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X-B

# First Term Exam – 2017-2018

## Class – X

## Subject – MATHEMATICS

**Time : 3 Hours** **Max. Marks : 80**

**General Instructions :**

(i) **All questions are compulsory, however there are two internal choices.**

(ii) **The question paper consists of 30 questions divided into four sections – A, B, C and D.**

(iii) **Section A contains 6questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each and Section D contains 8questions of 4 marks each.**

(iv) **Use of calculators is not permitted**

### SECTION - A

1. Give two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$

2. Find the value of k for which the equation $x^2 + 5kx + 16$ has no real roots.

3. Is 184 a term of the AP 3,7,11,…?

4. E and F are points on the sides PQ and PR respectively of a $\Delta$ PQR. State whether EF $\parallel$ QR if PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

5. Evaluate $2\sin^2 30° \tan 60° - 3\cos^2 60° \sec^2 30°$

6. Find the probability of exactly 52 Sundays in a leap year

### SECTION - B

7. If $\alpha$ and $\beta$ are the zeroes of the polynomial $x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

8. Solve the pair of linear equations graphically: $x + 3y = 6$ and $2x - 3y = 12$ and tell the type of solution obtained.

9. How many three digit are divisible by 7 ?

10. S and T are points on sides PR and QR of $\Delta$ PQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

11. If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\frac{(B+C)}{2} = \cos\frac{A}{2}$$

12. The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of the tower is $\sqrt{st}$ .

## SECTION - C

13. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

14. Verify that 3, -1, -1/3 are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the coefficients.

15. Solve :
$$a(x+y) + b(x-y) = a^2 - ab + b^2$$
$$a(x+y) - b(x-y) = a^2 + ab + b^2$$

16. Find the roots of the following quadratic equation by the method of completing the square

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

17. If m times the $m^{th}$ term of an AP is equal to n times its $n^{th}$ term, show that the $(m+n)^{th}$ term of the AP is zero.

18. In right triangle ABC, right angled at B, points D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$

**OR**

BL and CM are medians of triangle ABC right angled at A. Prove that $4(BL^2 + CM^2) = 5\,BC^2$

19. $\angle B$ and $\angle Q$ are acute angles of two right triangles $\Delta ABC$ and $\Delta PQR$ right angled at C and R such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

20. Prove that $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$ , using the identity $\sec^2 A = 1 + \tan^2 A$.

21. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the deck was shuffled well. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a heart (ii) a king

22. The annual rainfall record of a city for 66 days is given in the following table.

| Rainfall (in cm) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| Number of days | 22 | 10 | 8 | 15 | 5 | 6 |

Calculate the median rainfall using ogives (of more than type and of less than type)

## SECTION - D

23. Prove that $\sqrt{3}$ is irrational and hence show $2- \sqrt{3}$ is irrational.

24. Find all the zeroes of $2x^4 + 7x^3 - 19x^2 - 14x + 30$ , if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$ .

25. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

26. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

The man wanted to calculate the optimum speed to be allowed so that least pollution takes place , which quality is exhibited by him?

27. Two pipes together can fill a tank in $3\frac{1}{13}$ minutes. The pipe of larger diameter takes 3 minutes less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

28. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on . In how may rows are the 200 logs placed and how many logs are in the top row?

**OR**

150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more dropped the third day and so on. It takes eight more days to finish the work now. Find the number of days in which the work was completed.

29. **Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.**

30. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data

| **Monthly consumption (in units)** | **Number of consumers** |
|---|---|
| 65 – 85 | 4 |
| 85 – 105 | 5 |
| 105 – 125 | 13 |
| 125 – 145 | 20 |
| 145 – 165 | 14 |
| 165 – 185 | 8 |
| 185 – 205 | 4 |