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CLASS XI

MATHS

ASSIGNMENT

RELATIONS AND FUNCTIONS

HEAD OFFICE : B-1/30, MALVIYA NAGAR

PH. 26675331, 26675333, 26675334, 8527363750

CENTRES AT: H-36 B, KALKAJI PH. : 26228900, 40601840

E-555, 1ST FLOOR RAMPHAL CHOWK, SEC-7 DWARKA PH. 9560088728-29

CHAPTER 2

RELATIONS AND FUNCTIONS

ORDERED PAIR

Two elements a & b listed in a specific order form an ordered pair denoted by (a, b).

In (a, b): a is first element and b is second element.

NOTE:

a. An ordered pair is not a set consisting of two elements.

b. The order of two elements in an ordered pair is important and the two elements need not to be distinct. In general $(a, b) \neq (b, a)$

eg: The Position of a point in a two dimensional plane in cartesian system is represented by an ordered pair i.e., (1, 2), (3, 2), (3, 4) etc.

EQUALITY OF ORDERED PAIRS: Two ordered pairs (a_1, b_1), (a_2, b_2) are equal if and only if there corresponding elements are equal. i.e. $a_1 = a_2$ and $b_1 = b_2$.

eg: Find the values of a and b, if $(3a - 2, b + 3) = (2a - 1, 3)$.

Ans. $3a - 2 = 2a - 1$ and $b + 3 = 3 \Rightarrow a = 1$ & $b = 0$

CARTESIAN PRODUCT OF TWO SETS

If A & B are two non empty sets, then the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of A and B, and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$. If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$

EXAMPLE

1. If A = {2, 4, 6} and B = {1, 2}, then find $A \times B$ and $B \times A$

Ans. $A \times B = \{(2, 1), (2, 2), (4, 1), (4, 2), (6, 1), (6, 2)\}$; $B \times A = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6)\}$

2. A = {a, b}, B = {1, 2, 3} find $A \times B$, $B \times A$, $A \times A$, $B \times B$, $(A \times B \cap B \times A)$.

Ans. $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$; $B \times A = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$

$A \times A = \{(a, a), (a, b), (b, a), (b. b)\}$

$B \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

$(A \times B) \cap (B \times A) = \phi$

NUMBER OF ELEMENTS IN THE CARTESIAN PRODUCT OF TWO SETS

If there are p elements in set A and q elements in set B, then there will be pq elements in $A \times B$.

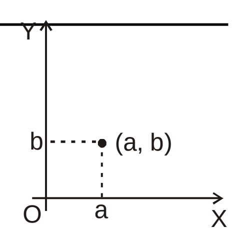
i.e. if $n(A) = p$ and $n(B) = q$. Then $n(A \times B) = pq$.

NOTE: If A and B are non empty sets and either A or B is an infinite set, then $A \times B$ is also infinite set.

GRAPHICAL REPRESENTATION OF CARTESIAN PRODUCT OF TWO SETS

a. Let A and B be two sets and (a, b) be an ordered pair, where $a \in A$ and $b \in B$. To represent it graphically, we draw two mutually perpendicular lines OX and OY intersecting each other at O and represent set A on horizontal line OX and set B on vertical line OY.

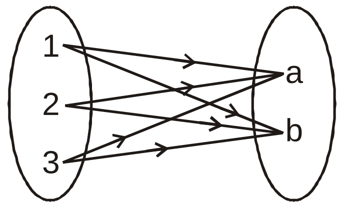
b. The point of intersection of a vertical line through a and horizontal line through b represent the ordered pair (a, b).



DIAGRAMATIC REPRESENTATION OF CARTESIAN PRODUCT OF TWO SETS

In order to represent A × B by an arrow diagram, we first draw venn diagrams representing sets A and B one opposite to the other as shown in figure and write the elements of sets.

Now, we draw line segments starting from each elements of set A & terminating to each element of set B.



ORDERED TRIPLET

If A, B, C are three sets, then (a, b, c); where $a \in A$, $b \in B$ and $c \in C$, is called an ordered triplet. Ordered triplet is also called 3-tuple.

CARTESIAN PRODUCT OF THREE SETS

If A, B, C are three sets. Then A × B × C is the set of all ordered triplets having first element from A, second element from B & third element from C. Thus, $A \times B \times C = \{(a, b, c): a \in A, b \in B \text{ and } c \in C\}$

In general, if $A_1, A_2, A_3, ..., A_n$ are n sets, then $(a_1, a_2, a_3, ..., a_n)$ is called a n-tuple, where $a_i \in A_i$, i = 1, 2, 3, ..., n and the set of all such n-tuples is called the cartesian product of $A_1, A_2, A_3, ..., A_n$ and is denoted by $A_1 \times A_2 \times A_3 \times ... \times A_n$.

In symbolic form $A_1 \times A_2 \times A_3 \times ... \times A_n = \{(a_1, a_2, a_3, ..., a_n): a_i \in A_i, 1 \le i \le n\}$

EXAMPLE

1. If A = {1, 2}, B = {3, 4}, C = {4, 5, 6}. Then find: **a.** A × B × C **b.** A × A × A

Ans. a. A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

A × B × C = {(1, 3, 4), (1, 4, 4), (2, 3, 4), (2, 4, 4), (1, 3, 5), (1, 4, 5), (2, 3, 5), (2, 4, 5), (1, 3, 6), (1, 4, 6), (2, 3, 6), (2, 4, 6)}

b. A × A = {(1, 1), (1, 2), (2, 1), (2, 2)}, A = {1, 2}

A × A × A = {(1, 1, 1), (1, 2, 1), (2, 1, 1), (2, 2, 1), (1, 1, 2), (1, 2, 2), (2, 1, 2), (2, 2, 2)}

SOME THEOREMS ON CARTESIAN PRODUCT OF SETS

In this section, we intend to study some results on cartesian product of sets which are given as theorems.

1. For any three sets A, B and C, we have

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

2. For any three sets A, B and C, we have $A \times (B - C) = (A \times B) - (A \times C)$

3. If A and B are any two non-empty sets, then $A \times B = B \times A \Leftrightarrow A = B$

4. If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

5. If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

6. If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

7. For any sets A, B, C and D, we have $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Corollary: For any sets A and B, $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

8. For any three sets A, B and C, we have

(i) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$ (ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

9. Let A and B be two non-empty sets having n elements in common, then A × B and B × A have n^2 elements in common.

10. Let A be a non-empty set such that A × B = A × C. Then B = C.

ILLUSTRATIONS

1. Find x and y, if $(x + 3, 5) = (6, 2x + y)$

Sol. $(x + 3, 5) = (6, 2x + y) \Rightarrow x + 3 = 6$ and $5 = 2x + y \Rightarrow x = 3$ and $5 = 6 + y \Rightarrow y = -1$

2. $P = \{7, 8\}$, $Q = \{5, 4, 2\}$, find $P \times Q$, $Q \times P$

Sol. $P \times Q = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$; $Q \times P = \{(5, 7), (4, 7), (2, 7), (5, 8), (4, 8), (2, 8)\}$

3. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B?

Sol. $A = \{a, b\}$, $B = \{(x, y)\}$

4. $A = \{-1, 1\}$, find $A \times A \times A$

Sol. $A \times A = \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

5. $A = \{1, 2)\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$, $D = \{5, 6, 7, 8\}$. Verify that:

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (ii) $A \times C$ is a subset of $B \times D$.

Sol. (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \phi$

$A \times \phi = \phi$ ..... (1)

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$(A \times B) \cap (A \times C) = \phi$ ..... (2)

Clearly, from equation (1) & (2), $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times C$ is a subset of $B \times D$.

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

Clearly, $A \times C \subset B \times D$

6. $A = \{1, 2\}$, $B = \{a, e, i, o, u\}$, number of subsets of $A \times B$?

Sol. $n(A \times B) = n(A)\, n(B) = 2 \times 5 = 10$

If 'n' number of elements in a set, then total number of subset are $= 2^n = 2^{10} = 1024$

7. Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ & $(0, 1)$ find set A & remaining elements of $A \times A$.

Sol. $n(A \times A) = n(A).\, n(A) = n(A)^2 = 9 \Rightarrow n(A) = 3$; $A \times A$ has elements $= (-1, 0), (0, 1)$

$\therefore\ -1, 0, 1 \in A$; $A = \{-1, 0, 1\}$

Other elements of $A \times A \Rightarrow (-1, -1), (-1, 1), (0, 1), (0, 0), (1, -1), (1, 0), (1, 1)$

8. If $A = \{1, 4\}$, $B = \{2, 3, 6\}$ and $C = \{2, 3, 7\}$, then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol. We have, $A = \{1, 4\}$, $B = \{2, 3, 6\}$ and $C = \{2, 3, 7\}$

(i) To find $A \times (B \cup C)$

$B \cup C = \{2, 3, 6\} \cup \{2, 3, 7\} \Rightarrow B \cup C = \{2, 3, 6, 7\}$

$\therefore A \times (B \cup C) = \{1, 4\} \times \{2, 3, 6, 7\} = \{(1, 2), (1, 3), (1, 6), (1, 7), (4, 2), (4, 3), (4, 6), (4, 7)\}$ .....(i)

To find $(A \times B) \cup (A \times C)$

$A \times B = \{(1, 2), (1, 3), (1, 6), (4, 2), (4, 3), (4, 6)\}$ and $A \times C = \{(1, 2), (1, 3), (1, 7), (4, 2), (4, 3), (4, 7)\}$

$\therefore$ (A × B) $\cup$ (A × C) = {(1, 2), (1, 3), (1, 6), (1, 7), (4, 2), (4, 3), (4, 6), (4, 7)} .....(ii)

From Eqs. (i) and (ii), we get, A × (B $\cup$ C) = (A × B) $\cup$ (A × C)

(ii) To find A × (B $\cap$ C)

B $\cap$ C = {2, 3}

$\therefore$ A × (B $\cap$ C) = {1, 4} × {2, 3} = {(1, 2), (1, 3), (4, 2), (4, 3)} ....(iii)

To find (A × B) $\cap$ (A × C)

A × B = {(1, 2), (1, 3), (1, 6), (4, 2), (4, 3), (4, 6)}

A × C = {(1, 2), (1, 3), (1, 7), (4, 2), (4, 3), (4, 7)}

$\therefore$ (A × B) $\cap$ (A × C) = {(1, 2), (1, 3), (4, 2), (4, 3)} ....(iv)

From Eqs. (iii) and (iv), we get, A × (B $\cap$ C) = (A × B) $\cap$ (A × C) Hence Verified.

9. If R is the set of all real numbers, what do the cartesian products R × R and R × R × R represent?

Sol. The cartesian product of the set R of all real numbers with itself i.e., R × R is the set of all ordered pairs (x, y) where x, y $\in$ R.

In other words, R × R = {(x, y) : x, y $\in$ R}

Clearly, R × R is the set of all points in XY-plane. The set R × R is also denoted by R^2.

Similarly, we have R × R × R = {(x, y, z) : x, y, z $\in$ R}

Clearly, it represents the set of all points in space.

The set R × R × R is also denoted by R^3.

10. Let A = {–1, 3, 4} and B = {2, 3}. Represent the following products graphically i.e., by lattices:

(i) A × B (ii) A × A

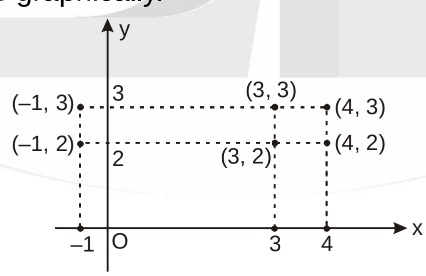
Sol. (i) We have, A × B = {(–1, 2), (–1, 3), (3, 2), (3, 3), (4, 2), (4, 3)}

In order to represent A × B graphically, we follow the following steps:

(a) Draw two mutually perpendicular lines one horizontal and other vertical.

(b) On the horizontal line represent the elements of set A and on the verticle line represent the elements of set B.

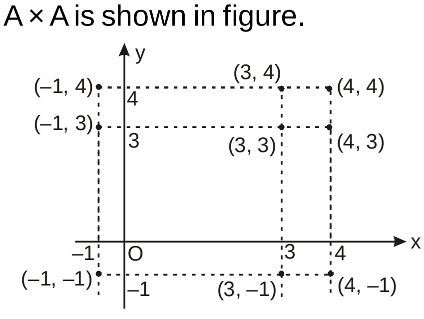
(c) Draw vertical dotted lines through points representing elements of A on horizontal line and horizontal lines through points representing elements of B on the vertical line. Points of intersection of these lines will represent A × B graphically.



(ii) We have, A × A = {–1, 3, 4} × {–1, 3, 4}

= {(–1, –1), (–1, 3), (–1, 4), (3, –1), (3, 3), (3, 4), (4, –1), (4, 3), (4, 4)}

Graphical representation of A × A is shown in figure.



PRACTICE WORKSHEET 1

1. Find x and y, if:

a. $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$ **b.** (2(x +1), y) = (2x + 2, 3) **c.** (2(x +3), y) = (2x + 2, 5)

2. **a.** If A = {– 1, 1}, B = {1, 2}, find A × B **b.** If A = {– 4, 3}, find A × A. **c.** If A = {– 1, 1}, find A × A × A.

3. Let A = {1, 2, 3} and B = {x : x $\in$ N, x is prime less than 5}. Find : **a.** A × B **b.** B × A.

4. Express A = {(a, b) : 2a + b = 5, a, b $\in$ W} as the set of ordered pairs.

5. If A = {1, 2, 3}, B = {3, 4}, C = {1, 3, 5} Find : **a.** A × (B $\cup$ C) **b.** (A × B) $\cap$ (A × C)

6. If A = {1, 2, 3}, B = {4}, C = {5}, verify that

a. $A \times (B \cup C) = (A \times B) \cup (A \times C)$ **b.** $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

c. $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **d.** $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

e. $A \times (B - C) = (A \times B) - (A \times C)$

7. Find sets A and B, If A×B= {(a,1), (b,1), (b,3), (a,3), (a,2), (b,2)}

8. Let A and B two sets such that n (A) = 3 and n (B) = 2. If A × B = {(x, 1), (y, 2), (z, 1)}. If x, y, z are distinct, then find the sets A & B.

9. Let A and B be two sets such that A × B consists of 6 elements. If three elements of A × B are: (1, 4), (2, 6), (3, 6). Find A × B and B × A.

10. If A = {1, 2} and B = {3, 4}. Find A × B. List how many subsets will A × B have?

RELATIONS

A relation R from a non empty set A to a non empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the orderd pairs in A × B i.e., R $\subseteq$ A × B.

NOTE:

a. If (a, b) $\in$ R, then we write it as a R b and we say that a is related to b by the relation R.

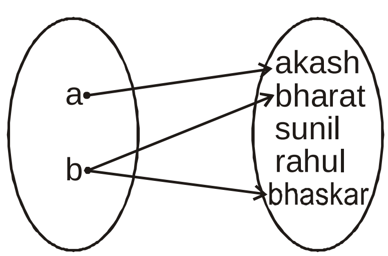
b. If (a, b) $\notin$ R, then we write it as a $\not R$ b and we say that a is not related to b by the relation R.

eg: Consider the two sets A = {a, b} and B = {akash, bharat, sunil, rahul, bhaskar}

The cartesian product of A and B has 10 ordered pairs i.e., A × B = {(a, akash), (a, bharat), (a, sunil), (a, rahul), (b, akash), (b, bharat), (b, sunil), (b, rahul), (a, bhaskar), (b, bhaskar)}

Now, we can obtain a subset of A × B by introducing a relation R between the first element a and the second element b of each ordered pair as

R = {(a, b) : a is the first letter of name b, $\forall$ a $\in$ A, b $\in$ B}



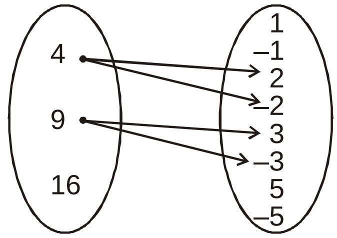
Then R = {(a, akash), (b, bharat), (b, bhaskar)}

EXAMPLE

1. $A = \{4, 9, 16\}$, $B = \{1, -1, 2, -2, 3, -3, 5, -5\}$

$R = \{(a, b) : a = b^2, a \in A, b \in B\}$

Sol. $R = \{(4, 2), (4, -2), (9, 3), (9, -3)\}$



2. If $A = \{a, b, c, d\}$, $B = \{p, q, r, s\}$, then which of the following are relations from A to B.

a. $R_1 = \{(a, p), (b, r), (c, s)\}$ **b.** $R_2 = \{(q, b), (c, s), (d, r)\}$

c. $R_3 = \{(a, p), (a, q), (d, p), (c, r), (b, r)\}$ **d.** $R_4 = \{(a, p), (q, a), (b, s), (s, b)\}$

Sol. **a.** $R_1 \subset A \times B$. Hence R_1 is a relation from A to B.

b. $(q, b) \notin A \times B$. Hence R_2 is not a relation from A to B.

c. $R_3 \subset A \times B$. Hence R_3 is a relation from A to B.

d. (q, a) & $(s, b) \notin A \times B$. Hence R_4 is not a relation from A to B.

REPRESENTATION OF A RELATION

A relation can be represented algebraically by roster form or by set-builder form and visually it can be represented by an arrow diagram which are given below

(I) **ROSTER FORM:** In this form, we represent the relation by the set of all ordered pairs belongs to R.

e.g., Let R is a relation from set $A = \{-3, -2, -1, 1, 2, 3\}$ to set $B = \{1, 4, 9, 10\}$ by the rule

$$aRb \Leftrightarrow a^2 = b$$

Then, in roster form, R can be written as $R = \{(-1, 1)\ (-2, 4)\ (1, 1)\ (2, 4)\ (-3, 9)\ (3, 9)\}$

(II) **SET-BUILDER FORM:** In this form, we represent the relation R from set A to set B as

$R = \{(a, b) : a \in A, b \in B$ and the rule which relates A and B$\}$

e.g., Let R is a relation from set $A = \{1, 2, 4, 5\}$, to set $B = \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{5}\right\}$

such that $R = \left\{(1,1), \left(2, \frac{1}{2}\right)\left(4, \frac{1}{4}\right)\left(5, \frac{1}{5}\right)\right\}$

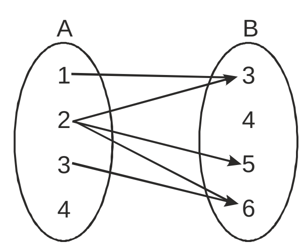
Then, in set-builder form, R can be written as $R = \left\{(a, b) : a \in A, b \in B \text{ and } b = \frac{1}{a}\right\}$

Note: We cannot write every relation from set A to set B in set-builder form.

(III) **ARROW DIAGRAM:** To represent a relation by an arrow diagram, we draw arrows from first element to second element of all ordered pairs belonging to relation R.

e.g.,Let $R=\{(1, 3), (2, 5), (3, 6), (2, 6), (2, 3)\}$ be a relation from set $A = \{1, 2, 3, 4\}$ to set $B = \{3, 4, 5, 6\}$.

Then by an arrow diagram, it can be represented as



Note:

(i) If $R = \phi$, then R is called an empty relation. (ii) If $R = A \times B$, then R is called the universal relation.

(iii) If R_1 and R_2 are two relations from A to B, then $R_1 \cup R_2$, $R_1 \cap R_2$ and $R_1 - R_2$ are also relations from A to B.

TOTAL NUMBER OF RELATIONS

Let A, B are two non-empty finite sets containing m and n elements, respectively then number of ordered pairs in (A × B) is mn.

$\therefore$ Total Number of subset of A × B is 2^{mn}

Hence total number of relation from set A to set B is 2^{mn}, as each relation from A to B is a subset of A × B.

Among these 2^{mn} relation the empty relation ϕ and universal relation A × B are trivial relation from A to B.

e.g. If A = {a, b} and B = {2, 3}, then find the number of relation from A to B.

Sol. Number of subsets of A × B = $2^{n(A\times B)} = 2^4$

So, the number of relations from A to B is 16.

DOMAIN, CO-DOMAIN, RANGE OF RELATION

Let us consider the relation from A to B, such that R = {(a, b): a $\in$ A and b $\in$ B}

DOMAIN

The set of all first elements of the ordered pair in a relation R from a set A to set B is called the domain of the relation R. i.e. Domain (R) = {a : (a, b) $\in$ R}

RANGE

The set of all the second elements of the ordered pair in a relation R from set A to set B is called the range of the relation, i.e. Range (R) = {b : (a, b) $\in$ R}

CO-DOMAIN

The set B is known as the co-domain of Relation R from set A to B.

Note: **Range $\subseteq$ Co-domain**

ILLUSTRATIONS

1. If A = {1, 2, 3}, B = {4, 5, 6}, which of the following are relations from A to B? Give reasons.

a. R_1 = {(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)} **b.** R_2 = {(4, 2), (2, 6), (5, 1), (2, 4)}

Sol. a. Clearly $R_1 \subseteq$ A × B.

So, it is a relation from A to B.

b. Since (4, 2) $\in R_2$ but (4, 2) $\notin$ A × B.

$\therefore$ R_2 is not a relation from A to B.

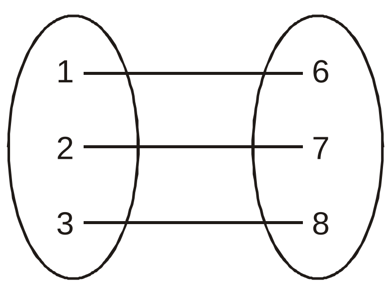
2. Define A relation R on the set N of natural number by R = {(x, y): y = x + 5, x < 4, x, y $\in$ N}.

Depict this relationship using: **a.** roster form **b.** arrow diagram **c.** Write domain and range

Sol. R = {(x, y): y = x + 5, x is a natural number less than 4, x, y $\in$ N}

a. In roster form, R = {(1, 6), (2, 7), (3, 8)}

b. arrow diagram



c. domain = {1, 2, 3}, range = {6, 7, 8}

3. A = {1, 2, 3, 4, 5, 6}. Let R be the relation on A defined by {(a, b) : a, b $\in$ A, a divides b}

a. Write R in roster form b. find the domain of R c. find the range of R.

Sol. A = {1, 2, 3, 4, 6} and R is a relation on A defined by {(a, b) : a $\in$ A, b $\in$ A, a divides b}

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

Domain (R) = {1, 2, 3, 4, 6}, Range (R) = {1, 2, 3, 4, 6}

4. If R = {(x, x + 5); x $\in$ (0, 1, 2, 3, 4, 5)}, find the domain and range of the given relation?

Sol. R = {(x, x + 5); x $\in$ {0, 1, 2, 3, 4, 5}}

Domain (R) = {0, 1, 2, 3, 4, 5}, Range (R) = {5, 6, 7, 8, 9, 10}

5. If A = {x, y, z}, B = {1, 2} are two sets then find the number of relations from A to B?

Sol. A × B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}

Total number of relations = 2^6 = 64

6. Let R be the relation on Z defined by R = {(a, b): a $\in$ Z, b $\in$ Z, a – b is an even integer}. Find the domain and range of R.

Sol. R = {(a, b): a $\in$ Z, b $\in$ Z, a – b is an even integer}

(a – b) is an even integer, provided a, b are both odd or both even

$\therefore$ R = {(a, b) : a and b are even integers} $\cup$ {(a, b) : a and b are odd integers}

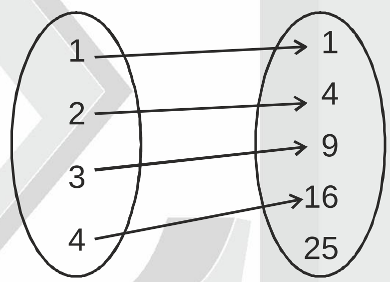
$\therefore$ Domain (R) = Z, Range (R) = Z

7. Let A = {1, 2, 3, 4} & B = {1, 4, 9, 16, 25} & R be a relation defined from A to B as R = {(x, y) : x $\in$ A, y $\in$ B and y = x^2}

a. depict this relation using arrow diagram b. find domain of R

c. find range of R d. write co-domain of R

Sol. R = {(x, y) : x $\in$ A, y $\in$ B and y = x^2}



Domain (R) = {1, 2, 3, 4}, Range (R) = {1, 4, 9, 16}, Co-domain (R) = {1, 4, 9, 16, 25}

8. Let R be a relation on N defined by R = {(a, b) : a, b $\in$ N and a = b^2} are the following true?

a. (a, a) $\in$ R for all a $\in$ N **b.** (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R **c.** (a, b) $\in$ R, (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R

Sol. **a.** It is true for a = 1 only

$\Rightarrow$ as 1 = 1^2 but for (2, 2), (3, 3) etc. It is not true.

$\therefore$ (a, a) $\notin$ R for all a $\in$ N

b. If (a, b) $\in$ R $\Rightarrow$ a = b^2 and if (b, a) $\in$ R

$\Rightarrow$ b = a^2

(2, 4) $\Rightarrow$ 2 $\neq$ 4^2

(4, 2) $\Rightarrow$ 4 = 2^2

$\therefore$ (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R is not ture.

c. If (a, b) $\in$ R $\Rightarrow$ a = b^2, (b, c) $\in$ R

$\Rightarrow$ b = c^2, (a, c) $\in$ R $\Rightarrow$ a = c^2

(16, 4) $\in$ R $\Rightarrow$ 16 = 4^2

(4, 2) $\in$ R $\Rightarrow$ 4 = 2^2 but (16, 2) $\notin$ R

$\therefore$ this statement is not true.

PRACTICE WORKSHEET 2

1. **a.** If A = {1, 2}, B = {3, 4}. Find the possible number of relation from A $\rightarrow$ B, hence list them.

b. If A = {1, 2, 3, 4} B = {2, 3, 5, 7}. Find the possible number of relations from B $\rightarrow$ A, hence list them.

c. If A = {0,1}, list all the relations on A.

2. Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x + 1}

a. Depict this relation using an arrow diagram. **b.** Write down the domain, co-domain, range.

3. Let A = {1, 2, 3, ……., 14}. Define a relation R from A to A by R = {(x, y): 3x – y = 0, where x, y $\in$ A}, find Domain & Range. Also represent the arrow diagram.

4. Let R be the relation on the set N of natural numbers defined by

R = {(a, b) : a + 3b = 12, a $\in$ N, b $\in$ N}. Find R, Domain and Range

5 A relation R is define on the set integers Z and the relation is defined by

R = {(x,y): $x^2 + y^2 = 25$}, find Domain, Range.

6. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by

R = {(x, y): the difference between x and y is odd, x $\in$ A, y $\in$ B}, write roster form and hence find Domain and Range.

7. A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows : (x, y) $\in$ R $\Leftrightarrow$ x is relatively prime to y. Express R as a set of ordered pairs and determine its domain and range.

8. Determine the R, domain and range for the given relation of R in the following cases:

a. R = {(x + 1, x + 5): x $\in$ {0, 1, 2, 3, 4, 5}} **b.** R={(x, x^3): x is a prime number less than 10}

9. Let R be a relation from Q to Q defined by R = {(a, b): a, b $\in$ Q and a – b $\in$ Z}. Show that :

a. (a, a) $\in$ R for all a $\in$ Q **b.** (a, b) $\in$ R implies that (b , a) $\in$ R

c. (a, b) $\in$ R and (b , c) $\in$ R implies that (a, c) $\in$ R.

10. Let a relation R_1 on the set R of all real numbers be defined as (a, b) $\in$ R $\Leftrightarrow$ 1 + ab > 0 for all a, b $\in$ R. Show that: (i) (a, a) $\in$ R_1 for all a $\in$ R. (ii) (a, b) $\in$ R_1 $\Rightarrow$ (b, a) $\in$ R_1 for all a, b $\in$ R

ANSWERS

PRACTICE WORKSHEET 1

1. a. x = 2; y = 1 **b.** x $\in$ R, y = 3 **c.** no value for x and y for which ordered pair equal

2. a. {(–1, 1), (–1, 2), (1, 1), (–1, 2)} **b.** {(– 4, – 4), (– 4, 3), (3, – 4), (3, 3)}

c. {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

3. a. {(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)} **b.** {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

4. {(0, 5), (1, 3), (2, 1)}

5. a. {(1, 1), (1, 3), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), (3, 1), (3, 3), (3, 4), (3, 5)}

b. {(1, 3), (2, 3), (3, 3)}

6. a. yes, **b.** yes, **c.** yes **d.** yes **e.** yes

7. A= {a,b}, B = {1,2,3}

8. A = {x, y, z}, B = {1, 2 }

9. A × B = {(1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6)} and B × A = {(4, 1), (6, 1), (4, 2), (6, 2), (4, 3), (6, 3)}

10. A × B = {(1, 3), (1, 4), (2, 3), (2, 4)} and 16 subsets.

PRACTICE WORKSHEET 2

1. a. 16, **b.** 2^{16}, **c.** 16 and possible relations are ϕ, {(0,0)}, {(0,1)}, {(1,0)}, {(1, 1)}, {(0, 0), (0, 1)}, {(0, 0), (1, 0)}, {(0, 0), (1, 1)}, {(0, 1), (1, 0)}, {(0, 1), (1, 1)}, {(1, 0), (1, 1)}, {(0, 0), (0, 1), (1, 0)}, {(0, 0), (0, 1), (1, 1)}, {(0, 0), (1, 0), (1, 1)}, {(0, 1), (1, 0), (1, 1)},. {(0, 0), (0, 1), (1, 0), (1, 1)}

2. Domain ={1, 2, 3, 4, 5}, Range = {2, 3, 4, 5, 6} & Co – domain=A

3. Domain = {1, 2, 3, 4}, Range = {3, 6, 9, 12}

4. R = {(9, 1), (6, 2), (3, 3)}, Domain ={9, 6, 3} & Range = {1, 2, 3}

5. R = {(0, 5), (0, –5), (3, 4), (–3, 4), (3, – 4), (–3,– 4), (4, 3), (– 4,3), (4, –3), (– 4, –3), (5, 0), (–5, 0) }
Domain = {0, 3, –3, 4, – 4, 5, –5}, Range : {0, 3, –3, 4, – 4, 5, –5}

6. R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}, Domain = {1, 2, 3, 5} and Range = {4, 6, 9}

7. R = {(2, 3), (2, 7), (3,7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 7)}, Domain = {2, 3, 4, 5} and Range = {3, 7, 10}

8. a. R = {(1, 5), (2, 6), (3,7), (4,8), (5,9), (6, 10)}, Domain = {1, 2, 3, 4, 5, 6} and Range = {5, 6, 7, 8, 9, 10}

b. R = {(2, 8), (3, 27), (5, 125), (7, 343)}, Domain = {2, 3, 5, 7} and Range = {8, 27, 125, 343}

FUNCTIONS

FUNCTION AS A RELATION

A relation f from a non empty set A to a non-empty set B is said to be a function, if every element of set A has one and only one image in the set B.

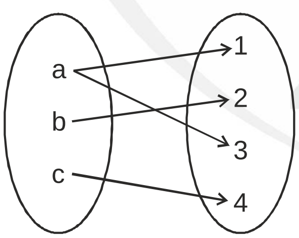
In other words, we can say that a function f is a relation from a non empty set A to a non empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element.

If f is a function from a set A to a set B, then we write

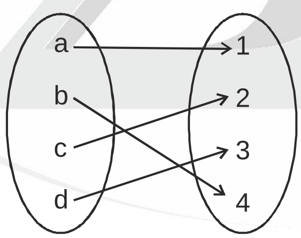
$f : A \to B$ or $A \xrightarrow{f} B$ and it is read as f is a function from A to B.

If $(a, b) \in f \Rightarrow f(a) = b$ here, 'b' is called the image of 'a' under f & 'a' is called the pre-image of 'b' under f.

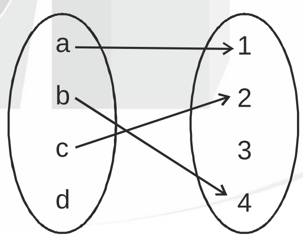
A function is also known as mapping or map.



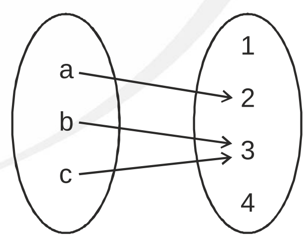
- Not a function
- 'a' element has two image.



- It's function
- Every element of set A has image one and only



- Not a function
- 'd' element of set A has no image



- It's a function
- every element has one and only one image
- b & c have same image.

Characteristics of a function $f : A \to B$

(i) For each elements $x \in A$, there is unique element $y \in B$.

(ii) The element $y \in B$ is called the image of x under the function f. Also, y is called the value of function f at x i.e., $f(x) = y$.

(iii) $f : A \to B$ is not a function if there is an element in A which has more than one image in B. But more than one element of A may be associated to the same element of B.

(iv) $f : A \to B$ is not a function, if an element in A does not have an image in B.

EXAMPLE

1. Let A = {2, 3, 4}, B = {3, 4, 5} and f_1, f_2, f_3 be three subset of A × B defined as

f_1 = {(2, 3), (3, 4), (4, 5)}

f_2 = {(2, 3), (2, 4), (3, 4), (4, 5)}

f_3 = {(2, 4), (3, 5)}

Sol. f_1 is a function from A to B, since every element of A has one and only one image in set B.

f_2 is not a function, since 2 $\in$ A has two image in B

f_3 is not a function, since 4 $\in$ A has no image in B.

2. Is the given relation a function? give reason for your answer.

a. h = {(4, 6), (3, 9), (–11, 6), (3, 11)}

b. f = {(x, x) | x is a real number}

c. g = {(x, 1/x) | x is a positive integer}

d. s = {(x, x^2) | x is a positive integer}

e. t = {(x, 3) | x is a real number}

Sol. a. Since, 3 has two image so it is not a function.

b. Since every element has unique image, hence it is a function.

c. Clearly, it is a function.

d. Clearly, it is a function.

e. It is a function, whether all the first elements of all ordered pairs have the same image but they all have one & only one image.

DOMAIN, CO-DOMAIN AND RANGE OF A FUNCTION

If a function is defined from a set A to set B. i.e., f : A $\to$ B

Then, Set A = Domain of function *f*

Set B = Co-domain of function *f*

The set containing the images of the elements of set A is called the range of *f*

i.e., Range of *f* = {f(x) : x $\in$ A}

Range $\subseteq$ Co-domain

EXAMPLE

1. If x, y $\in$ {1, 2, 3, 4}, then show that f_1, f_2 and f_3 are function or not where.

f_1 = {(x, y) : y = x + 1} and

f_2 = {(x, y : x + y = 5)} and

f_3 = {(x, y) : x + y > 4}

Sol. f_1 = {(1, 2), (2, 3), (3, 4)}

f_2 = {(1, 4), (2, 3), (3, 2), (4, 1)}

f_3 = {(1, 4), (2, 3), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3), (3, 2)}

Clearly, f_1 & f_3 are not function.

and f_2 is a function

Domain (*f*)= {1, 2, 3, 4}

Range (*f*)= {1, 2, 3, 4}

2. X = {1, 2, 3, 4, 5}, Y = {2, 4, 6, 8, 10, 12, 14} *f* : X $\to$ Y defined by *f* (x) = 2x. Find domain & range?

Sol. *f* = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)}

Domain (*f*) = X = {1, 2, 3, 4, 5}

Range (*f*) = {2, 4, 6, 8, 10}

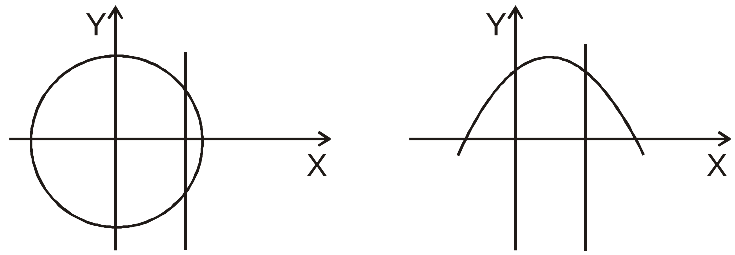
3. Let X = R, Y = R, then the subset $f = \{(x, x^3) : x \in R\}$ of R × R is a function R to R find domain & range?

Sol. Domain of (f) = R

Rnage of (f) = R

IDENTIFICATION OF A FUNCTION WITH ITS GRAPHS

If we draw a line parallel to y-axis. Then if the line intersect the graph of the expression in more than one point, then the expression is a relation else, if it intersect at only one point, then the expression is a function.



Relation Function

EQUAL FUNCTIONS

Two function f and g are said to be equal iff

a. domain of f = domain of g

b. Co-domain of f = co-domain of g

c. $f(x) = g(x)$ for every x belonging to their common domain and then we write $f = g$

EXAMPLE

1. Let A = {1, 2}, B = {3, 6} and $f : A \to B$ is given by $f(x) = x^2 + 2$ and $g : A \to B$ given by $g(x) = 3x$, Then, we observe that f and g have the same domain & codomain

$f(1) = 3 = g(1)$ and $f(2) = 6 = g(2)$. Hence, $f = g$

2. $f(x) = x$, $g(x) = \dfrac{x^2}{x}$ are not equal functions as domain of $f(x)$ = R ; Domain of $g(x) = R - \{0\}$

ILLUSTRATIONS

1. Express the following functions as set of ordered pairs and determine their range.

a. $f : A \to R$, $f(x) = x^2 + 1$, $\forall$ A = {–1, 0, 2, 4} **b.** $g : A \to N$, $g(x) = 2x$, $\forall$ A = $\{x : x \in N, x \le 10\}$

Sol. **a.** $f(-1) = (-1)^2 + 1 = 2$ $f(0) = (0)^2 + 1 = 1$ $f(2) = (2)^2 + 1 = 5$ $f(4) = (4)^2 + 1 = 17$

$f = \{(x, f(x)) : x \in A\} = \{(-1, 2), (0, 1), (2, 5), (4, 17)\}$

Hence, Range of $f(x)$ = {2, 15, 17}

b. g(1) = 2 g(2) = 4 g(3) = 6 g(4) = 8 g(5) = 10 g(6) = 12

g(7) = 14 g(8) = 16 g(9) = 18 g(10) = 20

$g = \{(x, g(x)): x \in A\} = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12), (7, 14), (8, 16), (9, 18), (10, 20)\}$

Range of g(x) = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

2. **a.** Let A = {1, 2}, find all the function from A to A.

b. Let N be the set of natural numbers and the relation R be defined on N such that $R = \{(x, y): y = 2x; x, y \in N\}$. What is the domain, co-domain and range of R. Is this relation a function?

Sol. **a.** If $f : A \to A$ is a function, then f(1) can be chosen in two ways as we can have either f(1)=1 or f(1) = 2

Similarly f(2) = 1 or f(2) = 2. Thus, f can be selected in four ways.

So we have, f = {(1, 1), (2, 2)} or f = {(1, 2), (2, 1)} ; f = {(1, 1), (2, 1)} or f = {(1, 2), (2, 2)}

b. R = {(1, 2), (2, 4), (3, 6), ...}

Since every elements of N is uniquely associated to an element of N under R, therefor R is a function from N to N.

Domain = {1, 2, 3, ...} = N ; Co-domain = N ; Range = {2, 4, 8, ...} = set of even natural numbers.

3. If *f* and g are functions defined by $f(x) = x^2 + 7$, $g(x) = 3x + 5$. Then find each of the following

a. $f(3) + g(-5)$ **b.** $f(1/2) \times g(14)$ **c.** $f(-2) + g(-1)$ **d.** $f(t) - f(-2)$ **e.** $\dfrac{f(t)-(5)}{t-5}$, if $t \neq 5$

Sol. **a.** $f(3) = (3)^2 + 7 = 16$, $g(-5) = -15 + 5 = 10$, $f(3) + g(-5) = 6$

b. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 7 = \frac{29}{4}$, $g(14) = 47$, $\left(\frac{1}{2}\right) f \times g(14) = \frac{29}{4} \times 47 \Rightarrow \frac{1363}{4}$

c. $f(-2) = (-2)^2 + 7 = 11$, $g(-1) = 3(-1) + 5 = 2$, $f(-2) + g(-1) = 2$

d. $f(-2) = 11$, $f(t) - f(-2) = t^2 + 7 - 11 = t^2 - 4$

e. $\dfrac{f(t)-f(5)}{t-5}$, if $t \neq 5 \Rightarrow \dfrac{t^2+7-32}{t-5} = \dfrac{t^2-25}{t-5} = \dfrac{(t+5)(t-5)}{(t-5)} \Rightarrow t+5 \ \{\because t \neq 5\}$

4. Find domain for which the function $f(x) = 2x^2 - 1$ & $g(x) = 1 - 3x$ are equal

Sol. $f(x) = g(x) \Rightarrow 2x^2 - 1 = 1 - 3x \Rightarrow 2x^2 + 3x - 2 = 0 \Rightarrow 2x^2 + 4x - x - 2 = 0 \Rightarrow 2x(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 1) = 0$ i.e., $x = -2, \frac{1}{2}$.

5. For what value of x $f(x) < g(x)$ $\forall$ $f(x) = 2x + 1$, $g(x) = 4x - 7$

Sol. $2x + 1 < 4x - 7 \Rightarrow 8 < 2x$ i.e., $4 < x$

$\Rightarrow x \in (4, \infty)$

6. Let $f : R \to R$ be given by $f(x) = x^2 + 3$. Find **a.** $\{x : f(x) = 28\}$ **b.** the pre-images of 39 and 2 under *f*.

Sol. **a.** $x^2 + 3 = 28 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

$\Rightarrow \{x : f(x) = 28\} = \{-5, 5\}$

b. $x^2 + 3 = 39$

(i) $x^2 = 36 \Rightarrow x = \pm 6$, Hence, Pre-images of 36 are ± 6

(ii) $x^2 + 3 = 2 \Rightarrow x^2 = -1$

Since no real value of x satisfying the equation.

$\therefore$ '2' does not have any pre-image under *f*.

7. Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function described by the formula $f(x) = ax + b$ for some integer a, b. Determine a, b and find $f(x)$

Sol. $f(1) = 1$, $f(2) = 3$, $f(0) = -1$, $f(-1) = -3$

$f(x) = ax + b$

at $x = 1$, $a + b = 1$ ...(1)

at $x = 2$, $2a + b = 3$ ...(2)

Solving (1) & (2) $\Rightarrow a = 2, b = -1$

$\Rightarrow f(x) = 2x - 1$

Thus, $f(0)$ & $f(-1)$ satisfy the equation

Hence, $a = 2, b = -1$

8. If $f : R \to R$ be defined as follows: $f(x) = \begin{Bmatrix} 1, \text{if } x \in Q \\ -1, \text{if } x \notin Q \end{Bmatrix}$. Find:

a. $f\left(\frac{1}{2}\right)$, $f(\pi)$, $f(\sqrt{2})$ **b.** range of f **c.** pre-images of 1 and –1

Sol. **a.** $\frac{1}{2} \in Q \Rightarrow f\left(\frac{1}{2}\right) = 1$ $\Rightarrow \pi \notin Q \Rightarrow f(\pi) = -1$ $\Rightarrow \sqrt{2} \notin Q \in f(\sqrt{2}) = -1$

b. Range of f = {–1, 1}

c. Pre-image of 1 = set of rational numbers = Q

Pre-image of –1 = set of irrational numbers. i.e. $x \notin Q$

PRACTICE WORKSHEET 1

1. Let X = {1,2, 3, 4} and Y = {1,5,9,11,15,16}. Determine which of the following sets are functions from X to Y :

a. f_1 = {(1, 1), (2, 11), (3, 1), (4, 15)} **b.** f_2 = {(1, 1), (2, 7), (3, 5)} **c.** f_3 = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

2. Write the following relations as sets of ordered pairs and find which of them are functions :

a. {(x, y) : y = 3x, x∈{1, 2, 3}, y ∈ {3, 6, 9, 12}} **b.** {(x, y) : y > x + 1 x = 1, 2 and y = 2, 4, 6}

c. {(x, y) : x + y = 3, x, y ∈{0, 1, 2, 3, 4, 5}}

3. Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)} Are the following true?

a. f is a relation from A to B. **b.** f is a function from A to B.

4. Which of the following relation are functions? In case of a function, determine its domain and range.

(i) {(1, 2), (1, 3), (2, 5)} (ii) {(2, 1), (3, 1), (5, 2)} (iii) {(1, 2), (2, 2), (3, 2)}

5. Let f = {(1, 4), (2, 2), (0, 5), (3, 1)} be a linear function from Z to Z. Find f(x).

6. Express the following function as sets of ordered pairs and determine their ranges:

Let A = {9, 10, 11, 12, 13} and let $f : A \to N$ be defined by $f(n)$ the highest prime factor of n.

7. If the function t which maps temperature in degree celcius into temperature in degree farrenheit is defined by $t(C) = \frac{9C}{5} + 32$, then find: i. t(0) ii. t(28) iii. t(–10) iv. the value of c, when t (c) = 212

8. Let A be the set of two positive intergers. Let $f : A \to Z^+$ (set of positive integers) be defined by $f(n) = p$, where p is he highest prime factor of n. If range of f = {3}. Find set A. Is A uniquely determined?

9. Let f be the subset of Q × Z defined by $f = \left\{\left(\frac{m}{n}, m\right) : m \in Z, n \in Z, n \neq 0\right\}$. Is f a function from Q into Z? Justify your answer.

REAL FUNCTION

A function which has either R or one of its subset as its range is called a real valued function.

A function which has either R or one of its subset as range and domain both is called a real function.

Let f be a function define by f(x) = 2x + 1 from N to N then.

We have,

x	1	2	3	4
$f(x) = y$	3	5	7	9

Domain = {1, 2, 3, 4, 5 ...} $\subset$ R
Range = {3, 5, 7, 9 ...} $\subset$ R
Hence, it is a real function.

DOMAIN OF REAL FUNCTIONS

The domain of the real function f(x) is the set of all those real numbers for which the expression for f(x) assumes real values only.

WORKING RULE FOR FINDING THE DOMAIN OF REAL FUNCTION

Polynomial function is always define for all real numbers, so its domain is R.

Steps for Rational Function

Step-I: Firstly put the denominator equal to zero and find the values of variable.

Step-II: The values of variable obtained in step-I are those values at which rational function is not define. So subtract the values from R to get the required domain.

EXAMPLE

1. $f(x) = \dfrac{3x-2}{x^2-1}$

Sol. $x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

Domain $\in$ R – {–1, 1}

2. $f(x) = \dfrac{1}{\sqrt{x-2}}$

Sol. Obviously, $x - 2 \geq 0$

but here $x - 2 \neq 0$ {as in denominator 0 is not allowed}

Hence $x - 2 > 0 \Rightarrow x > 2$

Domain of f(x) = $x \in (2, \infty)$

3. $f(x) = \dfrac{x^2+3x+5}{x^2-5x+4}$

Sol. $x^2 - 5x + 4 = 0$

$\Rightarrow x^2 - 4x - x + 4 = 0$

$\Rightarrow x(x - 4) - 1(x - 4) = 0$

$\Rightarrow x = 1, 4$

Domain = R – {1, 4}

RANGE OF REAL FUNCTIONS

The range of a real function of a real variable is the set of all real values taken by f(x) at points in its domain.

Working Rule for Finding Range of Real Function

Let y = f(x) be a real function then for finding the range we may use the following steps:

Step-I: Find the domain of the function y = f(x)

Step-II: Transform the equation y = f(x) as x = g(y) i.e., convert x in terms of y.

Step-III: Find the values of y from x = g(y) for which the values of x are real in the domain of f.

Step-IV: The set of values of y obtained in step-III be the range of function f. If ‘a’ is the least value of y and ‘b’ is the greatest value of y, then range of f = [a, b]

1. f(x) = $\frac{x-2}{3-x}$, find range?

Sol. $3 - x = 0 \quad \Rightarrow x = 3$

Domain of f(x) = R – {3}

$y = \frac{x-2}{3-x} \Rightarrow y(3 - x) = x - 2 \quad \Rightarrow 3y - xy = x - 2 \quad \Rightarrow 3y + 2 = (y + 1)x$

$\Rightarrow x = \frac{3y+2}{y+1} = g(y)$

Find values of y for which the values of x are real in the domain of f.

$x = g(y) = \frac{3y+2}{y+1} \qquad \Rightarrow$ x is real, if $y + 1 \neq 0 \Rightarrow y \neq -1$

So, x = g(y) takes real values in R – {–1}

Hence, range of f = R – {–1}

2. f(x) = $\sqrt{16-x^2}$ find the domain and range of f(x)?

Sol. f(x) is defind when, $16 - x^2 \geq 0 \qquad \Rightarrow 4^2 - x^2 \geq 0 \qquad \Rightarrow (4 - x)(4 + x) \geq 0$

⊖ve ⊕ve ⊖ve

–4 4

Domain $\in$ [–4, 4]

Range of f $\Rightarrow$ let $y = f(x) \Rightarrow y = \sqrt{16-x^2} \Rightarrow y^2 = 16 - x^2 \quad \Rightarrow x^2 = 16 - y^2 \quad \Rightarrow x = \sqrt{16-y^2}$

Clearly, x will take real values if $16 - y^2 \geq 0 \Rightarrow y^2 - 16 \leq 0 \quad \Rightarrow (y - 4)(y + 4) \leq 0$

⊕ve ⊖ve ⊕ve

–4 4

$y \in [-4, 4]$

Also $y = \sqrt{16-x^2} \geq 0$ for all $x \in [-4, 4]$.

Hence range of f(x) = [0, 4]

ILLUSTRATIONS

1. Find domain of the following functions:

i. $\frac{1}{\sqrt{1-\cos x}}$ ii. $\frac{x^3-x+3}{x^2-1}$ iii. $\frac{1}{\sqrt{x^2-3x+2}}$ iv. $\sqrt{4-x}+\frac{1}{\sqrt{x^2-1}}$

Sol. i. f(x) is defined when $1 - \cos x > 0$

as we know $-1 \leq \cos x \leq 1$

$1 \geq -\cos x \geq -1$

$2 \geq 1 - \cos x \geq 0$

$\therefore$ for f(x) to be defined, $1 - \cos x \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 2n\pi, \forall\, n \in z$

Hence, Domain of f(x) = $R - \{2n\pi : n \in z\}$

ii. $\dfrac{x^3 - x + 3}{x^2 - 1}$ f(x) is not defined when $x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = -1, 1$

$\therefore$ Domain of f(x) = R – {–1, 1}

iii. for f(x) to be defined

$x^2 - 3x + 2 > 0$

$x^2 - 2x - x + 2 > 0$

$x(x - 2) - 1(x - 2) > 0$

$(x - 1)(x - 2) > 0$

$\oplus$ve $\quad \ominus$ve $\quad \oplus$ve

1 $\quad$ 2

$x \in (-\infty, 1) \cup (2, \infty)$

iv. Clearly, f(x) is defined for

$4 - x \geq 0$ and $x^2 - 1 > 0$

$x - 4 \leq 0$ and $(x - 1)(x + 1) > 0$

$x \leq 4$ and $x \in (-\infty, -1) \cup (1, \infty)$

$x \in (-\infty, -1) \cup (1, 4]$

Hence, Domain (f) = $(-\infty, -1) \cup (1, 4]$

2. Find the range of the following functions?

i. $f(x) = \dfrac{3}{2 - x^2}$ $\quad$ ii. $f(x) = 1 + 3\cos 2x$ $\quad$ iii. $f(x) = \dfrac{x^2}{1 + x^2}$ $\quad$ iv. $f(x) = \dfrac{x}{1 + x^2}$

Sol. i. $f(x) = \dfrac{3}{2 - x^2}$ for f(x) to be real, we must have $2 - x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{2}$

$\therefore$ Domain of f(x) = R – $\left\{-\sqrt{2}, \sqrt{2}\right\}$

Let f(x) = y.

Then, $y = \dfrac{3}{2 - x^2} \Rightarrow 2y - yx^2 = 3 \Rightarrow y\,x^2 = 2y - 3 \Rightarrow x^2 = \dfrac{2y - 3}{y} \Rightarrow x = \pm\sqrt{\dfrac{2y - 3}{y}}$

Now, x will take real value other than $-\sqrt{2}$, $\sqrt{2}$ if, $\dfrac{2y - 3}{y} \geq 0$

$\oplus$ve $\quad \ominus$ve $\quad \oplus$ve

0 $\quad \dfrac{3}{2}$

$y \in (-\infty, 0) \cup \left[\dfrac{3}{2}, \infty\right)$

Range of f(x) = $(-\infty, 0) \cup \left[\dfrac{3}{2}, \infty\right)$

ii. $f(x) = 1 + 3\cos 2x$, Domain of f(x) = R,

$y = 1 + \cos 2x$

as we know, $-1 \le \cos 2x \le 1$

$-3 \le 3\cos 2x \le 3$

$-2 \le 1 + 3\cos 2x \le 4$

$-2 \le y \le 4$

Range of f = [–2, 4]

iii. $f(x) = \dfrac{x^2}{1+x^2}$ Clearly f(x) is defined for all x

$\therefore$ Domain of f = R

Now, $y = \dfrac{x^2}{1+x^2} \Rightarrow y + yx^2 = x^2 \Rightarrow y = x^2(1-y) \Rightarrow x^2 = \dfrac{y}{1-y} \Rightarrow x = \pm\sqrt{\dfrac{y}{1-y}}$

x will take real value, if $\dfrac{y}{1-y} \ge 0$

$\ominus$ve $\oplus$ve $\ominus$ve

0 1

$y \in [0, 1)$

Range of f = [0, 1)

iv. $f(x) = \dfrac{x}{1+x^2}$

Clearly, Domain of (f) = R

For x = 0, y = 0 and for all other $x \in R - \{0\}$

Let $y = \dfrac{x}{1+x^2} \Rightarrow y + yx^2 = x \Rightarrow yx^2 - x + y = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1-4y^2}}{2y}$

Clearly, x will take real values, if

$1 - 4y^2 \ge 0 \quad \Rightarrow 4y^2 - 1 \le 0 \quad \Rightarrow y^2 - \dfrac{1}{4} \le 0$

$\left(y - \dfrac{1}{2}\right)\left(y + \dfrac{1}{2}\right) \le 0$

$\oplus$ve $\ominus$ve $\oplus$ve

$-\dfrac{1}{2}$ $\dfrac{1}{2}$

$y \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

$\therefore$ Range of f = $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

3. Find Domain and Range of the following function.

i. $f(x) = \dfrac{x^2-9}{x-3}$ ii. $f(x) = \dfrac{1}{1-x^2}$ iii. $f(x) = \dfrac{1}{2-\sin 3x}$ iv. $f(x) = \dfrac{4-x}{x-4}$

Sol. i. Domain of $f(x) = R - \{3\}$

Let $f(x) = y$ Then, $y = \dfrac{x^2 - 9}{x - 3} \Rightarrow y = x + 3$ $\quad (\because\ x \neq 3)$

Clearly, y takes all real values except 6

Range of $f = R - \{6\}$

ii. $f(x) = \dfrac{1}{1 - x^2}$

Domain of f: f(x) is not defined when

$x^2 - 1 = 0 \quad \Rightarrow x^2 = 1 \quad$ i.e., $x = -1, +1$

$\therefore$ Domain of $f = R - \{-1, 1\}$

Let $f(x) = y \quad \Rightarrow y = \dfrac{1}{1 - x^2}$ i.e., $y - y\,x^2 = 1$

$\Rightarrow x = \pm\sqrt{\dfrac{y-1}{y}}$

Clearly, x will take real values, if $\dfrac{y-1}{y} \geq 0$

$\oplus$ve $\quad$ $\ominus$ve $\quad$ $\oplus$ve

0 $\quad$ 1

$y \in (-\infty, 0) \cup [1, \infty)$

Hence Range $f = (-\infty, 0) \cup [1, \infty)$

iii. As we know, $\quad -1 \leq \sin 3x \leq 1$ for all $x \in R$

$1 \geq -\sin 3x \geq -1$

$3 \geq 2 - \sin 3x \geq 1$

$\therefore\ 2 - \sin 3x \neq 0$

$\therefore$ f(x) is defined for all $x \in R$

Domain of $f = R$

Now, $1 \leq 2 - \sin 3x \leq 3$ for all $x \in R \quad \Rightarrow \dfrac{1}{3} \leq \dfrac{1}{2 - \sin 3x} \leq 1 \quad \Rightarrow \dfrac{1}{3} \leq f(x) \leq 1$

Range of $f = \left[\dfrac{1}{3}, 1\right]$

iv. $f(x) = \dfrac{4 - x}{x - 4}$ f(x) is not defined when $x - 4 = 0$ i.e., $x = 4$

$\therefore$ Domain of $f \in R - \{4\}$

Range

For any $x \in$ Domain (f), we have

$f(x) = \dfrac{4 - x}{x - 4} = \dfrac{-(x - 4)}{x - 4} = -1$

Range (f) = $\{-1\}$

PRACTICE WORKSHEET 2

1. Find the domain of each of the following real valued function :

a. f(x) = $\frac{x-1}{x-3}$ **b.** f(x) = $\frac{1}{\sqrt{x^2-1}}$ **c.** f(x) = $\frac{x^2+2x+1}{x^2-8x+12}$

d. f(x) = $\frac{2x+1}{x^2-1}$ **e.** f(x) = $\sqrt{x-2}$ **f.** f(x) = $\frac{1}{\sqrt{1-x}}$

g. f(x) = $\frac{\sqrt{x-2}}{3-x}$ **h.** f(x) = $\sqrt{4-x^2}$ **i.** f(x) = $\sqrt{x^2-9}$

2. Find the domain and range of each of the following real valued functions :

a. f(x) = x^3 **b.** f(x) = $\frac{x-2}{3-x}$ **c.** f(x) = $\frac{1}{2x-1}$

d. f(x) = $\sqrt{x^2-169}$ **e.** f (x) = $\sqrt{x-7}$ **f.** f (x) = $\frac{1}{\sqrt{x-5}}$

3. Find the domain and range of each of the following real valued functions :

a. f(x) = $\frac{4-x}{x-4}$ **b.** f(x) = $\frac{x-2}{2-x}$ **c.** f (x) = $4x^2 + 12x + 15$

4. If a function f : R $\rightarrow$ R be defined by $f(x)=\begin{cases} 3x-2, & x<0; \\ 1 & x=0; \\ 4x+1 & x>0 \end{cases}$ Find : $f(1)$, $f(-1)$, $f(0)$; $f(2)$.

5. If f(x) = x^2, find $\frac{f(1.1)-f(1)}{(1.1-1)}$

6. Given A = {–1, 0, 2, 5, 6, 11}, B = {–2, –1, 0, 18, 28, 108} and $f(x) = x^2 - x - 2$. Is f(A) = B?

7. If f(x) = $\frac{x+1}{x-1}$, show that : f [f(x)] = x.

8. If y = f(x) = $\frac{ax-b}{bx-a}$, show that : x = f(y)

9. Find the values of b and c for which the identity f(x + 1) – f(x) = 8x + 3 is satisfied where f(x) = $bx^2 + cx + d$.

10. If f(x) + 2f(1 – x) = $x^2 + 2$ $\forall x \in$ R, then determine f(x).

ALGEBRA OF REAL FUNCTIONS

Let f : $D_1 \rightarrow$ R and g : $D_2 \rightarrow$ R be two real function with domain D_1 and D_2, respectively. Then, algebraic operations as addition, subtraction, multiplication and division of two real functions are given below

1. **Addition of two real function:** The sum function (f + g) is defined by
   (f + g) (x) = f (x) + g(x), $\forall$ x $\in D_1 \cap D_2$.
2. **Subtraction of two real functions:** The difference function (f – g) is defined by
   (f – g) (x) = f(x) – g(x), $\forall$ x $\in D_1 \cap D_2$.
3. **Multiplication of two real functions:** The product function (f g) is defined by

$(fg)(x) = f(x).\, g(x),\ \forall\ x \in D_1 \cap D_2$.

4. **Quotient of two real functions:** The quotient function is defined by

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)},\ \forall\ x \in D_1 \cap D_2 - \{x : g(x) = 0\}$$

5. **Multiplication of a real function by a scale:** The scalar multiple function cf is defined by $(c f)(x) = c.\, f(x),\ \forall\ x \in D_1$ and c is a scalar (real number)

Note: a. These operations are defined for real functions only on their commin domain.

b. For any real function $f : D \to R$ and $n \in N$, we define

$$\underbrace{(f\,f\,f\,f\,........f)(x)}_{n\text{ times}} = \underbrace{(f(x)\,f(x)........f)(x)}_{n\text{ times}} = \{f(x)\}^n,\ \forall\ x \in D.$$

EXAMPLE

1. $f(x) = \sqrt{x}$ and $g(x) = x$ then, find $(f + g)(x)$, $(f - g)(x)$, $(f g)(x)$ and $\left(\frac{f}{g}\right)x$

Sol. $f(x) : D_1 \to R$

$g(x) : D_2 \to R$

$D_1 = x \geq 0 \Rightarrow x \in [0, \infty]$

$D_2 = x \in R$

$\therefore$ Domain of $(f + g)(x)$, $(f - g)(x)$, $(f g)(x)$, $(f g)(x)$ is $D_1 \cap D_2$ i.e., $x \geq 0$

$\therefore\ (f + g)(x) = \sqrt{x} + x \qquad \Rightarrow (f - g)(x) = \sqrt{x} - x \qquad \Rightarrow (f g)(x) = \sqrt{x}\, x = x^{3/2}$.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ \forall\ x \in D_1 \cap D_2 - \{x : g(x) = 0\} \qquad \Rightarrow g(x) = x = 0\ \forall\ x = 0$

$\therefore$ Domain of $\left(\frac{f}{g}\right)(x) = x > 0$, i.e, $x \in (0, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x} = x^{-½},\ \forall\ x > 0$

2. Let $f(x) = x$ & $g(x) = |x|$, find $\left(\frac{f}{g}\right)(x) = ?$

Sol. $f : R \to R$ is defined as $f(x) = x$

$g : R \to R$ is defined as $f(x) = |x|$

Clearly, f + g have the same domain.

also, $g(x) = 0 \Rightarrow |x| = 0 \Rightarrow x = 0$

Therefore the quotient of f by g, i.e., $\frac{f}{g}$ is a function from $R - \{0\} \to R$ and is defined as

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \begin{cases} \frac{x}{x} = 1, & x > 0 \\ \frac{x}{-x} = -1, & x < 0 \end{cases}$$

(i) Polynomial Function:

If a function f is defined by $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$ where n is a **non negative integer** and $a_0, a_1, a_2, \ldots\ldots\ldots, a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n.

(ii) Algebraic Function:

y is an algebraic function of x, if it is a function that satisfies an algebraic equation of the form, $P_0(x)\, y^n + P_1(x)\, y^{n-1} + \ldots\ldots. + P_{n-1}(x)\, y + P_n(x) = 0$ where n is a positive integer and $P_0(x), P_1(x) \ldots\ldots.$ are polynomials in x. e.g. $y = |x|$ is an algebraic function, since it satisfies the equation $y^2 - x^2 = 0$.

Note: ☞ All polynomial functions are algebraic but not the converse.

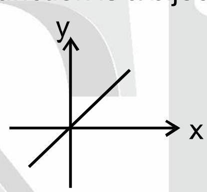
☞ A function that is not algebraic is called **Transcendental Function**.

(iii) Rational Function:

A rational function is a function of the form, $y = f(x) = \frac{g(x)}{h(x)}$, where g (x) & h (x) are polynomials.

(iv) Identity function:

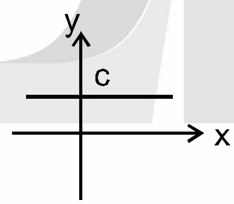
The function $f : A \to A$ defined by, $f(x) = x\ \forall\ x \in A$ is called the identity function on A and is denoted by I_A. It is easy to observe that identity function is a bijection.



Here, Domain $\in$ R and Range $\in$ R

(v) Constant function:

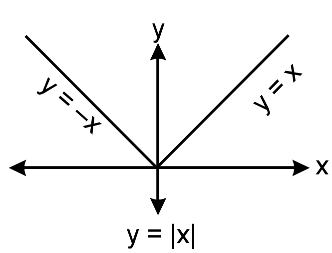
A function $f : A \to B$ is said to be a constant function, if every element of A has the same f image in B. Thus $f : A \to B;\ f(x) = c,\ \forall\ x \in A,\ c \in B$ is a constant function.



Here, Domain $\in$ R and Range $\in$ {c}

(vi) Absolute Value Function / Modulus Function:

The symbol of modulus function is $f(x) = |x|$ and is defined as: $y = |x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$.



Here, Domain $\in$ R and Range $\in [0, \infty)$

Properties of Modulus Function:

(a) For any real number x, we have $\sqrt{x^2} = |x|$

(b) If a and b are positive real numbers then:

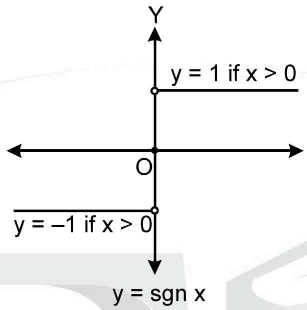
(i) $x^2 \le a^2 \Leftrightarrow |x| \le a \Leftrightarrow -a \le x \le a$

(ii) $x^2 \ge a^2 \Leftrightarrow |x| \ge a \Leftrightarrow x \le -a$ or $x \ge a$

(iii) $a^2 \le x^2 \le b^2 \Leftrightarrow a \le |x| \le b \Leftrightarrow x \in [-b, -a] \cup [a, b]$

(vii) Signum Function : (Also known as sign(x))

A function f (x) = sgn (x) is defined as follows : $f(x) = \text{sgn}(x) = \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$



It is also written as $\text{sgn}\, x = \begin{cases} \dfrac{|x|}{x}; & x \ne 0 \\ 0; & x = 0 \end{cases}$

Here, Domain $\in$ R and Range $\in \{-1, 0, 1\}$

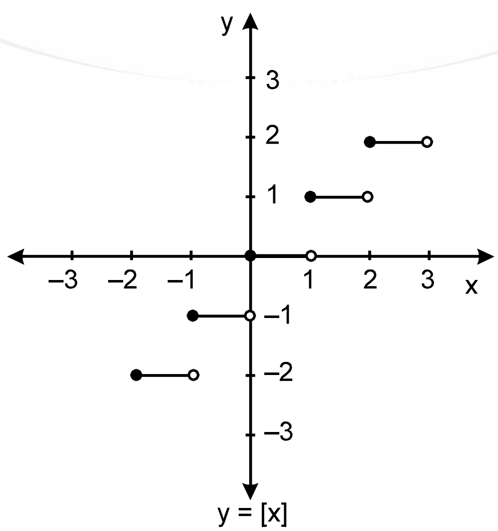
Note: $\text{sgn}\, f(x) = \begin{cases} \dfrac{|f(x)|}{f(x)}; & f(x) \ne 0 \\ 0; & f(x) = 0 \end{cases}$

(viii) Greatest Integer Function or Step Function :

The function y = f (x) = [x] is called the greatest integer function where [x] equals to the greatest integer less than or equal to x. For example :

for $-1 \le x < 0$; $[x] = -1$; for $0 \le x < 1$; $[x] = 0$

for $1 \le x < 2$; $[x] = 1$; for $2 \le x < 3$; $[x] = 2$ and so on.



Here, Domain $\in$ R and Range $\in$ Z (integers)

Properties of greatest integer function :

(a) $x - 1 < [x] \le x$

(b) If m is an integer, then $[x \pm m] = [x] \pm m$.

(c) $[-x] = -[x] - 1$

(d) $[x] + [-x] = \begin{cases} 0 & , \text{ if } x \text{ is an integer} \\ -1 & , \text{ if } x \text{ is not an integer} \end{cases}$

(e) $[x] + [y] \le [x + y] \le [x] + [y] + 1$

(f) $[x] \ge k \Rightarrow x \ge k$, where $k \in Z$

(g) $[x] \le k \Rightarrow x < k$, where $k \in Z$

(h) $[x] > k \Rightarrow x \ge k + 1$, where $k \in Z$

(i) $[x] < k \Rightarrow x < k$, where $k \in Z$

(j) $[x + y] = [x] + [y + x - [x]]$ for all $x, y \in R$

(k) $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \ldots + \left[x + \frac{n-1}{n}\right] = [nx]$, $n \in N$.

(ix) SMALLEST INTEGER FUNCTION (CEILING FUNCTION):

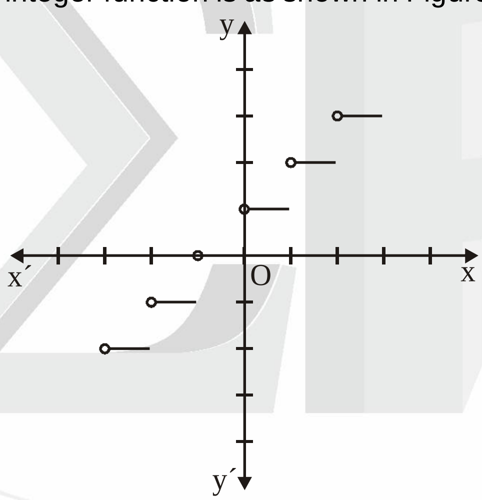
For any real number x, we use the symbol $\lceil x \rceil$ to denote the smallest integer greater than or equal to x.

For example: $\lceil 4.7 \rceil = 5$, $\lceil -7.2 \rceil = -7$ $\lceil 5 \rceil = 5$, $\lceil 0.75 \rceil = 1$ etc.

The function $f : R \to R$ defined by $f(x) = \lceil x \rceil$ for all $x \in R$ *is called the smallest integer function or the ceiling function*. It is also a step function.

Here, Domain $\in$ R and Range $\in$ Z (integers)

The graph of the smallest integer function is as shown in Figure.



PROPERTIES OF SMALLEST INTEGER FUNCTION: Following are some properties of smallest integer function:

(i) $\lceil -n \rceil = -\lceil n \rceil$, where $n \in Z$

(ii) $\lceil -x \rceil = -\lceil x \rceil + 1$, where $n \in R - Z$

(iii) $\lceil x + n \rceil = \lceil x \rceil + n$, where $x \in R - Z$ and $n \in Z$

(iv) $\lceil x \rceil + \lceil -x \rceil = \begin{cases} 1, & \text{if } x \notin Z \\ 0, & \text{if } x \in Z \end{cases}$

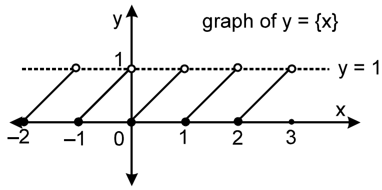
(v) $\lceil x \rceil + \lceil -x \rceil = \begin{cases} 2\lceil x \rceil - 1, & \text{if } x \in Z \\ 2\lceil x \rceil, & \text{if } x \notin Z \end{cases}$

(x) Fractional Part Function:

It is defined as, $y = \{x\} = x - [x]$.

e.g. the fractional part of the number 2.1 is $2.1 - 2 = 0.1$ and $\{-3.7\} = 0.3$.

The period of this function is 1 and graph of this function is as shown.

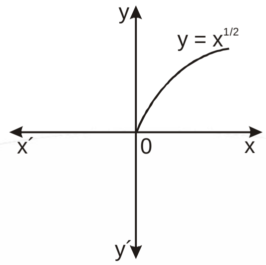


Here, Domain $\in$ R and Range $\in$ [0, 1)

(xi) Square Root Function:

The function f : $R^+ \to R$ defined by f (x) = $+\sqrt{x}$ is called the square root function.

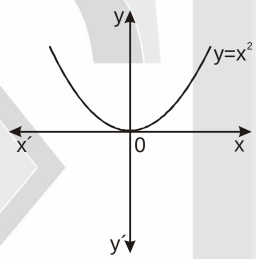
Domain of the square root function is R^+ i.e., [0, ∞).



(xii) Square Function:

The function f : R $\to$ R defined by f (x) = x^2 is called the square function.

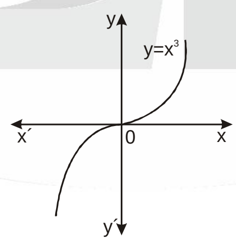
Domain of the square function is R and its range is the set of all non-negative real numbers i.e. [0, ∞).



(xiii) Cube Function:

The function f : R $\to$ R defined by f (x) = x^3 is called the cube function.

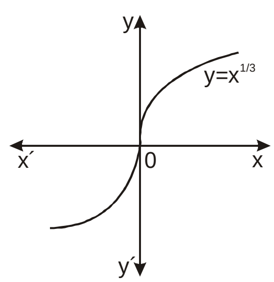
Domain function $\in$ R and Range function $\in$ R



(xiv) Cube Root Function:

The function f : R $\to$ R defined by f (x) = $x^{1/3}$ is called the cube root function.

Domain and range of the cube root function are both equal to R.



ILLUSTRATIONS

1. Find the quotient of the identity function by the modulus function.

Sol. Let f & g denote respectively, the identity function and the modulus function. Then,

$f : R \to R$ is defined as $f(x) = x$ and,

$g : R \to R$ in defined as $g(x) = |x|$

clearly f & g have the same domain.

also, $g(x) = 0 \Rightarrow |x| = 0 \Rightarrow x = 0$

$\therefore$ the quotient of f by g i.e., $\frac{f}{g}$ is a function from $R - \{0\} \to R$ and is defined as

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x}{|x|} = \begin{cases} \frac{x}{x} = 1, & x > 0 \\ \frac{x}{-x} = -1, & x < 0 \end{cases}$$

2. Find the product of the identity function and the reciprocal function.

Sol. Let f and g denote respectively the identity function and the reciprocal function. Then,

$f : R \to R$ is defined as $f(x) = x$ for all $x \in R$ and

$g : R - \{0\} \to R$ is defined as $g(x) = \frac{1}{x}$ for all $x \in R - \{0\}$

we have, Domain (f) $\cap$ Domain (g) $= R \cap R - \{0\} = R - \{0\}$

$\therefore$ $fg(x) = f(x)\, g(x) = x \times \frac{1}{x} = 1$ for all $x \in R - \{0\}$

Thus, $fg : R - \{0\} \to R$ is given by $fg(x) = 1$ for all $x \in R - \{0\}$

3. Let f and g be two real function defined by $f(x) = \frac{1}{x+4}$ and $g(x) = (x + 4)^3$. Find the following:

i. f + g ii. f – g iii. fg iv. $\frac{f}{g}$ v. 2f vi. $\frac{1}{f}$ vii. $\frac{1}{g}$

Sol. Clearly, Domain of $f(x) = R - \{-4\}$, Domain of $g(x) = R$

$\therefore$ Domain (f) $\cap$ Domain (g) $= R - \{-4\}$

i. $f + g : R - \{-4\} \to R$ is given by $(f + g)(x) = f(x) + g(x) \Rightarrow (f + g)(x) = \frac{1}{x+4} + (x + 4)^3 = \frac{(x+4)^4 + 1}{x+4}$

ii. $f - g : R - \{-4\} \to R$ is defined as $(f - g)(x) = f(x) - g(x) \Rightarrow (f - g)(x) = \frac{1}{x+4} - (x + 4)^3 = \frac{1-(x+4)^3}{(x+4)}$

iii. $fg : R - \{-4\} \to R$ is given by $(fg)(x) = f(x) \,.\, g(x) = \frac{1}{x+4} \times (x + 4)^3 = (x + 4)^2$

iv. $g(x) = (x + 4)^3$

$g(x) = 0 \Rightarrow (x + 4)^3 = 0 \Rightarrow x = -4$

$\therefore$ Domain $\left(\dfrac{f}{g}\right)$ = Domain (f) $\cap$ Domain (g) – {x : g(x) = 0} = R – {–4}

Thus, $\dfrac{f}{g}$: R – {–4} $\to$ R is given by $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{1}{(x+4)^4}$

v. (2f)(x) = 2(f(x)) = $\dfrac{2}{x+4}$ for all x $\in$ R – {–4}

vi. We observe that f(x) $\neq$ 0 for any x $\in$ R – {–4}

$\dfrac{1}{f}$: R – {–4} $\to$ R is given by $\left(\dfrac{1}{f}\right)(x) = \dfrac{1}{f(x)}$ = (x + 4)

4. If f is a real function defined by f(x) = $\dfrac{x-1}{x+1}$, then prove that: f(2x) = $\dfrac{3f(x)+1}{f(x)+3}$

Sol. we have, f(x) = $\dfrac{x-1}{x+1}$

$\dfrac{f(x)+1}{f(x)-1} = \dfrac{x-1+x+1}{x-1-x-1}$ (applying componendo and dividendo)

$\Rightarrow \quad x = \dfrac{f(x)+1}{1-f(x)}$

Now, f(2x) = $\dfrac{2x-1}{2x+1}$ $\Rightarrow$ f(2x) = $\dfrac{2\left\{\dfrac{f(x)+1}{1-f(x)}\right\}-1}{2\left\{\dfrac{f(x)+1}{1-f(x)}\right\}+1}$ $\Rightarrow$ f(2x) = $\dfrac{2f(x)+2-1+f(x)}{2f(x)+2+1-f(x)}$ $\Rightarrow$ f(2x) = $\dfrac{3f(x)+1}{f(x)+3}$

5. If y = f(x) = $\dfrac{1-x}{1+x}$, then show that x = f(y).

Sol. y = f(x) = $\dfrac{1-x}{1+x}$

Now, f(y) = $\dfrac{1-y}{1+y} = \dfrac{1-\dfrac{1-x}{1+x}}{1+\dfrac{1-x}{1+x}} = \dfrac{\dfrac{1+x-1+x}{1+x}}{\dfrac{1+x+1-x}{1+x}} = \dfrac{\dfrac{2x}{1+x}}{\dfrac{2}{1+x}} = x$ $\Rightarrow$ f(y) = x

x = f(y), when y = f(x) = $\dfrac{1-x}{1+x}$

6. If f(x) = $\dfrac{|x|}{x}$, x $\neq$ 0, then show that |f(α) – f(–α)| = 2, when $\alpha \neq 0$.

Sol. If α > 0, then |f(α) – f(–α)| = $\left|\dfrac{|\alpha|}{\alpha} - \dfrac{|-\alpha|}{-\alpha}\right|$ = |1 – (–1)| = 2

If α < 0, then |f(α) – f(–α)| = $\left|\dfrac{|\alpha|}{\alpha} - \dfrac{|-\alpha|}{-\alpha}\right| = \left|\dfrac{-\alpha}{\alpha} - \dfrac{-\alpha}{-\alpha}\right|$ = |–1–1| = 2

7. Find the Domain of the following functions.

(i) $f(x)=\dfrac{1}{\sqrt{x-|x|}}$ (ii) $\dfrac{1}{\sqrt{x+|x|}}$ (iii) $f(x)=\dfrac{1}{\sqrt{x-[x]}}$ (iv) $f(x)=\dfrac{1}{\sqrt{x+[x]}}$

Sol. (i) $|x|=\begin{cases} x, \text{if } x >, 0 \\ -x, \text{if } x<0 \end{cases}$

$x-|x| = \begin{cases} x-x=0, \text{if } x\geq 0 \\ x+x=2x, \text{if } x<0 \end{cases}$

$x-|x|\leq 0$ for all x

$\therefore$ Domain (f) = ϕ

(ii) $x+|x| = \begin{cases} 2x, \text{if } x>0 \\ 0, \text{if } x<0 \end{cases}$

assumes real-values if x > 0

$\therefore$ Domain (f) = $(0, \infty)$

(iii) $x-[x]=0$ for $x \in Z$

$0 < x-[x] < 1$ for $x\in R-Z$

$\therefore$ Domain of (f) = R – Z

(iv) $x+[x]>0$ for all $x>0$

$x+[x]=0$ for $x=0$

$x+[x]<0$ for all $x<0$

f(x) is defind for all x > 0

$\therefore$ Domain (f) = $(0, \infty)$

8. Find the range of each of the following functions:

(i) $f(x) = |x-3|$ (ii) $f(x) = 1-|x-2|$ (iii) $f(x)=\dfrac{|x-4|}{x-4}$ (iv) $f(x)=\dfrac{\sin(\pi[x^2+1])}{x^4+1}$

(v) $\dfrac{1}{x-[x]}$

Sol. (i) Clearly f(x) is defined for all $x \in R$

Domain (f) = R

Now, $|x-3| \geq 0$ for all $x\in R$

$0 \leq |x-3| < \infty$

$0 \leq f(x) < \infty$

$\therefore$ Range (f) = $[0, \infty)$

(ii) Domain (f) = R

Now, $|x-2|\geq 0$

$-|x-2|\leq 0$

$1-|x-2|\leq 1$

$f(x)\leq 1$

Range (f) = $(-\infty, 1]$

(iii) Domain (f) = R – {4}

$$f(x)=\begin{cases}\dfrac{x-4}{x-4}=1, & \text{if } x>4\\ \dfrac{-(x-4)}{x-4}=-1, & \text{if } x<4\end{cases}$$

Range (f) = {–1, 1}

(iv) Value of sine function at multiple of π will be 0.

Range (f) = 0

(v) as x – [x] = 0 for all $x \in Z$

0 < x – [x] < 1 for all $x \in R - Z$

Domain of (f) = R – Z

Now, $0 < x - [x] < 1$ for all $x \in R - Z$

$$0 < \sqrt{x-[x]} < 1$$

$$1 < \frac{1}{\sqrt{x-[x]}} < \infty \text{ for all } x \in R - Z$$

$$1 < f(x) < \infty$$

$\therefore$ Range (f) = $(1, \infty)$

9. If $[x]^2 - 5[x] + 6 = 0$ (Where [.] denote the greatest Integer function). Then find the value of x.

Sol. Let [x] = t

We have, $t^2 - 5t + 6 = 0$

$t^2 - 3t - 2t + 6 = 0$

$t(t-3) - 2(t-3) = 0$

$t = 2, 3$

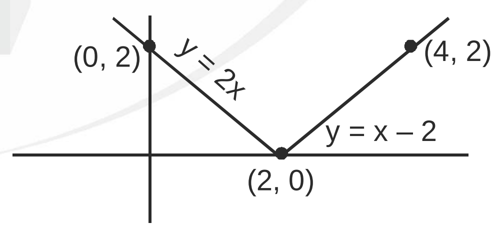
[x] = 2 or [x] = 3

$x \in [2, 3)$ or $x \in [3, 4)$

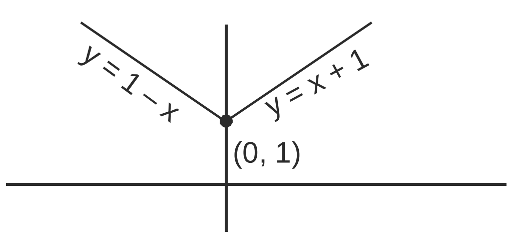
$\therefore$ $x \in [2, 4)$

10. Draw the graph of the function. $f : R \to R$ such that f(x) = |x – 2|

Sol. Clearly, $y = |x-2| = \begin{cases} x-2, & \text{if } x \ge 2\\ 2-x, & \text{if } x<2\end{cases}$



11. The function f is defined by $f(x) = \begin{cases}1-x, & x<0\\ 1, & x=0\\ x+1, & x>0\end{cases}$. Draw the graph of f(x).



PRACTICE WORKSHEET 3

1. **Draw the graph for the following functions and hence find the domain and range:**

   a. constant function
   b. identity function
   c. polynomial function of degree 2 and 3
   d. rational function
   e. modulus function
   f. signum function
   g. greatest integer function
   h. smallest integer function
   i. fractional part function.
   j. all trigonometric fuctions

2. **Draw the graph for the following functions and hence find domain and range:.**

   a. f(x) = $\frac{x-2}{2-x}$
   b. f(x) = $\frac{x^2-4}{x+2}$

3. **Draw the graph for the following functions and hence find domain and range:**

   a. f(x) = $\begin{cases} 1-x, & x<0 \\ 1, & x=0 \\ 1+x, & x>0 \end{cases}$
   b. f(x) = $\begin{cases} 3-x, & x>1 \\ 1, & x=1 \\ 2x, & x<1 \end{cases}$
   c. f(x) = $\begin{cases} 1, & x\ge 1 \\ x, & -1<x<1, \\ -1, & x\le -1 \end{cases}$

4. Let f, g : R $\to$ R be defined, respectively by f(x) = x + 1, g(x) = 2x –3.

   Find f + g, f – g, 2f, fg, and $\frac{1}{f}$ and $\frac{f}{g}$

5. Let *f* and *g* be real functions defined by *f*(x) = $\sqrt{x+2}$ and g(x) = $\sqrt{4-x^2}$. Then find each of the following functions:

   a. f + g **b.** f – g **c.** fg **d.** f / g **e.** ff **f.** gg

6. Let *f* : [2, ∞) $\to$ R and g : [–∞ , 2) $\to$ R be two real functions defined by

   f(x) = $\sqrt{x-2}$ and g(x) = $\sqrt{x+2}$. Find *f* + g and *f* – g.

7. Find the sum and difference of the identity function and the modulus function.
8. What are the sum and difference of the identify function and the reciprocal function?
9. Find the product of the identity function and the modulus function.
10. Find domain of the following function: $\sqrt{\frac{x-2}{x+2}}+\sqrt{\frac{1-x}{1+x}}$
11. Find range of following function.

    (i) $\frac{x}{|x|}$ (ii) $\frac{x+2}{|x+2|}$ (iii) |x – 1| (iv) $e^{x-[x]}$, x $\in$ R

12. Find Domain & range at the following functions

    (i) $\sqrt{x-[x]}$ (ii) $\sqrt{[x]-x}$ (iii) $\frac{1}{\sqrt{[x]-x}}$ (iv) $\frac{1}{\sqrt{|x|-x}}$

13. If f (x) = $\frac{1}{2x+1}$, $x \neq -\frac{1}{2}$, then show that f {(fx)} = $\frac{2x+1}{2x+3}$, provided that $x \neq -\frac{3}{2}$.

ANSWERS

PRACTICE WORKSHEET 1

1. a. Function, **b.** Not a function, **c.** Not a function

2. a. {(1,3), (2,6), (3,9)}; Function

b. {(1,4), (1,6), (2,4), (2,6)}; Not a function **c.** {(0,3), (1,2), (2, 1), (3, 0)}; Function

3. a. Yes **b.** No

4. (i) Not a function (ii) Yes, function Domain = {2, 3, 5}, Range = {1, 2}

(iii) Yes, function Domain = {1, 2, 3}, Range = {2}

5. f(x) = – 2x + 6

6. f = {(9, 3), (10, 5), (11, 11), (12, 3), (13, 13)}, Domain = {9, 10, 11, 12, 13}, Range = {3, 5, 11, 13}

7. i. 32 ii. $\frac{412}{5}$ iii. 14 iv. c = 100 **8.** No **9.** No

PRACTICE WORKSHEET 2

1. a. R – {–2}, **b.** $(-\infty, -1) \cup (1, \infty)$, **c.** R – {2, 6}, **d.** R – {–1, 1}, **e.** $[2, \infty)$, **f.** $(-\infty, 1]$, **g.** [2, 3), **h.** [–2, 2], **i.** $(-\infty, -3] \cup [3, \infty)$

2. a. Domain = R, Range = R

b. Domain = R – {3}, Range = R – {–1}

c. Domain = R – $\left\{\frac{1}{2}\right\}$, Range = R – {0}

d. Domain = $(-\infty, -13] \cup [13, \infty)$, Ramge = $[0, \infty)$

e. Domain = $[7, \infty)$ Range =

f. Domain = $(5, \infty)$, Range = $(0, \infty)$

3. a. Domain = R – {4}, Range = {–1}

b. Domain = R – {2}, Range = {–1}

4. *f*(1) = 5, *f*(–1) = – 5, *f*(0) = 1, *f*(2) = 9 **5.** 2.1 **6.** No

9. b = 4, c = –1 **10.** $\frac{(x-2)^2}{3}$

PRACTICE WORKSHEET 3

1. a. Domain=R, Range = k, **b.** Domain=R, Range =R

c. i. Domain = R, Range=$[0, \infty)$

ii. Domain = R, Range=k

d. Domain = R –{0}, Range = R –{0}

e. Domain = R, Range = Non negative real no.

f. Domain = R, Range ={–1, 0 1}

g. Domain = R, Range=Z **h.** Domain = R, Range = Z

i. Domain = R, Range = [0, 1)

j. sin x : Domain = R, Range = [–1, 1], **cosx** : Domain = R, Range = [–1, 1]

tan x : Domain = R – $\left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$, Range = R **cot x :** Domain = R – $\{2n\pi : n \in Z\}$, Range = R

sec x : Domain = R – $\left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$, Range = $(-\infty, 1] \cup [1, \infty)$

cosec x : Domain = R – $\{2n\pi : n \in Z\}$, Range = $(-\infty, 1] \cup [1, \infty)$

2. a. Domain = R –{2}, Range = {– 1},

b. Domain (f) = R –{– 2}, Range (f) = R –{– 4}

3. a. Domain = R, Range = $[1, \infty)$

b. Domain = R, Range = $(\infty, 1]$

c. Domain = R, Range = [–1,1]

4. (f + g) x = 3x – 2, (f – g) x = – x + 4, 2f (x) = 2x + 2, (f.g) (x) = $2x^2 - x - 3$, $\frac{1}{f}$(x) = $\frac{1}{x+1}; x \neq -1$ $\left(\frac{f}{g}\right)$ x = $\frac{x+1}{2x-3}$, x $\neq \frac{3}{2}$

5. a. f+g= $\sqrt{x-2} + \sqrt{4-x^2}$

b. f – g = $\sqrt{x-2}$ $\sqrt{4-x^2}$ **c.** fg = (x + 2)$\sqrt{x-2}$

d. $\frac{f}{g} = \frac{1}{\sqrt{x-2}}$ **e.** ff = x + 2 **f.** gg = $4 - x^2$

6. *f* + g , *f* – g both does not exist

7. $f+g = \begin{cases} 2x, & If\ x \geq 0 \\ 0 & If\ x < 0 \end{cases}$ and $f-g = \begin{cases} 0, & If\ x \geq 0 \\ 2x & If\ x < 0 \end{cases}$

8. $f+g = x + \frac{1}{x}$ and $f-g = x - \frac{1}{x}$

9. $f.g = \begin{cases} x^2 & if\ x \geq 0 \\ -x^2 & if\ x < 0 \end{cases}$ **10.** ϕ

11. (i) {–1, 1} (ii) {–1, 1} (iii) $\{0, \infty\}$ (iv) [1, e)

12. (i) D(f) = R, R(f) = [0, 1) (ii) D(f) = Z, R(f) = {0} (iii) D(f) = ϕ = R(f) (iv) D(f) = $(-\infty, 0)$, R(f) = $(0, \infty)$

MISCELLANEOUS PROBLEMS

1. The function *f* is defined by f (x) = $\begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$. The relation *g* is defined by

g (x) = $\begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$. Show that f (x) is a function and g(x) is a relation.

2. Let *f* be the sub set of Z × Z. Define as *f* = {(ab, a + b) : a, b $\in$ z}. is *f* a function from Z $\rightarrow$ Z? Justify your answer.

3. Which of the following are functions?

a. $\{(x, y) : y^2 = x, x, y \in R\}$ b. $\{(x, y) : y = |x|, x, y \in R\}$

c. $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$ d. $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$

4. If f is a function such that f(0) = 2, f(1) = 3 and f(x + 2) = (2x) – f(x + 1) for every real x then evaluate f(5).

5. If $a_0 = x$ $a_{n+1} = f(a_n)$, n = 0, 1, 2, ... find a_n where $f(x) = \frac{1}{1-x}$.

6. Find the domain of the function $f(x) = \sqrt{5|x| - x^2 - 6}$.

7. Find the domain of the function f(x) defined by $f(x) = \frac{\sqrt{1-|x|}}{\sqrt{2-|x|}}$

8. Find domain and range of following functions.

(i) $f(x) = \sqrt{x^2 - 5x + 6}$ (ii) $f(x) = \frac{x}{x^2 + 5x + 6}$ (iii) $f(x) = \frac{1}{\sqrt{16 - x^2}}$

9. Write the range of the function f (x) = sin [x], where $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$.

10. If f (x) = cos $[\pi^2]$ x + cos $[-\pi^2]$ x, where [x] denotes the greatest integer less than or equal to x, then write the value of f (π).

11. Write the range of the function f(x) = cos[x], where $\frac{-\pi}{2} < x < \frac{\pi}{2}$.

12. If f: R $\rightarrow$ [–1, 1] where f(x) = sin π/2[x] (where [*] denotes the greatest integer function) then find the range of f(x).

13. Find the range of $f(x) = \frac{2x-2}{x^2 - 2x + 3}$

14. The function f : R $\rightarrow$ R is defined by f (x) = $\cos^2 x + \sin^4 x$. Then find the range of f (x).

15. If f(x) be defined on [–2, 2] and is given by $f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$ & g(x) = f(|x|) + |f(x)|. Find g(x).

16. Let $f(x) = \frac{\alpha x}{x+1}$, x $\neq$ –1. Then write the value of α satisfying f(f(x)) = x for all x $\neq$ –1.

17. Draw the graph of the given function $f(x) = \frac{|(x-1)(x-2)|}{(x-1)(x-2)}$.

EXEMPLAR EXERCISE

OBJECTIVE TYPE QUESTIONS

1. Let n(A) = m, and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is

(A) m^n (B) $n^m - 1$ (C) mn – 1 (D) $2^{mn} - 1$

2. Range of f(x) = $\frac{1}{1-2\cos x}$ is

(A) $\left[\frac{1}{3}, 1\right]$ (B) $\left[-1, \frac{1}{3}\right]$ (C) $(-\infty, -1) \cup \left[\frac{1}{3}, \infty\right)$ (D) $\left[-\frac{1}{3}, 1\right]$

3. Let f(x) = $\sqrt{1+x^2}$, then

(A) f(xy) = f(x) . f(y) (B) f(xy) $\geq$ f(x) . f(y) (C) f(xy) $\leq$ f(x) . f(x) (D) None

4. Domain of $\sqrt{a^2 - x^2}$ (a > 0) is

(A) (–a, a) (B) [–a, a] (C) [0, a] (D) (–a, 0]

5. If f (x) = ax + b, where a and b are integers, f (–1) = – 5 and f (3) = 3, then a and b are equal to

(A) a = – 3, b = –1 (B) a = 2, b = – 3 (C) a = 0, b = 2 (D) a = 2, b = 3

6. The domain of the function f given by f (x) = $\frac{x^2+2x+1}{x^2-x-6}$.

(A) R – {3, –2} (B) R – {–3, 2} (C) R – {3, –2} (D) R – (3, –2)

7. The domain and range of the function f given by f (x) = 2 – |x – 5| is

(A) Domain = R^+, Range = ($-\infty$, 1] (B) Domain = R, Range = ($-\infty$, 2]
(C) Domain = R, Range = ($-\infty$, 2) (D) Domain = R^+, Range = ($-\infty$, 2]

8. The domain for which the functions defined by f(x) = $3x^2 - 1$ and g(x) = 3 + x are equal to:

(A) $\left[-1, \frac{4}{3}\right]$ (B) $\left[1, \frac{4}{3}\right]$ (C) $\left[-1, -\frac{4}{3}\right]$ (D) $\left[-2, -\frac{4}{3}\right]$

SHORT ANSWER TYPE QUESTIONS

9. If P = {x : x < 3, x $\in$ N}, Q = {x : x = 2, x $\in$ W}. Find (P $\cup$ Q) × (P $\cap$ Q), where W is the set of whole numbers.

10. If A = {x : x $\in$ W, x < 2} B = {x : x $\in$ N, 1 < x < 5} C = {3, 5} then find: (i) A × (B $\cap$ C) (ii) A × (B $\cup$ C)

11. In each of the following cases, find a and b. (i) (2a + b, a – b) = (8, 3) (ii) $\left(\frac{a}{4}, a-2b\right)$ = (0, 6 + b).

12. A = {1, 2, 3, 4, 5}, S = {(x, y) : x $\in$ A, y $\in$ A}, then find the ordered pairs which satisfy the conditions given below:

(i) x + y = 5 (ii) x + y < 5 (iii) x + y > 8

13. If R_1 = {(x, y) | y = 2x + 7, where x $\in$ R and –5 $\leq$ x $\leq$ 5} is a relation. Then find the domain and Range of R_1.

14. If R_2 = {(x, y) | x and y are integers and $x^2 + y^2 = 64$} is a relation, find the value of R_2.

15. If R_3 = {(x, |x|) | x is a real number} is a relation, then find domain and range of R_3.

16. Express the following functions as set of ordered pairs and determine their range.

f :x $\to$ R, f (x) = $x^3 + 1$, where X = {–1, 0, 3, 9, 7}

LONG ANSWER TYPE QUESTIONS

17. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? Justify. If this is described by the relation, g (x) = $\alpha x + \beta$, then what values should be assigned to α and β?

18. Re define the function: f(x) = |x – 2| + |2 + x|, $-3 \le x \le 3$.

19. If f(x) = $\frac{x-1}{x+1}$, then show that: (i) $f\left(\frac{1}{x}\right) = -f(x)$ (ii) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

20. Find the domain and Range of the function f (x) = $\frac{1}{\sqrt{x-5}}$.

MISCELLANEOUS PROBLEMS ANSWERS

2. No **3.** a. No, b. Yes, c. No, d. No **4.** 13

5. $a_n = \begin{cases} x \text{ if } n = 3m, m \in I \\ \frac{1}{1-x} \text{ if } n = 3m+1, m \in I \\ \frac{x-1}{x} \text{ if } n = 3m+2, m \in I \end{cases}$ **6.** [–3, –2] ∪ [2, 3] **7.** Domain (f) = (–∞, –2) ∪ (2, ∞) ∪ [–1, 1]

8. (i) D(f) = (–∞, 2] ∪ [3, ∞), R(f) = [0, ∞) (ii) D(f) = (–∞, –3) ∪ (–2, ∞), R(f) = (–∞, 5 – $2\sqrt{6}$] ∪ [5 + $2\sqrt{6}$, ∞]

(iii) D(f) = (–4, 4), R(f) = [1/4, ∞)

9. {–sin 1, 0, sin 1} **10.** 0 **11.** [1, cos 1, cos 2] **12.** {–1, 0, 1} **13.** $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

14. $\left[\frac{3}{4}, 1\right]$ **15.** $g(x) = \begin{cases} -x-1+1 = -x, & -2 \le x \le 0 \\ (x-1)-(x-1) = 0, & 0 < x < 1 \\ (x-1)+(x-1) = 2(x-1), & 1 \le x \le 2 \end{cases}$ **16.** $\alpha = -1$.

EXEMPLAR EXERCISE ANSWERS

1. D **2.** C **3.** C **4.** B **5.** B **6.** A **7.** B **8.** A **9.** {(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)}

10. {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)} **11.** (i) a = $\frac{11}{3}$ and b = $\frac{2}{3}$ (ii) a = 0, b = –2

12. (i) The set of ordered pairs satisfying x + y = 5 is, {(1, 4), (2, 3), (3, 2), (4, 1)}.

(ii) The set of ordered pairs satisfying x + < 5 is {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}.

(iii) The set of ordered pairs satisfying x + y > 8 is {(4, 5), (5, 4), (5, 5)}.

13. [–3, 17]. **14.** R_2 = {(0, 8), (0, –8), (8, 0), (–8, 0)} **15.** Range of R_3 = $R^+ \cup \{0\}$ or (0, ∞)

16. f = {0, 1, 28, 730, 344}. **17.** $\alpha = 2, \beta = -1$. **18.** $= \begin{cases} -2x, & -3 \le x < -2 \\ 4, & -2 \le x < 2 \\ 2, & 2 \le x < 3 \end{cases}$ **20.** R(f) = R^+.