EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Write the following sets in the roster form:

(*i*) $A = \{x : x \in R, 2x + 11 = 15\}$ (*ii*) $B = \{x \mid x^2 = x, x \in R\}$ (*iii*) $C = \{x \mid x$ is a positive factor of a prime number $P\}$

Sol. (*i*) Given that: $A = \{x : x \in R, 2x + 11 = 15\}$

$\therefore\ 2x + 11 = 15 \Rightarrow 2x = 15 - 11 \Rightarrow 2x = 4 \Rightarrow x = 2$

Hence, $A = \{2\}$

(*ii*) Given that: $B = \{x \mid x^2 = x, x \in R\}$

$\therefore\ x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x - 1) = 0$

$\therefore\ x = 0, x = 1$

Hence, $B = \{0, 1\}$

(*iii*) Given that: $C = \{x \mid x$ is a positive factor of a prime number $P\}$

So, the positive factors of prime number P are 1 and P.

Hence, $C = \{1, P\}$

Q2. Write the following sets in the roster form:

(*i*) $D = \{t \mid t^3 = 1, t \in R\}$ (*ii*) $E = \left\{w \,\middle|\, \frac{w-2}{w+3} = 3, w \in R\right\}$

(*iii*) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$

Sol. (*i*) Given that: $D = \{t \mid t^3 = 1, t \in R\}$

$\therefore\ t^3 = t$

$\Rightarrow\ t^3 - t = 0 \Rightarrow t(t^2 - 1) = 0$

$\Rightarrow\ t(t - 1)(t + 1) = 0 \Rightarrow t = 0, t = 1, t = -1$

Hence, $D = \{-1, 0, 1\}$

(*ii*) Given that: $E = \left\{w \,\middle|\, \frac{w-2}{w+3} = 3, w \in R\right\}$

$\therefore\ \frac{w-2}{w+3} = 3$

$\Rightarrow\ 3w + 9 = w - 2 \Rightarrow 3w - w = -2 - 9$

$\Rightarrow\ 2w = -11 \Rightarrow w = \frac{-11}{2} \in R$

Hence, $w = \left\{\frac{-11}{2}\right\}$

(*iii*) Given that: $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$

$\therefore \quad x^4 - 5x^2 + 6 = 0 \Rightarrow x^4 - 3x^2 - 2x^2 + 6 = 0$

$\Rightarrow \quad x^2(x^2 - 3) - 2(x^2 - 3) = 0 \Rightarrow (x^2 - 2)(x^2 - 3) = 0$

$\Rightarrow \quad x^2 - 2 = 0$ and $x^2 - 3 = 0$

$\Rightarrow \quad x = \pm\sqrt{2}$ and $x = \pm\sqrt{3}$

Hence, $F = \{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\}$

Q3. If $Y = \{x \mid x$ is a positive factor of the number $2^{P-1}(2^P - 1)$, where $2^P - 1$ is a prime number. Write Y in the roster form.

Sol. Given that:

$$Y = \{x \mid x \text{ is a positive factor of the number } 2^{P-1}(2^P - 1)\}$$

The factors of $2^{P-1}(2^P - 1)$ are $1, 2, 2^2, 2^3, 2^4, \ldots, 2^{P-1}(2^P - 1)$

Hence, $Y = \{1, 2, 2^2, 2^3, 2^4, \ldots, 2^{P-1}(2^P - 1)\}$

Q4. State which of the following statements are true and which are false. Justify your answer.

(*i*) $35 \in \{x \mid x$ has exactly four positive factors$\}$

(*ii*) $128 \in \{y \mid$ the sum of all positive factors of y is $2y\}$

(*iii*) $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$

(*iv*) $496 \notin \{y \mid$ the sum of all the positive factors of y is $2y\}$

Sol. (*i*) Given that: $35 \in \{x \mid x$ has exactly four positive factors$\}$

$\therefore$ Factors of 35 are 1, 5, 7, 35

Hence, the statement (*i*) is 'True'.

(*ii*) Given that: $128 \in \{y \mid$ the sum of all positive factors of y is $2y\}$

$\therefore$ Factors of 128 are 1, 2, 4, 8, 16, 32, 64 and 128.

Sum of all the factors $= 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128$

$= 255 \neq 2 \times 128$

Hence, the given statement is 'False.'

(*iii*) Given that: $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$

$\therefore \quad x^4 - 5x^3 + 2x^2 - 112x + 6 = 0$

Now for $x = 3$, we have

$(3)^4 - 5(3)^3 + 2(3)^2 - 112(3) + 6$

$\Rightarrow 81 - 135 + 18 - 336 + 6 \Rightarrow -366 \neq 0$

Hence, statement (*iii*) is 'True'.

(*iv*) Given that:

$496 \notin \{y \mid$ the sum of all the positive factors of y is $2y\}$

$\therefore$ The positive factors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, 248 and 496

$\therefore$ The sum of all the positive factors of 496

$= 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 + 496 = 992 = 2 \times 496$

Hence, the given statement is 'False'.

Q5. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

Verify that $L - (M \cup N) = (L - M) \cap (L - N)$

Sol. Given that: L = {1, 2, 3, 4}, M = {3, 4, 5, 6} and N = {1, 3, 5}

To verify that $L - (M \cup N) = (L - M) \cap (L - N)$

$$\text{L.H.S.} = L - (M \cup N) = \{1, 2, 3, 4\} - \{\{3,4,5,6\} \cup \{1,3,5\}\}$$
$$= \{1, 2, 3, 4\} - \{1, 3, 4, 5, 6\} = \{2\}$$

$$\text{R.H.S.} = (L - M) \cap (L - N)$$
$$(L - M) = \{1, 2, 3, 4\} - \{3, 4, 5, 6\} = \{1, 2\}$$
$$(L - N) = \{1, 2, 3, 4\} - \{1, 3, 5\} = \{2, 4\}$$
$$\therefore \quad (L - M) \cap (L - N) = \{1, 2\} \cap \{2, 4\} = \{2\}$$
$$\text{L.H.S.} = \text{R.H.S.}$$

Hence, verified.

Q6. If A and B are subsets of the universal set U, then show that:

(*i*) $A \subset A \cup B$ (*ii*) $A \subset B \Leftrightarrow A \cup B = B$ (*iii*) $(A \cap B) \subset A$

Sol. (*i*) Given that: $A \subset U$ and $B \subset U$

Let $x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

Hence, $A \subset (A \cup B)$

(*ii*) If $A \subset B$

Then let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow \quad A \cup B \subset B \quad$ ...(1)

But $\quad B \subset A \cup B \quad$ ...(2)

From eqn. (1) and (2), we get

$$A \cup B = B$$

Let $y \in A \Rightarrow y \in (A \cup B) \Rightarrow y \in B$

$\Rightarrow y \in B \Leftrightarrow A \cup B = B$

(*iii*) Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B \Rightarrow x \in A$

So $A \cap B \subset A$.

Q7. Given that:

N = {1, 2, 3, ..., 100}, then write

(*i*) The subset of N whose elements are even number.

(*ii*) The subset of N whose elements are perfect square numbers.

Sol. We are given that: N = {1, 2, 3, 4, 5, ..., 100}

(*i*) Required subset whose elements are even

$$= \{2, 4, 6, 8, ..., 100\}$$

(*ii*) Required subset whose elements are perfect squares

$$= \{1, 4, 9, 16, 25, 36, ..., 100\}$$

Q8. If X = {1, 2, 3}, if n represents any number of X, write the following sets containing all numbers represented by

(*i*) $4n$ (*ii*) $n + 6$ (*iii*) $\frac{n}{2}$ (*iv*) $n - 1$

Sol. Given that: X = {1, 2, 3}

(*i*) $\{4n \mid n \in X\} = \{4, 8, 12\}$ (*ii*) $\{n + 6 \mid n \in X\} = \{7, 8, 9\}$

(*iii*) $\left\{\frac{n}{2} \middle| n \in X\right\} = \left\{\frac{1}{2}, 1, \frac{3}{2}\right\}$ (*iv*) $\{(n - 1) \mid n \in X\} = \{0, 1, 2\}$

Q9. If Y = {1, 2, 3, ..., 10} and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions:

(*i*) $a \in Y$ but $a^2 \notin Y$ (*ii*) $a + 1 = 6, a \in Y$

(*iii*) a is less than 6 and $a \in Y$

Sol. Given that: Y = {1, 2, 3, ..., 10}

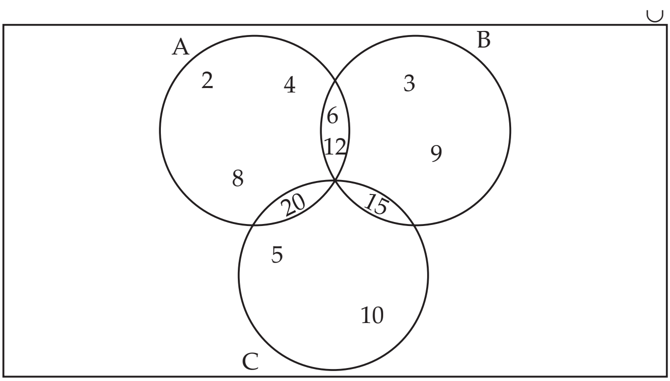
(*i*) $\{a \in Y \text{ but } a^2 \notin Y\} = \{4, 5, 6, 7, 8, 9, 10\}$

(*ii*) $\{a + 1 = 6, a \in Y\} = \{5\}$

(*iii*) $\{a < 6 \text{ and } a \in Y\} = \{1, 2, 3, 4, 5\}$

Q10. A, B and C are subsets of universal set U. If A = {2, 4, 6, 8, 12, 20}, B = {3, 6, 9, 12, 15} and C = {5, 10, 15, 20} and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

Sol. Given that: A, B and C are the subsets of a universal set U.



Where A = {2, 4, 6, 8, 12, 20}

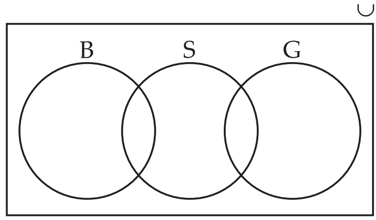
B = {3, 6, 9, 12, 15}

and C = {5, 10, 15, 20}

Q11. Let U be the set of all boys and girls in a school. G be the set of all girls in the school, B be the set of all boys in the school and S be the set of all students in the school who take swimming. Some but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationships among sets U, G, B and S.

Sol. Given that:

U = Set of universal
G = Set of girls
B = Set of boys
S = Set of all students, who take swimming.



Q12. For all sets A, B and C, show that $(A - B) \cap (A - C) = A - (B \cup C)$.

Sol. To prove $(A - B) \cap (A - C) = A - (B \cup C)$

L.H.S. Let $x \in (A - B) \cap (A - C)$
$\Rightarrow x \in (A - B)$ and $x \in (A - C)$
$\Rightarrow (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$
$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$
$\Rightarrow x \in A - (B \cup C)$
So $(A - B) \cap (A - C) \subset A - (B \cup C)$ ...(*i*)
R.H.S. Let $y \in A - (B \cup C)$
$\Rightarrow y \in A$ and $y \notin (B \cup C)$
$\Rightarrow y \in A$ and $(y \notin B$ and $y \notin C)$
$\Rightarrow (y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$
$\Rightarrow y \in (A - B)$ and $y \in (A - C)$
So $A - (B \cup C) \subset (A - B) \cap (A - C)$ ...(*ii*)
From eqn. (*i*) and (ii), we get
$$A - (B \cup C) = (A - B) \cap (A - C)$$

Determine whether each of the statement in exercise 13–17 is true or false. Justify your answer.

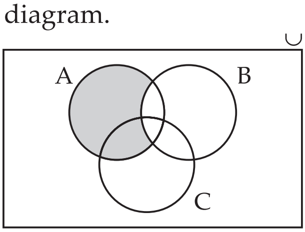
Q13. For all sets A and B, $(A - B) \cup (A \cap B) = A$

Sol. L.H.S. $= (A - B) \cup (A \cap B)$
$= [(A - B) \cup A] \cap [(A - B) \cup B]$
$= A \cap (A \cup B) = A =$ R.H.S.

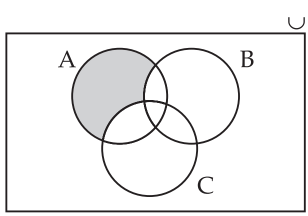
Hence, the given statement is 'True'.

Q14. For all sets A, B and C, $A - (B - C) = (A - B) - C$.

Sol. Let us solve the given statement by the following Venn diagram.



$A - (B - C)$



$(A - B) - C$

Clearly from the above diagram, we calculate that

$$A - (B - C) \neq (A - B) - C$$

Hence, the given statement is not 'True'.

Q15. For all sets A, B and C, if $A \subset B$ then $A \cap C \subset B \cap C$.

Sol. Let $x \in A \cap C$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in B$ and $x \in C$

$\Rightarrow x \in (B \cap C) \Rightarrow (A \cap C) \subset (B \cap C)$

Hence, the given statement is 'True'.

Q16. For all sets A, B and C, if $A \subset B$ then $A \cup C \subset B \cup C$.

Sol. Let $x \in A \cup C$

$\Rightarrow x \in A$ or $x \in C$

$\Rightarrow x \in B$ or $x \in C$ $[\because A \subset B]$

$\Rightarrow x \in (B \cup C) \Rightarrow (A \cup C) \subset (B \cup C)$

Hence, the given statement is 'True'.

Q17. For all sets A, B and C, if $A \subset C$ and $B \subset C$ then $A \cup B \subset C$

Sol. Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in C$ or $x \in C$ $[\because A \subset C$ and $B \subset C]$

$\Rightarrow x \in C \Rightarrow A \cup B \subset C$

Hence, the given statement is 'True'.

Q18. For all sets A and B, $A \cup (B - A) = A \cup B$.

Sol. L.H.S. $= A \cup (B - A) = A \cup (B \cap A')$ $[\because A - B = A \cap B']$

$= (A \cup B) \cap (A \cup A')$

$= A \cup B \cap U$ $[\because A \cup A' = U]$

$= A \cup B =$ R.H.S. $[\because A \cup U = A]$

Hence, the given statement is proved.

Q19. For all sets A and B, $A - (A - B) = A \cap B$.

Sol. L.H.S. $= A - (A - B) = A - (A \cap B')$ $[\because A - B = A \cap B']$

$= A \cap (A \cap B')' = A \cap [A' \cup (B')']$

$[\because (A \cap B)' = A' \cup B']$

$= A \cap [A' \cup B]$ $[\because (A')' = A]$

$= (A \cap A') \cup (A \cap B) = \phi \cup (A \cap B)$

$= A \cap B =$ R.H.S.

L.H.S. = R.H.S. Hence proved.

Q20. For all sets A and B, $A - (A \cap B) = A - B$.

Sol. L.H.S. $= A - (A \cap B) = A \cap (A \cap B)'$

$= A \cap (A' \cup B') = (A \cap A') \cup (A \cap B')$

$= \phi \cup (A - B)$ $[\because A - B = A \cap B']$

$= A - B =$ R.H.S.

L.H.S. = R.H.S. Hence proved.

Q21. For all sets A and B, (A ∪ B) – B = A – B.

Sol. L.H.S. $(A \cup B) - B = (A \cup B) \cap B'$ $[\because A - B = A \cap B']$

$= (A \cap B') \cup (B \cap B')$ $[\because A \cap A' = \phi]$

$= (A - B) \cup \phi$

$= A - B$ R.H.S.

L.H.S. = R.H.S. Hence proved.

Q22. Let $T = \left\{x \middle| \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Is T is an empty set? Justify your answer.

Sol. Given that: $T = \left\{x \middle| \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$

$$\Rightarrow \quad \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}$$

$$\Rightarrow \quad \frac{x+5}{x-7} - \frac{4x-40}{13-x} = 5$$

$$\Rightarrow \quad \frac{(x+5)(13-x)-(x-7)(4x-40)}{(x-7)(13-x)} = 5$$

$$\Rightarrow \quad \frac{13x - x^2 + 65 - 5x - 4x^2 + 40x + 28x - 280}{13x - x^2 - 91 + 7x} = 5$$

$$\Rightarrow \quad -5x^2 + 76x - 215 = 5(13x - x^2 + 7x - 91)$$

$$\Rightarrow \quad -5x^2 + 86x - 215 = 65x - 5x^2 + 35x - 455$$

$$\Rightarrow \quad 76x - 100x = -455 + 215 \Rightarrow -24x = -240$$

$$x = \frac{240}{24} = 10$$

$$\therefore \quad T = 10$$

Hence, T is not an empty set.

LONG ANSWER TYPE QUESTIONS

Q23. Let A, B and C be sets. Then show that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Sol. Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and ($x \in B$ or $x \in C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

$\Rightarrow (x \in A \cap B)$ or $(x \in A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$ ...(*i*)

Now let $y \in (A \cap B) \cup (A \cap C)$

$\Rightarrow y \in (A \cap B)$ or $y \in (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) or ($y \in A$ and $y \in C$)

$\Rightarrow y \in A$ and ($y \in B$ or $y \in C$)

$\Rightarrow y \in$ A and $y \subset$ (B $\cup$ C)

$\Rightarrow y \in$ A $\cap$ (B $\cup$ C) ...(*ii*)

From eqn. (*i*) and (*ii*) we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

L.H.S. = R.H.S. Hence proved.

Q24. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed

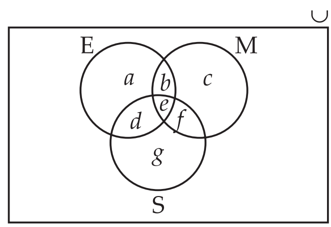
(*i*) in English and Mathematics but not in Science.

(*ii*) in Mathematics and Science but not in English.

(*iii*) in Mathematics only.

(*iv*) in more than one subject only.

Sol. Let the number of students passed in Mathematics M, E be in English and S be in Science.



Then n(U) = 100, n(M) = 12, n(E) = 15, n(S) = 8, n(E $\cap$ M) = 6, n(M $\cap$ S) = 7, n(E $\cap$ S) = 4 and n(E $\cap$ M $\cap$ S) = 4

Let us draw a Venn diagram. According to the Venn diagram, we get

$$a + b + d + e = 15 \quad ...(i)$$

$$b + c + e + f = 12 \quad ...(ii)$$

$$d + e + f + g = 8 \quad ...(iii)$$

$$n(E \cap M) = 6 \quad \therefore b + e = 6 \quad ...(iv)$$

$$n(M \cap S) = 7 \quad \therefore e + f = 7 \quad ...(v)$$

$$n(E \cap S) = 4 \quad \therefore d + e = 4 \quad ...(vi)$$

$$n(E \cap M \cap S) = 4 \quad \therefore e = 4 \quad ...(vii)$$

From eqn. (*iv*) and (*vii*) we get $b + 4 = 6 \quad \therefore b = 2$

From eqn. (*v*) and (*vii*) we get $4 + f = 7 \quad \therefore f = 3$

From eqn. (*vi*) and (*vii*) we get $d + 4 = 4 \quad \therefore d = 0$

From eqn. (*i*) we get

$$a + b + d + e = 15 \Rightarrow a + 2 + 0 + 4 = 15 \Rightarrow a = 9$$

From eqn. (*ii*) $b + c + e + f = 12 \Rightarrow 2 + c + 4 + 3 = 12 \Rightarrow c = 3$

From eqn. (*iii*) $d + e + f + g = 8 \Rightarrow 0 + 4 + 3 + g = 8 \Rightarrow g = 1$

$\therefore$ (*i*) Number of students who passed in English and Mathematics but not in Science, $b = 2$.

(*ii*) Number of students who passed in Mathematics and Science but not in English, $f = 3$.

(*iii*) Number of students who passed in Mathematics only, $c = 3$.

(*iv*) Number of students who passed in more than one subject $= b + e + d + f = 2 + 4 + 0 + 3 = 9$.

Q25. In a class of 60 students, 25 play cricket and 20 students play tennis and 10 students play both the games. Find the number of students who play neither.

Sol. Total number of students = 60 $\Rightarrow n(\text{U}) = 60$

Number of students who play cricket = 25 $\Rightarrow n(\text{C}) = 25$

Number of students who play tennis = 20 $\Rightarrow n(\text{T}) = 20$

Number of students who play both the games = 10 $\Rightarrow n(\text{C} \cap \text{T}) = 10$

$$\therefore \quad n(\text{C} \cup \text{T}) = n(\text{C}) + n(\text{T}) - n(\text{C} \cap \text{T})$$
$$= 25 + 20 - 10 = 35$$

Number of students who play neither

$$= n(\text{U}) - n(\text{C} \cup \text{T})$$
$$= 60 - 35 = 25$$

Q26. In a survey of 200 students of a school, it was found that 120 study mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

Sol. Total number of student = 200 $\Rightarrow$ $n(\text{U}) = 200$

Number of students who study Mathematics = 120 $\Rightarrow n(\text{M}) = 120$

Number of students who study Physics = 90 $\Rightarrow n(\text{P}) = 90$

Number of students who study Chemistry = 70 $\Rightarrow n(\text{C}) = 70$

Number of students who study Mathematics and Physics

$$= 40 \Rightarrow n(\text{M} \cap \text{P}) = 40$$

Number of students who study Physics and Chemistry

$$= 30 \Rightarrow n(\text{P} \cap \text{C}) = 30$$

Number of students who study Chemistry and Mathematics

$$= 50 \Rightarrow n(\text{C} \cap \text{M}) = 50$$

Number of students who study none of the subjects = 20

$$\Rightarrow \quad n(\text{M}' \cap \text{P}' \cap \text{C}') = 20$$

$$\therefore n(\text{U}) - n(\text{M}' \cap \text{P}' \cap \text{C}') = n(\text{M} \cup \text{P} \cup \text{C}) = 200 - 20 = 180$$

Now $\quad n(\text{M} \cup \text{P} \cup \text{C}) = n(\text{M}) + n(\text{P}) + n(\text{C}) - n(\text{M} \cap \text{P}) - n(\text{P} \cap \text{C})$
$$- n(\text{M} \cap \text{C}) + n(\text{M} \cap \text{P} \cap \text{C})$$

$$\Rightarrow \quad 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(\text{M} \cap \text{P} \cap \text{C})$$

$$\Rightarrow \quad 180 - 160 = n(\text{M} \cap \text{P} \cap \text{C})$$

$$\Rightarrow \quad n(\text{M} \cap \text{P} \cap \text{C}) = 20$$

Hence, the number of students who study all the three subjects = 20.

Q27. In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10%

families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find

(*i*) The number of families which buy newspaper A only.

(*ii*) The number of families which buy none of A, B and C.

Sol. Total number of families = 10000 $\Rightarrow$ $n(\text{U}) = 10000$

Number of families who buy newspaper A = 40% $\Rightarrow n(\text{A}) = 40\%$

Number of families who buy newspaper B = 20% $\Rightarrow n(\text{B}) = 20\%$

Number of families who buy newspaper C = 10% $\Rightarrow n(\text{C}) = 10\%$

Number of families who buy newspapers A and B = 5%

$\Rightarrow n(\text{A} \cap \text{B}) = 5\%$

Number of families who buy newspapers B and C = 3%

$\Rightarrow n(\text{B} \cap \text{C}) = 3\%$

Number of families who buy newspapers A and C = 4%

$\Rightarrow n(\text{A} \cap \text{C}) = 4\%$

Number of families who buy all the three newspapers = 2%

$\Rightarrow n(\text{A} \cap \text{B} \cap \text{C}) = 2\%$

(*i*) Number of families who buy newspaper A only

$= n(\text{A}) - n(\text{A} \cap \text{B}) - n(\text{A} \cap \text{C}) + n(\text{A} \cap \text{B} \cap \text{C})$

$$= \frac{40}{100} - \frac{5}{100} - \frac{4}{100} + \frac{2}{100} = \frac{33}{100}$$

$$= 10000 \times \frac{33}{100} = 3300 \text{ families}$$

(*ii*) Number of families who buy none of A, B and C

Newspaper in percent (%) = $n(\text{U}) - n(\text{A} \cup \text{B} \cup \text{C})$

$$\Rightarrow n(\text{U}) - [n(\text{A}) + n(\text{B}) + n(\text{C}) - n(\text{A} \cap \text{B}) - n(\text{B} \cap \text{C}) - n(\text{A} \cap \text{C}) + n(\text{A} \cap \text{B} \cap \text{C})]$$

$\Rightarrow [100 - (40 + 20 + 10 - 5 - 3 - 4 + 2)]\%$

$\Rightarrow (100 - 60)\% = 40\%$

$\therefore$ Number of families, who buy none of A, B and C newspaper out of 10000 families are

$$= 10000 \times \frac{40}{100} = 4000 \text{ families.}$$

Q28. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows:

French = 17, English = 13, Sanskrit = 15, French and English = 9, English and Sanskrit = 4, French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study

(*i*) Only French (*ii*) Only English

(*iii*) Only Sanskrit (*iv*) English and Sanskrit but not French

(*v*) French and Sanskrit but not English

(*vi*) French and English but not Sanskrit (*vii*) Atleast one of the three languages.

(*viii*) None of the three languages.

Sol. Let us use Venn diagram method.

Total number of students = 50 $\Rightarrow n(U) = 50$

Number of students who study French = 17 $\Rightarrow n(F) = 17$

Number of students who study English = 13 $\Rightarrow n(E) = 13$

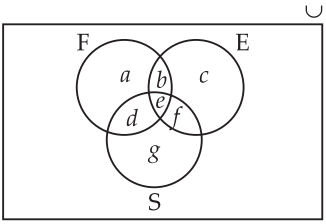
Number of students who study Sanskrit = 15 $\Rightarrow n(S) = 15$

Number of students who study French and English = 9

$\Rightarrow n(F \cap E) = 9$

Number of students who study English and Sanskrit = 4

$\Rightarrow n(E \cap S) = 4$



Number of students who study French and Sanskrit = 5

$\Rightarrow n(F \cap S) = 5$

Number of students who study French, English and Sanskrit = 3

$\Rightarrow n(F \cap E \cap S) = 3$

$$n(F) = 17$$
$$a + b + d + e = 17 \quad ...(i)$$
$$n(E) = 13$$
$$b + c + e + f = 13 \quad ...(ii)$$
$$n(S) = 15$$
$$d + e + f + g = 15 \quad ...(iii)$$
$$n(F \cap E) = 9 \quad \therefore b + e = 9 \quad ...(iv)$$
$$n(E \cap S) = 4 \quad \therefore e + f = 4 \quad ...(v)$$
$$n(F \cap S) = 5 \quad \therefore d + e = 5 \quad ...(vi)$$
$$n(E \cap F \cap S) = 3 \quad \therefore e = 3 \quad ...(vii)$$

From (*iv*) $b + 3 = 9 \Rightarrow b = 9 - 3 = 6$

From (*v*) $3 + f = 4 \Rightarrow f = 4 - 3 = 1$

From (vi) $d + 3 = 5 \Rightarrow d = 5 - 3 = 2$

Now from eqn. (*i*) $a + 6 + 2 + 3 = 17 \Rightarrow a = 17 - 11 \Rightarrow a = 6$

Now from eqn. (*ii*) $6 + c + 3 + 1 = 13 \Rightarrow c = 13 - 10 \Rightarrow c = 3$

From eqn. (*iii*) $2 + 3 + 1 + g = 15 \Rightarrow g = 15 - 6 \Rightarrow g = 9$

(*i*) Number of students who study French only, $a = 6$

(*ii*) Number of students who study English only, $c = 3$

(*iii*) Number of students who study Sanskrit only, $g = 9$

(*iv*) Number of students who study English and Sanskrit but not French, $f = 1$

(*v*) Number of students who study French and Sanskrit but not English, $d = 2$

(*vi*) Number of students who study French and English but not Sanskrit, $b = 6$

(*vii*) Number of students who study at least one of the three languages

$$= a + b + c + d + e + f + g$$
$$= 6 + 6 + 3 + 2 + 3 + 1 + 9 = 30$$

(*viii*) Number of students who study none of the three language

$= 50 - 30 = 20.$

OBJECTIVE TYPE QUESTIONS

Choose the correct answer out of the given four options in each of the Exercises from 29 to 43 (M.C.Q.)

Q29. Suppose $A_1, A_2, ..., A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, ..., B_n$ are n sets each with 3 elements. Let $\bigcup_{i=1}^{30} A_i$ $\Rightarrow \bigcup_{j=1}^{n} B_i = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's then n is equal to

(*a*) 15 (*b*) 3 (*c*) 45 (*d*) 35

Sol. Number of elements is $A_1 \cup A_2 \cup A_3 ... \cup A_{30}$

$= 30 \times 5 = 150$ (Repetition is not done)

But each element is used 10 times

$$\therefore \qquad S = \frac{30 \times 5}{10} = 15$$

If the elements $B_1, B_2, ..., B_n$ are not repeated. Then the total number of elements = 3.

But each element is repeated 9 times

$$\therefore \qquad S = \frac{3n}{9} \Rightarrow 15 = \frac{n}{3} \Rightarrow n = 15 \times 3 = 45$$

Hence, the correct option is (*c*).

Q30. Two finite sets having m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively

(*a*) 4, 7 (*b*) 7, 4 (*c*) 4, 4 (*d*) 7, 7

Sol. Number of subsets of a given set having m element = 2^m and the number of subsets of set containing n elements = 2^n

As per the given condition, we have

$$2^m - 2^n = 112$$
$$\Rightarrow \quad 2^n (2^{m-n} - 1) = 112 \Rightarrow 2^n.(2^{m-n} - 1) = 2^4.7$$
$$\therefore \quad 2^n = 2^4 \text{ and } 2^{m-n} - 1 = 7$$
$$\Rightarrow \quad n = 4 \text{ and } 2^{m-n} = 1 + 7 = 8 = 2^3$$
$$\Rightarrow \quad n = 4 \text{ and } m - n = 3 \Rightarrow m - 4 = 3 \Rightarrow m = 7$$

Hence, the correct option (*b*).

Q31. The set $(A \cap B')' \cup (B \cap C)$ is equal to

(*a*) $A' \cup B \cup C$ (*b*) $A' \cup B$ (*c*) $A' \cup C'$ (*d*) $A' \cap B$

Sol. We know that: $(A \cap B)' = A' \cup B'$

$$\therefore \quad (A \cap B')' \cup (B \cap C) = [A' \cup (B')'] \cup (B \cap C)$$
$$= (A' \cup B) \cup (B \cap C) \qquad [\because (B')' = B]$$
$$= A' \cup B$$

Hence, the correct option is (*b*).

Q32. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapezium in a plane. Then F_1 may be equal to

(*a*) $F_2 \cap F_3$ (*b*) $F_3 \cap F_4$

(*c*) $F_2 \cup F_5$ (*d*) $F_2 \cup F_3 \cup F_4 \cup F_1$

Sol. We know that rectangles, rhombus and square in a plane is a parallelogram but trapezium is not a parallelogram.

$$\therefore \quad F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$$

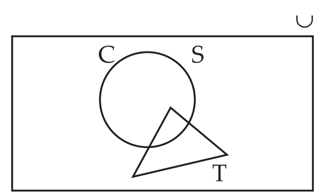
Hence, the correct option is (*d*).

Q33. Let S = Set of points inside the square, T = Set of points inside the triangle and C = Set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then,

(*a*) $S \cap T \cap C = \phi$ (*b*) $S \cup T \cup C = C$

(*c*) $S \cup T \cup C = S$ (*d*) $S \cup T = S \cap C$

Sol. The given conditions of the question may be represented by the following Venn diagram.



From the given Venn diagram, we clearly conclude that

$$S \cup T \cup C = S$$

Hence, the correct option is (*c*).

Q34. Let R be the set of points inside a rectangle of sides *a* and *b* ($a, b > 1$) with two sides along the positive direction of X-axis and Y-axis, then

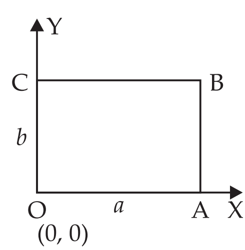
(*a*) $R = \{(x, y) : 0 \le x \le a, 0 \le y \le b\}$

(*b*) $R = \{(x, y) : 0 \le x < a, 0 \le y \le b\}$

(*c*) $R = \{(x, y) : 0 \le x \le a, 0 < y < b\}$

(*d*) $R = \{(x, y) : 0 < x < a, 0 < y < b\}$

Sol. Let OABC be a rectangle whose sides *a* and *b* are along the positive direction of X and Y respectively.



$\therefore$ Clearly,

$R = \{(x, y) : 0 < x < a \text{ and } 0 < y < b\}$

Hence, the correct option is (*d*).

Q35. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games. Then, the number of students who play neither is

(*a*) 0 (*b*) 25 (*c*) 35 (*d*) 45

Sol. Total number of students = 60 $\Rightarrow n(\text{U}) = 60$

Number of students who play Cricket = 25 $\Rightarrow n(\text{C}) = 25$

Number of students who play Tennis = 20 $\Rightarrow n(\text{T}) = 20$

Number of students who play Cricket and Tennis both = 10

$\Rightarrow n(\text{C} \cap \text{T}) = 10$

$$\therefore \quad n(\text{C} \cup \text{T}) = n(\text{C}) + n(\text{T}) - n(\text{C} \cap \text{T})$$
$$= 25 + 20 - 10 = 45 - 10 = 35$$
$$\therefore \quad n(\text{C}' \cap \text{T}') = n(\text{U}) - n(\text{C} \cup \text{T})$$
$$= 60 - 35 = 25$$

Hence, the correct option is (*b*).

Q36. In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then the number of persons who read neither is

(*a*) 210 (*b*) 290 (*c*) 180 (*d*) 260

Sol. Total number of persons in a town = 840 $\Rightarrow n(\text{U}) = 840$

Number of persons who read Hindi = 450 $\Rightarrow n(\text{H}) = 450$

Number of persons who read English = 300 $\Rightarrow n(\text{E}) = 300$

Number of persons who read both = 200 $\Rightarrow n(\text{H} \cap \text{E}) = 200$

$$\therefore \quad n(\text{H} \cup \text{E}) = n(\text{H}) + n(\text{E}) - n(\text{H} \cap \text{E})$$
$$= 450 + 300 - 200 = 550$$
$$n(\text{H}' \cap \text{E}') = n(\text{U}) - n(\text{H} \cup \text{E})$$
$$= 840 - 550 = 290$$

Hence, the correct option is (*b*).

Q37. If $\text{X} = \{8^n - 7n - 1 \mid n \in \text{N}\}$ and $\text{Y} = \{49n - 49 \mid n \in \text{N}\}$, then

(*a*) $\text{X} \subset \text{Y}$ (*b*) $\text{Y} \subset \text{X}$ (*c*) $\text{X} = \text{Y}$ (*d*) $\text{X} \cap \text{Y} = \phi$

Sol. Given that: $\text{X} = \{8^n - 7n - 1 \mid n \in \text{N}\} = \{0, 49, 490, ...\}$

and $\text{Y} = \{49n - 49 \mid n \in \text{N}\} = \{0, 49, 98, ...\}$

Here, it is clear that every element belonging to X is also present in Y.

$\therefore$ $\text{X} \subset \text{Y}$. Hence, the correct option is (*a*).

Q38. A survey shows that 63% of the people watch a news channel where as 76% watch another channel. If x% of the people watch both channel, then

(*a*) $x = 35$ (*b*) $x = 63$ (*c*) $39 \le x \le 63$ (*d*) $x = 39$

Sol. Let p% of the people watch a channel and q% of the people watch another channel

$$\therefore \quad n(p \cap q) = x\% \text{ and } n(p \cup q) \le 100$$

So $$n(p \cup q) \ge n(p) + n(q) - n(p \cap q)$$

$100 \geq 63 + 76 - x$

$100 \geq 139 - x \Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$

Now $n(p) = 63$

$\therefore$ $n(p \cap q) \leq n(p) \Rightarrow x \leq 63$

So $39 \leq x \leq 63$. Hence, the correct option is (*c*).

Q39. If sets A and B are defined as

$A = \left\{(x, y) \mid y = \frac{1}{x}, 0 \neq x \in R\right\}$, $B = \{(x, y) \mid y = -x, x \in R\}$ then

(*a*) $A \cap B = A$ (*b*) $A \cap B = B$ (*c*) $A \cap B = \phi$ (*d*) $A \cup B = A$

Sol. Given that: $A = \left\{(x, y) \mid y = \frac{1}{x}, 0 \neq x \in R\right\}$,

and $B = \{(x, y) \mid y = -x, x \in R\}$

It is very clear that $y = \frac{1}{x}$ and $y = -x$

$\because$ $\frac{1}{x} \neq -x$

$\therefore$ $A \cap B = \phi$

Hence, the correct option is (*c*).

Q40. Let A and B are two sets then $A \cap (A \cup B)$ equals to

(*a*) A (*b*) B (*c*) ϕ (*d*) $A \cap B$

Sol. Given that: $A \cap (A \cup B)$

Let $x \in A \cap (A \cup B)$

$\Rightarrow x \in A$ and $x \in (A \cup B)$

$\Rightarrow x \in A$ and ($x \in A$ or $x \in B$)

$\Rightarrow$ ($x \in A$ and $x \in A$) or ($x \in A$ and $x \in B$)

$\Rightarrow x \in A$ or $x \in (A \cap B)$

$\Rightarrow x \in A$. Hence the correct option is (*a*).

Q41. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, ..., 18\}$ and N the set of natural numbers is the universal set, then $[A' \cup (A \cup B) \cap B']$ is

(*a*) ϕ (*b*) N (*c*) A (*d*) B

Sol. Given that: $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$

$B = \{2, 4, ..., 18\}$

$U = N = \{1, 2, 3, 4, 5, ...\}$

$$\begin{aligned}
A' \cup (A \cup B) \cap B' &= A' \cup [(A \cap B') \cup (B \cap B')] \\
&= A' \cup (A \cap B') \cup \phi && [\because A \cap A' = \phi] \\
&= A' \cup (A \cap B') \\
&= (A' \cup A) \cap (A' \cup B') \\
&= N \cap (A' \cup B') && [\because A' \cup A = N] \\
&= A' \cup B' \\
&= (A \cap B)' = (\phi)' = N && [\because A \cap B = \phi]
\end{aligned}$$

Hence, the correct option is (*b*).

Q42. Let S = {x | x is a positive multiple of 3 less than 100}
P = {x | x is a prime number less than 20}. Then n(S) + n(T) is
(a) 34 (b) 41 (c) 33 (d) 30

Sol. Given that: S = {x | x is a positive multiple of 3 < 100}

$\therefore \quad S = \{3, 6, 9, 12, 15, 18, ..., 99\}$

$\Rightarrow \quad n(S) = 33$

$T = \{x \mid x \text{ is a prime number} < 20\}$

$\therefore \quad T = \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow n(T) = 8$

So, $n(S) + n(T) = 33 + 8 = 41$

Hence, the correct option is (b).

Q43. If X and Y are two sets and X′ denotes the compliment of X, then $X \cap (X \cup Y)'$ is equal to
(a) X (b) Y (c) ϕ (d) $X \cap Y$

Sol. Let $x \in X \cap (X \cup Y)'$

$\Rightarrow x \in X \cap (X' \cap Y')$

$\Rightarrow x \in (X \cap X') \cap (X \cap Y')$

$\Rightarrow x \in \phi \cap (X \cap Y')$ $[\because A \cap A' = \phi]$

$\Rightarrow x \in \phi$

Hence, the correct option is (c).

Fill in the Blanks in Each of the Exercises 44 to 52.

Q44. The set $\{x \in R : 1 \leq x < 2\}$ can be written as ....

Sol. The set $\{x \in R : 1 \leq x < 2\}$ can be written as [1, 2)
Hence, the filler is **[1, 2)**.

Q45. When A = ϕ then number of elements is P(A) is ....

Sol. Here A = ϕ $\therefore$ $n(A) = 0$

$\because \quad n[P(A)] = 2^{n(A)} = 2^0 = 1$

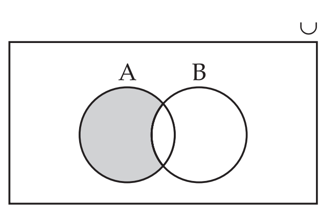
Hence, the filler is **1**.

Q46. If A and B are finite sets such that $A \subset B$, then $n(A \cup B)$ = ...

Sol. Since $A \subset B$ $\therefore$ $n(A \cup B) = n(B)$
Hence, the filler is ***n*(B)**.

Q47. If A and B are any two sets, then A – B is equal to ....

Sol. From the Venn diagram it is clear that
A – B = A ∩ B′.
Hence the filler is **A ∩ B′**.



A – B = A ∩ B′

Q48. Power set of the set A = {1, 2} is ....

Sol. Power set of A = P(A) = $\{\{1\}, \{2\}, \{1, 2\}, \phi\}$
Hence, the filler is **$\{\{1\}, \{2\}, \{1, 2\}, \phi\}$**.

Q49. Given the set A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}. Then the universal set of all the three sets A, B and C can be ....

Sol. Given that: A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}

∴ Universal set of all the given sets is

U = {0, 1, 2, 3, 4, 5, 6, 8}

Hence, the filler is **{0, 1, 2, 3, 4, 5, 6, 8}**.

Q50. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8}, then

(*i*) $(B \cup C)'$ is .... (*ii*) $(C - A)'$ is ....

Sol. Given that: U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8}

(*i*) $(B \cup C)' = \{2, 3, 4, 6, 7, 8\}' = \{1, 5, 9, 10\}$

Hence, the filler is **{1, 5, 9, 10}**.

(*ii*) $(C - A)' = \{4, 8\}' = \{1, 2, 3, 5, 6, 7, 9, 10\}$

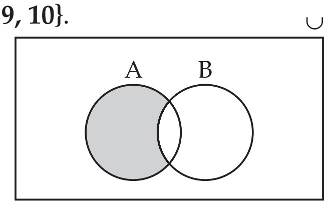
Hence, the filler is **{1, 2, 3, 5, 6, 7, 9, 10}**.

Q51. For all set A and B, $A - (A \cap B)$ is equal to ....

Sol. Since $A - B = A \cap B'$

$\Rightarrow A - (A \cap B) = A \cap B'$

Hence, the filler is $\mathbf{A \cap B'}$.



$A - B = A \cap B'$

Q52. Match the following sets for all sets A, B and C

(*i*)	$((A' \cup B') - A)'$	(*a*)	$A - B$
(*ii*)	$[B' \cup (B' - A)]'$	(*b*)	A
(*iii*)	$(A - B) - (B - C)$	(*c*)	B
(*iv*)	$(A - B) \cap (C - B)$	(*d*)	$(A \times B) \cap (A \times C)$
(*v*)	$A \times (B \cap C)$	(*e*)	$(A \times B) \cup (A \times C)$
(*vi*)	$A \times (B \cup C)$	(*f*)	$(A \cap C) - B$

Sol. (*i*)

$$\begin{aligned}((A' \cup B') - A)' &= [(A' \cup B') \cap A']' && [\because A - B = A \cap B']\\ &= [(A \cap B)' \cap A']' && [\because A' \cup B' = (A \cap B)']\\ &= [(A \cap B)']' \cup (A')' && [\because (A')' = A]\\ &= (A \cap B) \cup A\\ &= A.\end{aligned}$$

So (*i*) is match with (*b*).

(*ii*)

$$\begin{aligned}[B' \cup (B' - A)]' &= [B' \cup (B' \cap A')]' && [\because A - B = A \cap B']\\ &= (B')' \cap (B' \cap A')' && [\because A' \cap B' = (A \cup B)']\\ &= B \cap (B \cup A) = B\end{aligned}$$

So (*ii*) is matched with (*c*).

(*iii*)

$$\begin{aligned}(A - B) - (B - C) &= (A \cap B') - (B \cap C') && [\because A - B = (A \cap B')]\\ &= (A \cap B') \cap (B \cap C')'\\ &= (A \cap B') \cap (B' \cup (C')') && [(A \cap B)' = A' \cup B']\end{aligned}$$

$= (A \cap B') \cap (B' \cup C)$
$= [A \cap (B' \cup C)] \cap [B' \cap (B' \cup C)]$
$= [A \cap (B' \cup C)] \cap B'$
$= (A \cap B') \cap (B' \cup C) \cap B'$
$= (A \cap B') \cap B' = (A \cap B') = A - B$

So, (*iii*) is matched with (*a*).

(*iv*) $(A - B) \cap (C - B) = (A \cap B') \cap (C \cap B')$ $[\because A - B = (A - B')]$
$= (A \cap C) \cap B'$
$= (A \cap C) - B$ $[\because A \cap B' = A - B]$

Hence, (*iv*) is matched with (*f*).

(*v*) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

So, (*v*) is matched with (*d*).

(*vi*) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

So, (*vi*) is matched with (*e*).

State True or False for the Statements in Each of the Exercises 53 to 58.

Q53. If A is any set, then $A \subset A$.

Sol. Since every set is a subset of itself. So, it is 'True'.

Q54. Given that $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and if $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ then $B \not\subset M$.

Sol. $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Since M and B has same elements
$\therefore$ $M = B$, so $B \not\subset M$ is 'False'.

Q55. The sets $\{1, 2, 3, 4\}$ and $\{3, 4, 5, 6\}$ are equal.

Sol. False, since the two sets do not contain the same elements.

Q56. $Q \cup Z = Q$, where Q is the set of rational numbers and Z is the set of integers.

Sol. True, since every integer is a rational number.
$\therefore Z \subset Q$, so $Q \cup Z = Q$.

Q57. Let sets R = T be defined as
$R = \{x \in Z \mid x \text{ is divisible by } 2\}$
$T = \{x \in Z \mid x \text{ is divisible by } 6\}$. Then $T \subset R$.

Sol. We can written the given sets is Roster form
$R = \{..., -8, -6, -4, -2, 0, 2, 4, 6, 8, ...\}$
and $T = \{..., -18, -12, -6, 0, 6, 12, 18, ...\}$
Since every element of T is present in R. So, $T \subset R$.
Hence, the statement is 'True'.

Q58. Given $A = \{0, 1, 2\}$, $B = \{x \in R \mid 0 \le x \le 2\}$ then $A = B$.

Sol. Here $A = \{0, 1, 2\}$, B is a set having all real numbers from 0 to 2. So $A \neq B$.
Hence, the given statement is 'False'.