

THE INDIAN SCHOOL
HALF YEARLY EXAMINATION 2017-18
MATHEMATICS
CLASS X

Time: 3 Hrs.

Max Marks: 80

General instructions:

- (i) All questions are compulsory.
- (ii) The question paper consists of 30 questions divided into 4 sections, A, B, C and D. **Section-A** comprises of 6 questions of 1 mark each. **Section-B** comprises of 6 questions of 2 marks each. **Section-C** comprises of 10 questions of 3 marks each and **Section-D** comprises of 8 questions of 4 marks each.
- (iii) All questions in **Section-A** are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice in this question paper.
- (v) Use of calculator is not permitted.

SECTION-A

Q1. Is $7 \times 11 \times 13 \times 15 + 15$ a prime number? Justify your answer.

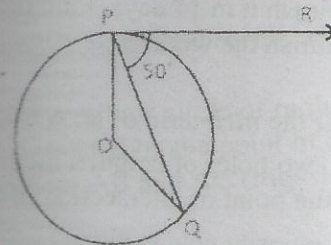
Q2. If α, β are the zeroes of the polynomial $f(x) = x^2 - p(x+1) - c$, then find the value of $(\alpha+1)(\beta+1)$.

Q3. Solve for a and b : $2^a + 3^b = 17$ and $2^{a+b} - 3^{b+1} = 5$.

Q4. If $a_n = 5 - 11n$, find the common difference.

Q5. A die is thrown twice. What is the probability that 5 will not come up either time?

Q6. In the given figure, if O is the centre of the circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ , then find $\angle POQ$.

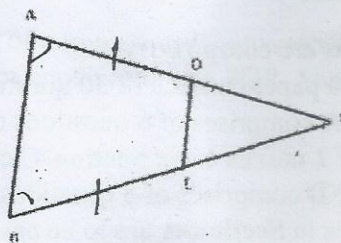


SECTION-B

Q7. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$. find the value of $\alpha^3 + \beta^3$.

Q8. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Q9. In the given figure, $\angle A = \angle B$ and $AD = BE$, show that $DE \parallel AB$.



Q10. Evaluate:

$$\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$$

Q11. The LCM of two numbers is 14 times their HCF. The sum of HCF and LCM is 600. If one number is 280, find the other number.

Q12. If $\cos A + \cos^2 A = 1$, then prove that $\sin^2 A + \sin^4 A = 1$.

SECTION-C

Q13. What must be added to $x^4 + 2x^3 - 2x^2 - 2x - 1$ to obtain a polynomial which is exactly divisible by $x^2 + 2x + 3$.

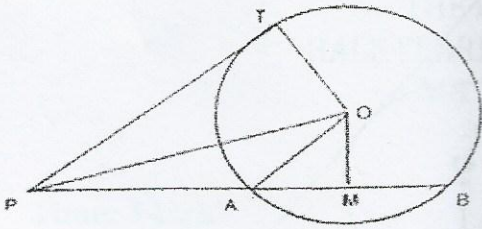
Q14. Find the H.C.F. of 65 and 117 and express it in the form $65m + 117n$.

Q15. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Q16. If the m th term of an A.P. is $\frac{1}{n}$ and n th term is $\frac{1}{m}$, then show that its (mn) th term is 1.

Q17. Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

Q18. In the given figure, PT is a tangent and PAB is a secant. If $PT = 6\text{cm}$, $AB = 5\text{cm}$, find the length of PA . (figure on the next page).



Q19. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2) = 4\sqrt{mn}$.

Q20 Offices in Delhi are open for five days a week (Monday to Friday). Two employees of an office remain absent for one day in the same particular week. Find the probability that they remain absent on

- i) the same day
- ii) consecutive days
- iii) different days

Q21. ABC is an isosceles triangle in which $AB=AC$, circumscribing about a circle. Show that BC is bisected at the point of contact.

Q22. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

SECTION-D

Q23. If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, prove that $l^2 m^2 (l^2 + m^2 + 3) = 1$.

Q24. If $\tan A = n \tan B$ and $\sin A = m \sin B$, Prove that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$.

Q25. Find the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeroes are in A.P.

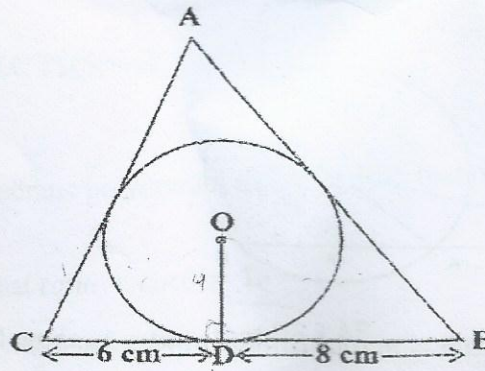
Q26. Prove that for any positive integer n , $n^3 - n$ is divisible by 6.

Q27. Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$$\begin{aligned} 2x - 3y &= 7 \\ (a + b)x - (a + b - 3)y &= 4a + b \end{aligned}$$

Q28. How many terms are identical in the two APs 2,4,6,8,...upto 100 terms and 3,6,9,...upto 80 terms?

Q29. In the following figure triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC. (Figure on the next page)



30. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.