Solution ~~Set~~ 3
S.P-3 (Std)

Section A

① In ΔAOB
$OA = OB \quad \angle OAB = \angle OBA$
$= x$

$\angle AOB + \angle OBA + 116° = 180°$
$2x = 180° - 116°$
$x = \frac{64}{2} = 32°$
$\angle BAS$
$= 90° - 32 = 58°$ (c)

② one (a)

③ ~~P(not ...)~~
outcomes = 51 (d)

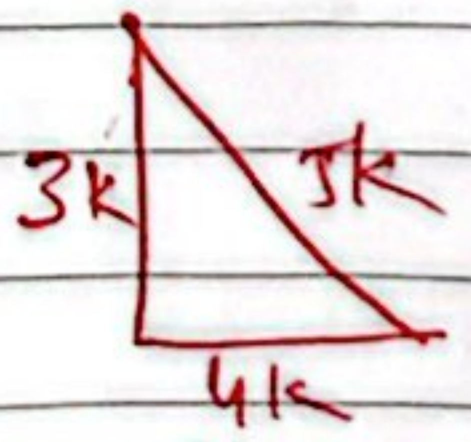
④ $x^2 - 3x - m(m+3)$
$x^2 - 3x - m^2 - 3m$
$x^2 - m^2 - 3(x+m)$
$(x+m)(x-m) - 3(x+m)$
$(x+m)(x-m-3)$
$x = -m, \; m+3$ (b)

⑤ $\dfrac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$

$\frac{1}{2} - 1 + 2$

$\frac{1}{2} + 1 = 1\frac{1}{2} = \frac{3}{2}$ (a)

⑥ $\tan\theta = \frac{3}{4}$



$\sin\theta \cdot \cos\theta = \frac{3k}{5k} \times \frac{4k}{5k}$
$= \frac{12}{25}$ (c)

⑦ (b) $\frac{5}{3}$

⑧ $25 \times 4 = 100$ (c)

⑨ $-81 = 21 + (n-1)(-3)$
$\frac{-81-21}{-3} = n-1$

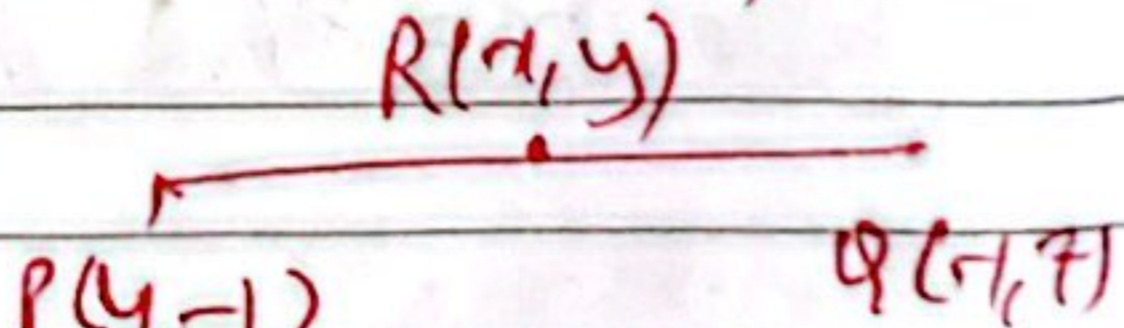
$\frac{-102}{-3} = n-1$
$34 + 1 = n$
$35 = n$ (c)

⑩ $A(-1, 7) \quad C(6, 7)$
$B(-5, 6) \quad D(P, 4)$
Diagonal AC
Let O be the intersection of diagonal
$O(x, y)$
AC
$x = \frac{-1+6}{2} = \frac{5}{2}$

BD
$x = \frac{-5+P}{2}$

ie $\frac{5}{\cancel{2}} = \frac{-5+P}{\cancel{2}}$
$P = 10$ (a)

⑪



$x = \frac{2 \times 4 + 3 \times (-1)}{3+2}$
$= \frac{8-3}{5} = \frac{5}{5} = 1$ (d)

(12) $\triangle ABC \sim \triangle DEF$

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{AB}{2AB} = \frac{8}{EF}$$

$EF = 16 cm$ (a)

(13) $x^2 - (\text{Sum})\, x + \text{product}$

$$x^2 - \left[\sqrt{\frac{5}{2}} - \sqrt{\frac{5}{2}}\right]x + \left(\sqrt{\frac{5}{2}}\right)\left(-\sqrt{\frac{5}{2}}\right)$$

$$x^2 - 0x - \frac{5}{2}$$

$= 2x^2 - 5$ (b).

(14) $x^2 - 4x + 3\sqrt{2} = 0$

$b^2 - 4ac = (-4)^2 - 4 \times 1 \times 3\sqrt{2}$

$= 16 - 16.97$

$-0.97 < 0$

(a)

(15) $\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$h = 4 \times 5$

$h = 20$ (B)

(16) $3x - Ky = 7$

$6x + 10y = 3$

$$\frac{a_1}{a_2} = \frac{3}{6} \qquad \frac{b_1}{b_2} = \frac{-K}{10}$$

$$\frac{3}{6} = \frac{-K}{10}$$

$-K = 5$ $\boxed{K = -5}$

(B)

(17) (a) Median

(18) Diameter of ball 2a

$r = a$

Volume $= \frac{4}{3}\pi a^3$ (D)

(19) (D)
Assertion is False
Reason is true.

(20) (a) Assertion is true, reason is true and Reason is the correct explanation

Section B

(21) 70, 40

$70 = 7 \times 2 \times 5$

$40 = 2 \times 2 \times 2 \times 5$

LCM $= 7 \times 2 \times 2 \times 5 \times 2$
$= 280$

HCF $= 2 \times 5 = 10$

Product $= 70 \times 40 = 2800$

HCF × LCM $= 10 \times 280$
$= 2800$.

Verified

OR

LCM = Product of Numbers

$221 = 13 \times 17$

$p = 17,\ q = 13$

$3p - q = 3 \times 17 - 3$
$= 51 - 13$
$= 38$

22

$x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}$

$= \frac{2\times 4 + 3\times(-1)}{2+3}$

$= \frac{5}{5} = 1$

$y = \frac{2\times(-3) + 3\times 7}{2+3}$

$= \frac{15}{5} = 3$

$(1, 3)$

23

$AB = \sqrt{(7-2)^2 + (3+2)^2}$
$= \sqrt{25+25} = \sqrt{50}$

$BC = \sqrt{(11-7)^2 + (-1-3)^2}$
$= \sqrt{16+16} = \sqrt{32}$

$CD = \sqrt{(6-11)^2 + (-6+1)^2}$
$= \sqrt{25+25} = \sqrt{50}$

$DA = \sqrt{(6-2)^2 + (-6+2)^2}$
$= \sqrt{16+16} = \sqrt{32}$

Diagonal

$AC = \sqrt{(11-2)^2 + (-1+2)^2}$
$= \sqrt{81+1} = \sqrt{82}$

$BD = \sqrt{(6-7)^2 + (-6-3)^2}$
$= \sqrt{1+81}$
$= \sqrt{82}$

Rectangle

24 $\sin\alpha = \frac{1}{2}$

$\alpha = 30^\circ$

$3\cos 30^\circ - 4\cos^3 30$

$3\times\frac{\sqrt{3}}{2} - 4\times\left(\frac{\sqrt{3}}{2}\right)^3$

$\frac{3\sqrt{3}}{2} - \cancel{4}\times\frac{3\sqrt{3}}{\cancel{8}2}$

$\frac{3\sqrt{3}}{2}\left[1 - 1\right]$

$= 0$

25 Total No. of balls $= 24$

~~balls = 24+4+11 19~~

No. of white balls

$24 - (4+11) = 24 - 15$
$= 9$

P(white ball) $= \frac{9}{24}$
$= \frac{3}{8}$

OR

No. of red balls $= 5$

Let No. of blue balls $= x$

Total $= 5+x$

P(blue ball) $= 3$ P(red ball)

$\frac{x}{5+x} = 3\times\frac{5}{5+x}$

$x = 15$

Section C

(26) $3+2\sqrt{5} = \frac{a}{b}$ $b \neq 0$

a, b coprime

$2\sqrt{5} = \frac{a}{b} - 3$

$2\sqrt{5} = \frac{a-3b}{b}$

$\sqrt{5} = \frac{a-3b}{2b}$

(27) $3x^2+5x-2$

$= 3x^2+6x\cancel{+1}-x-2$

$3x(x+2)-1(x+2)$

$(3x-1)(x+2)$

$x = \frac{1}{3}$ and -2

$2\alpha = \frac{2}{3}$ $2\beta = -4$

$x^2 - (\text{sum})x + \text{product}$

$x^2 - \left(\frac{2}{3} - 4\right)x + \frac{2}{3} \times (-4)$

$= x^2 - \left(\frac{-10}{3}\right)x - \frac{8}{3}.$

$x^2 + \frac{10}{3}x - \frac{8}{3}.$

$= \frac{1}{3}\left[3x^2+10x-8\right]$

(28) $x, 18-x$

$\frac{1}{x} + \frac{1}{18-x} = \frac{9}{40}$

$\frac{18-\cancel{x}+\cancel{x}}{(18-x)x} = \frac{9}{40}$

$18 \times 40 = 9x(18-x)$

$720 = 9x(18-x)$

$80 = 18x - x^2$

$-x^2+18x-80=0$

$x^2-18x+80=0$

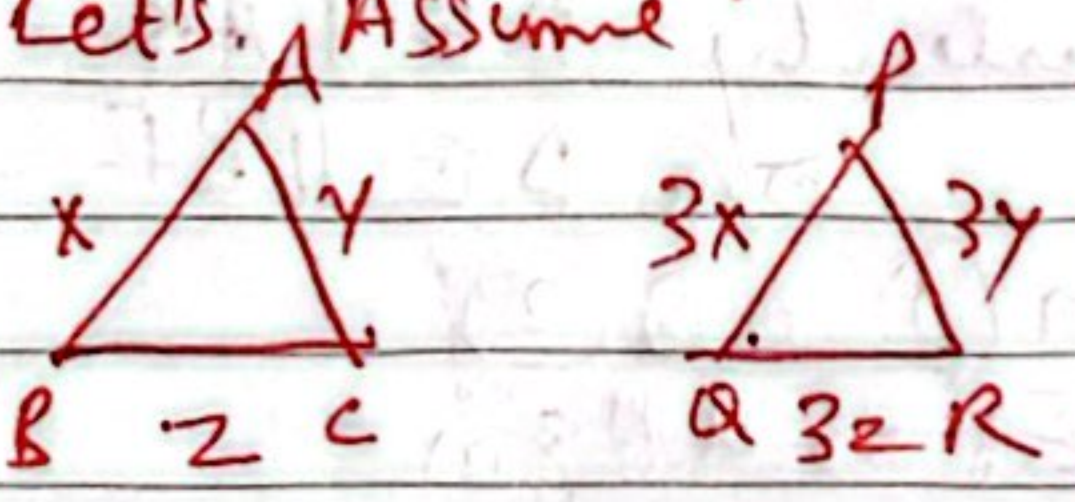
$(x-10)(x-8)=0$

$x = 10, \cancel{18}8$

$18-x = \cancel{18-x}\ 8, 10.$

(29) The perimeter of a Δ is the sum of 3 sides.

Let's Assume



Perimeter ΔABC $X+Y+Z$

Perimeter Δ PQR $3X+3Y+3Z$
$= 3(X+Y+Z)$

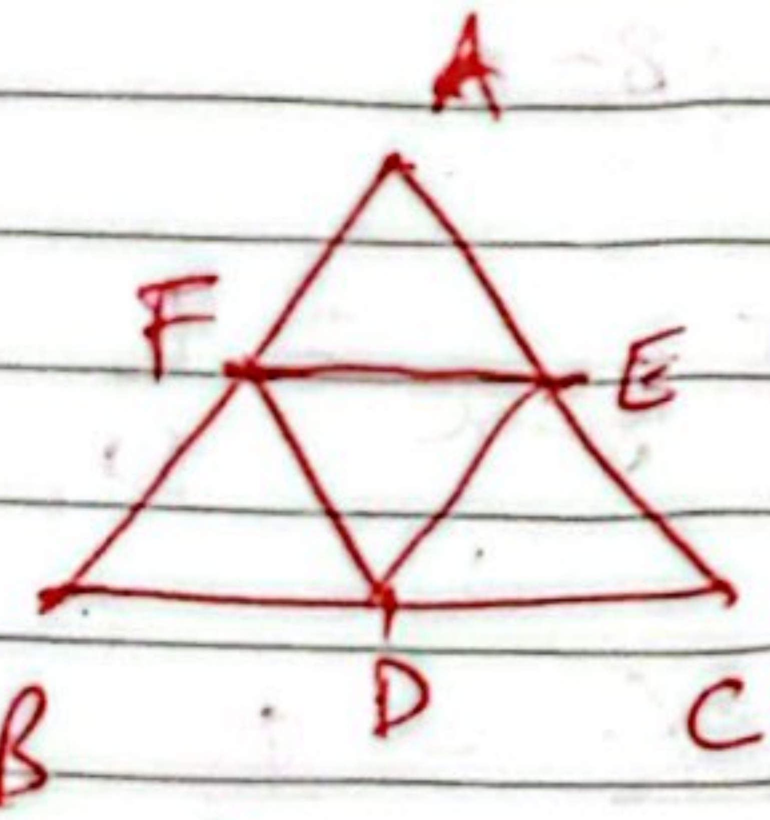
Sides of Δ PQR are 3 times sides of ΔABC.

Hence perimeter is also proportional

∴ ΔABC ~ ΔPQR by SSS.

OR

(29)



To prove

(1) $\Delta FBD \sim \Delta DEF$

(2) $\Delta DEF \sim \Delta ABC$

Proof

A line Joining mid points of two sides of Δ is parallel to 3rd side

In ΔABC

F and E are mid points

$\therefore$ $FE \parallel BC$

or $FE \parallel BD \longrightarrow (1)$

Similarly $DE \parallel AB$

or $DE \parallel BF \longrightarrow (2)$

From (1) and (2)

DBEF is a llgm

Similarly DCEF is llgm

ie $\angle DEF = \angle ABC$

Also DCEF will also be a llgm

$\angle DFE = \angle ACB$

ie $\Delta DEF \sim \Delta ABC$

In ΔDEF and ΔABC

$\angle DFE = \angle ACB$

$\angle DEF = \angle ABC$

by AA Similarity

ie $\Delta DEF \sim \Delta ABC$

In ΔFBD and ΔDEF

$\angle FED = \angle FBD$ (proved above)

$\angle EFD = \angle FDB$ (alternate $\angle$s)

$\therefore$ $\Delta FBD \sim \Delta DEF$ by AA.

Q30

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2\operatorname{cosec}\theta$$

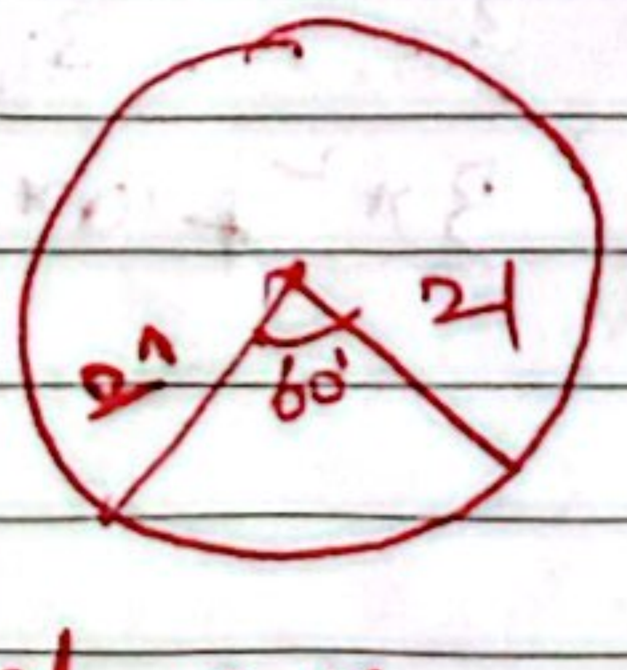
$$\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$\frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\sin\theta}$$

$$\frac{1+\cos\theta + 1-\cos\theta}{\sin\theta}$$

$$= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$$

(31)



length of arc

$$= \frac{\theta}{360°} \times 2\pi r$$

$$\frac{60}{360} \times 2 \times \frac{22}{7} \times 2$$

$\frac{1 \times \cancel{2} \times 22 \times \cancel{3}}{\cancel{6}\,\cancel{2}} = 22\text{ cm}$

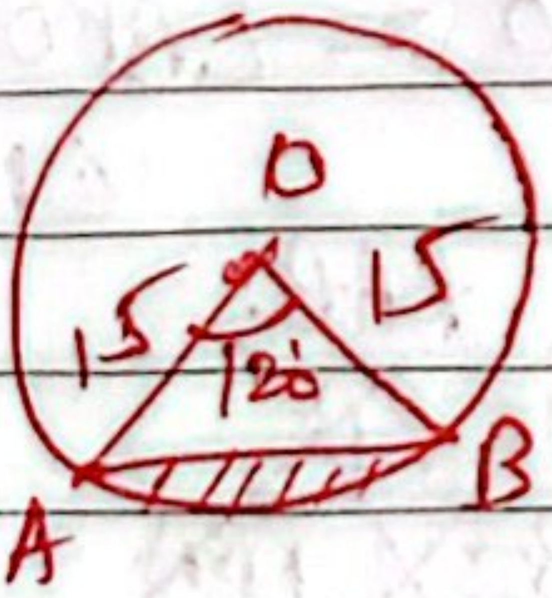
Area of sector

$\frac{\theta}{360} \times \pi r^2$

$\frac{60}{360} \times \frac{22}{7} \times 21 \times 21$

$\frac{1}{6} \times 11 \times 21 = 231\text{ cm}^2$

OR



Area of segment

Area of sector − Ar△AOB.

$\frac{120}{360} \times \pi \times 15 \times 15 - r^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$

$\frac{1}{3} \times \frac{22}{7} \times 15 \times 15 - 15 \times 15 \times \sin 60° \cos 60°$

$15^2 \left[\frac{22}{3 \times 7} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right]$

$225 \left[\frac{22}{21} - \frac{\sqrt{3}}{4}\right]$

$225 \left[1.047 - 0.433\right]$

136.039 aprox

SECTION D

(32)

(A)

$x + 2y = 3$

x	3	1	5
y	0	1	−1

$2x - 3y = -8$

x	−4	−1	−2.5
y	0	2	+1

On graph sheet

(B) $\boxed{x > y}$

x km/hr ————————— y km/hr

A 180 km B

$D = S \times t$

Case 1

$9x - 9y = 180$

Case 2

$1x + 1y = 180$

$x - y = 20$

$x + y = 180$

$2x = 200$

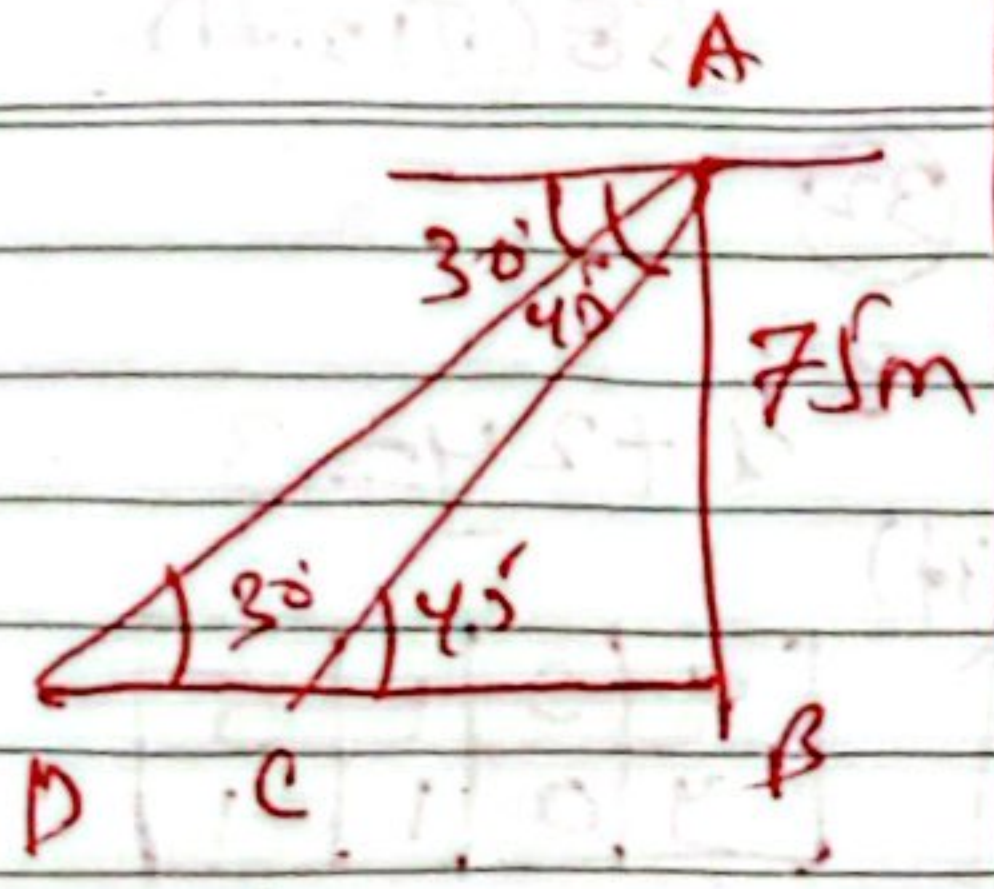
$x = 100$ km/hr

$x + y = 180$

$100 + y = 180$

$y = 80$ km/hr

(33)



In ΔABC

$\tan 45 = \frac{75}{CB}$

$1 = \frac{75}{CB}$

$CB = 75$ m

In ΔADB

$\tan 30 = \frac{75}{CD + 75}$

$\frac{1}{\sqrt{3}} = \frac{75}{CD + 75}$

$CD + 75 = 75\sqrt{3}$

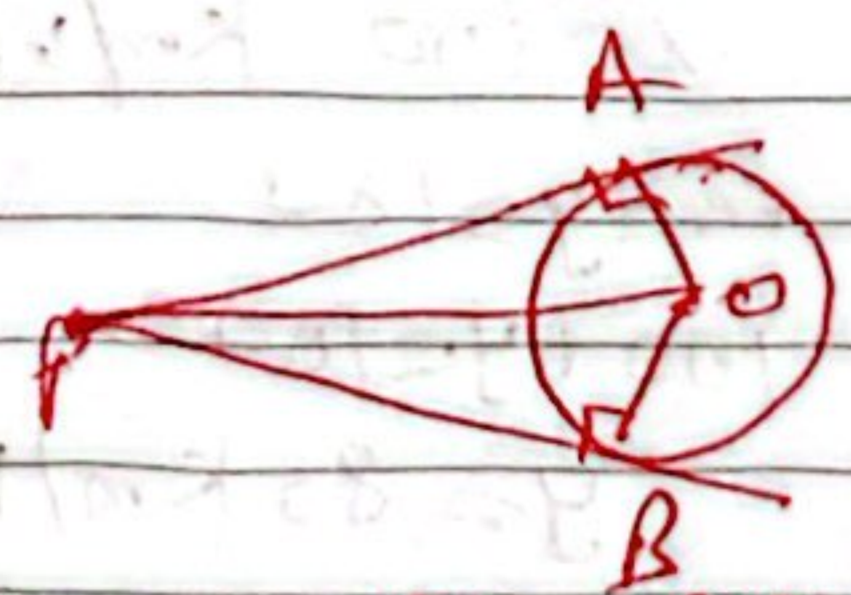
$CD = 75(\sqrt{3} - 1)$ m

$= 75(1.732 - 1)$

$= 75 \times 0.732$

$= 54.9$ m

(34)



To prove $PA = PB$

Proof- In ΔAOP and ΔBOP

$OP = OP$ (common)

$\angle OAP = \angle OBP$ (each 90°)

$OA = OB$ (radius)

$\Delta AOP \cong \Delta BOP$ by RHS

$PA = PB$ (C.P.C.T)

(2) In ΔPOA and ΔPOB.

$PA = PB$ (tangents)

$OP = OP$ (common)

$OA = OB$ (radii)

$\Delta POA \cong \Delta POB$ by SSS

$\angle APO = \angle BPO$ by CPCT

~~In ΔPAB.~~
~~$PA = PB$~~
~~$\angle PAB = \angle PBA$~~
~~(sides opp equal angles)~~

In ΔPAQ and ΔPBQ

$PA = PB$

$PQ = PQ$ (common)

$\angle APO = \angle BPO$ (proved above)

$\Delta PAQ \cong \Delta PBQ$ by SAS

$AQ = QB$ C.P.C.T

$\angle PQA = \angle PQB$ CPCT

$\angle PQA + \angle PQB = 180$ (linear pair)

$2\angle PQA = 180$

$\angle PQA = 90$

35)

CI	f	x_i	fx
85-90	15	87.5	1312.5
90-95	22	92.5	2035
95-100	20	97.5	1950
100-105	18	102.5	1845
105-110	20	107.5	2150
110-115	25	112.5	2812.5
	120		12105

$\bar{x} = \frac{12105}{120} = 100.87$

$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 110 + \frac{25-20}{50-20-0} \times 5$

$= 110 + \frac{5}{30} \times 5$

$= 110 + \frac{5}{6} = 110.83$

OR.

Median =

CI	f	cf
0-10	5	5
10-20	x	$5+x$
20-30	20	$25+x$
30-40	15	$40+x$
40-50	y	$40+x+y$
50-60	5	$45+x+y$
	60	

$45 + x + y = 60$

$x + y = 15$

Modal class 20-30

$Median = l + \frac{\frac{n}{2} - cf}{f} \times h$

$20 + \frac{\frac{60}{2} - (5+x)}{20} \times 10$

$28.5 - 20 = \frac{30 - 5 - x}{20} \times 10$

$8.5 \times 2 = 25 - x$

$17 - 25 = -x$

$-8 = -x$

$x = 8$

$y = 7$

Section E

36 As he takes 2 sec. less ~~than~~ each day

① ∴ AP will be

51, 49, 47 . . . 31 seconds

② $a_n = a_1 + (n-1)d$

$31 = 51 + (n-1)(-2)$

$\frac{-20}{-2} = n-1$

$10 + 1 = n$

11 days.

③ $a_n = 2n+3$

$a_1 = 2+3 = 5$

$a_2 = 2 \times 2 + 3$

$= 4 + 3 = 7$

$d = 7 - 5 = 2$

OR

$(x+10) - 2x = (3x+2) - (x+10)$

$x+10-2x = 3x+2-x-10$

$-x+10 = 2x-8$

$-3x = -18$

$x = 6$

(37)

(1) In $\triangle CAB$ and $\triangle CLO$

$\angle CAB = \angle CLO = 90°$

$\angle C = \angle C$ (common)

$\triangle CAB \sim \triangle CLO$ AA

(2) In $\triangle DCA$ and $\triangle BAC$

$DC = BA$ ($x = y$ given)

$\angle DCA = \angle BAC$ ($90°$)

$CA = AC$.

$\triangle DCA \sim \triangle BAC$ by SAS

$\frac{DA}{BC} = \frac{DC}{BA} = \frac{y}{x}$

$\frac{BC}{DA} = \frac{x}{y} = \frac{x}{x} = \frac{1}{1}$ or $1:1$

(3) $\triangle CAB \sim \triangle CLO$

$\frac{CA}{CL} = \frac{AB}{LO}$

$\Rightarrow \frac{z}{a} = \frac{x}{d} \Rightarrow a = \frac{zd}{x}$

OR

In $\triangle ALO$ and $\triangle ACD$

$\angle ALO = \angle ACD = 90°$

$\angle A = \angle A$ (common)

$\triangle ALO \sim \triangle ACD$ AA

$\frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{b}{z} = \frac{d}{y}$

$b = \frac{zd}{y}$

(38)

$\cancel{l = \sqrt{24^2+3.5^2}}$

$\cancel{= \sqrt{576+12.25}}$

$\cancel{= 24.25}$

(1) $l = \sqrt{24^2+7^2}$

$= \sqrt{625}$

$= 25$

(2) $C.S.A = \pi r l$

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 25$

$= 550\ cm^2$

(3) C.S.A of cylinder

$2\pi rh + \pi r^2$

$\pi r(2h + r)$

$\frac{22}{\cancel{7}} \times \cancel{7}(2\times 24 + 7)$

22×55

$= 1210\ cm^2$

OR

Volume of cylinder − Volume of cone

$\pi r^2 h - \frac{1}{3}\pi r^2 h$

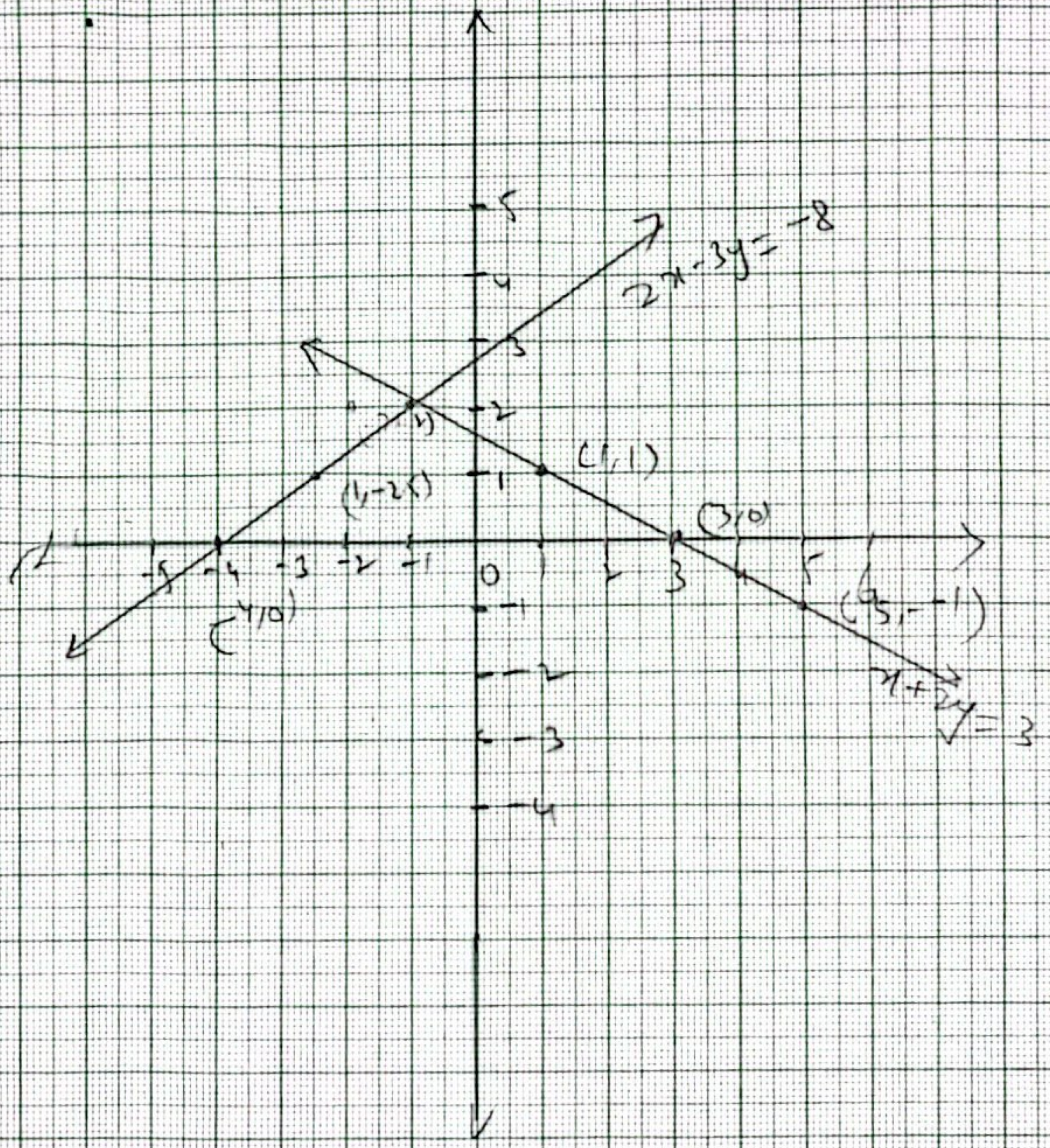
$\pi r^2 h\left(1 - \frac{1}{3}\right)$

$\frac{2}{3}\pi r^2 h$

$\frac{2}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times \overset{8}{\cancel{24}}$

$2464\ cm^3$

Topic : ______ Date : ______



Axis ______

Axis ______