7. Find the value of k for which the given systems of equations will be inconsistent or have no solution.

   a. 2x + ky = k + 2 & kx + 8y = 3k **b.** kx + 3y = 3 & 12x + ky = 6

• SPECIAL CASES OF LINEAR EQUATIONS •

A. Equation of the form: $ax + by = c$ and $bx + ay = d$

ILLUSTRATION

Q.7 Solve the pair of equations: $41x + 37y = 86$ and $37x + 41y = 70$

Sol. $41x + 37y = 86$ .... (1) ; $37x + 41y = 70$ .... (2)

Adding eq (1) and (2) we get, $78x + 78y = 156$ on simplifying, $x + y = 2$ .... (3)

Subtracting eq (1) and (2) we get, $4x - 4y = 16$ on simplifying, $x - y = 4$ ....(4)

Solving eq (3) and (4), we get $x = 3$ and $y = -1$.

B. Equations reducible to linear equation

ILLUSTRATION

Q.8 Solve for x and y $\frac{2}{x} + \frac{3}{y} = 13$, $\frac{5}{x} + \frac{2}{y} = 16$

Sol. $\frac{2}{x} + \frac{3}{y} = 13$ ....(1) $\frac{5}{x} + \frac{2}{y} = 16$ ....(2)

Putting $\frac{1}{x} = a$, & $\frac{1}{y} = b$ in eq (1) & (2) ; we get, $2a + 3b = 13$ .... (3)

$5a + 2b = 16$ ....(4)

Solving eq (3) and (4), we get $a = 2$ and $y = 3$.

Substituting the values of a & b, we get, $\frac{1}{x} = 2$ and $\frac{1}{y} = 3$. Hence, $x = \frac{1}{2}$ and $y = \frac{1}{3}$

C. Decimal coefficients

ILLUSTRATION

Q.9 Solve the pair of equations $0.2x + 0.3y = 1.1$, $0.7x - 0.5y = -0.8$

Sol. $0.2x + 0.3y = 1.1$ ....(1) $0.7x - 0.5y = -0.8$ ....(2)

Multiplying eq (1) and (2) by 10, $2x + 3x = 11$ ....(3) $7x - 5y = -8$ ....(4)

Solving eq (3) and (4), we get $x = 1$ and $y = 3$.

D. Square root coefficients

ILLUSTRATION

Q.10 Solve for x and y: $3\sqrt{2}x - 5\sqrt{3}y + \sqrt{5} = 0$ and $2\sqrt{3}x + 7\sqrt{2}y - 2\sqrt{5} = 0$

Sol. $3\sqrt{2}x - 5\sqrt{3}y + \sqrt{5} = 0$ ....(1)

$2\sqrt{3}x + 7\sqrt{2}y - 2\sqrt{5} = 0$ ....(2)

Multiply eq (1) by $2\sqrt{3}$ & eq (2) by $3\sqrt{2}$

$6\sqrt{6}x - 30y + 2\sqrt{15} = 0$ ....(3)

$6\sqrt{6}x + 42y - 6\sqrt{10} = 0$ ....(4)

Subtracting eq (3) and (4), we get $-72y = -6\sqrt{10} - 2\sqrt{15}$

$$y=\frac{-6\sqrt{10}-2\sqrt{15}}{-72}=\frac{3\sqrt{10}+\sqrt{15}}{36}$$

Multiply eq (1) by $7\sqrt{2}$ & eq (2) by $5\sqrt{3}$ $42x-35\sqrt{6}y+7\sqrt{10}=0$ ....(5)

$30x+35\sqrt{6}y+10\sqrt{15}=0$ ....(6)

Adding eq (5) and (6), we get $72x=-7\sqrt{10}-10\sqrt{15}$; $x=\frac{-7\sqrt{10}-10\sqrt{15}}{72}$

E. Variable coefficients

ILLUSTRATION

Q.11 Solve for x and y: ax + by = a –b and bx –ay = a + b

Sol. ax + by = a –b ....(1)

bx –ay = a + b ....(2)

Multiply eq(1) by b & eq (2) by a,

$abx + b^2y = ab - b^2$ ....(3)

$abx - a^2y = a^2 + ab$ ....(4)

Subtracting eq (1) and (2), we get $(b^2 + a^2)y = -b^2 - a^2$, $y=\frac{-(b^2+a^2)}{b^2+a^2}=-1$

Multiply eq(1) by a & eq (2) by b,

$a^2x + aby = a^2 - ab$ ....(5)

$b^2x - aby = ab + b^2$ ....(6)

Adding eq (5) and (6), we get $(a^2 + b^2)x = a^2 + b^2$; $x=\frac{(b^2+a^2)}{b^2+a^2}=1$

PRACTICE PROBLEMS

Solve for x and y :

8. 103 x + 51y = 617 & 97x + 49 y=583.

9. $\frac{2}{x}+\frac{2}{3y}=\frac{1}{6},\ \frac{3}{x}+\frac{2}{y}=0,$ $x\neq 0$ and hence find the value of m for which y = mx –4.

10. $\frac{x}{a}+\frac{y}{b}=2$ and $ax - by = a^2 - b^2$.

11. $\frac{22}{x+y}+\frac{15}{x-y}=5$ and $\frac{55}{x+y}+\frac{45}{x-y}=14$

12. ax + by = c and bx + ay = 1 + c

13. 0.5 x + 0.5 y = 4 & 0.4 x – 0.2 y = 1.4

Equation Type : ax + by + c = 0

ILLUSTRATION

Q.12 Solve the following system of linear equations graphically: $x - y = 1$, $2x + y = 8$. Shade the area bounded by these two lines and y- axis. Also, determine this area.

Sol. (i) $x - y = 1$; $x = y + 1$

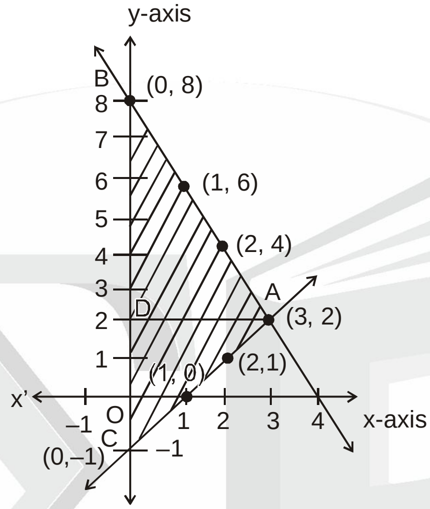
x	0	1	2
y	-1	0	1

(ii) $2x + y = 8$; $y = 8 - 2x$

X	0	1	2
Y	8	6	4

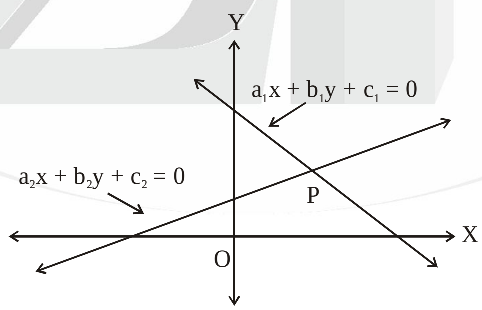
Solution is x = 3 and y = 2

Area of DABC = $\frac{1}{2} \times BC \times AD = \frac{1}{2} \times 9 \times 3 = 13.5$ Sq. unit.

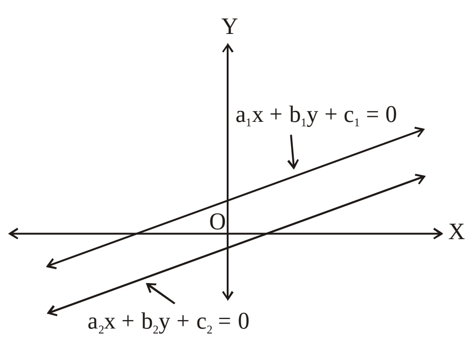


Nature of Graphical Solution: Let equations of two lines are $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$.

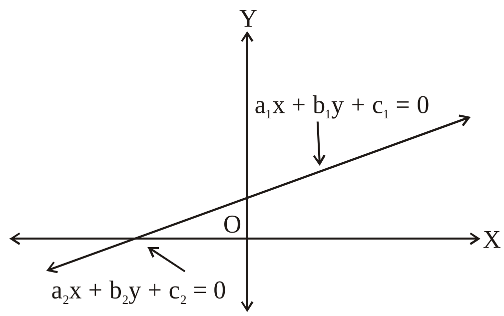
(i) Lines are consistent (**unique solution)** i.e. they meet at one point condition is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$



(ii) Lines are inconsistent **(no solution)** i.e. they do not meet at one point condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



(iii) Lines are coincident **(infinite solution)** i.e. overlapping lines condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



PRACTICE PROBLEMS

14. Solve graphically for x and y, 2x + y = 6 and 2x – y + 2 = 0.

15. Solve graphically for x and y, 3x + y – 11 = 0 and x – y –1 = 0.

• LINEAR EQUATIONS BASED ON WORD PROBLEMS •

For solving daily - life problems with the help of simultaneous linear equation in two variables or equations reducible to them proceed as follows:-

(i) Represent the unknown quantities by same variable x and y, which are to be determined.

(ii) Find the conditions given in the problem and translate the verbal conditions into a pair of simultaneous linear equations.

(iii) Solve these equations & obtain the required quantities with appropriate units.

NUMBER PROBLEMS

ILLUSTRATI0N

Q.13 The coach of a cricket team buys 7 bats and 6 balls for 3800 Rs. Later, she buys 3 bats and 5 balls for 1750 Rs. Find the cost of each bat and each ball.

Sol. Let cost of one bat = x Rs. and cost of one ball = y Rs.

According to the question, Cost of 7 bats and 6 balls = 3800 Rs.

i.e., $7x + 6y = 3800$ ...(i)

Cost of 3 bats and 5 balls = 1750 Rs.

i.e., $3x + 5y = 1750$ ...(ii)

From Eq. (ii), $5y = 1750 - 3x \Rightarrow y = \frac{1750 - 3x}{5}$ ...(iii)

On substituting y from Eq. (iii) in Eq. (i), we get $7x + 6 \times \frac{(1750 - 3x)}{5} = 3800$

On multiplying both sides by 5, we get $35x + 6 \times (1750 - 3x) = 5 \times 3800$

$\Rightarrow 35x + 10500 - 18x = 19000 \quad \Rightarrow 35x - 18x = 19000 - 10500 \quad \Rightarrow 17x = 8500 \quad \Rightarrow x = 500$

On substituting x = 500 in Eq. (iii), we get $y = \frac{1750 - 3 \times 500}{5} \Rightarrow y = \frac{1750 - 1500}{5} = \frac{250}{5} \Rightarrow y = 50$

Hence, cost of one bat is 500 Rs. and cost of one ball is 50 Rs.

Q.14 Find the cost of 2 T-shirts and 3 pants, if the cost of 2 T-shirts and one pant is 625 Rs. and 3 T-shirts and 2 pants together costs 1125 Rs.

Sol. Let the cost of one T-shirt be x Rs. and that of one pant be y Rs.

According to the question, $2x + y = 625$ ...(i) and $3x + 2y = 1125$ ...(ii)

On multiplying Eq. (i) by 2, we get, $4x + 2y = 1250$ ...(iii)
On subtracting Eq. (ii) from Eq. (iii), we get, $x = 125$
On substituting the value of x in Eq. (i), we get, $250 + y = 625 \Rightarrow y = 375$
Therefore, cost of one T-shirt is 125 Rs. and the cost of one pant is 375 Rs.
The cost of 2 T-shirts and 3 pants = $2x + 3y = 2 \times 125 + 3 \times 375 = 250 + 1125 = 1375$ Rs.

PRACTICE PROBLEMS

16. 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, then find the number of boys and girls who took part in the quiz.

17. 5 pencils and 7 pens together cost 50 Rs. whereas 7 pencils and 5 pens together cost 46 Rs. Find the cost of one pencil and that of one pen.

AGE PROBLEMS

ILLUSTRATION

Q.15 5 yr, hence the age of Jacob will be three times that of his son. 5 yr ago, Jacob's age was seven times that of his son. What are their present ages?

Sol. Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob.
After 5 yr, Jacobs son age = $(x + 5)$ yr, Jacob's age = $(y + 5)$ yr
According to the question, $(y + 5) = 3(x + 5) \Rightarrow y + 5 = 3x + 15 \Rightarrow 3x - y + 10 = 0$ ...(i)
5 yr ago, Jacob's son age = $(x - 5)$ yr, Jacob's age = $(y - 5)$ yr
According to the question, $(y - 5) = 7(x - 5)$
$\Rightarrow 7x - y - 30 = 0$ ...(ii)
From Eq. (i), $y = 3x + 10$ ...(iii)
On substituting $y = 3x + 10$ in Eq. (ii), we get $7x - (3x + 10) - 30 = 0 \Rightarrow 4x - 40 = 0 \Rightarrow x = 10$
On substituting $x = 10$ in Eq. (iii), we get, $y = 3 \times 10 + 10 \Rightarrow y = 40$
Hence, the present age of Jocob is 40 yr and his son is 10 yr.

Q.16 Father's age is 3 times the sum of ages of his two children. After 5 yr, his age will be twice the sum of ages of the two children. Find the age of father.

Sol. Let father's age be x year and let sum of ages of his two children be y year.
For I Condition, $x = 3y$ ...(i)
After 5 yr, age of father = $(x + 5)$, and sum of ages of two children = $(y + 10)$
For II Condition, $x + 5 = 2(y + 10)$ ...(ii)
On putting the value of x from Eq. (i) in Eq. (ii), we get, $3y + 5 = 2(y + 10) \Rightarrow 3y + 5 = 2y + 20$
$\Rightarrow 3y - 2y = 20 - 5 \Rightarrow y = 15$
On putting the value of y in Eq. (i), we get, $x = 3(15) = 45$. Hence, age of father is 45 yr.

PRACTICE PROBLEMS

18. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

19. I am three times times as old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son?

DIGITS PROBLEMS

Let us assume unit's place digit be x, and ten's place digit be y then, a two digit number in two variable is represented as **10y + x.**
Sum of digits = **x + y**.
Difference of the digits = **x – y or y – x**.
Number formed by reversing the digits = **10x + y**.

ILLUSTRATION

Q.17 The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Sol. Let x be the digit at units place and y be the digit at tens place. Number = 10y + x.

According to the given question, $x + y = 9$ ...(i)

When we reverse the order of the digits, the new number = 10x + y

According to the given question, $9 \times (x + 10y) = 2 \times (y + 10x)$

$\Rightarrow 9x + 90y = 2y + 20x \quad \Rightarrow 88y = 11x \quad \Rightarrow x = 8y$

On substituting x = 8 y from Eq. (ii) in Eq.(i), we get $8y + y = 9 \quad \Rightarrow 9y = 9 \quad \Rightarrow y = 1$

On substituting y = 1 in Eq. (ii), we get $x = 8 \times 1 \Rightarrow x = 8$

Hence, the number is $x + 10y = 8 + 10 \times 1 = 18$

Q.18 The sum of digits of a two-digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Sol. Let the digit in the units place be x and digit in the ten's place by y. Then, number formed = 10y + x

Number formed by reversing the digit = 10x + y

According to the question, $x + y = 15$ ...(i)

and $10x + y = (10y + x) + 9$

$\Rightarrow 10x - x + y - 10y = 9 \quad \Rightarrow 9x - 9y = 9 \quad \Rightarrow 9(x - y) = 9$

$\Rightarrow x - y = 1$ [dividing by 9] ...(ii)

On adding Eqs. (i) and (ii), we get, $2x = 15 + 1 \Rightarrow 2x = 16 \Rightarrow x = \frac{16}{2} = 8$

On putting x = 8 in Eq. (i), we get $8 + y = 15 \Rightarrow y = 15 - 8 = 7$

Hence, the required number is, $10y + x = 10 \times 7 + 8 = 78$

PRACTICE PROBLEMS

20. The sum of a two-digit number and number obtained by reversing the order of digits is 99. If the digits of the number differ by 3. Then, find the numbers.

21. Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.

FRACTION PROBLEMS

ILLUSTRATION

Q.19 A fraction becomes 9/11, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes 5/6. Find the fraction.

Sol. Let numerator be x and denominator be y. Fraction = $\frac{x}{y}$

According to the first condition, $\frac{x+2}{y+2} = \frac{9}{11}$ ...(i)

and second condition, $\frac{x+3}{y+3} = \frac{5}{6}$ ...(ii)

From Eq. (i), $11 \times (x + 2) = 9 \times (y + 2) \quad \Rightarrow 11x + 22 = 9y + 18$

$\Rightarrow 11x - 9y + 4 = 0$ ....(iii)

From Eq. (ii), $6 \times (x + 3) = 5 \times (y + 3)$ and $6x + 18 = 5y + 15$

$\Rightarrow 6x - 5y + 3 = 0$ ...(iv)

From Eq. (iv), 5y = 6x + 3 $\Rightarrow y=\frac{6x+3}{5}$ ...(v)

On substituting y from Eq. (v) in Eq. (iii), we get $11x-9\times\left(\frac{6x+3}{5}\right)+4=0$ $\Rightarrow$ 55x – 9 × (6x + 3) + 20 = 0

[$\because$ multiplying by 5]

$\Rightarrow$ 55x – 54x – 27 + 20 = 0 $\Rightarrow$ x = 7

On substituting x = 7 in Eq. (v), we get $y=\frac{6\times7+3}{5}\Rightarrow y=\frac{45}{5}$ $\Rightarrow$ y = 9

Hence, the fraction $\frac{x}{y}$ is $\frac{7}{9}$.

Q.20 If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$, if we only add 1 to the denominator. What is the fraction?

Sol. Let numerator be x and denominator be y. Fraction = $\frac{x}{y}$

According to the Ist condition, $\frac{x+1}{y-1}=1$; x + 1 = y – 1 $\Rightarrow$ x – y = –2 ...(i)

According to the IInd condition, $\frac{x}{y+1}=\frac{1}{2}$ $\Rightarrow$ 2x = y + 1 $\Rightarrow$ 2x – y = 1 ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get (2x – y) – (x – y) = 1 + 2 $\Rightarrow$ x = 3

On substituting x = 3 in Eq. (i), we get, 3 – y = –2 $\Rightarrow$ y = 5

Hence, the fraction $\frac{x}{y}$ is $\frac{3}{5}$.

PRACTICE PROBLEMS

22. A fraction becomes 1/3, when 1 is subtracted from the numerator and it becomes 1/4, when 8 is added to its denominator. Find the fraction.

23. The sum of a numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to 1/3. Find the fraction.

SPEED PROBLEMS AND UPSTREAM & DOWNSTREAM PROBLEMS

ILLUSTRATION

Q.21 Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 h. If they travel towards each other, they meet in 1 h. What are the speeds of two cars?

Sol. Let speed of car I = x km/h and speed of car II = y km/h

Car I starts from point A and car II starts from point B.

First case: Two cars meet at C after 5 h.

AC = Distance travelled by car I in 5 h = 5 × km

BC = Distance travelled by car II in 5 h = 5 y km

We know that, AC – BC = AB

$\Rightarrow$ 5x – 5y = 100 [$\because$ AB = 100 km] $\Rightarrow$ x – y = 20 [$\because$ dividing by 5) ...(i)

Second case: Two cars meet at C after one hour x + y = 100 ...(ii)

On adding Eqs. (i) and (ii), we get, 2x = 120 $\Rightarrow$ x = 60

On substituting x = 60 in Eq. (ii), we get, 60 + y = 100 $\Rightarrow$ y = 40

Hence, the speed of the two cars are respectively 60 km/h and 40 km/h.

Q.22 Nisha can row downstream 20 km in 2 h and upstream 4 km in 2 h. Find her speed of rowing in still water and the speed of the current?

Sol. Let speed of boat in still water = x km/h, Speed of current = y km/h

Then, speed downstream = (x + y) km/h and Speed upstream = (x – y) km/h

According to the given question, $\frac{20}{x+y} = 2$ $\left[\because \text{ speed} = \frac{\text{distance}}{\text{time}}\right]$

and $\frac{4}{x-y} = 2$ $\Rightarrow$ x + y = 10 ...(i)

and x – y = 2 ...(ii)

On adding Eqs. (i) and (ii), we get, 2x = 12 $\Rightarrow$ x = 6

On substituting x = 6 in Eq. (ii), we get, 6 – y = 2 $\Rightarrow$ y = 4

Hence, speed of boat in still water is 6 km/h and speed of current is 4 km/h.

Q.23 Ruchi travels 300 km to her home partly by train and partly by bus. She takes 4 h, if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 min longer. Find the speed of the train and the bus separately.

Sol. Let the speed of the train be x km/h and the speed of the bus be y km/h.

In first case, Ruchi travels 60 km by train and 240 km by bus in 4 h.

Thus, $\frac{60}{x} + \frac{240}{y} = 4$ i.e., $\frac{15}{x} + \frac{60}{y} = 1$ [$\because$ dividing by 4] ...(i)

Similarly, in second case $\frac{100}{x} + \frac{200}{y} = 4 + \frac{1}{6}$ $\left[\because 10 \text{ min} - \frac{1}{6}\text{h}\right]$

$\Rightarrow \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ i.e., $\frac{24}{x} + \frac{48}{y} = 1$ $\left[\because \text{ multiplying eq by } \frac{6}{25}\right]$ ...(ii)

On putting $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, we get, 15u + 60v = 1 ...(iii) and 24u + 48v = 1 ...(iv)

On multiplying Eq. (iii) by 4 and Eq. (iv) by 5, then subtracting Eq. (iv) from Eq. (iii), we get

4(15u + 60v) – 5(24u + 48v) = 4 × 1 – 5 × 1

$\Rightarrow$ 60u – 120u = –1 $\Rightarrow$ –60u = –1 $\Rightarrow u = \frac{1}{60}$

On substituting $u = \frac{1}{60}$ in Eq. (iii), we get $15 \times \frac{1}{60} + 60v = 1$ $\Rightarrow 60v = 1 - \frac{1}{4} = \frac{3}{4}$ $\Rightarrow v = \frac{1}{80}$

Now, $u = \frac{1}{60}$ and $v = \frac{1}{80}$ $\Rightarrow \frac{1}{x} = \frac{1}{60}$ and $\frac{1}{y} = \frac{1}{80}$ $\left[\text{putting } u = \frac{1}{x} \text{ and } v = \frac{1}{y}\right]$

$\Rightarrow$ x = 60 and y = 80

Hence, the speed of train is 60 km/h and the speed of bus is 80 km/h.

PRACTICE PROBLEMS

24. Anil is sitting on a boat which goes 30 km upstream and 44 km downstream in 10 h. In 13 h, he can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

25. Points P and Q are 70 km apart on a highway. A car starts from P and another car starts from Q at the same time. If they travel in the same direction, they meet in 7 h but if they travel towards each other they meet in 1 h. What are their respective speeds?

ILLUSTRATION

Q.24 2 Women and 5 men can together finish an embroidery work 4 days, while 3 women and 6 men can finish in 3 days. Find the time taken by one woman alone to finish the work and also that taken by one man alone.

Sol. Let one woman finish the work in x days and one man finish the work in y days.

Work of one woman in one day = $\frac{1}{x}$; Work of one man in one day = $\frac{1}{y}$

Work of 2 women and 5 men in one day = $\frac{2}{x}+\frac{5}{y}=\frac{5x+2y}{xy}$

$\therefore \frac{2}{x}+\frac{5}{y}=\frac{1}{4}$ 1

Work of 3 women and 6 men in a day $\Rightarrow \frac{3}{x}+\frac{6}{y}=\frac{1}{3}$ 2

Let $\frac{1}{x}$ = p, $\frac{1}{y}$ = 2 $\qquad \therefore$ 2p + 5q = $\frac{1}{4}$

$\Rightarrow$ 8p + 20q = 1 ....3 $\qquad \therefore$ 3p + 6q = $\frac{1}{3}$ 4

p = $\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$ = $\frac{20\times(-1)-18\times(-1)}{8\times18-9\times20}$ = $\frac{-20+18}{144-180}$ = $\frac{-2}{-36}$ = 18

$\therefore$ p = $\frac{1}{18} \Rightarrow \frac{1}{x}=\frac{1}{18}$ $\therefore$ x = 18

q = $\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$ = $\frac{(-1)\times9-(-1)8}{8\times18-9\times20}$ = $\frac{-9+8}{144-180}$; q = $\frac{+1}{+36}$ $\therefore$ q = $\frac{1}{y}$ = $\frac{1}{36}$ $\therefore$ y = 36

PRACTICE PROBLEMS

26. 5 men the 2 women can complete a work in 4 days, while 6 men and 3 women can complete the same work in 3 days. In how many days, will be the work be completed by 1 man alone and in how many days, will the work be completed by 1 woman alone?

27. 8 men and 12 boys can do a piece of work in 5 days while 6 men and 8 boys can finish it in 7 days. Find the time taken by one man alone and one day alone to finish the work.

MIXED PROBLEMS

ILLUSTRATION

Q.25 Meena went to a bank to withdraw 2000 Rs. She asked the cashier to give her 50 Rs. and 100 Rs. notes only. Meena got 25 notes in all. Find how many notes of 50 Rs. and 100 Rs., she received?

Sol. Let number of 50 Rs. notes = x , Number of 100 Rs. notes = y

According to the given question, x + y = 25

and 50 × x + 100 × y = 2000 ...(i)

$\Rightarrow$ x + 2y = 40 [$\because$ dividing by 50] ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get, (x + 2y) – (x + y) = 40 – 25 $\Rightarrow$ y = 15

On substituting by = 15 in Eq. (i), we get, x + 15 = 25 $\Rightarrow$ x = 10

Hence, number of 50 Rs. notes is 10 and number of 100 Rs. notes is 15.

Q.26 The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is 105 Rs. and for a journey of 15 km, the charge paid is 155 Rs. What are the fixed charges and charge per kilometre? How much does a person have to pay for travelling a distance of 25 km?

Sol. Let fixed charge be x Rs. and charge per kilometre be y Rs.

According to the question, x + 10 y = 105 ...(i) and x + 15 y = 155 ...(ii)

From Eq. (i), x = 105 – 10 y ...(iii)

On substituting x from Eq. (iii) in Eq. (ii), we get, 105 – 10y + 15y = 155 ⇒ 5y = 155 – 105 ⇒ $y = \frac{50}{5}$

⇒ y = 10

On substituting y = 10 in Eq. (iii), we get, x = 105 – 10 × 10 = 105 – 100 = 5 ⇒ x = 5

Hence, fixed charges is 5 Rs., Rate per kilometre is 10 Rs.

Amount to be paid for travelling 25 km = Fixed charge + 10 Rs. × 25 = 5 Rs. + 250 Rs. = 255 Rs.

Hence, the amount paid by the person for travelling 25 km is 255 Rs.

Q.27 Harish wants to invest certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum respectively, so as to earn an annual interest of 3720 Rs. His friend Hamida advised him to interchange the amount of investments in the two schemes to get 40 Rs. more as annual interest. How much money did Harish plan to invest in each scheme in the beginning?

Sol. Let Harish wants of invest x Rs. in scheme A and y Rs. in scheme B.

Since, annual interest = 3720 Rs.

$$\Rightarrow \frac{x\times 8\times 1}{100}+\frac{y+9\times 1}{100}=3720 \left[\because I=\frac{PRT}{100}\right] \Rightarrow \frac{8x}{100}+\frac{9y}{100}=3720 \Rightarrow 8x + 9y = 372000 \text{ ...(i)}$$

After interchanging the amount in two schemes, the annual interest = 40 + 3720

$$\therefore \frac{x\times 9\times 1}{100}+\frac{y\times 8\times 1}{100}=3760 \quad \Rightarrow 9x + 8y = 376000 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 9 and Eq. (ii) by 8 and then subtracting, we get

$$\begin{array}{r} 72x + 81y = 3348000 \\ 72x + 64y = 3008000 \\ - \quad - \quad\quad - \\ \hline 17y = 340000 \end{array}$$

$$\Rightarrow y = \frac{340000}{17} = 20000$$

On putting the value of y in Eq. (i), we get, 8x + 9 (20000) = 372000

$$\Rightarrow 8x = 372000 - 180000 \Rightarrow x = \frac{192000}{8} = 24000$$

Hence, Harish invests 24000 Rs. in scheme A and 20000 Rs. in scheme B.

Q.28 The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Then, find the number of students in the class.

Sol. Let No. of rows be x and no. of columns be y.

∴ Total no. of students = xy

If 3 students are extra in a row then there are 1 row less.

∴ No. of students = (x – 1) (y + 3)

∴ (x – 1) (y + 3) = xy ⇒ xy + 3x – y – 3 = xy ⇒ 3x – y = 3 1

If 3 students are less in a row then there are 2 row more

(x + 2) (y – 3) = xy ⇒ xy – 3x + 2y – 6 = xy ⇒ 3x – 2y + 6 = 0

3x – 2y = –6 2

Subtracting eq^n 2 from eq^n 1

3x – y = 3 1

3x – 2y = –6 2

– + +

y = 9

3x – y = 3 ⇒ 3x – 9 = 3 ⇒ 3x = 12 ⇒ x = 4

∴ No. of students in the class ⇒ xy = 4 × 9 = 36.

Q.29 The area of a rectangle gets reduced by 9 sq units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 sq units. Find the dimensions of the rectangle.

Sol. Let x be the length and y be the breadth of a rectangle, then area of rectangle = xy sq unit

In first case, area is reduced by 9 sq units, when length = x – 5 units and breadth = y + 3 units

Then area of rectangle = (x – 5) (y + 3) sq units

According to the given question, xy – (x – 5) × (y + 3) = 9

⇒ xy – (xy + 3x – 5y – 15) = 9 ⇒ –3x + 5y + 15 = 9 ⇒ 3x – 5y = 6

In second case, area is increased by 67 sq units, when length = x + 3 and breadth = y + 2

According to the given question, (x + 3) × (y + 2) – xy = 67

⇒ xy + 2x + 3y + 6 – xy = 67 ⇒ 2x + 3y = 61 ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by , we get, 9x – 15y = 18 ...(iii)

and 10x + 15y = 305 ...(iv)

On adding Esq. (iii) and (iv), we get, 19x = 323 ⇒ x = 17

On substituting x = 17 in Eq. (ii), we get, 34 + 3y = 61 ⇒ 3y = 27 ⇒ y = 9

Hence, length is 17 units and breadth is 9 units.

PRACTICE PROBLEMS ANSWERS

1. a. x = 10, y = 13 **b.** x = $\frac{18}{5}$, y = $\frac{-17}{15}$ **c.** x = 3, y = –2 **d.** x = $\frac{4a+5b}{2a+b}$, y = $\frac{6}{2a+b}$

2. a. x = 80, y = 30 **b.** x = $\frac{35}{2}$, y = $\frac{-15}{2}$ **c.** x = $\frac{-42}{2a}$, y = $\frac{46}{2a}$ **d.** x = 6, y = 4 **e.** x = 4, y = 0

3. a. Equations have unique solution. **b.** Equations do not have any solution. **c.** Equations have infinite solution.

4. a. k can take any value except $\frac{-20}{3}$ **5.** k can take any value except 49. **6. a.** k = 6 **b.** k = 4

7. a. k = –4 **b.** k = $\pm$ 6 **8.** x = 5, y = 2 **9.** x = 6, y = –4 **10.** x = a, y = b **11.** x = 8, y = 3

12. x = $\frac{ac-b(1+c)}{(a^2-b^2)}$, y = $\frac{bc-a(1+c)}{(b^2-a^2)}$ **13.** x = –1, y = 9 **14 & 15** DO YOUR SELF

16. 3, 7 **17.** Rs. 3, Rs. 5 **18.** Nuri = 50 yr & Sonu = 20 yr.

19. Hence, my present age is 45 years and my son's present age is 15 years **20.** 63 or 36. **21.** 36 **22.** 5/12

23. 5/13 **24.** Speed of car A = 40 km/h, speed of car B = 30 km/h **25.** 40km/h and 30km/h

26. Man = 36 days, woman = 18 days