ΣXEMPLAR POINT® PVT. LTD.

XI MATHS SETS THEORY & ASSIGNMENT

INTRODUCTION OF SETS

DEFINITION

Well defined collection of objects is called a set.

Well defined collection - means that given a set and an object, it must be possible to decide whether or not the object belongs to set.

(i) Objects are called the member (elements) of the set.

(ii) Sets are usually denoted by capital letters A, B, C etc. & their elements are denoted by smallletters a, b, c etc.

(iii) If a is the member of A then we write as $a \in A$ (a belongs to A) otherwise, $a \notin A$ (a does not belongs to A)

EXAMPLES

1. Collection of even natural numbers less than 12.

Ans. 2, 4, 6, 8, 10

2. Collection of vowels in the English alphabet.

Ans. a, e, i, o, u

3. Collection of good cricket players of India.

Ans. It is not a well defined collection. Because it is difficult to decide who is good or not?

PRACTICE WORKSHEET 1

1. Which of the following collections are sets? Justify your answer :

a. A collection of all natural numbers less than 50.

b. The collection of all girls in your class.

c. The collection of good cricket players of India.

d. A collection of novels written by Munshi Prem Chand.

e. The collection of all even integers.

f. The collection of all lines in a particular plane.

g. The collection of most dangerous animals.

h. The collection of most talented writers of India.

i. The collection of prime integers.

REPRESENTATION OF SETS

There are two ways to represent a given set.

1. ROSTER OR TABULAR FORM: In this method all the elements of the set are listed and separated by commas and are enclosed within braces {}.

EXAMPLES

1. The set of prime natural numbers less than 10 in the tabular form.

Ans. A = {2, 3, 5, 7}

2. Set N of natural numbers.

Ans. N = {1,2, 3 ...} dots indicating infinitely many positive integer.

<u>REMARK:</u> (i) In roster form order of element is immaterial.

(ii) In roster form same elements are taken once.

EXAMPLES

1. A = {1, 2, 3, 4} and B = {1, 4, 2, 3} are same set.

2. Set of letters forming the word "SCHOOL".

Ans. A = {S, C, H, O, L}

2. SET-BUILDER FORM: In this form, write one or more variable (x, y etc.) representing an arbitary member of the set, this is followed by a statement or a property which must be satisfied by each member of the set. And no element outside the set possess that property.

EXAMPLE

1. Set of prime numbers less than 20.

Ans. Set builder form = $\left\{x : \underbrace{x \text{ is a prime no. less than } 20}_{\text{Property}}\right\}$

In roster form = {2, 3, 5, 7, 11, 13, 17, 19}

2. Set of natural numbers in set builder form is written as N = {x: x is a natural number}

SOME STANDARD SETS

We enlist below some sets of numbers which are most commonly used in the study of set:

(i) The set of natural numbers. It is usually denoted by **N**, i.e., **N** = {1, 2, 3, 4, ...}

(ii) The set of whole numbers. It is usually denoted by **W,** i.e., **W** = {0, 1, 2, 3, ...}

(iii) The set of integers. It is usually denoted by **Z** or **I**, i.e., **Z** = {..., –3, –2, –1, 0, 1, 2, 3, ...}

(iv) The set of rational numbers. It is usually denoted by **Q**, i.e., **Q** = {x : x is a rational number}

or $\mathbf{Q} = \left\{x : x = \dfrac{m}{n}, \text{ where m and n are integers and } n \neq 0\right\}$

(v) The set of real numbers. It is usually denoted by **R,** i.e., **R** = {x : x is a real number}

or **R** = {x : x is either a rational number or an irrational number}.

(vi) The set of positive integers. It is usually denoted by $\mathbf{Z}^+$, i.e., $\mathbf{Z}^+$ = {1 2, 3, 4, ...}

(vii) The set of positive real numbers. It is usually denoted by $\mathbf{R}^+$, i.e., $\mathbf{R}^+$ = {x : x is a positive real number}

(viii) The set of positive rational numbers. It is denoted by $\mathbf{Q}^+$, i.e., $\mathbf{Q}^+$ = {x : x is a positive rational no.}

(ix) The set of irrational numbers, is composed of all other real numbers which are not rational. It is usually denoted by **T.**, i.e., **T** = {x : $x \in$ R and $x \notin$ Q}

PRACTICE WORKSHEET 2

1. Write the following sets in roster form or tabular form :

a. A = {x : x is an integer and – 3 < x < 7}

b. A = {x : x is a natural number less than 6}

c. A = {x : x is a two digit natural number such that sum of the digits is 8}

d. A = {x : x is a prime number which is divisor of 60}

e. A = {The set of all letters in the word TRIGONOMETRY}

f. A = {The set of all letters in the word BETTER}

g. A = {x : x is $x \in$ Z and x^2 <25}

h. A = {x : x is a prime number & 10 < x < 20}

i. A = {x : x = 2n and $n \in$ N}

j. {x : x is a vowel in the word MATHEMATICS}

k. {x : x = odd prime number & x £10, x Î W}

2. **Write the following in the set builder form :**

a. $\{3, 6, 9, 12\}$ **b.** $\{2, 4, 8, 16, 32\}$ **c.** $\{5, 25, 125, 625\}$ **d.** $\{2, 4, 6, \ldots\}$

e. $\{1, 4, 9, \ldots 100\}$ **f.** $\{1, 2, 3, 6\}$ **g.** $\{2, 3\}$ **h.** $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\right\}$

i. $\{M, I, S, P\}$ **j.** $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ **k.** $\{4, 11, 30, 67, 128\}$ **l.** $\{8, 15, 22, 29, 36, 43, 50\}$

TYPES OF SETS

1. **EMPTY SET:** A set which does not contain any element is called the empty set /null set /void set.
It is Denoted by the symbol ϕ or { }.

EXAMPLES

1. Collection of integer whose square is less than 0.

Ans. $\because$ Square of an integer can not be negative, hence it is an empty set.

2. Collection of all the real roots of the equation $x^2 + 5 = 0$.

Ans. $\because$ $x^2 = -5$, as the equation has no real roots, hence it is an empty set.

2. **FINITE SET:** A set in which the number of elements can be counted i.e. the counting procedure ends at a certain point is known as finite set.

CARDINAL NUMBER: Number of distinct elements in a finite set A is called cardinal number and it is denoted by n(A).

REMARK

● Empty set is always considered finite.

● A singleton set is a set which contains only one distinct element.

3. **INFINITE SET:** A Set in which counting of the number of elements do not end, is known as infinite set.

EXAMPLES

1. $S = \{2, 4, 6, 8\}$, no. of elements in Set S $\Rightarrow$ n(s) = 4(finite number), hence it is a finite set.
2. Set of all student studying in a particular school is a finite set.
3. Set of all natural Numbers N is an infinite set as there are infinite natural number.
4. Set of all people 'A' living in a particular city is a finite set whether n(A) = C (say) is a very big number.

4. **EQUIVALENT SETS:** The finite sets A & B are said to be equivalent written as A ~ B if they contain the same number of distinct elements i.e. their cardinal numbers are equal. [i.e. n(A) = n(B)]

EXAMPLES

1. $A = \{1\}, B = \{2, 2, 2\} \rightarrow$ are equivalent because both set has one distinct element.
2. $A = \{3, 4\}, B = \{x : x^2 = 4\}$ are equivalent sets.

5. **EQUAL SETS :** Two sets A & B are said to be equal written as A = B, if every element of A is the element of B & every element of B is the element of A.

REMARK- Equal sets are always equivalent but equivalent sets may not be equal.

EXAMPLES:

1. $A = \{-1, 1\}, B = \{x : x^2 = 1\}$ are equal.
2. $\{0, 0\}$ & $\{3\}$ are equivalent not equal.

PRACTICE WORKSHEET 3

1. **Determine the empty sets, singleton sets.**

   a. $A = \{x : x^2 = 16 \text{ and } 2x = 5\}$.

   b. $B = \{x : x^2 = 2, x \text{ is a rational number}\}$.

   c. $C = \{x : 3 \le x \le 5 \text{ and } 5 \le x \le 6$, where 'x' is an integer$\}$.

   d. $D = \{x : x > 0 \text{ and } x^2 = 25\}$.

   e. $E = \{x : x^3 + 1 = 0 \text{ and x is an integer}\}$.

   f. Set of point of intersection of two parallel lines.

   g. Set of odd natural numbers divisible by 2.

2. **Which of the following sets are finite and infinite sets?**

   a. The set of all lines parallel to x-axis.

   b. The set of all circles passing through the origin.

   c. The set of positive integers less than 100.

   d. The set of natural numbers which are multiples of 5.

   e. The set of prime numbers less than 99.

   f. The set of straight lines passing through a fixed point.

   g. The set of all natural numbers which divide 42.

   h. $\{x : x \in Z \text{ and } x > 6\}$

   i. {x : x is people of India speaking Hindi}

   j. The set of animals on this earth.

3. **From the sets given below, select the equivalent sets :**

   $A = \{1, 2, 3, 5\}$

   $B = \{x : x^2 - 5x + 4 = 0\}$,

   C = {x : x is a letter of the word LOVE}.

   D = {x : x is a letter of the word WOLF}.

4. **Are the following sets equal?**

   a. $A = \{2, 1\}$, ; $B = \{x : x^2 - 3x + 2 = 0\}$.

   b. $A = \{x : x^3 - 8 = 0 \text{ and x is a real number}\}$; $B = \{x : x^2 + 7x - 18 = 0 \text{ and } x > 0\}$

   c. A = The set of letters in the word 'ALLOY'. ; B = The set of letters in the word 'LOYAL'.

   d. $A = \{x : x \in Z \text{ and } x^2 \le 4\}$. ; $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$.

   e. A = {x : x is a multiple of 10}. ; B = {10, 20, 30, ...}.

SUBSETS AND POWER SET

1. **SUBSETS:** Let A, B be two sets such that every member of A is a member of B, then A is called a subset of B, written as $A \subseteq B$.

   If A is subset of B and if a is an element of A implies that a is also the element of B.

   i.e. $A \subseteq B$, if $x \in A \Rightarrow x \in B$ for each $x \in A$

   If there exist any $x \in A$ which is not contained in B ($x \notin B$) then $A \not\subseteq B$ (A is not the subset of B)

2. **SUPERSET:** If A is a subset of B, We say that A is contained in B or B contains A, it is written as $B \supseteq A$ and we say that B is a superset of A.

3. **PROPER SUBSET:** Let A be a subset of B. We say, A is a proper subset of B if $A \neq B$ and It is denoted as $A \subset B$.

   ● Every set is an improper subset of iself. ● If a A is non-empty, then null set is a proper subset of A.

REMARK:

(1) Two sets A & B are equal if and only if (iff) $A \subset B$ & $B \subset A$

(2) Every element of a set A belong to A, $\Rightarrow$ Every set is a subset of itself.

(3) Empty set is also a subset of every set.

EXAMPLES:

1. $A = \{-1, 2, 5\}, B = \{3, -1, 2, 7, 5\} \rightarrow$ Clearly, $A \subset B$

2. Set of all even natural numbers is a subset of the set of natural numbers.

Ans. Set of all even natural numbers (A) = {2, 4, 6, ...}, Set of natural numbers (B) = {1, 2, 3, 4, 5, ...}

Clearly, $A \subset B$.

3. $N \subset W \subset Z \subset Q \subset R$, $T \subset R$ (T is a set of irrational numbers).

4. $A = \{-3, 0, 5\}$, $B = \{-3, -1, 0, 2, 5, 7\}$.

Ans. A is proper subset of B and it is written as $A \subset B$.

4. **POWER SET:** The set formed by all the subset, of a given set A is called the power set of A, it is usually denoted by P(A).

EXAMPLES:

1. Let $A = \{0\}$, then $P(A) = \{\phi, \{0\}\} \rightarrow n(P(A)) = 2 = 2^1$
2. $A = \{1, 2\}$, then $P(A) = \{\{1\}, \{2\}, \{1, 2\}, \phi\} \rightarrow n(P(A) = 4 = 2^2$

Note: (1) no. of elements in power set = $2^{(\text{numbers of element in set A})}$ i.e. $n(P(A)) = 2^{n(A)}$

(2) Every element of power set is a set.

3. Find $P(P(P(\phi)))$, where P is powerset?

Ans. As ϕ is a set {}, having no elements,

$n(P(\phi)) = 2^0 = 1$

$\therefore$ $P(\phi) = \{\phi\}$

$n[P(P(\phi))] = 2^1 = 2$

$\therefore$ $P(P(\phi))] = \{\phi, \{\phi\}\}$

$n[P(P(P(\phi)))] = 2^2 = 4$

$\therefore$ $P(P(P(\phi))) = \{\{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}, \phi\}$

5. **SET OF SETS:** A set in which all elements are sets, is called set of sets.

6. **COMPARABLE SETS:** Two sets A & B are said to be comparable iff either $A \subset B$ or $B \subset A$

EXAMPLES:

1. $A = \{\{2\}, \{3, 5\}, \{7\}\ \phi\}$, Determine whether Set A is the set of sets or not.

Ans. Every element of set A is a set, hence it is set of sets.

2. Are the following sets comparable sets or not -

a. $A = \{1, 2\}$, $B = \{1, 2, 3\} \rightarrow$ are comparable as $A \subset B$.

b. $A = \{0, 1, 3\}$, $B = \{0, 1\} \rightarrow$ are comparable as $B \subset A$.

c. $A = \{0, 1\}$, $B = \{0, 6\} \rightarrow$ are not comparable as neither $A \subset B$ nor $B \subset A$.

d. $A = \{-1, 1\}$, $B = \{x : x^2 = 1\} \rightarrow$ are comparable as $A \subset B$ and also $B \subset A$.

REMARK: Equal sets are always comparable. However, comparable sets may not be equal.

7. **UNIVERSAL SET:** In particular problem, all sets under investigation are regarded as subsets of a fixed set. We call this set, the universal set, it is denoted by U.

Ex. For the set of all integers, the universal set can be the set of rational numbers or the set R of real numbers can be taken as universal set.

8. **INTERVAL AS SUBSETS OF R:**

a. **OPEN INTERVAL:** If a & b are two fixed real numbers such that a < b then all values of x lying between a and b constitutes open interval. In open interval a, b themselves do not belong to interval.

It is denoted by] a, b [or (a, b).

On the real number line it is represented as

$(a, b) = \{x : a < x < b \ \& \ x \in R\}$

b. **CLOSED INTERVAL:** If a & b are two fixed real numbers such that a < b then all values of x lying between a and b and including a, b both constitutes closed interval.

It is denoted by [a, b]

On the real number line it is represented as

$[a, b] = \{x : a \le x \le b \ \& \ x \in R\}$

c. SEMI OPEN/SEMI CLOSED INTERVAL

(i) If a & b are two fixed real numbers such that $a < b$ then the value of all x lying between a & b, including a but excluding b constitutes interval closed from left & open from right.

It is denoted by [a, b).

On the real number line it is represented as

$[a, b) = \{x : a \le x < b \ \& \ x \in R\}$

(ii) If a & b are two fixed real numbers such that $a < b$ then the value of all x lying between a & b, excluding a but including b constitutes interval open from left & closed from right.

It is denoted by (a, b].

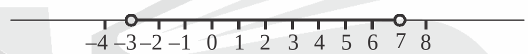
On the real number line it is represented as

$(a, b] = \{x : a < x \le b \ \& \ x \in R\}$

EXAMPLES:

1. $A = \{x : x$ is a real no & $-3 < x < 7\}$

Ans. $A = (-3, 7)$

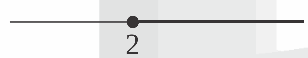


2. $A = \{x : x$ is a real no. less than $6\}$

Ans. $A = (-\infty, 6)$

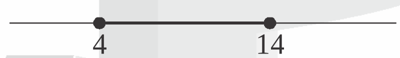
3. $A = \{x : x$ is a real no. greater than $2\}$

Ans. $A = [2, \infty)$



4. $A = \{x : x$ is real no. more than equal to 4 & less than equal to $14\}$

Ans. $A = [4, 14]$



PRACTICE WORKSHEET 4

1. Write down the number of sub sets, proper subsets, hence list them. Also write their power sets.

a. $\{a\}$ **b.** $\{0, 1\}$ **c.** $\{a, b, c\}$ **d.** $\{1, 2, 3, 4\}$ **e.** $\{\phi\}$ **f.** $\{1, \{1\}\}$ **g.** ϕ

2. Let $A = \{1, 2, \{3, 4\}, 5, \{6\}, \{7\}\}$. Which of the following statements are correct and incorrect :

a. $1 \in A$ **b.** $1 \subset A$ **c.** $6 \in A$ **d.** $\{6\} \subset A$ **e.** $\{6\} \in A$ **f.** $\{3, 4\} \in A$ **g.** $\{3, 4\} \subset A$

h. $\{\{3, 4\}\} \subset A$ **i.** $\{1, 2, 5\} \subset A$ **j.** $\{1, 2\ 5\} \in A$ **k.** $\{1, 2, 3\} \subset A$ **l.** $\phi \in A$

m. $\phi \subset A$ **n.** $\{\phi\} \subset A$ **o.** $\{\{3, 4\}, \{6\}, \{7\}\} \in A$ **p.** $\{\{3, 4\}, \{6\}, \{7\}\} \subset A$

3. If $A = \{\phi, \{\phi\}, 1, \{1, \phi\}, 7\}$, then which of the following are true?

a. $7 \subset A$ **b.** $\{\{\phi\}\} \subset A$ **c.** $\{\{\{7\}\}, \{1\}\} \not\subset A$ **d.** $\{\phi, \{\phi\}, \{1, \phi\}\} \subset A$ **e.** $\{1\} \in A$.

4. Which of the following statements are correct :

a. $\{\phi\} = \{0\}$ **b.** $\{a, b, c\} = \{b, a, c\}$ **c.** $\{1, 2, \{3\}\} = \{\{1\}, 2, 3$ **d.** $\{\{1\}, 2, \{3\}\} = \{\{1\}, \{3\}, 2\}$

e. $\{\phi\} \in \{\{\phi\}\}$ **f.** $\{\{1, 2\}, \{2\}, \{2, 3\}\} = \{\{1\}, \{1, 2\}, \{3\}\}$ **g.** $\{3\} \in \{2, 3, 4\}$

h. $\phi \subset \{a, \{a\}\}$ **i.** $a \in \{\{a\}, \{\{a\}\}\}$ **j.** $\{a\} \in \{a, \{a\}\}$.

5. If $A = \{x : x$ is a letter in the word 'able'$\}$; $B = \{x : x$ is a letter in the word 'lable'$\}$;

$C = \{x : x$ is a letter in the word 'lab'$\}$ which of the following are true?

a. $A = B$ **b.** $A \subset B$ **c.** $C \subset B$ **d.** $A = C$

6. Let A = {a, b, c, d}, B = {a, b, c} and C = {b, d}. Find all sets X such that :
a. $X \subset B$ and $X \subset C$ **b.** $X \subset A$ and $X \not\subset B$

7. Let A = {1, 2, 3, 4}, B = {1, 2, 3} and C = {2, 4}. Find all sets X satisfying each pair of conditions :
a. $X \subset B$ and $X \not\subset C$ **b.** $X \subset B$, $X \neq B$ and $X \not\subset C$ **c.** $X \subset A$, $X \subset B$ and $X \subset C$

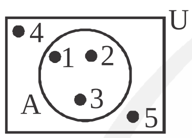
8. Let A = {1, 2, 3, 4}, B = {1, 2, 3} C = {2, 4} Find all sets X satisfying each pair of condition:
a. $X \subseteq A$, $X \subseteq B$ & $X \subseteq C$ **b.** $X \subseteq A$ & $X \not\subseteq C$.

9. **Write the following intervals in the set-builder form :**
a. (–7, 0) **b.** [6, 12] **c.** (6, 12] **d.** [–20, 3)

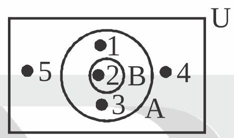
VENN DIAGRAMS

In venn Diagram the universal set 'U' is represented by points within a rectangle & its subset, are represent by points in closed curve (usualy circles) within the rectangle.

Ex. U = {1, 2, 3, 4, 5}
A = {1, 2, 3}

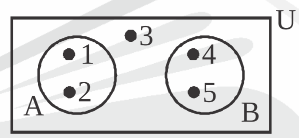


Ex. U = {1, 2, 3, 4, 5}
A = {1, 2, 3}, B = {2}



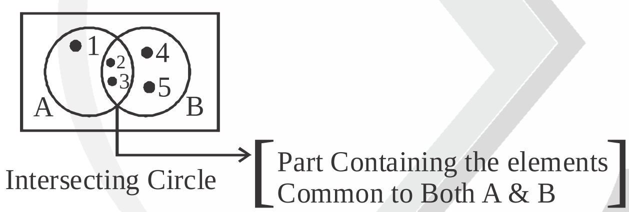
B⊂A

Ex. U = {1, 2, 3, 4, 5}
A = {1, 2}, B = {4, 5}



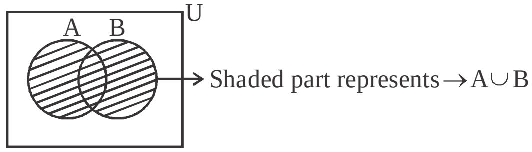
Nonintersecting circle
As A & B have nothing in comman

Ex. U = {1, 2, 3, 4, 5}
A = {1, 2, 3), B = {2, 3, 4, 5}



OPERATIONS ON SETS

1. **UNION OF TWO SETS ($A \cup B$):** Let A, B be any two sets, then the set containing the elements which belongs to A or to B or to both is called union of A & B.
It is written as $A \cup B$ (read as A union B)
Symbolically, $A \cup B = \{x;\ x \in A \text{ or } x \in B\}$
Clearly, if $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ and, $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$



REMARKS: (1) $A \subseteq A \cup B$, (2) $B \subseteq A \cup B$
(3) If $A \subseteq B$ then $A \cup B = B$ (4) If $B \subseteq A$ then $A \cup B = A$

Note: If $A_1, A_2, A_3, \ldots, A_n$ are sets then their union is denoted by $\bigcup_{i=1}^{n} A_i$ or $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$

EXAMPLES:

1. $A = \{1, 2, 3\}$ and $B = \{1, 3, 5, 7\}$ then $A \cup B = \{1, 2, 3, 5, 7\}$

2. $A = \{x : x = 2n + 1, n \in Z\}$ & $B = \{x : x = 2n, n \in Z\}$, then find $A \cup B$?

Ans. $A \cup B = \{x : x \text{ is an odd integer}\} \cup \{x : x \text{ is an even integer}\}$

$A \cup B = \{x : x \text{ is an integer}\} = Z$

PROPERTIES OF UNION:

(1) $A \cup B = B \cup A$ (Commutative Law)

(2) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative Law)

(3) $A \cup A = A$ (Idempotent Law)

(4) $A \cup \phi = A$ (Identity Law)

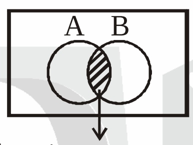
(5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributive Law)

Union is distributive over intersection.

2. INTERSECTION OF SETS: Let A & B be two sets. The intersection of A & B is the set of all those elements that belongs to both A and B.

Symbolically, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Clearly, if $x \in A \cap B \Leftrightarrow x \in A$ & $x \in B$ and $x \notin (A \cap B) \Rightarrow x \notin A$ or $x \notin B$



Shaded region represents $A \cap B$

Note: If $A_1, A_2, A_3, \ldots, A_n$ are sets then their intersection is denoted by $\bigcap_{i=1}^{n} A_i$ or $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$

<u>REMARKS:</u> (1) $A \cap B \subseteq A$ (2) $A \cap B \subseteq B$

(3) If $A \subseteq B$ then $A \cap B = A$ (4) If $B \subseteq A$ then $A \cap B = B$

EXAMPLES:

1. $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 9\}$, $C = \{1\}$, $(A \cap B) \cap C = ?$

Ans. $A \cap B \Rightarrow \{1, 3\}$

$(A \cap B) \cap C = \{1\}$

2. $A = \{x : x = 3n, n \in Z\}$, $B = \{x : x = 4n, n \in Z\}$ then find $A \cap B$?

Ans. $X \in A \cap B \Leftrightarrow x = 3n, n \in Z$ & $x = 4n, n \in Z$

$\Leftrightarrow$ x is multiple of 3 & 4 both

$\Leftrightarrow$ x is multiple of 12 $\Leftrightarrow x = 12n, n \in Z$

$A \cap B = \{x : x = 12n, n \in Z\}$

PROPERTIES OF INTERSECTION:

(1) $A \cap B = B \cap A$ (Commutative Law)

(2) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative Law)

(3) $A \cap A = A$ (Idempotent Law)

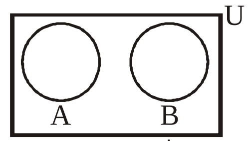
(4) $A \cap U = A$ (Identity Law)

(5) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Intersection is distributive over union)

IMPORTANT POINTS **a.** $x \in (A \cup B) \Leftrightarrow x \in A$ or $x \in B$ **b.** $x \in (A \cap B) \Leftrightarrow x \in A$ and $x \in B$

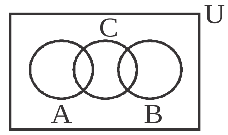
c. $x \notin (A \cup B) \Leftrightarrow x \notin A$ and $x \notin B$ **d.** $x \notin (A \cap B) \Leftrightarrow x \notin A$ or $x \notin B$

3. DISJOINT SETS: Two set A & B are said to be disjoint, if $A \cap B = \phi$ i.e. if they have no element in Common.



$A \cap B = \phi$

Ex. A = {1, 2, 3, 4, 5, 6}, B = {7, 8, 9}, C = {2, 4, 8, 9}



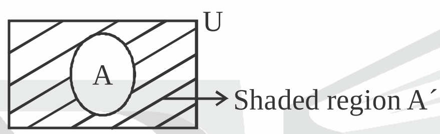
Ans. A & B are disjoint $\Rightarrow A \cap B = \phi$

(A & C) and (B & C) are intersecting circles.

4. COMPLEMENT OF A SET: Let U be the universal set and A be a set such that $A \subset U$, then the complement of A with respect to U is denoted by A' or A^c or U – A and is defined as the set of all elements of U which are not in A.

i.e. $A' = \{x;\ x \in U \text{ and } x \notin A\}$

Obviously $A' = U - A$



Ex. U = {1, 2, 3, 4, 5, 6}, A = {1, 3, 4}, find A´

Ans. $A' = \{2, 5, 6\}$

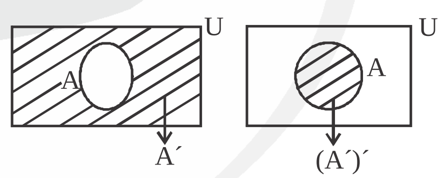
PROPERTIES:

(1) $U' = \{x;\ x \in U \text{ and } x \notin U\} = \phi$

(2) $\phi' = \{x;\ x \in U \ \&\ x \notin \phi'\} = U$

(3) Involution Law

$(A')' = \{x;\ x \in U \ \&\ x \notin A'\} = A$



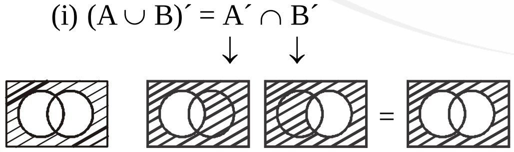
(4) Complement Law

(i) $A \cup A' = \{x;\ x \in U \ \&\ x \in A\} \cup \{x;\ x \in U \ \&\ x \notin A\} = U$

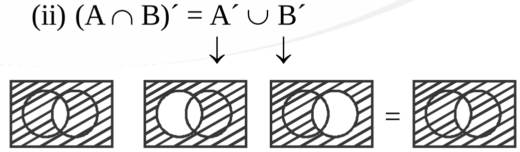
(ii) $A \cap A' = \{x;\ x \in U \ \&\ x \in A\} \cap \{x;\ x \in U,\ x \notin A\} = \phi$

(5) Demorgan's Law

(i) $(A \cup B)' = A' \cap B'$



(ii) $(A \cap B)' = A' \cup B'$

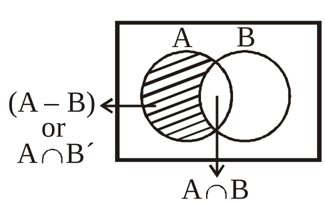


5. DIFFERENCE OF SETS : Let A & B be two sets. The difference of A & B written as A–B, is the set of all those element which are in A but not in B.

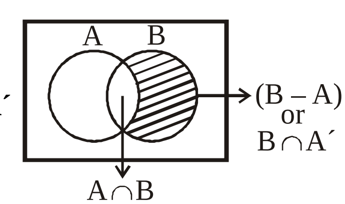
i.e. $A - B = \{x : x \in A \ \&\ x \notin B\}$ and $B - A = \{x : x \in B \ \&\ x \notin A\}$

Also, $A - (A \cap B) \Rightarrow A \cap B'$ Also, $B - (A \cap B) \Rightarrow B \cap A'$

Hence, $\mathbf{A - B = A \cap B'}$



Hence, $\mathbf{B - A = B \cap A'}$



EXAMPLES

1. $A = \{a, b, c\}, B = \{b, c\} \rightarrow A - B = \{a\}$

2. $A = \{2, 3, 4, 5, 6, 7\}, B = \{3, 5, 7, 9, 11, 13\}$

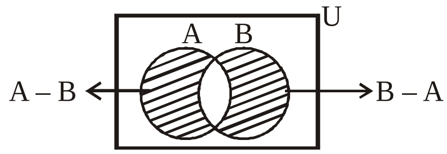
then $A - B = \{2, 4, 6\}$

and $B - A = \{9, 11, 13\}$

Note: $(A - B) \cap (B - A) = \phi$

6. SYMMETRIC DIFFERENCE OF TWO SETS: Let A, B are two sets. Then symmetric difference of set A & B is the set $(A - B) \cup (B - A)$ and it is denoted by $A \Delta B$

i.e. $A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$



$(A - B) \cup (B - A) = A \Delta B$

EXAMPLES

1. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 6, 7, 9\}$, $A \Delta B = ?$

Ans. $A - B = \{2, 4, 8\}$ $B - A = \{9\}$ $A \Delta B = \{2, 4, 8, 9\}$

2. If $A = \{x \in R : 0 < x < 3\}$ and $B = \{x \in R : 1 \le x \le 5\}$, find $A \Delta B = ?$

Ans. $A - B = \{x \in R : 0 < x < 1\}$ $B - A = \{x \in R : 3 < x \le 5\}$

$A \Delta B = \{x \in R : 0\ x < 1 \text{ or } 3 < x \le 5\}$

ILLUSTRATIONS

1. Let $x = \{1, 2, 3, ... 10\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 4, 8, 10\}$ then verify the following-

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (ii) $A - B = A \cap B'$ (iii) $(A \cup B)' = A' \cap B'$

Ans. (i) $(A \cap B) = \{1, 3, 5\}$, $(A \cap C) = \{2, 4\}$, $(B \cup C) = \{1, 2, 3, 4, 5, 7, 8, 9, 10\}$

LHS:– $A \cap (B \cup C) = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5, 7, 8, 9, 10\} = \{1, 2, 3, 4, 5\}$

RHS:– $(A \cap B) \cup (A \cap C) = \{1, 3, 5\} \cup \{2, 4\} = \{1, 2, 3, 4, 5\}$

(ii) $A - B = A \cap B'$

$B' = \{2, 4, 6, 8, 10\}$

LHS: $A - B = \{2, 4\}$

RHS: $\{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} = \{2, 4\}$

(iii) $(A \cup B)' = A' \cap B'$

LHS: $(A \cup B) = \{1, 2, 3, 4, 5, 7, 9\}$, $(A \cup B)' = \{6, 8, 10\}$

RHS: $A' = \{6, 7, 8, 9, 10\}$, $B' = \{2, 4, 6, 8, 10\}$

$A' \cap B' = \{6, 8, 10\}$

2. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$; find

(i) A', (ii) B', (iii) C', (iv) $(A - B)'$, (v) $(A')'$,

(vi) $(C - A)'$, (vii) $(B \cup C)'$, (viii) $(C \cap A)'$, (ix) $A \cup B \cup C$, (x) $(A \cap B \cap C)'$.

Ans. Here, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$.

(i) $A' = \{x : x \in U \text{ and } x \notin A\} = \{4, 6, 7, 8, 9, 10\}$.

(ii) $B' = \{x : x \in U \text{ and } x \notin B\} = \{1, 3, 5, 8, 9, 10\}$.

(iii) $C' = \{x : x \in U \text{ and } x \notin C\} = \{1, 5, 6, 7, 9, 10\}$.

(iv) $(A - B)' = \{1, 3, 5\}' = \{2, 4, 6, 7, 8, 9, 10\}$.

(v) $(A')' = \{4, 6, 7, 8, 9, 10\}' = \{1, 2, 3, 5\} = A$.

(vi) $(C - A)' = \{4, 8\}' = \{1, 2, 3, 5, 6, 7, 9, 10\}$.

(vii) $(B \cup C)' = \{2, 3, 4, 6, 7, 8\}' = \{1, 5, 9, 10\}$.

(viii) $(C \cap A)' = \{2, 3\}' = \{1, 4, 5, 6, 7, 8, 9, 10\}$.

(ix) $A \cup B \cup C = (A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7\} \cup \{2, 3, 4, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

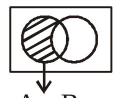
(x) $(A \cap B \cap C)' = \{(A \cap B) \cap C\}' = (\{2\} \cap \{2, 3, 4, 8\})' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$.

3. Let A & B be two sets, using properties of sets prove that:

(i) $(A - B) \cup B = A \cup B$

Ans.
$\Rightarrow (A \cap B') \cup B$
$\Rightarrow (A \cup B) \cap (B' \cup B)$
$\Rightarrow (A \cup B) \cap U$
$\Rightarrow A \cup B$

(ii) $(A - B) \cup A = A$



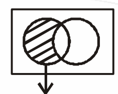
A – B

$(A - B) \cup A = A$

(iii) $(A - B) \cap B = \phi$

$(A - B) \cap B = (A \cap B') \cap B$
$\Rightarrow A \cap (B' \cap B)$
$\Rightarrow A \cap \phi$
$\Rightarrow \phi$

(iv) $(A - B) \cap A = A - B$



A – B

$(A - B) \cap A = A - B$

4. For any two sets A & B, Prove that $A \cup B = A \cap B \Leftrightarrow A = B$

Ans. Let $A = B$

$A \cup B = A$ and $A \cap B = A$

$\Rightarrow A \cup B = A \cap B$

Thus, $A = B \Rightarrow (A \cup B) = (A \cap B)$ ...(1)

Conversely,

Let $A \cup B = A \cap B$

For any $x \in A \Rightarrow x \in A \cup B \Rightarrow x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in B$

$\therefore A \subset B$ ...(2)

Now, for any $y \in B \Rightarrow y \in A \cup B$
$\Rightarrow y \in A \cap B$
$\Rightarrow y \in A$ and $y \in B$
$\Rightarrow y \in A$

$\therefore B \subset A$ ...(3)

from (2), (3) We get $A = B$, Thus, $A \cup B = A \cap B \Rightarrow A = B$ ...(4)

from (1) & (4) $A \cup B = A \cap B \Leftrightarrow A = B$

5. $(A - B) \cup B = A$ if and only if $B \subset A$

Ans. Let $(A - B) \cup B = A$

Then, we have to Prove that $B \subset A$

$\Rightarrow (A - B) \cup B = A$

$\Rightarrow (A \cap B') \cup B = A \{A - B = A \cap B'\}$

$\Rightarrow (A \cup B) \cap (B' \cup B) = A \quad \Rightarrow (A \cup B) \cap U = A \quad \Rightarrow A \cup B = A \quad \Rightarrow B \subset A$

Conversely, Let $B \subset A$, Then we have to prove that $(A - B) \cup B = A$

$(A - B) \cup B = (A \cap B') \cup B$
$= (A \cup B) \cap (B' \cup B) \quad = (A \cup B) \cap U \quad = A \cup B = A$

PRACTICE WORKSHEET 5

1. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8}, C = {3, 4, 5, 6}, then prove that :

a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **b.** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2. If U = {1, 2, 3, 4, 5, 6, 7}, A = {1, 2, 3, 6} and B = {2, 4, 6}, then show that : –

a. $(A \cup B)' = A' \cap B'$ **b.** $(A \cap B)' = A' \cup B'$

3. If U = {a, e, i, o, u}, A = {a, e, i}, B = {e, o, u} and C = {a, i, u}then verify that: -

a. $A \cap (B - C) = (A \cap B) - (A \cap C)$ **b.** $A \cup (B - C) = (A \cup B) - (A \cup C)$

c. $A - (B \cup C) = (A - B) \cap (A - C)$ **d.** $A - (B \cap C) = (A - B) \cup (A - C)$

4. Draw Venn diagrams for the following sets

a. U **b.** $A \cup B$ **c.** $A \cap B$ **d.** $A - B$ **e.** $B - A$

f. Symmetric difference of A & B ($A \Delta B$) **g.** A' **h.** $(A \cup B)'$

i. $(A \cap B)'$ **j.** $(A - B)'$ **k.** $(B - A)'$ **l.** $A \cup B \cup C$

m. $A \cap B \cap C$ **n.** $A \cap B \cap C'$ **o.** $B \cap C \cap A'$ **p.** $A \cap C \cap B'$

q. $(A \cap B \cap C)'$ **r.** $A \cap B' \cap C'$ **s.** $B \cap A' \cap C'$ **t.** $C \cap A' \cap B'$

5. If A and B are any two sets, then prove that

(i) $A - B = A \cap B'$ (ii) $(A - B) \cap B = \phi$ (iii) $(A \cup B) - B = A - B$.

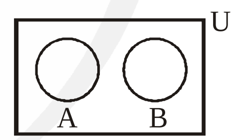
6. Using laws of set algebra, show that

(i) $(A \cup B) \cap (A \cup B') = A$ (ii) $(A - B) \cup (A \cap B) = A$

(iii) If $A \cap B' = \phi$, show that $A \subset B$ (iv) If $A' \cup B = U$, show that $A \subset B$.

APPLICATION OF SETS

1. (i) if A & B are finite sets & $A \cap B = \phi$ then $\mathbf{n(A \cup B) = n(A) + n(B)}$

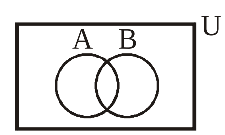


Clearly, $n(A \cup A') = n(A) + n(A')$ {As A & A′ are disjoint sets}

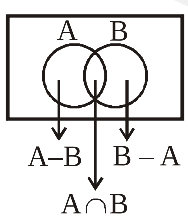
$\Rightarrow$ $n(U) = n(A) + n(A')$

$\mathbf{n(A') = n(U) - n(A)}$

(ii) If A & B are finite sets such that $A \cap B \neq \phi$ then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Proof:



$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$ (as they are disjoint sets)

$+ n(A \cap B) - n(A \cap B)$ (add and subtract $n(A \cap B)$)

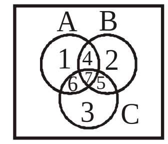
$\mathbf{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

also, $(A - B)$ & $(A \cap B)$ are disjoint sets & $(B - A)$ & $(A \cap B)$ are disjoint sets

$n(A) = n(A - B) + n(A \cap B)$ $n(B) = n(B - A) + n(A \cap B)$

$\mathbf{n(A - B) = n(A) - n(A \cap B)}$ $\mathbf{n(B - A) = n(B) - n(A \cap B)}$

2. If A, B & C are finite sets, then



$n(A \cup B \cup C) = n((A \cup B) \cup C)$

$= n(A \cup B) + n(C) - n((A \cup B) \cap C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cup B) \cap C)$ ...(1)

Now, $n((A \cup B) \cap C) = n[A \cap C) \cup (B \cap C)]$

$\Rightarrow n(A \cap C) + n(B \cap C) - n((A \cap C)(B \cap C)]$

$\Rightarrow n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)$ ....(2)

From (1) & (2)

$\mathbf{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}$

Clearly, $n(A \cup B \cup C) = n(A \cap B' \cap C') + n(A' \cap B \cap C') + (A' \cap B' \cap C) + n(A \cap B \cap C') + n(B \cap C \cap A') + n(C \cap A \cap B') + n(A \cap B \cap C)$

ILLUSTRATIONS

1. (i) A & B are two sets such that $n(A) = 38$, $n(B) = 42$ (ii) $n(A \cup B) = 60$, find $n(A \cap B) = ?$

Ans. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $60 = 38 + 42 - n(A \cap B) \Rightarrow n(A \cap B) = 80 - 60 = 20$

(ii) X & Y are two sets such that $X \cup Y$ has 50 elements, X has 28 element, Y has 32 elements, How many elements $X \cap Y$ have?

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$50 = 28 + 32 - x \Rightarrow x = 10$

2. In a group of 800 people, 550 can speak Hindi and 450 can speak English. How many can speak both Hindi and English?

Ans. Let H denoe the set of people speaking Hindi and E denote the set of people speaking English. We are given that

$n(H) = 550$, $n(E) = 450$ and $n(H \cup E) = 800$.

We have to find $n(H \cap E)$.

We know that

$n(H \cup E) = n(H) + n(E) - (H \cap E)$

$\Rightarrow n(H \cap E) = n(H) + n(E) - (H \cup E)$

$\Rightarrow n(H \cap E) = 550 + 450 - 800 = 200$

Hence, 200 persons can speak both Hindi and English.

3. In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all the people speak at least one of the two languages. How many people speak only English and not Hindi? How many people speak English?

Ans. Let H denote the set of people speaking Hindi and E the set of people speaking English. Then, it is given that

$n(H \cup E) = 50$, $n(H) = 35$, $n(H \cap E) = 25$.

Now, $n(E - H) = n(H \cup E) - n(H) = 50 - 35 = 15$

Thus, the number of people speaking English but not Hindi is 15.

We have, $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\Rightarrow 50 = 35 + n(E) - 25 \Rightarrow n(E) = 40$.

Hence, the number of people who speak English is 40.

4. There are 200 individuals with a skin disorder, 120 has been exposed to chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to (i) chemical C_1 or chemical C_2 (ii) chemical C_1 but not chemical C_2 (iii) chemical C_2 but not chemical C_1.

Ans. Let U denote the universal set consisting of individuals suffering from the skin disorder, A denote the set of individuals exposed to chemical C_1 and B denote the set of individuals exposed to chemical C_2.

We have, $n(U) = 200$, $n(A) = 120$, $n(B) = 50$ and $n(A \cap B) = 30$.

(i) Number of individuals exposed to chemical C_1 or chemical C_2

$= n(A \cup B)$

$= n(A) + n(B) - n(A \cap B) = 120 + 50 - 30 = 140$

(ii) Number of individuals exposed to chemical C_1 but not chemical C_2

$= n(A \cap B')$

$= n(A) - (A \cap B)$

$= 120 - 30 = 90.$

(iii) Number of individuals exposed to chemical C_2 but not chemical C_1

$= n(A' \cap B)$

$= n(B) - n(A \cap B)$

$= 50 - 30 = 20.$

5. Out of 500 car owners investigated, 400 owned Maruti car and 200 owned Hyundai car; 50 owned both cars. Is this data correct?

Ans. Let U be the set of all car owners investigated, M be the set of persons who owned Maruti cars and H be the set of persons who owned Hyundai cars.

It is given that

$n(U) = 500$, $n(M) = 400$, $n(H) = 200$ and $n(M \cap H) = 50$.

$\therefore \quad n(M \cup H) = n(M) + n(H) - n(M \cap H)$

$= 400 + 200 - 50 = 550$

But, $M \cup H \subseteq U$. Therefore,

$n(M \cup H) \le n(U)$

$\Rightarrow \quad n(M \cup H) \le 500$

This is a contradication. So, the given data is incorrect.

6. If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the minimum number of element in $A \cup B$.

Ans. We have, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

CASE I When $n(A \cap B)$ is minimum, i.e. $n(A \cap B) = 0$.

This is possible only when $A \cap B = \phi$. In this case,

$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9.$

So, maximum number of elements in $A \cup B$ is 9.

CASE II When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$. In this case, $n(A \cap B) = 3$.

$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B) = (3 + 6 - 3) = 6$

So, minimum number of elements in $A \cup B$ is 6.

7. In a survey of 700 students in a college, 180 were listed as drinking Limca, 275 as drinking Miranda and 95 were listed as both drinking Limca as well as Miranda. Find how many students were drinking neither Limca nor Miranda.

Ans. Let U be the set of all surveyed students, A denote the set of students drinking Limca and B be the set of students drinking Miranda.

In is given that

$n(U) = 700$, $n(A) = 180$, $n(B) = 275$ and $n(A \cap B) = 95$

We have to find,

$n(A' \cap B')$

Now,

$$n(A' \cap B') = n(A \cup B)'$$
$$= n(U) - n(A \cup B)$$
$$= n(U) - \{n(A) + n(B) - n(A \cup B)\}$$
$$= 700 - (180 + 275 - 95)$$
$$= 700 - 360$$
$$= 340$$

8. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three news papers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.

Ans. Let P, Q and R be the set of families buying newspaper A, B and C respectively. Let U be the universal set. Then,

$n(P) = 40\%$ of $10{,}000 = 4000$, $n(Q) = 20\%$ of $10{,}000 = 2000$, $n(R) = 10\%$ of $10{,}000 = 1000$,

$n(P \cap Q) = 5\%$ of $10{,}000 = 500$, $n(Q \cap R) = 3\%$ of $10{,}000 = 300$, $n(R \cap P) = 4\%$ of $10{,}000 = 400$

$n(P \cap Q \cap R)$ 2% of $10{,}000 = 200$, $n(U) = 10{,}000$.

(i) Required number $= n(P \cap Q' \cap R') = n(P \cap (Q \cup R)')$

$= n(P) - n[P \cap (Q \cup R)]$ $[\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(P) - n[(P \cap Q) \cup (P \cap R)]$

$= n(P) - [n(P \cap Q) + n(P \cap R) - n\{(P \cap Q) \cap (P \cap R)\}]$

$= n(P) - [n(P \cap Q) + n(P \cap R) - n(P \cap Q \cap R)]$

$= 4000 - [500 + 400 - 200] = 3300$

(ii) Required number $= n(P' \cap Q \cap R') = n(Q \cap P' \cap R')$

$= n(Q \cap (P \cup R)')$

$= n(Q) - n(Q \cap (P \cup R))$ $[\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(Q) - n[(Q \cap P) \cup (Q \cap R)]$

$= n(Q) - [n(Q \cap P) + n(Q \cap R) - n\{(Q \cap P) \cap (Q \cap R)\}]$

$= n(Q) - [n(P \cap Q) + n(Q \cap R) - n(P \cap Q \cap R)]$

$= 2000 - [500 + 300 - 200] = 1400$

(iii) Required number $= n(P' \cap Q' \cap R') = n[(P \cup Q \cup R)']$

$= n(U) - n(P \cup Q \cup R)$

$= n(U) - [n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)]$

$= 10000 - [4000 + 2000 + 1000 - 500 - 300 - 400 + 200] = 4000$

PRACTICE WORKSHEET 6

1. **a.** Find $n(X \cup Y)$, if $n(X) = 15$, $n(Y) = 20$, $n(X \cap Y) = 7$.

b. Find $n(X \cap Y)$, if $n(X) = 10$, $n(Y) = 8$, $n(X \cup Y) = 15$.

c. Find $n(X \cap Y')$ and $n(Y \cap X')$, if $n(X) = 35$, $n(Y) = 25$, $n(X \cap Y) = 10$.

2. In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach Physics, Only Physics, Only Mathematics?

3. In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games.

a. How many students like to play both cricket and football? **b.** How many students like to play only cricket?

c. How many students like to play only football?

4. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French.

a. How many people speak at least one of these two languages?

b. How many people speak only French? **c.** How many people speak only Spanish?

d. Also draw the venn diagram?

5. There are 40 students in a chemistry class and 60 students in a physics class. Find the number of students, which are either in physics class or in chemistry class in the following cases:

a. The two classes meet at the same hour.

b. The two classes meet at different hours and 20 students are enrolled in both the subjects.

6. In a town with a population of 5000, 3200 people are egg–eaters, 2500 are meat eaters and 1500 eat both egg & meat.

a. How many are pure vegetarians? **b.** Also draw the Venn diagram?

7. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice.

a. How many are taking only orange juice. **b.** How many are taking only apple juice.

c. How many are taking neither orange nor apple juice.

8. From amongst the 3000 literate individuals of a city 55% read newspaper A, 60% newspaper B and 25% neither A nor B. How many individuals read both the newspaper A as well as B.

9. In a mathematics class, 20 children had forgotten their rulers and 17 had forgotten their pencils. "Go and borrow them from some one at once", said the teacher. 24 children left the room. Draw a Venn diagram and find how many children had forgotten both?

10. In a survey of 25 students, it was found that 15 had taken Maths, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Maths and Chemistry, 9 had taken Maths and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students who have taken :

a. Only Chemistry. **b.** Only Maths. **c.** Only Physics.

11. In a survey of 60 people, it was found that 25 people read newspaper 'H', 26 read newspaper 'T', 26 read newspaper 'I', 9 read both 'H' and 'I', 11 read both 'H' and 'T', 8 read both 'T' and 'I', 3 read all three newspaper. Find the people who read:

a. 'H' and 'T' but not 'I'. **b.** 'T' and 'I' but not 'H'. **c.** 'I' and 'H' but not 'T'. **d.** None of given newspapers.

12. In a survey of 45, athletic teams in a certain school, 21 are in Basketball team, 26 in Hockey team and 29 in the Football team. 14 play Hockey and Basketball, 15 play Hockey and Football, 12 play Football and Basketball and 8 play all the three games. How many members are:

a. In at least one the team. **b.** In exactly one team **c.** in exactly in two teams

d. In at least two teams **e.** In at most two teams

13. A survey of 500 television viewers produced the following information's; 285 watch Football, 195 watch Hokey, 115 watch Basketball, 45 watch Football and Basketball, 70 watch Football and Hokey, 50 watch Hokey and Basketball, 50 do not watch any of the three games. How many watch:

a. At least one of the game. **b.** Only Football.

c. Only Hockey. **d.** Only Basketball.

e. Football and Hockey but not Basketball. **f.** Hokey and Basketball but not Football.

g. Basketball and Football but not Hockey. **h.** Exactly one game. **i.** Exactly two games.

14. A TV survey gives the following data for TV viewing :

60% see programme A; 50% programme B; 50% programme C; 30% programmes A and B; 20% programmes B and C; 30% programmes A and C; 10% do not view any programmes.

Draw a venn-diagram and find the following :

a. What percent view only A, B and C ? **b.** What percent view exactly 2 programmes?

15. In a survey of 100 students, the number of students studying the various languages were found to be: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find :

a. How many students were studying Hindi? **b.** How many students were studying English and Hindi?

16. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students that had

(i) only chemistry. (ii) only mathematics. (iii) only physics.

(iv) physics and chemistry but not mathematics. (v) mathematics and physics but not chemistry.

(vi) only one of the subjects. (vii) at least one of the three subjects.

(viii) none of the subjects.

MISCELLANEOUS WORKSHEET

1. In each of the following, determine whether the statement is true or false. If it is true or false, prove it.

a. If $x \in A$ and $A \in B$, then $x \in B$. **b.** If $A \subset B$ and $B \in C$, then $A \in C$.

c. If $A \subset B$ and $B \subset C$, then $A \subset C$. **d.** If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$.

e. If $x \in A$ and $A \not\subset B$, then $x \in B$. **f.** If $A \subset B$ and $x \notin B$, then $x \notin A$.

2. Prove the following properties of sets:

a. Idempotent Law **b.** Identity Law **c.** Associative Law **d.** Distributive Law **e.** De – Morgan's Law

3. Show that the following four are equivalent set:

a. $A \subset B$ **b.** $A - B = \{\}$ **c.** $A \cup B = B$ **d.** $A \cap B = A$

4. Show that if $A \subset B$, then $C - B \subset C - A$

5. Show that $A \cap B = A \cap C$ need not imply $B = C$.

6. Using properties of sets, show that : **a.** $A \cup (A \cap B) = A$ **b.** $A \cap (A \cup B) = A$.

7. Show that for any sets A and B, $A \cup (B - A) = (A \cup B)$ and $A = (A \cap B) \cup (A - B)$

8. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

9. Let A, B, and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

10. Assume that $P(A) = P(B)$. Show that $A = B$.

11. For any sets A and B, show that : **a.** $P(A \cap B) = P(A) \cap P(B)$ **b.** $P(A \cup B) \neq P(A) \cup P(B)$

12. Let B be a subset of A and let $P(A : B) = \{X \in P = (A) : X \supset B\}$, then prove that: $P(A : \phi) = P(A)$.

13. For any two sets A and B prove by using properties of sets that :

(i) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ (ii) $(A \cap B) \cup (A - B) = A$ (iii) $(A \cup B) - A = B - A$

14. For sets A, B and C using properties of sets, prove that :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$ (ii) $A - (B \cap C) = (A - B) \cup (A - C)$ (iii) $(A \cup B) - C = (A - C) \cup (B - C)$

15. For sets A, B and C using properties of sets, prove that :

(i) $A - (B - C) = (A - B) \cup (A \cap C)$ (ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

16. Two finite sets A and B contain 'm' and 'n' elements. If P(A) contains 192 elements more then P(B). Then find 'm' and 'n'.

NCERT EXEMPLAR QUESTIONS

A. OBJECTIVE TYPE QUESTIONS

1. Suppose, $A_1, A_2, ..., A_{30}$ are thirty sets each having 5 elements and B_1, B_2, B_n are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ & each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's. Then, n is equal to

a. 15 b. 3 c. 45 d. 35

2. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively

a. 4, 7 b. 7, 4 c. 4, 4 d. 7, 7

3. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then, F_1 may be equal to

a. $F_2 \cap F_3$ b. $F_3 \cap F_4$ c. $F_2 \cup F_5$ d. $F_2 \cup F_3 \cup F_4 \cup F_1$

4. Let S = set of points inside the square, T = set of points inside the triangle and C = set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then,

a. $S \cap T \cap C = \phi$ b. $S \cup T \cup C = C$ c. $S \cup T \cup C = S$ d. $S \cup T = S \cap C$

5. If $X = \{8^n - 7n - 1 \mid n \in N\}$ and $y = \{49n - 49 \mid n \in N\}$. Then,

a. $X \subset Y$ b. $Y \subset X$ c. $X = Y$ d. $X \cap Y = \phi$

6. A survey shows that 63% of the people watch a news channel whereas 76% watch another channel. If x% of the people watch both channel, then

a. x = 35 b. x = 63 c. $39 \le x \le 63$ d. x = 39

7. If A = {1, 3, 5, 7, 9, 11, 13, 15, 17}, B = {2, 4, ..., 18} and N the set of natural numbers is the universal set, then $(A' \cup (A \cup B) \cap B')$ is

a. ϕ b. N c. A d. B

8. If X and Y are two sets and X´ denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to

a. X b. Y c. ϕ d. $X \cap Y$

B. SHORT ANSWER TYPE QUESTIONS

1. Write the following sets in the roster form.

(i) $A = \{x : x \in R, 2x + 11 = 15\}$ (ii) $B = \{x \mid x^2 = x, x \in R\}$

(iii) C = {x | x is a positive factor of a prime number p}

2. Write the following sets in the roster form.

(i) $D = \{t \mid t^3 = t, t \in R\}$ (ii) $E = \{W \mid \frac{w-2}{w+3} = 3, w \in R\}$

(iii) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$

3. If Y = {x | x is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number}. Write Y in the roster form.

4. State which of the following statements are true and which are false. Justify your answer.

(i) 35 ∈ {x | x has exactly four positive factors}. (ii) 128 ∈ {y | the sum of all the positive factors of y is 2y}.

(iii) $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$. (iv) 496 ∉ {y | the sum of all the positive factors of y is 2y}.

5. If A and B are subsets of the universal set U, then show that

(i) $A \subset A \cup B$ (ii) $A \subset B \Leftrightarrow A \cup B = B$ (iii) $(A \cap B) \subset A$

6. If Y = {1, 2, 3, ..., 10} and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions.

(i) $a \in Y$ but $a^2 \notin Y$ (ii) $a + 1 = 6$, $a \in Y$ (iii) a is less than 6 and $a \in Y$

7. Let U be the set of all boys and girls in a school, G be the set of all girls in the school, B be the set of all boys in the school and S be the set of all students in the school who take swimming. Some but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationship among sets U, G, B and S.

8. For all sets A, B and C, show that $(A - B) \cap (A - C) = A - (B \cup C)$.

9. For all sets A and B, $(A - B) \cup (A \cap B) = A$

10. For all sets A, B and C, $A - (B - C) = (A - B) - C$.

11. For all sets A, B and C, if $A \subset B$, then $A \cap C \subset B \cap C$.

12. For all sets A, B and C, if $A \subset B$, then $A \cup C \subset B \cup C$.

13. For all sets A, B and C, if $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.

14. For all sets A and B, $A \cup (B - A) = A \cup B$.

15. For all sets A and B, $A - (A - B) = A \cap B$.

16. For all sets A and B, $A - (A \cap B) = A - B$.

17. For all sets A and B, $(A \cup B) - B = A - B$.

C. LONG ANSWER TYPE QUESTIONS

1. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed

(i) in English and Mathematics but not in Science. (ii) in Mathematics and Science but not in English.

(iii) in Mathematics only. (iv) in more than one subject only.

2. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

3. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows French = 17, English = 13, Sanskrit = 15 French and English = 09, English and Sanskrit = 4, French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study

(i) only French. (ii) only English

(iii) only Sanskrit. (iv) English and Sanskrit but not French.

(v) French and Sanskrit but not English. (vi) French and English but not Sanskrit.

(vii) atleast one of the three languages. (viii) none of the three languages.

ANSWERS

PRACTICE WORKSHEET 1

1. a. Y **b.** Y **c.** N **d.** Y **e.** Y **f.** N **g.** N **h.** Y **i.** Y

PRACTICE WORKSHEET 2

1. a. Y **b.** Y **c.** N **d.** Y **e.** Y **f.** N **g.** N **h.** Y **i.** Y

2. a. A = {x : x = 3 n, n $\in$ N and n $\leq$ 4}

b. A = {x : x = 2^n, n $\in$ N, n $\leq$ 5}

c. A = {x : x = 5^n, n $\in$ N, n $\leq$ 4}

d. A = {x : x = 2n, where n $\in$ N}

e. A = {x : x = n^2, n $\in$ N and n $\leq$ 10}

f. A = {x : x are the divisor's of 6}

g. A = {x : x is the prime factor's of 6}

h. A = {x : x = $\frac{1}{2^n}$, n $\in$ N }

i. A = {x : x is a letter of the word MISSISSIPPI}

j. A = {x : x = $\frac{n}{n+1}$, n $\in$ N and n $\leq$ 6}

k. A= {x : x= n^3 +3, n $\in$ N and n $\leq$ 5}

l. A ={x: x=7n+1, n $\in$ N & n $\leq$ 7 }}

PRACTICE WORKSHEET 3

1. a. Empty **b.** Empty **c.** singleton **d.** singleton **e.** singleton **f.** Empty **g.** Empty

2. a. Infinite **b.** Infinite **c.** finite **d.** Infinite **e.** finite **f.** Infinite **g.** finite **h.** Infinite **i.** finite **j.** finite

3. A, C, and D

4. a. Equal **b.** Equal **c.** Equal **d.** Not Equal **e.** Not Equal

PRACTICE WORKSHEET 4

1. a. 2 ; 1; {f, {a}} **b.** 4; 3; {f, {0}, {1}, {0, 1}}

c. 8; 7; {f,{a},{b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

d. 16;15; {f{1},{2},{3},{4},{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4},{2,3,4},{1,3,4},{1,2,3, 4}}

e. 2; 1; {ϕ, {ϕ}} **f.** 4; 3; { ϕ,{1},{{1}}, {1,{1}} **g.** 1; 0; {ϕ}

2. a. True **b.** false **c.** false **d.** false **e.** True **f.** True **g.** false **h.** True **i.** false **j.** True **k.** false **l.** false **m.** True **n.** false **o.** false **p.** True

3. a. false **b.** true **c.** true **d.** true

4. a. false **b.** true **c.** false **d.** true **e.** false **f.** false **g.** false **h.** true **i.** false **j.** true

5. a. true **b.** true **c.** true **d.** false

6. a. ϕ , {b} **b.** {d}, {a,d}, {b,d}, {c,d}, {a,b,d}, {b,c, d}, {a, c, d}, {a, b, c, d}

7. a. {1}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}

b. {1}, {3}, {1, 2}, {1, 3}, {2, 3} **c.** ϕ , {2}

8. a. X = ϕ or {2}

b. {1}, {3},{1,2}, {1,4}, {3,2}, {3,4}, {1,3}, {1,2, 4}, {2, 3, 4}, {1, 3, 4}, {1, 2, 3}, {1, 3, 2, 4}

9. a. {x: x $\in$ R, $-7 < x < 0$} **b.** {x: x $\in$ R, $6 \leq x \leq 12$}

c. {x: x $\in$ R, $6 < x \leq 12$} **d.** {x: x $\in$ R, $-20 \leq x < 3$}

PRACTICE WORKSHEET 6

1. a. 28 **b.** 3 **c.** 25, 15

2. a. 12 **b.** 8 **c.** 8

3. a. 5 **b.** 19 **c.** 11

4. a. 60 **b.** 40 **c.** 10

5. a. 100 **b.** 80 **6. a.** 800

7. a. 75 **b.** 25 **c.** 225

8. 39.17% **or** 39% to 40% **9.** 13

10. a. 5 **b.** 4 **c.** 2

11. **a.** 8 **b.** 5 **c.** 6 **d.** 8

12. **a.** 43 **b.** 18 **c.** 17 **d.** 25 & 37

13. **a.** 450 **b.** 190 **c.** 95 **d.** 40

e. 50 **f.** 30 **g.** 25 **h.** 325 **i.** 105

14. **a.** 30% **b.** 50%

15. **a.** 18 **b.** 3

16. (i) 5 (ii) 4 (iii) 2 (iv) 1 (v) 6

(vi) 11 (vii) 23 (viii) 2

MISCELLANEOUS WORKSHEET

2. **a.** F, **b.** F, **c.** T, **d.** F, **e.** F, **f.** T

14. m = 8, n = 6

NCERT EXEMPLAR QUESTIONS

A. OBJECTIVE TYPE QUESTIONS

1. c **2.** b **3.** d **4.** c **5.** a

6. c **7.** b **8.** c

B. SHORT ANSWER TYPE QUESTIONS

1. (i) A = {2} (ii) B = {0, 1} (iii) C = {1, P}

2. (i) D = {–1, 0, 1} (ii) E = $\left\{\frac{-11}{2}\right\}$

(iii) F = $\left\{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\right\}$

3. Y = $\{1, 2, 2^2, 2^3, \ldots, 2^{P-1}, 2^P-1\}$

4. (i) True (ii) False (iii) True (iv) False

6. (i) {4, 5, 6, 7, 8, 9, 10}

(ii) {5} (iii) {1, 2, 3,4, 5}

9. True **10.** False

11. True **12.** True

13. True **14.** True

15. True **16.** True **17.** True

C. LONG ANSWER TYPE QUESTIONS

1. (i) 2 (ii) 3 (iii) 3 (iv) 9

2. 20

3. (i) 6 (ii) 3 (iii) 9 (iv) 1 (v) 2

(vi) 6 (vii) 30 (viii) 20