EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Find the mean deviation about the mean of the distributions:

Size	20	21	22	23	24
Frequency	6	4	5	1	4

Sol.

Size (x_i)	Frequency (f_i)	$f_i x_i$	$d_i = \lvert x_i - \overline{x} \rvert$	$f_i d_i$
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
Total	20	433	6.35	25.00

$$\text{Mean } \overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{433}{20} = 21.65$$

$$\text{Mean deviation MD} = \frac{\sum f_i d_i}{\sum f_i} = \frac{25}{20} = 1.25$$

Here, the required MD = 1.25

Q2. Find the mean deviation about the median of the following distribution:

Marks obtained	10	11	12	14	15
Number of students	2	3	8	3	4

Sol.

Marks obtained	f_i	$c.f.$	$d_i = \lvert x_i - \text{Med} \rvert$	$f_i d_i$
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
Total	20			25

Here $\sum f_i = \text{N} = 20$ and $\sum f_i d_i = 25$

$$\text{Median} = \frac{1}{2}\left[\left(\frac{\text{N}}{2}\right)\text{th observation} + \left(\frac{\text{N}}{2}+1\right)\text{th observation}\right]$$

$$= \frac{1}{2}\left[\left(\frac{20}{2}\right)\text{th observation} + \left(\frac{20}{2}+1\right)\text{th observation}\right]$$

$$= \frac{1}{2}\left[10\text{th observation} + 11\text{th observation}\right] = \frac{1}{2}[12+12]$$

$\therefore$ Median $= 12$

$$\therefore \quad \text{M.D.} = \frac{\sum f_i d_i}{\sum f_i} = \frac{25}{20} = 1.25$$

Hence, the required MD = 1.25

Q3. Calculate the mean deviation about the mean of the set of first n natural numbers, when n is an odd number.

Sol. First n natural numbers are 1, 2, 3, ..., n. Here, n is odd.

$$\therefore \quad \text{Mean } \overline{x} = \frac{1+2+3+\cdots+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

The deviations of numbers from mean $\left(\frac{n+1}{2}\right)$ are

$$1-\frac{n+1}{2}, 2-\frac{n+1}{2}, 3-\frac{n+1}{2}, \ldots, n-\frac{n+1}{2}$$

$$i.e., \; -\frac{n-1}{2}, -\frac{n-3}{2}, \ldots, -2, -1, 0, 1, 2, \ldots, \frac{n-1}{2}.$$

The absolute values of deviation from the mean i.e. $|x_i - \overline{x}|$ are

$$\frac{n-1}{2}, \frac{n-3}{2}, \ldots, 2, 1, 0, 1, 2, \ldots, \frac{n-1}{2}.$$

The sum of absolute values of deviations from the mean i.e. $|x_i - \overline{x}|$

$$= 2\left(1+2+3+\ldots \text{ to } \frac{n-1}{2} \text{ terms}\right)$$

$$= 2.\frac{\frac{n-1}{2}\left(\frac{n-1}{2}+1\right)}{2} = \frac{n-1}{2}.\frac{n+1}{2} = \frac{n^2-1}{4}.$$

$\therefore$ Mean deviation about the mean

$$= \frac{\sum|x_i - \bar{x}|}{n} = \frac{\frac{n^2-1}{4}}{n} = \frac{n^2-1}{4n}.$$

Q4. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an even number.

Sol. First n natural numbers are 1, 2, 3, 4, 5, 6, …, n (even)

$$\therefore \quad \text{Mean } \bar{x} = \frac{1+2+3+4+\cdots+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\therefore \text{ MD} = \frac{1}{n}\left[\left|1-\frac{n+1}{2}\right| + \left|2-\frac{n+1}{2}\right| + \left|3-\frac{n+1}{2}\right| + \ldots + \left|\frac{n-2}{2}-\frac{n+1}{2}\right|\right.$$

$$\left. + \left|\frac{n}{2}-\frac{n+1}{2}\right| + \left|\frac{n+2}{2}-\frac{n+1}{2}\right| \ldots + \left|n-\frac{n+1}{2}\right|\right]$$

$$= \frac{1}{n}\left[\left|\frac{1-n}{2}\right| + \left|\frac{3-n}{2}\right| + \left|\frac{5-n}{2}\right| + \ldots + \left|\frac{-3}{2}\right| + \left|-\frac{1}{2}\right|\right.$$

$$\left. + \left|\frac{1}{2}\right| + \ldots + \left|\frac{n-1}{2}\right|\right]$$

$$= \frac{1}{n}\left[\frac{1}{2} + \frac{3}{2} + \cdots + \frac{n-1}{2}\right]\left(\frac{n}{2}\right) \text{terms}$$

$$= \frac{1}{n}\left(\frac{n}{2}\right)^2 = \frac{1}{n}\cdot\frac{n^2}{4} = \frac{n}{4}$$

[$\because$ Sum of first odd n natural numbers = n^2]

Hence, the required MD = $\frac{n}{4}$.

Q5. Find the standard deviation of first n natural numbers.

x_i	1	2	3	4	5	—	—	n
x_i^2	1	4	9	16	25	—	—	n^2

Sol.

$$\sum x_i = 1+2+3+4+5+\cdots+n = \frac{n(n+1)}{2}$$

$$\sum x_i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore \text{ S.D. } (\sigma) = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{n(n+1)(2n+1)}{6n} - \frac{n^2(n+1)^2}{4n^2}}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}}$$

$$= \sqrt{\frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}}$$

$$= \sqrt{\frac{4n^2+6n+2-3n^2-6n-3}{12}} = \sqrt{\frac{n^2-1}{12}}$$

Hence, the required SD = $\sqrt{\frac{n^2-1}{12}}$

Q6. The mean and standard deviation of some data for the time taken to complete a test are calculated with the following result:
Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25. Further, another set of 15 observations $x_1, x_2, x_3, \ldots, x_{15}$, also is seconds, is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$. Calculate the standard deviation based on all 40 observation.

Sol. Given that $n_1 = 25,\ \bar{x}_1 = 18.2$ and $\sigma_1 = 3.25$

and $n_2 = 15,\ \sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

For the first set, we have

$$\sum x_1 = 25 \times 18.2 = 455$$

$$\therefore \quad \sigma_1^2 = \frac{\sum x_i^2}{25} - (18.2)^2$$

$$\Rightarrow \quad (3.25)^2 = \frac{\sum x_i^2}{25} - 331.24$$

$$\Rightarrow 10.5625 + 331.24 = \frac{\sum x_i^2}{25} \quad \Rightarrow \quad \sum x_i^2 = 25 \times (10.5625 + 331.24)$$

$$= 25 \times 341.8025 = 8545.06$$

For the combined standard deviation of the 40 observation, $n = 40$

and $\sum x_i^2 = 5524 + 8545.06 = 14069.06$

$$\Rightarrow \qquad \sum x_i = 455 + 279 = 734$$

$$\therefore \qquad SD = \sqrt{\frac{14069.06}{40} - \left(\frac{734}{40}\right)^2} = \sqrt{351.7265 - (18.35)^2}$$

$$= \sqrt{351.7265 - 336.7225} = \sqrt{15.004} = 3.87$$

Hence, the required SD = 3.87

Q7. The mean and standard deviation of a set of n_1 observations are $\bar{x}_1$ and s_1 respectively while the mean and standard deviation of another set of n_2 observations are $\bar{x}_2$ and s_2 respectively. Show that the standard deviation of the combined set of $(n_1 + n_2)$ observations is given by

$$SD = \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{n_1 + n_2} + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

Sol. Let $x_i, i = 1, 2, 3, 4, \ldots, n_1$
and $y_j, j = 1, 2, 3, 4, \ldots, n_2$

$$\therefore \qquad \bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n} x_i \quad \text{and} \quad \bar{x}_2 = \frac{1}{n_2}\sum_{j=1}^{n} y_j$$

$$\Rightarrow \qquad \sigma_1^2 = \frac{1}{n_1}\sum_{i=1}^{n_1}(x_i - \bar{x}_1)^2 \quad \text{and} \quad \sigma_2^2 = \frac{1}{n_2}\sum_{j=1}^{n_2}(y_i - \bar{x}_2)^2$$

Now mean of the combined series is given by

$$\bar{x} = \frac{1}{n_1 + n_2}\left[\sum_{i=1}^{n_1} x_i + \sum_{j=1}^{n_2} y_j\right] = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Therefore, σ^2 of the combined series is

$$\sigma^2 = \frac{1}{n_1 + n_2}\left[\sum_{i=1}^{n_1}(x_i - \bar{x})^2 + \sum_{j=1}^{n_2}(y_j - \bar{x})^2\right]$$

$$\text{Now,} \sum_{i=1}^{n_1}(x_i - \bar{x})^2 = \sum_{i=1}^{n_1}(x_i - \bar{x}_j + \bar{x}_j - \bar{x})^2$$

$$= \sum_{i=1}^{n_1}(x_i - x_j)^2 + n_1(\bar{x}_j - \bar{x})^2$$

$$+ 2(\bar{x}_j - \bar{x})\sum_{i=1}^{n_1}(x_i - \bar{x}_j)^2$$

$$\text{But} \quad \sum_{i=1}^{n}(x_i - \bar{x}_i) = 0$$

[$\because$ The algebraic sum of the deviation of values of first series from their mean is zero]

Also $\sum_{i=1}^{n_1}(x_i - \bar{x})^2 = n_1 s_1^2 + n_1(\bar{x}_1 - \bar{x})^2$

$= n_1 s_1^2 + n_1 d_1^2$

where $d_1 = (\bar{x}_1 - \bar{x})$

Similarly, we have

$$\sum_{j=1}^{n_2}(y_j - \bar{x})^2 = \sum_{j=1}^{n_2}(y_j - \bar{x}_i + \bar{x}_i - \bar{x})^2 = n_2 s_2^2 + n_2 d_2^2$$

where $d_2 = (\bar{x}_2 - \bar{x})$

Now combined Standard Deviation (SD)

$$\sigma = \sqrt{\frac{n_1(s_1^2 + d_1^2) + n_2(s_2^2 + d_2^2)}{n_1 + n_2}}$$

where $d_1 = \bar{x}_1 - \bar{x} = \bar{x}_1 - \left(\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}\right) = \frac{n_2(\bar{x}_1 - \bar{x}_2)}{n_1 + n_2}$

and $d_2 = \bar{x}_2 - \bar{x} = \bar{x}_2 - \left(\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}\right) = \frac{n_1(\bar{x}_2 - \bar{x}_1)}{n_1 + n_2}$

$$\therefore \sigma^2 = \frac{1}{n_1 + n_2}\left[n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2^2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2} + \frac{n_2 n_1^2(\bar{x}_2 - \bar{x}_1)^2}{(n_1 + n_2)^2}\right]$$

so, $\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$. Hence proved.

Q8. Two sets each of 20 observations, have the same standard deviation 5. The first set has a mean 17 and the second mean 22. Determine the standard deviation of the *x* sets obtained by combining the given two sets.

Sol. Given that $n_1 = 20, \sigma_1 = 5, \bar{x}_1 = 17$

and $n_2 = 20, \sigma_2 = 5, \bar{x}_2 = 22$

Now we know for combined two series that

$$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

$$= \sqrt{\frac{20 \times (5)^2 + 20 \times (5)^2}{20 + 20} + \frac{20 \times 20\,(17 - 22)^2}{(20 + 20)^2}}$$

$$= \sqrt{\frac{1000}{40} + \frac{400 \times 25}{1600}} = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}}$$

$$= \sqrt{31.25} = 5.59$$

Hence, the required SD = 5.59

Q9. The frequency distribution

x	A	2A	3A	4A	5A	6A
f	2	1	1	1	1	1

where A is a positive integer, has a variance of 160. Determine the value of A.

Sol.

x	f_i	$f_i x_i$	$f_i x_i^2$
A	2	2A	$2A^2$
2A	1	2A	$4A^2$
3A	1	3A	$9A^2$
4A	1	4A	$16A^2$
5A	1	5A	$25A^2$
6A	1	6A	$36A^2$
	$n = 7$	$\sum f_i x_i = 22A$	$\sum f_i x_i^2 = 92A^2$

$$\therefore \quad \text{Variance } \sigma^2 = \frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2$$

$$\Rightarrow \quad 160 = \frac{92A^2}{7} - \left(\frac{22A}{7}\right)^2 \quad \Rightarrow \quad 160 = \frac{92A^2}{7} - \frac{484A^2}{49}$$

$$\Rightarrow \quad 160 = \frac{644A^2 - 484A^2}{49} \quad \Rightarrow \quad 160 = \frac{160A^2}{49}$$

$$\Rightarrow \quad A^2 = 49 \quad \Rightarrow \quad A = 7$$

Hence, the value of A is 7.

Q10. For the frequency distribution

x	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard deviation.

Sol.

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	4	8	16
3	9	27	81
4	16	64	256
5	14	70	350
6	11	66	396
7	6	42	294
	N = 60	$\sum f_i x_i = 277$	$\sum f_i x_i^2 = 1393$

$$\therefore \quad SD(\sigma) = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2} = \sqrt{\frac{1393}{60} - \left(\frac{277}{60}\right)^2}$$

$$= \sqrt{23.23 - (4.62)^2} = \sqrt{23.21 - 21.34}$$

$$= \sqrt{1.87} = 1.37$$

Hence, the required SD = 1.37

Q11. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test.

Marks	0	1	2	3	4	5
Frequency	$x - 2$	x	x^2	$(x+1)^2$	$2x$	$(x+1)$

where x, is positive integer. Determine the mean and standard deviation of the marks.

Sol. Given that $\sum f_i = 60$

$$\therefore \quad x - 2 + x + x^2 + (x+1)^2 + 2x + (x+1) = 60$$

$$\Rightarrow \quad 4x - 2 + x^2 + x^2 + 2x + 1 + x + 1 = 60$$

$$\Rightarrow \quad 2x^2 + 7x - 60 = 0$$

$$\Rightarrow \quad 2x^2 + 15x - 8x - 60 = 0$$

$$\Rightarrow \quad x(2x+15) - 4(2x+15) = 0$$

$$\Rightarrow \quad (2x+15)(x-4) = 0$$

$$\Rightarrow \quad 2x + 15 = 0$$

$$\therefore \quad x = -\frac{15}{2} \text{ Rejected}$$

$$\therefore \quad x = 4 \qquad [\because \quad x \in I^+]$$

Now put $x = 4$ in the frequency distribution table

x_i	f_i	$d_i = x_i - 3$	$f_i d_i$	$f_i d_i^2$
0	2	−3	−6	18
1	4	−2	−8	16
2	16	−1	−16	16
3	25	0	0	0
4	8	1	8	8
5	5	2	10	20
	N = 60		$\sum f_i x_i = -12$	$\sum f_i x_i^2 = 78$

Let assumed mean A = 3

$$\text{Mean} = A + \frac{\sum f_i d_i}{N} = 3 + \left(\frac{-12}{60}\right) = 3 - \frac{1}{5} = \frac{14}{5} = 2.8$$

and $\quad SD(\sigma) = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} = \sqrt{\frac{78}{60} - \left(\frac{-12}{60}\right)^2}$

$= \sqrt{1.3 - 0.04} = \sqrt{1.26} = 1.12$

Hence, the required mean = 2.8 and SD = 1.12

Q12. The mean life of a sample of 60 bulbs are 650 hrs and the standard deviation was 8 hrs. If a second sample of 80 bulbs has a mean life of 660 hrs and the standard deviation 7 hrs, then find the over all standard deviation.

Sol. Given that $n_1 = 60,\ \bar{x}_1 = 650,\ s_1 = 8$
and $n_2 = 80,\ \bar{x}_2 = 660,\ s_2 = 7$

we know that for a combined series.

$$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

$$= \sqrt{\frac{60 \times (8)^2 + 80 \times (7)^2}{60 + 80} + \frac{60 \times 80(650 - 660)^2}{(60 + 80)^2}}$$

$$= \sqrt{\frac{6 \times 64 + 8 \times 49}{14} + \frac{60 \times 80 \times 100}{140 \times 140}}$$

$$= \sqrt{\frac{192 + 196}{7} + \frac{1200}{49}} = \sqrt{\frac{388}{7} + \frac{1200}{49}}$$

$$= \sqrt{\frac{2716 + 1200}{49}} = \sqrt{\frac{3916}{49}} = \frac{62.58}{7} = 8.9$$

Hence, the required SD = 8.9

Q13. If mean and standard deviation of 100 items are 50 and 4 respectively then find the sum of all the items and the sum of the square of items.

Sol. Given that $\bar{x} = 50,\ n = 100$ and SD $(\sigma) = 4$

$$\bar{x} = \frac{\sum x_i}{N} \Rightarrow 50 = \frac{\sum x_i}{100} \Rightarrow \sum x_i = 5000$$

and $\quad$ variance $\sigma^2 = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$

$$(4)^2 = \frac{\sum f_i x_i^2}{100} - (50)^2 \Rightarrow 16 = \frac{\sum f_i x_i^2}{100} - 2500$$

$\therefore \quad \sum f_i x_i^2 = (2500 + 16) \times 100$

$\Rightarrow \quad \sum f_i x_i^2 = 2516 \times 100 = 251600$

Hence, the required sum are 5000 and 251600.

Q14. If for distribution $\sum(x-5)=3, \sum(x-5)^2=43$ and total number of items is 18. Find the mean and standard deviation.

Sol. Given that $n = 18, \sum(x-5)=3, \sum(x-5)^2=43$

$$\therefore \quad \text{Mean} = A + \frac{\sum(x-5)}{n} = 5 + \frac{3}{18} = \frac{93}{18} = 5.166 = 5.17$$

$$\text{and} \quad \text{SD} = \sqrt{\frac{\sum(x-5)^2}{N} - \left[\frac{\sum(x-5)}{N}\right]^2} = \sqrt{\frac{43}{18} - \left(\frac{3}{18}\right)^2}$$

$$= \sqrt{2.39-(0.166)^2} = \sqrt{2.39-0.027} = 1.54$$

Hence, the required mean is 5.17 and SD = 1.54

Q15. Find the mean and variance of the frequency distribution given below

x	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f	6	4	5	1

Sol.

x	f_i	x_i	$f_i x_i$	$f_i x_i^2$
1–3	6	2	12	24
3–5	4	4	16	64
5–7	5	6	30	180
7–10	1	8.5	8.5	72.25
	N = 16		$\sum f_i x_i = 66.5$	$\sum f_i x_i^2 = 340.25$

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{66.5}{16} = 4.15$$

$$\text{Variance } (\sigma^2) = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum fx}{N}\right)^2$$

$$= \frac{340.25}{16} - (4.15)^2 = 21.26 - 17.22 = 4.04$$

Hence, the required mean = 4.15 and variance = 4.04

LONG ANSWER TYPE QUESTIONS

Q16. Calculate the mean deviation about the mean for the following frequency distribution.

Class-interval	0–4	4–8	8–12	12–16	16–20
Frequency	4	6	8	5	2

Sol.

Class-interval	f_i	x_i	$f_i x_i$	$d_i = \lvert x_i - \bar{x} \rvert$	$f_i d_i$
0–4	4	2	8	7.2	28.8
4–8	6	6	36	3.2	19.2
8–12	8	10	80	0.8	6.4
12–16	5	14	70	4.8	24.0
16–20	2	18	36	8.8	17.6
	N = 25		$\sum f_i x_i = 230$		$\sum f_i d_i = 96.0$

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{230}{25} = 9.2$$

and $$\text{Mean deviation} = \frac{\sum f_i d_i}{N} = \frac{96}{25} = 3.84$$

Hence, the required MD = 3.84

Q17. Calculate the mean deviation from the median of the following data:

Class-interval	0–6	6–12	12–18	18–24	24–30
Frequency	4	5	3	6	2

Sol.

Class-interval	f_i	x_i	$c.f.$	$d_i = \lvert x_i - \text{Med} \rvert$	$f_i d_i$
0–6	4	3	4	11	44
6–12	5	9	9	5	25
12–18	3	15	12	1	3
18–24	6	21	18	7	42
24–30	2	27	20	13	26
	N = 20				$\sum f_i d_i = 140$

$$\text{Median class} = \left(\frac{N}{2}\right)\text{th term} = \frac{20}{2}\text{th term} = 10\text{th term i.e. } 12-18$$

$$\therefore \quad \text{Median} = l + \frac{N/2 - cf}{f} \times h$$

$$= 12 + \frac{10-9}{3} \times 6 = 12 + \frac{1}{3} \times 6 = 12 + 2 = 14$$

and $$\text{MD} = \frac{\sum f_i d_i}{N} = \frac{140}{20} = 7$$

Hence, the required MD = 7

Q18. Determine the mean and standard deviation of the following distribution.

Marks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

Sol.

x	f_i	$f_i x_i$	$d_i = x_i - \overline{x}$	$f_i d_i$	$f_i d_i^2$
2	1	2	–4	–4	16
3	6	18	–3	–18	54
4	6	24	–2	–12	24
5	8	40	–1	–8	8
6	8	48	0	0	0
7	2	14	1	2	2
8	2	16	2	4	8
9	3	27	3	9	27
10	0	0	4	0	0
11	2	22	5	10	50
12	1	12	6	6	36
13	0	0	7	0	0
14	0	0	8	0	0
15	0	0	9	0	0
16	1	16	10	10	100
	N = 40	$\sum f_i x_i = 239$		$\sum f_i d_i = -1$	$\sum f_i d_i^2 = 325$

$$\text{Mean } \overline{x} = \frac{\sum f_i x_i}{N} = \frac{239}{40} = 5.9 = 6$$

$$\therefore \quad SD = \sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} = \sqrt{\frac{325}{40} - \left(\frac{-1}{40}\right)^2}$$

$$= \sqrt{8.125 - 0.000625}$$

$$= \sqrt{8.124375} = 2.85$$

Here, the required mean = 6 and MD = 2.85

Q19. The weight of coffee in 70 jars is shown in the following table.

Weight (in g)	Frequency
200–201	13
201–202	27
202–203	18
203–204	10
204–205	1
205–206	1

Determine variance and standard deviation of the above distribution.

Sol.

Class-interval	f_i	x_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
200–201	13	200.5	–2	–26	52
201–202	27	201.5	–1	–27	27
202–203	18	202.5(A)	0	0	0
203–204	10	203.5	1	10	10
204–205	1	204.5	2	2	4
205–206	1	205.5	3	3	9
	N = 70			$\sum f_i d_i = -38$	$\sum f_i d_i^2 = 102$

$$\therefore \quad \text{Variance} = \sigma^2 = \frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2$$

$$= \frac{102}{70} - \left(\frac{-38}{70}\right)^2 = 1.457 - 0.292 = 1.165$$

$$\therefore \quad \text{SD} = \sigma = \sqrt{1.165} = 1.08 \text{ g.}$$

Hence, the required variance = 1.165 and SD = 1.08 g

Q20. Determine mean and standard deviation of first n terms of an A.P. whose first term is a and common difference is d.

Sol.

x_i	$x_i - a$	$(x_i - a)^2$
a	0	0
$a + d$	d	d^2
$a + 2d$	$2d$	$4d^2$
—	—	—
—	—	—
—	—	—
$a + (n-1)d$	$(n-1)d$	$(n-1)^2 d^2$

We know that $\sum x_i = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad \text{Mean} = \frac{\sum x_i}{n} = \frac{1}{n}\left[\frac{n}{2}\{2a + (n-1)d\}\right] = \frac{1}{2}[2a + (n-1)d]$$

$$= a + \frac{n-1}{2}d$$

$$\therefore \quad \sum (x_i - a) = d[1 + 2 + 3 + \cdots + (n-1)] = \frac{d(n-1)n}{2}$$

and $\sum (x_i - a)^2 = d^2[1^2 + 2^2 + 3^2 + \cdots + (n-1)^2]$

$$= d^2 \cdot \frac{n(n-1)(2n-1)}{6}$$

$$\therefore \qquad \sigma = \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n}\right)^2}$$

$$= \sqrt{\frac{d^2 n(n-1)(2n-1)}{6n} - \left(\frac{dn(n-1)}{2n}\right)^2}$$

$$= \sqrt{\frac{d^2 (n-1)(2n-1)}{6} - \frac{d^2 (n-1)^2}{4}}$$

$$= d\sqrt{\frac{n-1}{2}\left(\frac{2n-1}{3} - \frac{n-1}{2}\right)}$$

$$= d\sqrt{\frac{n-1}{2}\left[\frac{4n-2-3n+3}{6}\right]}$$

$$= d\sqrt{\left(\frac{n-1}{2}\right)\left(\frac{n+1}{6}\right)} = d\sqrt{\frac{n^2-1}{12}}$$

Hence, the required SD = $d\sqrt{\frac{n^2-1}{12}}$

Q21. Following are the marks obtained, out of 100, by two students Ravi and Hashina in 10 tests.

Ravi	25	50	45	30	70	42	36	48	35	60
Hashina	10	70	50	20	95	55	42	60	48	80

who is more intelligent and who is more consistent?

Sol. Case I: For Ravi

x_i	$d_i = x_i - 45$	d_i^2
25	– 20	400
50	5	25
45 = A	0	0
30	– 15	225
70	25	625
42	– 3	9
36	– 9	81
48	3	9
35	– 10	100
60	15	225
Total	$\sum d_i = -9$	$\sum d_i^2 = 1699$

$\therefore \qquad \sigma = \sqrt{\frac{\sum d_i^2}{N} - \left(\frac{\sum di}{N}\right)^2}$

$= \sqrt{\frac{1699}{10} - \left(\frac{-9}{10}\right)^2} = \sqrt{169.09} = 13.003$

and $\qquad \overline{x} = A + \frac{\sum di}{N} = 45 - \frac{9}{10} = 44.1$

Now for Hashina

x_i	$d_i = x_i - 55$	d_i^2
10	– 45	2025
70	15	625
50	– 5	25
20	– 35	1225
95	40	1600
55 = A	0	0
42	– 13	169
60	5	25
48	– 7	49
80	25	625
Total	$\sum d_i = -20$	$\sum d_i^2 = 6368$

Assumed mean A = 55

$\therefore \qquad \sigma = \sqrt{\frac{\sum d_i^2}{N}} = \sqrt{\frac{6368}{10}} = 25.2$

and $\overline{x} = A + \frac{\sum d_i}{N} = 55 - \frac{20}{10} = 53$

For Ravi CV $= \frac{\sigma}{\overline{x}} \times 100 = \frac{13.003}{44.1} \times 100 = 29.48$

For Hashina CV $= \frac{\sigma}{\overline{x}} \times 100 = \frac{25.2}{53} \times 100 = 47.55$

Hence, Hashina is more consistent and intelligent.

Q22. Mean and Standard deviation of 100 observations were found to be 40 and 10 respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively. Then find the correct standard deviations.

Sol. Given that $n = 100$, $\overline{x} = 40$, $\sigma = 10$

$$\therefore \quad \bar{x} = \frac{\sum x_i}{N} \Rightarrow 40 = \frac{\sum x_i}{100} \Rightarrow \sum x_i = 4000$$

Corrected $\sum x_i = 4000 - 30 - 70 + 3 + 27 = 3930$

and Corrected mean $= \frac{3930}{100} = 39.3$

Now $\sigma^2 = \frac{\sum x_i^2}{n} - (40)^2 \Rightarrow 100 = \frac{\sum x_i^2}{100} - 1600$

$$\Rightarrow \quad \sum x_i^2 = 1700 \times 100 \Rightarrow \sum x_i^2 = 170000$$

$$\therefore \text{ Corrected } \sum x_i^2 = 170000 - (30)^2 - (70)^2 + (3)^2 + (27)^2$$

$$= 170000 - 900 - 4900 + 9 + 729 = 164938$$

$$\therefore \quad \text{Correct SD} = \sqrt{\frac{164938}{100} - (39.3)^2}$$

$$= \sqrt{1649.38 - 1544.49}$$

$$= \sqrt{104.89} = 10.24$$

Hence, the required SD = 10.24.

Q23. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Sol. Given that $n = 10$, $\bar{x} = 45$ and $\sigma^2 = 16$

$$\therefore \quad \bar{x} = \frac{\sum x_i}{n} \Rightarrow 45 = \frac{\sum x_i}{10} \Rightarrow \sum x_i = 450$$

Corrected $\sum x_i = 450 - 52 + 25$

$= 423$

$\therefore$ Correct Mean $\bar{x} = \frac{423}{10} = 42.3$

and $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \Rightarrow 16 = \frac{\sum x_i^2}{10} - (45)^2$

$$\Rightarrow \quad 16 = \frac{\sum x_i^2}{10} - 2025 \Rightarrow \frac{\sum x_i^2}{10} = 2041$$

$$\therefore \quad \sum x_i^2 = 20410$$

$$\therefore \quad \text{Correct } \sum x_i^2 = 20410 - (52)^2 + (25)^2$$

$$= 20410 - 2704 + 625$$

$$= 18331$$

and corrected variance

$$\sigma^2 = \frac{18331}{10} - (42.3)^2$$
$$= 1833.1 - 1789.3 = 43.8$$

Hence the required mean = 42.3 and variance = 43.8

OBJECTIVE TYPE QUESTIONS

Q24. The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is

(*a*) 2 (*b*) 2.57 (*c*) 3 (*d*) 3.75

Sol. Observations are given by 3, 10, 10, 4, 7, 10 and 5

$$\therefore \quad \bar{x} = \frac{3+10+10+4+7+10+5}{7} = \frac{49}{7} = 7$$

x_i	$d_i = \lvert x_i - \bar{x} \rvert$
3	4
10	3
10	3
4	3
7	0
10	3
5	2
Total	$\sum d_i = 18$

$$\text{MD} = \frac{\sum d_i}{n} = \frac{18}{7} = 2.57$$

Hence, the correct option is (*b*).

Q25. Mean deviation of x observations $x_1, x_2, x_3, \ldots, x_n$ from their mean $\bar{x}$ is given by

(*a*) $\sum_{i=1}^{n}(x_i - \bar{x})$ (*b*) $\frac{1}{n}\sum_{i=1}^{n}\lvert x_i - \bar{x}\rvert$

(*c*) $\sum_{i=1}^{n}(x_i - \bar{x})^2$ (*d*) $\frac{1}{n}\sum_{i=1}^{n}(xi - \bar{x})^2$

Sol. $\text{MD} = \frac{1}{n}\sum_{i=1}^{n}\lvert x_i - \bar{x}\rvert$

Hence, the correct option is (*b*).

Q26. When tested the lines (in hours) of 5 bulbs were noted as follows: 1357, 1090, 1666, 1494, 1623

The mean deviation (in hours) from their mean is

(*a*) 178 (*b*) 179 (*c*) 220 (*d*) 356

Sol. The lines of 5 bulbs are given by

1357, 1090, 1666, 1494, 1623

$$\therefore \quad \text{Mean} = \frac{1357 + 1090 + 1666 + 1494 + 1623}{5}$$

$$\Rightarrow \quad \bar{x} = \frac{7230}{5} = 1446$$

x_i	$d_i = \lvert x_i - \bar{x} \rvert$
1357	89
1090	356
1666	220
1494	48
1623	177
Total	$\sum d_i = 890$

$$\therefore \quad \text{MD} = \frac{\sum d_i}{n} = \frac{890}{5} = 178$$

Hence, the correct option is (*a*).

Q27. Following are the marks obtained by 9 students in a Mathematics test 50, 69, 20, 33, 53, 39, 40, 65, 59.
The mean deviation from the median is
(*a*) 9 (*b*) 10.5 (*c*) 12.67 (*d*) 14.76

Sol. Marks obtained are 50, 69, 20, 33, 53, 39, 40, 65 and 59
Let us write in ascending order
20, 33, 39, 40, 50, 53, 59, 65, 69.
Here $n = 9$

$$\therefore \quad \text{Median} = \frac{9+1}{2}\text{th term} = 5\text{th term i.e. } 50$$

$$\therefore \quad \text{Median} = 50$$

Now

x_i	$d_i = \lvert x_i - \text{Med} \rvert$
20	30
33	17
39	11
40	10
50	0
53	3
59	9
65	15
69	19
Total	$\sum d_i = 114$

$n = 9$ and $\sum d_i = 114$

$$\therefore \text{MD} = \frac{\sum d_i}{n} = \frac{114}{9} = 12.67$$

Hence, the correct option is (*c*).

Q28. The standard deviation of data 6, 5, 9, 13, 12, 8 and 10 is
(*a*) $\sqrt{\frac{52}{7}}$ (*b*) $\frac{52}{7}$ (*c*) $\sqrt{6}$ (*d*) 6

Sol. Given data are 6, 5, 9, 13, 12, 8 and 10

$$\therefore \quad n = 7$$

x_i	x_i^2
6	36
5	25
9	81
13	169
12	144
8	64
10	100
$\sum x_i = 63$	$\sum x_i^2 = 619$

$$\therefore \text{SD} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{619}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{\frac{619}{7} - (9)^2}$$

$$= \sqrt{\frac{619}{7} - 81}$$

$$= \sqrt{\frac{619 - 567}{7}} = \sqrt{\frac{52}{7}}$$

Hence, the correct option is (*a*).

Q29. If $x_1, x_2, \ldots, x_n$ be n observations and $\bar{x}$ be their arithmetic mean. Then, formula for the standard deviations is given by

(*a*) $\sum (x_i - \bar{x})^2$ (*b*) $\dfrac{\sum (x_i - \bar{x})^2}{x}$

(*c*) $\sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n}}$ (*d*) $\dfrac{\sum x_i^2}{n} - (\bar{x})^{-2}$

Sol. The formula for S.D = $\sigma = \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n}}$

Hence, the correct option is (*c*).

Q30. If the mean of 100 observations is 50 and their standard deviation is 5, then the sum of all the squares of all the observations is

(*a*) 50,000 (*b*) 250000 (*c*) 252500 (*d*) 255000

Sol. Here $\bar{x} = \dfrac{\sum x_i}{n}$

$$50 = \frac{\sum x_i}{100} \Rightarrow \sum x_i = 5000$$

$$\therefore \quad \text{SD} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$5 = \sqrt{\frac{\sum x_i^2}{100} - \left(\frac{5000}{100}\right)^2} \Rightarrow 25 = \frac{\sum x_i^2}{100} - 2500$$

$$\Rightarrow \frac{\sum x_i^2}{100} = 2500 + 25 \Rightarrow \frac{\sum x_i^2}{100} = 2525$$

$\therefore \quad \sum x_i^2 = 2525 \times 100 = 252500$

Hence, the correct option is (*c*)

Q31. If *a*, *b*, *c*, *d* and *e* be the observations with mean *m* and standard deviations S, then find the standard deviation of the observations *a* + K, *b* + K, *c* + K, *d* + K and *e* + K is

(*a*) S (*b*) KS (*c*) S + K (*d*) $\frac{S}{K}$

Sol. Given observation are *a*, *b*, *c*, *d* and *e*

$$\therefore \quad \text{Mean} = m = \frac{a+b+c+d+e}{5}$$

$$\therefore \quad \sum x_i = 5m$$

Now mean of *a* + K, *b* + K, *c* + K, *d* + K and *e* + K is

$$= \frac{a+K+b+K+c+K+d+K+e+K}{5}$$

$$= \frac{(a+b+c+d+e)+5K}{5} = \frac{5m+5K}{5} = m+K$$

$$\therefore \quad SD = \sqrt{\frac{\sum(x_i+K)^2}{N} - \left[\frac{\sum x_i + K}{N}\right]^2}$$

$$= \sqrt{\frac{\sum(x_i^2+K^2+2x_iK)}{N} - (m+K)^2}$$

$$= \sqrt{\frac{\sum x_i^2}{N} + \frac{\sum K^2}{N} + \frac{2K\sum x_i}{N} - m^2 - K^2 - 2mK}$$

$$= \sqrt{\frac{\sum x_i^2}{N} + K^2 + 2Km - m^2 - K^2 - 2mK}$$

$$= \sqrt{\frac{\sum x_i^2}{N} - m^2} \qquad \left[\because \frac{\sum x_i}{N} = m\right]$$

$= S$

Hence, the correct option is (*a*)

Q32. If x_1, x_2, x_3, x_4 and x_5 be the observations with mean *m* and standard deviations S then, the standard deviation of the observations Kx_1, Kx_2, Kx_3, Kx_4 and Kx_5 is

(*a*) K + S (*b*) S/K (*c*) KS (*d*) S

Sol. Here $\quad m = \frac{\sum x_i}{N}, \quad S = \sqrt{\frac{\sum x_i^2}{5} - \left(\frac{\sum x_i}{5}\right)^2}$

$$\therefore \qquad \text{SD} = \sqrt{\frac{K^2\sum x_i^2}{5} - \left(\frac{K\sum x_i}{5}\right)^2}$$

$$= \sqrt{\frac{K^2\sum x_i^2}{5} - K^2\left(\frac{\sum x_i}{5}\right)^2}$$

$$= K\sqrt{\frac{\sum x_i^2}{5} - \left(\frac{\sum x_i}{5}\right)^2}$$

$$= K \cdot S$$

Here, the correct option is (*c*).

Q33. Let $x_1, x_2, x_3, \ldots, x_n$ be n observations. Let $w_i = lx_i + k$ for $i = 1, 2, \ldots, n$ where l and k are constants. If the mean of x_i's is 48 and their standard deviation is 12, the mean of w_i's is 55 and standard deviations of w_i's is 15, then the values of l and k should be

(*a*) $l = 1.25, k = -5$ (*b*) $l = -1.25, k = 5$
(*c*) $l = 2.5, k = -5$ (*d*) $l = 2.5, k = 5$

Sol. Given that $w_i = x_i + k, \bar{x}_i = 48, \text{SD}(x_i) = 12,$
$w_i = 55$ and $\text{SD}(w_i) = 15$

then $\bar{w}_i = \bar{x}_i + k$

($\bar{w}_i$ = mean of w_i's and $\bar{x}_i$ is the mean of x_i's)

$\Rightarrow \qquad 55 = 48 + k \qquad (i)$

SD of w_i = SD of x_i

$15 = l \times 12$

$$\Rightarrow \qquad l = \frac{15}{12} = 1.25 \qquad (ii)$$

from eq. (*i*) and (*ii*) we have

$$k = \bar{w}_i - \bar{x}_i = 55 - 1.25 \times 48 = 55 - 60 = -5$$

Here, the correct option is (*a*).

Q34. The standard deviations for first 10 natural numbers is

(*a*) 5.5 (*b*) 3.87 (*c*) 2.97 (*d*) 2.87

Sol. We know that SD of first n natural numbers $\sqrt{\frac{n^2-1}{12}}$

Here $n = 10$

$$\therefore \qquad \text{SD} = \sqrt{\frac{(10)^2-1}{12}} = \sqrt{\frac{99}{12}} = \sqrt{8.25} = 2.87$$

Hence, the correct option is (*d*)

Q35. Consider the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. If 1 is added to each numbers, the variance of the numbers, so obtained is

(*a*) 6.5 (*b*) 2.87 (*c*) 3.87 (*d*) 8.25

Sol. Given numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Numbers obtained when 1 is added to the above numbers is 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11.

$$\therefore \quad \sum x_i = 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11$$

$$= \frac{10}{2}[2 \times 2 + (10 - 1) \cdot 1]$$

$$= 5[4 + 9] = 5 \times 13 = 65$$

$$\text{Now} \quad \sum x_i^2 = 2^2 + 3^2 + 4^2 + \ldots + 11^2$$

$$= (1^2 + 2^2 + 3^2 + 4^2 + \cdots + 11^2) - (1)^2$$

$$= \frac{n(n+1)(2n+1)}{6} - 1 = \frac{11 \times 12 \times 23}{6} - 1$$

$$= 22 \times 23 - 1 = 506 - 1 = 505$$

$$\therefore \quad \text{Variance } (\sigma^2) = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2 = \frac{505}{10} - \left(\frac{65}{10}\right)^2$$

$$= \frac{505}{10} - \left(\frac{65}{10}\right)^2 = 50.5 - (6.5)^2$$

$$= 50.5 - 42.25 = 8.25$$

Hence, the correct option is (*d*).

Q36. Consider the first 10 positive integers. If we multiply each number by – 1 and then add 1 to each number, the variance of the numbers, so obtained is

(*a*) 8.25 (*b*) 6.5 (*c*) 3.85 (*d*) 2.87

Sol. First 10 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

on multiplying each number by – 1, we get

– 1, – 2, – 3, – 4, – 5, – 6, – 7, – 8, – 9, – 10

on adding 1 to each of the number, we get

0, – 1, – 2, – 3, – 4, – 5, – 6, – 7, – 8, – 9

$$\therefore \quad \sum x_i = 0 - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9$$

$$= -45$$

$$\text{and} \quad \sum x_i^2 = 0^2 + (-1)^2 + (-2)^2 + (-3)^2 + (-4)^2 + \cdots + (-9)^2$$

$$= \frac{9 \times 10 \times 19}{6} = 285 \quad \left[\because \quad \sum n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$\therefore \quad \text{SD} = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2} = \sqrt{\frac{285}{10} - \left(\frac{-45}{10}\right)^2}$$

$$= \sqrt{\frac{285}{10} - \frac{2025}{100}} - \sqrt{\frac{2850 - 2025}{100}} = \sqrt{8.25}$$

$\therefore$ Variance = $(\text{SD})^2 = \left(\sqrt{8.25}\right)^2 = 8.25$

Hence, the correct option is (*a*).

Q37. The following information relates to a sample of size 60 $\sum x^2 = 18000$ and $\sum x = 960$, then the variance is

(*a*) 6.63 (*b*) 16 (*c*) 22 (*d*) 44

Sol. We know that variance $(\sigma^2) = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

$$= \frac{18000}{60} - \left(\frac{960}{60}\right)^2 = 300 - 256 = 44$$

Hence, the correct option is (*d*).

Q38. If the coefficient of variation of two distribution are 50, 60 and their Arithmetic means are 30 and 25 respectively, then the difference of their standard deviation is

(*a*) 0 (*b*) 1 (*c*) 1.5 (*d*) 2.5

Sol. Here, we have $CV_1 = 50, CV_2 = 60$

$\bar{x}_1 = 30$ and $\bar{x}_2 = 25$

$\therefore \quad CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100 \Rightarrow 50 = \frac{\sigma_1}{30} \times 100 \Rightarrow \sigma_1 = \frac{50 \times 30}{100} = 15$

and $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100 \Rightarrow 60 = \frac{\sigma_2}{25} \times 100 \Rightarrow \sigma_2 = \frac{60 \times 25}{100} = 15$

$\therefore$ Difference $\sigma_1 - \sigma_2 = 15 - 15 = 0$

Hence, the correct option is (*a*).

Q39. The standard deviations of some temperature data in °C is 5. If the data were converted into °F, then the variance would be

(*a*) 81 (*b*) 57 (*c*) 36 (*d*) 25

Sol. Given that $\sigma_C = 5$

We know that $C = \frac{5}{9}(F - 32) \Rightarrow F = \frac{9C}{5} + 32$

$\therefore \quad \sigma_F = \frac{9}{5}\sigma_C = \frac{9}{5} \times 5 = 9$

$\therefore \quad \sigma_F^2 = (9)^2 = 81$

Hence, the correct option is (*a*)

FILL IN THE BLANKS

Q40. Coefficient of variation = $\frac{-----}{\text{Mean}} \times 100$

Sol. $\text{CV} = \frac{\text{SD}}{\text{Mean}} \times 100$

Hence, the value of the filler is SD.

Q41. If $\overline{x}$ is the mean of n values of x, then $\sum_{i=1}^{n}(x_i - \overline{x})$ is always equal to ________. If a has any value other than $\overline{x}$ then $\sum_{i=1}^{n}(x_i - \overline{x})^2$ is ________ than $\sum(x_i - a)^2$

Sol. If $\overline{x}$ is the mean of n observations of x, then $\sum_{i=1}^{n}(x_i - \overline{x}) = 0$ and if 'a' has the value other than $\overline{x}$, then $\sum_{i=1}^{n}(x_i - \overline{x})^2$ is less than $\sum(x_i - a)^2$.

Hence, the value of the fillers are 0 and less.

Q42. If the variance of a data is 121, then the standard deviations of the data is ________.

Sol. We know that SD = $\sqrt{\text{variance}} = \sqrt{121} = 11$

Hence, the value of the filler is 11.

Q43. The standard deviation of a data is ________ of any change in origin but is ________ of change of scale.

Sol. Since the standard deviation of any data is independent of any change in origin but is dependent of any change of scale.

Hence, the value of the fillers are independent and dependent.

Q44. The sum of squares of the deviations of the values of the variable is ________ when taken about their arithmetic mean.

Sol. The sum of the squares of the deviations of the value of variable is minimum when taken about their arithmetic mean.

Hence, the value of the filler is minimum.

Q45. The mean deviations of the data is ________ when measured from the median.

Sol. The mean deviation of the data is least when measured from the median.

Hence, the value of the filler is least.

Q46. The standard deviations is ________ to the mean deviations taken from the arithmetic mean.

Sol. The standard deviations is greater than or equal to the mean deviation taken from the arithmetic mean.

Hence, the value of the filler is greater than or equal.