

CHINMAYA VIDYALAYA, NEW DELHI
PROFICIENCY EXAMINATION (2023-24)
CLASS X
SUBJECT-MATHEMATICS

TIME ALLOWED: 3 HOURS

MAXIMUM MARKS: 80

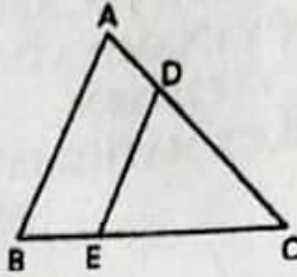
General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice of 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Qs of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION -A

- ✓ Q1. The prime factorisation of 96 is:
- (a) $2^5 \times 3$
 - (b) 2^6
 - (c) $2^4 \times 3$
 - (d) $2^3 \times 32$
- ✓ Q2. The value of x and y, $x + y = 3$ and $7x + 6y = 2$ are:
- (a) $x = 16, y = 9$
 - (b) $x = 8, y = 5$
 - (c) $x = 2, y = 15$
 - (d) $x = -16, y = 19$
- ✓ Q3. Form a quadratic polynomial, whose zeros are -3 and 5.
- (a) $x^2 - 2x - 15$
 - (b) $3x^2 + 5$
 - (c) $x^2 - 3x - 14$
 - (d) $x^2 + 2x + 15$
- ✓ Q4. For what values of 'a', does the quadratic equation $x^2 - ax + 1 = 0$ not have real roots?
- (a) (2,1)
 - (b) (0, 1)
 - (c) (-2, 2)
 - (d) (-1,1)

✓ Q5. In the figure, if $\angle A = \angle B$ and $AD = BE$. Then DE is Parallel to:



- (a) BE
- (b) EC
- (c) AC
- (d) AB

✓ Q6. The distance between two parallel tangents to a circle of radius 5 cm is:

- (a) 5 cm
- (b) 8 cm
- (c) 10 cm
- (d) 9 cm

✓ Q7. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation?

- (a) 30°
- (b) 45°
- (c) 60°
- (d) 90°

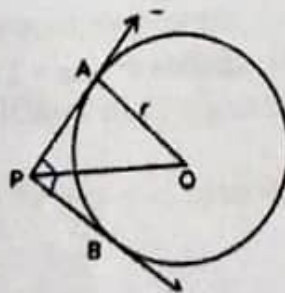
✓ Q8. The first negative term of the AP: $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ is:

- (a) 27
- (b) 24
- (c) 25
- (d) 28

✓ Q9. The pairs of equations $x + 2y + 5 = 0$ and $5x + 10y + 25 = 0$ have: (1)

- (a) unique solution
- (b) exactly two solutions
- (c) Infinitely many solutions
- (d) No solution

✓ Q10. In the figure, $\angle APB = 90^\circ$. The length of OP is:



- (a) $2r$
- (b) $\sqrt{3}r$
- (c) $4r$
- (d) $\sqrt{2}r$

Q11. The mid-point of the line segment joining the points $(-2, 4)$ and $(6, 10)$ is:

- (a) $(2, 7)$
- (b) $(5, 2)$
- (c) $(3, 5)$
- (d) $(4, 5)$

Q12. If the ratio between the volumes of two spheres is $8 : 27$, then the ratio between their surface areas is:

- (a) $2 : 3$
- (b) $1 : 2$
- (c) $25 : 16$
- (d) $4 : 9$

Q13. If $\operatorname{cosec} A - \cot A = 13$, then the value of $\operatorname{cosec} A + \cot A$ is:

- (a) $\frac{1}{12}$
- (b) $\frac{2}{13}$
- (c) $\frac{1}{14}$
- (d) $\frac{1}{13}$

Q14. In a rectangle OABC the length of its diagonal is, if its three vertices are O $(0, 0)$, A $(0, 3)$ and B $(5, 0)$:

- (a) 5 units
- (b) $\sqrt{34}$ units
- (c) 4 units
- (d) 3 units

Q15. The chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord is:

- (a) 10 cm
- (b) 20 cm
- (c) $10\sqrt{2}$ cm
- (d) $10\sqrt{3}$ cm

Q16. $\triangle ABC \sim \triangle DEF$ such that $DE = 3$ cm, $EF = 2$ cm, $DF = 2.5$ cm and $BC = 4$ cm. The perimeter of $\triangle ABC$ is:

- (a) 15 cm
- (b) 10 cm
- (c) 9 cm
- (d) 8 cm

Q17. If $S_n = 5n^2 + 3n$, then its n th term is:

- (a) $5n - 1$
- (b) $10n^2$

(c) $10n - 2$

(d) $8n - 3$

Q18. The value of k for which the equation $x^2 + 4x + k = 0$ has real roots is:

(a) $k = 4$

(b) $k < 4$

(c) $k > 4$

(d) $k \leq 4$

DIRECTION: In question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).

Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q19. Statement A (Assertion): If the circumference of a circle is 176 cm, then its radius is 28 cm.

Statement R (Reason): Circumference = $2\pi \times$ radius.

Q20. Statement A (Assertion): The point (0, 6) lies on y-axis.

Statement R (Reason): The x-coordinate of the point on y-axis is zero.

SECTION - B

Q21. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

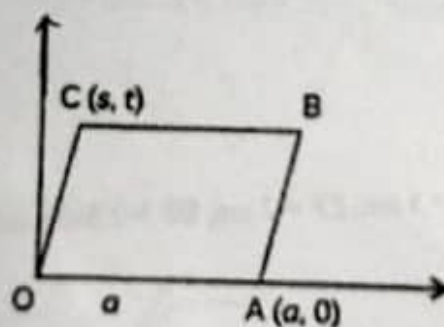
Q22. Find whether the following pair of linear equations is consistent or inconsistent:

$$3x + 2y = 8$$

$$6x - 4y = 9$$

OR

In the figure, OABC is a rhombus, where O is the origin. The coordinates of A and C are (a, 0) and (s, t) respectively.



(i) Write down the coordinates of B in terms of a, s and t.

(ii) Find the length of OC in terms of a and t.

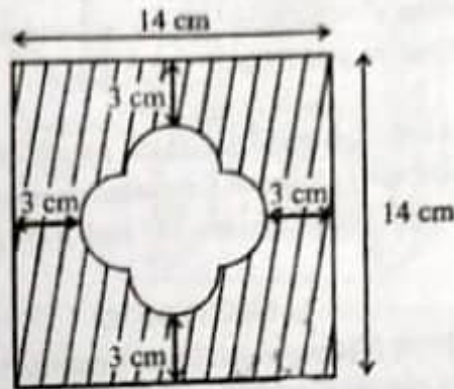
Q23. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the mid-point of PQ, find the coordinates of P and Q.

Q24. The length of a tangent from a point A at distance 5cm from the centre of the circle is 4cm. Find the radius of the circle.

OR

There are two circles with radii 20 cm and 7 cm, respectively. Find the radius of the circle whose circumference equals the sum of the circumference of the two circles.

Q25. Find the area of the unshaded region shown in the given figure.



SECTION-C

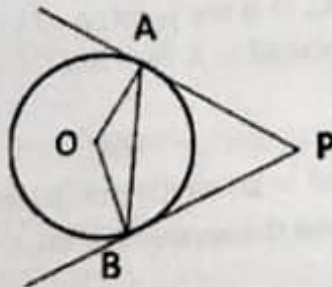
Q26. If $\sin \theta + \cos \theta = \sqrt{3}$, then prove that $\tan \theta + \cot \theta = 1$.

OR

In a $\triangle ABC$ right-angled at B, if $AB: AC = 1: \sqrt{2}$ Then find the value of: $\frac{2 \tan A}{1 + \tan^2 A}$

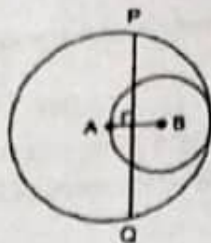
Q27. The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

Q28. PA and PB are tangents drawn to a circle of centre O from an external point P. Chord AB makes an angle of 30° with the radius at the point of contact. If length of the chord is 6 cm, find the length of the tangent PA and the length of the radius OA.



OR

In the figure, two circles with centres A and B and radii 5 cm and 3 cm touching each other internally. If the perpendicular bisector of segment AB, meets the bigger circle at P and Q, find the length of PQ.



Q29. If the roots of the equation $x^2 + 2cx + ab = 0$ are real and unequal, prove that the equation $x^2 - 2(a+b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots.

Q30. An ice cream cone has a hemispherical top. If the height of the conical portion is 9 cm and base radius is 2.5 cm, determine the volume of ice-cream contained in the ice cream cone.

Q31. Evaluate the greatest number which divides 245 and 1029 and leaves the 5 as remainder in each case.

SECTION-D

Q32. From the first floor of Qutab Minor, which 25 m from the level ground, a man observes the top of a building at an angle of elevation of 30° and the angle of depression of the base of the building is 60° . Calculate the height of the building.

OR

From the top of a tower 40 m tall, the angle of depression of the nearer of the two points A and B on the ground diametrically opposite sides of the tower is 45° . Show that the angle of depression θ of the other point if the distance of the two points from the base of the tower are in the ratio 1:2, is given by $\tan \theta = \frac{1}{2}$

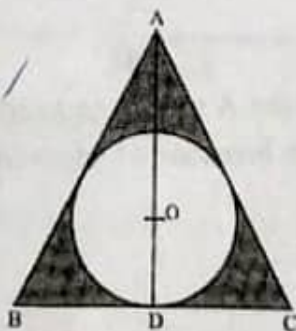
Q33. A circle touches all four sides of quadrilateral ABCD. Prove that $AB + CD = AD + BC$.

Q34. A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.

Q35. AD is the median of $\triangle ABC$, O is any point on AD. BO and CO produced meet AC and AB in E and F respectively. AD is produced to X such that $OD = DX$. Prove that:
 $AO : AX = AF : AB$.

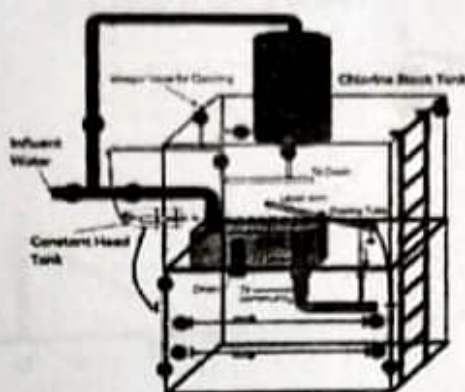
OR

In the figure, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius, r of the inscribed circle and the area of the shaded region. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]



SECTION- E

Q36. Selvi is setting up a water purifier system in her house which includes setting up an overhead tank in the shape of a right circular cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid.



The underground water tank (sump) is a sturdy single moulded piece built to withstand underground pressure and is available in the storage capacity of 2000L

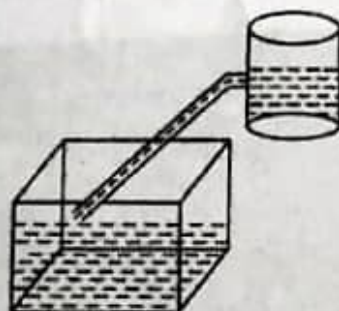
These, along with hassle-free installation and minimum maintenance needs make it the ideal water storage solution.

Dimensions (sump):

1.57 m × 1.44 m × 95 cm.

Dimensions (overhead tank):

Radius is 60 cm and Height is 95 cm



Water flow conditions at the required overload capacity should be checked for critical pressure drop to ensure that valves are adequately sized.

Based on the above information, answer the following questions:

(A) Find the ratio of the capacity of the sump to the capacity of the overhead tank.

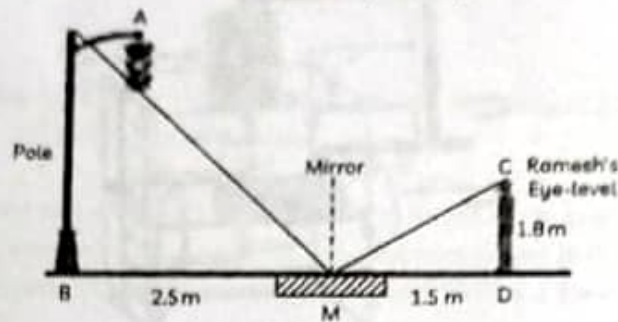
(B) If the overhead tank needs to be painted to save it from corrosion, how much area needs to be painted?

(C) If water is filled in the overhead tank at the rate of 20 litre per minute, the tank will be completely filled in how much time?

OR

If the amount of water in the sump, at an instant, is 1500 litres, then find the water level in the sump at that instant?

Q37. Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fixed on it) (see the figure). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.8 m above the ground. The distance of Ramesh and the pole from the mirror are 1.5 m and 2.5 m respectively.



Based on the above information, answer the following questions:

(A) Which criterion of similarity is applicable to similar triangles?

(B) Find the height of the pole.

(C) If Ramesh's eye level is 1.2 m above the ground, then find the height of the pole.

OR

If the distance of Ramesh and the pole from the mirror are 2.5 m and 1.5 m respectively, then find the height of the pole.

Q38. Mr. Punit arranged a lunch party for some of his friends. The expenses of the lunch are partly constant and partly proportional to the number of guests. The expense amount for 7 guests is ₹ 650 and ₹ 970 for 11 guests.



Let the constant expenses be denoted by ₹ x and proportional expenses per person by ₹ y

Based on this information, answer the following questions:

(A) How the situation stated above will be translated into a pair of linear equation?

(B) What are the proportional expenses per person?

OR

What are the constant expenses? If there are 15 guests at the party launch, thus what amount Mr. Punit has to pay?

(C) Find the value of k for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent.