## EXERCISE

### SHORT ANSWER TYPE QUESTIONS

**Q1.** If the letters of the word ALGORITHM are arranged at random in a row, what is the probability the letter GOR must remain together as a unit?

**Sol.** Word ALGORITHM has 9 letters.
If GOR remain together, then it will remain together.
$\therefore$ Number of letters = AL$\boxed{\text{GOR}}$ITHM = 6 + 1 = 7
Number of words = 7!
and the total number of words from ALGORITHM = 9!

So, the required probability = $\dfrac{7!}{9!} = \dfrac{7!}{9\cdot 8\cdot 7!} = \dfrac{1}{72}$

Hence, the required probability = $\dfrac{1}{72}$.

**Q2.** Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?

**Sol.** Number of desk occupied by one couple = 1
Only (4 + 1) = 5 persons to be assigned.
$\therefore$ Number of ways of assigning these 5 persons
$= 5! \times 2!$
Total number of ways of assigning 6 persons = 6!
$\therefore$ Probability that a couple has adjacent desk

$$= \frac{5!\times 2!}{6!} = \frac{1}{3}$$

So, the probability that the married couple will have no-adjacent desks = $1 - \dfrac{1}{3} = \dfrac{2}{3}$.

Hence, the required probability = $\dfrac{2}{3}$.

**Q3.** If an integer from 1 through 1000 is chosen at random then find the probability that the integer is a multiple of 2 or a multiple of 9.

**Sol.** We have multiples of 2, from 1 to 1000 are

2, 4, 6, 8, ..., 1000

Let $n$ be the number of terms

$a = 2$, $d = 2$, $a_n = 1000$

$$a_n = a + (n-1)\,d$$
$$1000 = 2 + (n-1)\,2$$
$$\Rightarrow \quad 1000 = 2n \quad \Rightarrow \quad n = 500$$

Now multiple of 9 from 1 to 1000 are

9, 18, 27, ..., 999

Here $a = 9$, $d = 9$ and $a_m = 999$ [$m$ is the number of terms]

$$a_m = a + (m-1)\,d$$
$$\Rightarrow \quad 999 = 9 + (m-1)\,9$$
$$\Rightarrow \quad 999 = 9m \quad \Rightarrow \quad m = 111$$

Now number multiples of 2 and 9 are

18, 36, 54, ..., 990

Here $a = 18$, $a_p = 990$, $d = 18$ [$p$ is the number of terms]

$$\therefore \quad a_p = a + (p-1)\,d$$
$$990 = 18 + (p-1)\,18$$
$$\Rightarrow \quad 990 = 18 + 18p - 18$$
$$\Rightarrow \quad 18p = 990 \quad \Rightarrow \quad p = 55$$

$\therefore$ Number of multiples of 2 or 9 $= 500 + 111 - 55$
$= 556$

$$\therefore \quad \text{Required probability} = \frac{\text{P(E)}}{n(\text{E})} = \frac{556}{1000} = 0.556$$

Hence, the required probability = 0.556.

**Q4.** An experiment consists of rolling a die until a 2 appears.

(*i*) How many elements of the sample space correspond to the event that the 2 appears on the $k$th roll of the die?

(*ii*) How many elements of the sample space correspond to the event that the 2 appears not later than the $k$th roll of the die?

**Sol.** Number of sample space = 6

(*i*) Given that 2 appears on the $k$th roll of the die.

So first $(k-1)$th roll have 5 out comes each and $k$th roll results 2 i.e. only 1 outcome.

$\therefore$ Number of element of sample space correspond to the event that 2 appears on the $k$th roll of the die $= 5^{k-1}$.

(*ii*) In this case, 2 appears not later than $k$th roll of the die, then it is possible that 2 comes in first throw i.e. 1 outcome. If 2 does not appear in first throw, then outcomes will be 5 and 2 outcomes in second throw i.e. 1 outcome.

$\therefore$ Possible out come $= 5 \times 1 = 5$

Similarly, if 2 does not appear in second throw and appears in third throw

$\therefore$ Possible outcome $= 5 \times 5 \times 1$

Now we have the series:

$$= 1 + 5 + 5 \times 5 + 5 \times 5 \times 5 + \ldots + 5^{k-1}$$
$$= 1 + 5 + 5^2 + 5^3 + \ldots + 5^{k-1}$$
$$= \frac{1.(r^k - 1)}{r - 1} = \frac{5^k - 1}{5 - 1}$$
$$= \frac{5^k - 1}{4}$$

Hence, the required answer $= \dfrac{5^k - 1}{4}$.

**Q5.** A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

**Sol.** Given that probability of even numbers

$$= \frac{1}{2} \times \text{Probability of odd numbers}$$

$\Rightarrow$ P(Odd) : P(Even) = 2 : 1

$\therefore$ P(odd number) $= \dfrac{2}{2+1} = \dfrac{2}{3}$

and P(even number) $= \dfrac{1}{2+1} = \dfrac{1}{3}$

Also given that, G the event that a number greater than 3 occurs in a single throw of die.

$\therefore$ The possible outcome are 4, 5 and 6 out of which two are even and one is odd.

$\therefore$ Required probability = P(G)

$$= 2 \times P(\text{even}) \times P(\text{odd})$$
$$= 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}$$

Hence, the required probability is $\dfrac{4}{9}$.

**Q6.** In a large metropolitan area, the probabilities 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set or both kind of sets. What is the probability that a family owns either anyone or both kinds of sets?

**Sol.** Let $E_1$ be the event that a family owns colour television set $E_2$ be the event that the family owns black and white television set.

Given that $P(E_1) = 0.87, P(E_2) = 0.36$

and $P(E_1 \cap E_2) = 0.30$

$\therefore$ The probability that a family owns either colour television set or black and white television set

$$\therefore \quad P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= 0.87 + 0.36 - 0.30$$
$$= 0.93$$

Hence, the required probability = 0.93.

**Q7.** If A and B are mutually exclusive events, P(A) = 0.35 and P(B) = 0.45, then find

(*i*) $P(A')$ (*ii*) $P(B')$ (*iii*) $P(A \cup B)$

(*iv*) $P(A \cap B)$ (*v*) $P(A \cap B')$ (*vi*) $P(A' \cap B')$

**Sol.** Given that P(A) = 0.35 and P(B) = 0.45

Since the events A and B are mutually exclusive then

$P(A \cap B) = 0$

(*i*) $P(A') = 1 - P(A) = 1 - 0.35 = 0.65$

(*ii*) $P(B') = 1 - P(B) = 1 - 0.45 = 0.55$

(*iii*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.35 + 0.45 - 0$
$= 0.80$

(*iv*) $P(A \cap B) = 0$

[$\because$ A and B are mutually exclusive events]

(*v*) $P(A \cap B') = P(A) - P(A \cap B)$
$= 0.35 - 0 = 0.35$

(*vi*) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.80 = 0.20.$

**Q8.** A team of medical students doing their internship have to assist during surgeries at a city hospital. The probability of surgeries rate as very complex, complex, routine, simple or very simple are respectively, 0.15, 0.20, 0.31, 0.26 and 0.08. Find the probabilities that a particular surgery will be rated

(*i*) complex or very complex

(*ii*) neither very complex nor very simple

(*iii*) routine or complex

(*iv*) routine or simple.

**Sol.** Let $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$ be the events that the surgeries are rated as very complex, complex, routine, simple and very simple respectively.

$\therefore P(E_1) = 0.15, P(E_2) = 0.20, P(E_3) = 0.31, P(E_4) = 0.26$
and $P(E_5) = 0.08$

(*i*) P(complex or very complex) = $P(E_1 \text{ or } E_2)$

$$\Rightarrow P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= 0.15 + 0.20 - 0$$
$$= 0.35 \quad [\because \text{ All event are independent}]$$

(*ii*) P(neither very complex nor Very simple) = $P(E_1' \cap E_5')$

$$= P(E_1 \cup E_5)' = 1 - P(E_1 \cup E_5)$$
$$= 1 - [P(E_1) + P(E_5)]$$
$$= 1 - [0.15 + 0.08] = 1 - 0.23 = 0.77$$

(*iii*) P(routine or complex) = $P(E_3 \text{ or } E_2)$

$$= P(E_3 \cup E_2) = P(E_3) + P(E_2)$$
$$= 0.31 + 0.20 = 0.51$$

(*iv*) P(routine or simple) = $P(E_3 \cup E_4) = P(E_3) + P(E_4)$

$$= 0.31 + 0.26 = 0.57$$

**Q9.** Four candidates A, B, C and D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that

(*i*) C will be selected?

(*ii*) A will not be selected?

**Sol.** Given that A is twice as likely to be selected as B

i.e. $P(A) = 2P(B)$

and C is twice as likely to be selected as D

$\therefore \quad P(C) = 2P(D) \quad \Rightarrow \quad P(B) = 2P(D)$

$$\Rightarrow \quad \frac{P(A)}{2} = 2P(D) \quad \Rightarrow \quad P(D) = \frac{1}{4}P(A)$$

Now B and C are given about the same chance

$\therefore \quad P(B) = P(C)$

Since, sum of all probabilities = 1

$\therefore \quad P(A) + P(B) + P(C) + P(D) = 1$

$$\Rightarrow \quad P(A) + \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(A)}{4} = 1$$

$$\Rightarrow \quad 4P(A) + 2P(A) + 2P(A) + P(A) = 4$$

$$\Rightarrow \quad 9P(A) = 4 \quad \Rightarrow \quad P(A) = \frac{4}{9}$$

(*i*) $$P(\text{C will be selected}) = P(C) = P(B) = \frac{P(A)}{2}$$

$$= \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$$

(*ii*) $P(\text{A will not be selected}) = P(A') = 1 - P(A)$

$$= 1 - \frac{4}{9} = \frac{5}{9}$$

**Q10.** One of the four persons John, Rita, Aslam and Gurpreet will be promoted next month. Consequently the sample space consists of four elementary outcomes, S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}. You are told the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as John's. Aslam's chances are four times that of John.

(*i*) Determine P (John promoted)
P (Rita promoted)
P (Aslam promoted)
P (Gurpreet promoted)

(*ii*) If A = {John promoted or Gurpreet promoted}, find P(A).

**Sol.** Let $E_1$, $E_2$, $E_3$ and $E_4$ be the events that John promoted, Rita promoted, Aslam promoted and Gurpreet promoted respectively.

$\therefore$ Sample space $S = \{E_1, E_2, E_3, E_4\}$

Given that probability of John's promotion is same as that of Gurpreet

$$\therefore \quad P(E_1) = P(E_4)$$

Rita's chances of promotion are twice as likely as John

$$\therefore \quad P(E_2) = 2P(E_1)$$

and Aslam's chances of promotion are 4 times that of John

$$\therefore \quad P(E_3) = 4P(E_1)$$

Since, the sum of all the probabilities = 1

$$\therefore \quad P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$$

$$\Rightarrow \quad P(E_1) + 2P(E_1) + 4P(E_1) + P(E_1) = 1$$

$$\Rightarrow \quad 8P(E_1) = 1$$

$$\Rightarrow \quad P(E_1) = \frac{1}{8}$$

(*i*) $P(\text{John promoted}) = P(E_1) = \frac{1}{8}$

$$P(\text{Rita promoted}) = P(E_2) = 2P(E_1) = 2 \times \frac{1}{8} = \frac{1}{4}$$

$$P(\text{Aslam promoted}) = P(E_3) = 4P(E_1) = 4 \times \frac{1}{8} = \frac{1}{2}$$

$$P(\text{Gurpreet promoted}) = P(E_4) = P(E_1) = \frac{1}{8}$$

(*ii*) P(John promoted or Gurpreet promoted) = $P(E_1 \cup E_4)$

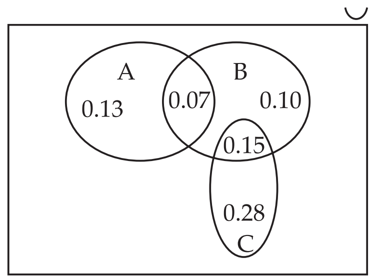
$$\Rightarrow \quad P(E_1 \cup E_4) = P(E_1) + P(E_4) - P(E_1 \cap E_4)$$

$$= \frac{1}{8} + \frac{1}{8} - 0 \qquad [\because P(E_1 \cap E_4) = 0]$$

$$= \frac{1}{4}$$

**Q11.** The accompanying Venn diagram shows three events A, B and C and also the probabilities of the various intersection

[For instance $P(A \cap B) = 0.07$]
Determine:




(*i*) P(A) (*ii*) $P(B \cap \bar{C})$ (*iii*) $P(A \cup B)$
(*iv*) $P(A \cap \bar{B})$ (*v*) $P(B \cap C)$
(*vi*) Probability of exactly one of the three occurs.

**Sol.** From the given Venn diagram

(*i*) $P(A) = 0.13 + 0.07 = 0.20$

(*ii*) $P(B \cap \bar{C}) = P(B) - P(B \cap C)$
$= 0.07 + 0.10 + 0.15 - 0.15$
$= 0.07 + 0.10$
$= 0.17$

(*iii*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.13 + 0.07 + 0.07 + 0.10 + 0.15 - 0.07$
$= 0.13 + 0.07 + 0.10 + 0.15$
$= 0.45$

(*iv*) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
$= 0.13 + 0.07 - 0.07 = 0.13$

(*v*) $P(B \cap C) = 0.15$

(*vi*) P(exactly one of the three occurs) $= 0.13 + 0.10 + 0.28$
$= 0.51$

## LONG ANSWER TYPE QUESTIONS

**Q12.** One urn contain two black balls (labelled $B_1$ and $B_2$) and one white ball. A second urn contain one black ball and two white balls (labelled $W_1$ and $W_2$). Suppose the following experiment is performed. One of the two urns is choosen at random. Next a ball is randomly choosen from the urn. Then a second ball is choosen at random from the same urn without replacing the first ball.

(*i*) Write the sample space showing all possible outcomes.

(*ii*) What is the probability that two black balls are choosen?

(*iii*) What is the probability that two balls of opposite colours are choosen?

**Sol.** Given that one of the two urns is choosen, then a ball is randomly choosen from the urn, then a second ball is choosen at random from the same urn without replacing the first ball

(*i*) Sample space S = $\{B_1B_2, B_1W, B_2B_1, B_2W, WB_1, WB_2, BW_1, BW_2, W_1B, W_1W_2, W_2B, W_2W_1\}$

Total number of Sample space, S = 12

(*ii*) If two black balls are choosen

then the favourable events are $B_1B_2$, $B_2B_1$ i.e. 2

$\therefore$ Required probability = $\dfrac{2}{12} = \dfrac{1}{6}$

(*iii*) If two balls of opposite colours are choosen then, the required probability = $\dfrac{8}{12} = \dfrac{2}{3}$

**Q13.** A bag contain 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:

(*i*) All the three balls are white.

(*ii*) All the three balls are red.

(*iii*) One ball is red and two balls are white.

**Sol.** Given that: Number of red balls = 8

Number of white balls = 5

(*i*) P (all the three balls are white)

$$= \frac{{}^5C_3}{{}^{13}C_3} = \frac{\frac{5!}{3!2!}}{\frac{13!}{3!10!}} = \frac{5!}{3!2!} \times \frac{3!10!}{13!}$$

$$= \frac{5!}{2!} \times \frac{10!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{5 \times 4 \times 3 \times 2!}{2!} \times \frac{1}{13 \times 12 \times 11}$$

$$= \frac{5 \times 4 \times 3}{13 \times 12 \times 11} = \frac{5}{143}$$

(*ii*) P (all the three balls are red)

$$= \frac{{}^8C_3}{{}^{13}C_3} = \frac{\frac{8!}{3!5!}}{\frac{13!}{3!10!}} = \frac{8!}{3!5!} \times \frac{3!10!}{13!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!} \times \frac{10!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{8 \times 7 \times 6}{13 \times 12 \times 11} = \frac{28}{143}$$

(*iii*) P (one ball is red and two balls are white)

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{8 \times 10}{\frac{13!}{3! \times 10!}}$$

$$= \frac{8 \times 10}{13!} \times \frac{3! \times 10!}{1} = \frac{8 \times 10 \times 3 \times 2 \times 10!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{8 \times 10 \times 6}{13 \times 12 \times 11} = \frac{40}{143}$$

**Q14.** If the letters of the word ASSASSINATION are arranged at random. Find the probability that

(*i*) Four S's come consecutively in the word.
(*ii*) Two I's and two N's come together.
(*iii*) All A's are not coming together.
(*iv*) No two A's are coming together.

**Sol.** Total number of word is ASSASSINATION are 13.
Where, we have 3A's, 4S', 2I's, 2N's, 1T's and 1O's.

(*i*) If 4 S's come consecutively in the word, then arrangement may be as follows:

$$\underbrace{\boxed{\text{S S S S}}}_{\text{1 Group}} \quad \underbrace{\text{A A A I I N N T O}}_{\text{9 others}}$$

$\therefore$ Number of words when all S's are together = $\dfrac{10!}{3!2!2!}$

and the total number of word formed from the words

$$\text{ASSASSINATION} = \frac{13!}{3!4!2!2!}$$

$$\therefore \text{Required probability} = \frac{\frac{10!}{3!2!2!}}{\frac{13!}{3!4!2!2!}} = \frac{10!}{3!2!2!} \times \frac{3!4!2!2!}{13!}$$

$$= \frac{10!4!}{13!} = \frac{10! \times 4 \times 3 \times 2}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{2}{143}$$

(*ii*) If 2I's and 2N's come together then there are 10 alphabets
Number of words when 2I's and 2N's are come together

$$= \frac{10!}{3!4!} \times \frac{4!}{2!2!}$$

$$\therefore \text{Required probability} = \frac{\frac{10!}{3!4!} \times \frac{4!}{2!2!}}{\frac{13!}{3!4!2!2!}}$$

$$= \frac{4!10!}{2!2!3!4!} \times \frac{3!4!2!2!}{13!}$$

$$= \frac{4!10!}{13!} = \frac{4 \times 3 \times 2 \times 10!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{4 \times 3 \times 2}{13 \times 12 \times 11} = \frac{2}{143}$$

(*iii*) If all A's are coming together, then three are 11 alphabets
Number of words when all A's come together

$$= \frac{11!}{4!2!2!}$$

$\therefore$ Probability when all A's come together

$$= \frac{\frac{11!}{4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{11!}{4!2!2!} \times \frac{4!3!2!2!}{13!}$$

$$= \frac{11! \times 3!}{13!} = \frac{6}{13 \times 12} = \frac{1}{26}$$

$\therefore$ Required probability when all A's do not come together

$$= 1 - \frac{1}{26} = \frac{25}{26}$$

(*iv*) If no two A's are together, then arranging the alphabets except A's

– S – S – S – S – I – N – T – I – O – N –

Number of ways of arranging all alphabets except A's

$$= \frac{10!}{4!2!2!}$$

There are 11 vacant places between these alphabets.

$\therefore$ 3 A's can be placed in 11 places in ${}^{11}C_3$ ways

$$= \frac{11!}{3!8!}$$

$\therefore$ Total number of words when no two A's together

$$= \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$$

$$\therefore \text{Required probability} = \frac{11! \times 10!}{3!8!4!2!2!} \times \frac{4!3!2!2!}{13!}$$

$$= \frac{10!}{8! \times 13 \times 12} = \frac{10 \times 9 \times 8!}{8! \times 13 \times 12}$$

$$= \frac{10 \times 9}{13 \times 12} = \frac{15}{26}$$

**Q15.** If a card is drawn from a deck of 52 cards, then find the probability of getting a king or a heart or a red card.

**Sol.** Total number of cards = 52

Favourable events = 4 kings + 13 hearts + 26 red – 13 – 2 = 28

$$\therefore \quad \text{Required probability} = \frac{28}{52} = \frac{7}{13}.$$

**Q16.** A sample space consists of 9 elementary outcomes $e_1, e_2, ..., e_9$ whose probabilities are

$P(e_1) = P(e_2) = 0.08$, $P(e_3) = P(e_4) = P(e_5) = 0.1$

$P(e_6) = P(e_7) = 0.2$, $P(e_8) = P(e_9) = 0.07$

Suppose $A = \{e_1, e_5, e_8\}$, $B = \{e_2, e_5, e_8, e_9\}$

(*i*) Calculate P(A), P(B) and $P(A \cap B)$.

(*ii*) Using the addition law of probability, calculate $P(A \cup B)$.

(*iii*) List of composition of the event $A \cup B$ and compute $P(A \cup B)$ by adding the probabilities of the elementary outcomes.

(*iv*) Calculate $P(\overline{B})$ from P(B). Also calculate $P(\overline{B})$ from the elementary outcomes of $\overline{B}$.

**Sol.** Given that: $S = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$

$A = \{e_1, e_5, e_8\}$ and $B = \{e_2, e_5, e_8, e_9\}$

$P(e_1) = P(e_2) = 0.08$

$P(e_3) = P(e_4) = P(e_5) = 0.1$

$P(e_6) = P(e_7) = 0.2$, $P(e_8) = P(e_9) = 0.07$

(*i*) $P(A) = P(e_1) + P(e_5) + P(e_8)$

$= 0.08 + 0.1 + 0.07 = 0.25$

(*ii*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ...(*i*)

But $P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$

$= 0.08 + 0.1 + 0.07 + 0.07$

$= 0.32$

and $P(A \cap B) = \{e_5, e_8\}$

$= P(e_5) + P(e_8) = 0.1 + 0.07 = 0.17$

Putting the values in eq. ($i$) we get

$P(A \cup B) = 0.25 + 0.32 - 0.17$

$= 0.40$

($iii$) $A \cup B = \{e_1, e_2, e_5, e_8, e_9\}$

$\therefore$ $P(A \cup B) = P(e_1) + P(e_2) + P(e_5) + P(e_8) + P(e_9)$

$= 0.08 + 0.08 + 0.1 + 0.07 + 0.07 = 0.40$

($iv$) $P(\overline{B}) = 1 - P(B) = 1 - 0.32 = 0.68$

**Q17.** Determine the probability P, for each of the following events:

($i$) An odd number appears in a single throw of a fair die.

($ii$) Atleast one head appears in two tosses of a fair coin.

($iii$) 4 kings, 9 of hearts or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.

($iv$) The sum of 6 appears in a single toss of a pair of fair dice.

**Sol.** ($i$) Possible outcomes of a single throw of die

S = {1, 2, 3, 4, 5, 6} out of which 1, 3, 5 are odd

$\therefore$ Required probability $= \frac{3}{6} = \frac{1}{2}$

($ii$) When a fair coin is tossed twice, then the sample space

S = {HH, HT, TH, TT}

$\therefore$ Probability of getting atleast one head (HH, HT, TH)

$= \frac{3}{4}$

($iii$) Favourable events are 4 kings + 2 of hearts + 3 of spades

$= 4 + 1 + 1 = 6$

$= \frac{6}{52} = \frac{3}{26}$

($iv$) When a pair of dice is rolled, then total number of sample space = 36 out of which (1, 5), (5, 1), (2, 4), (4, 2) and (3, 3) are the favourable events

$\therefore$ Required probability $= \frac{5}{36}$.

## OBJECTIVE TYPE QUESTIONS

Q18. In a non-leap year, probability of having 53 Tuesday or 53 Wednesday is (*a*) $\frac{1}{7}$ (*b*) $\frac{2}{7}$

(*c*) $\frac{3}{7}$ (*d*) None of these

**Sol.** There are 365 days in a non-leap year and there are 7 days in a week

$$\therefore \quad 365 \div 7 = 52 \text{ weeks} + 1 \text{ day}$$

So, this day may be Tuesday or Wednesday.

So, the required probability = $\frac{1}{7}$.

Hence, the correct option is (*a*).

**Q19.** Three numbers are choosen from 1 to 20. Find the probability that they are not consecutive

(*a*) $\frac{186}{190}$ (*b*) $\frac{187}{190}$ (*c*) $\frac{188}{190}$ (*d*) $\frac{18}{^{20}C_3}$

**Sol.** Set of three consecutive numbers from 1 to 20 are 1, 2, 3; 2, 3, 4; 3, 4, 5; ..., 18, 19, 20.

So, the probability that the numbers are consecutive

$$= \frac{18}{^{20}C_3} = \frac{18}{\frac{20!}{3!17!}} = \frac{18 \cdot 3! \cdot 17!}{20!}$$

$$= \frac{18 \times 3 \times 2 \times 17!}{20 \times 19 \times 18 \times 17!} = \frac{3 \times 2}{20 \times 19} = \frac{3}{190}$$

$\therefore$ P (three numbers are not consecutive)

$$= 1 - \frac{3}{190} = \frac{187}{190}$$

Hence, the correct option is (*b*).

**Q20.** While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be different colours.

(*a*) $\frac{29}{52}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{26}{51}$ (*d*) $\frac{27}{51}$

**Sol.** We know that out of 52 playing cards 26 are of red and 26 are of black colour.

$\therefore$ P(both cards of different colour)

$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$$

$$= 2 \times \frac{26}{52} \times \frac{26}{51} = \frac{26}{51}$$

Hence, the correct option is (*c*).

**Q21.** If seven persons are to be seated in a row, then the probability that two particular persons sit next to each other is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{6}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{1}{2}$

**Sol.** The two particular persons to be seated next each other then, they form one group.

Now the permutation of 6 persons = $6! \times 2!$

and Total number of permutations of 7 persons = $7!$

$$\therefore \text{ Required probability } = \frac{6! \times 2!}{7!}$$

$$= \frac{6! \times 2}{7 \times 6!} = \frac{2}{7}$$

Hence, the correct option is (*c*).

**Q22.** Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

(*a*) $\frac{1}{5}$ (*b*) $\frac{4}{5}$ (*c*) $\frac{1}{30}$ (*d*) $\frac{5}{9}$

**Sol.** Four digit number using the digits 0, 2, 3, 5 with out repetition and divisible by 5 with the given conditions is

If unit place be filled with 0

| | | | |
|---|---|---|---|
| 3 | 2 | 1 | 1 |

Then the number of ways = $3 \times 2 \times 1 \times 1 = 6$

If unit place be filled with 5

| | | | |
|---|---|---|---|
| 2 | 2 | 1 | 1 |

Then the number of ways = $2 \times 2 \times 1 \times 1 = 4$

$\therefore$ Total number of ways = $6 + 4 = 10$

Total number of ways of arranging the digits 0, 2, 3, 5 to form 4-digit numbers without repetition is $3 \times 3 \times 2 \times 1 = 18$

$$\therefore \text{ Required probability } = \frac{10}{18} = \frac{5}{9}$$

Hence, the correct option is (*d*).

**Q23.** If A and B are mutually exclusive events, then

(*a*) $P(A) \leq P(\overline{B})$ (*b*) $P(A) \geq P(\overline{B})$

(*c*) $P(A) < P(\overline{B})$ (*d*) None of these

**Sol.** For mutually exclusive events,

$$P(A \cap B) = 0$$

$$\therefore \quad P(A \cup B) = P(A) + P(B) \qquad [\because P(A \cap B) = 0]$$

$$\Rightarrow \quad P(A) + P(B) \le 1$$

$$\Rightarrow \quad P(A) + 1 - P(\overline{B}) \le 1 \qquad [P(B) = 1 - P(\overline{B})]$$

$$\Rightarrow \quad P(A) - P(\overline{B}) \le 0$$

$$\Rightarrow \quad P(A) \le P(\overline{B})$$

Hence, the correct option is (*a*).

**Q24.** If $P(A \cup B) = P(A \cap B)$ for any two events A and B then,

(*a*) $P(A) = P(B)$ (*b*) $P(A) > P(B)$

(*c*) $P(A) < P(B)$ (*d*) None of these

**Sol.** Given that: $P(A \cup B) = P(A \cap B)$

$$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = P(A \cap B)$$

$$\Rightarrow \quad [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$$

But $\quad P(A) - P(A \cap B) \ge 0 \qquad \ldots(i)$

$[\because P(A \cap B) \le P(A) \text{ or } P(B)]$

and $\quad P(B) - P(A \cap B) \ge 0 \qquad \ldots(ii)$

From eq. (*i*) and (*ii*) we get

$$P(A) = P(B)$$

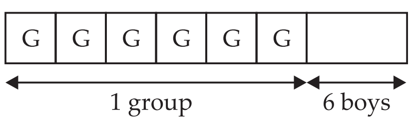
Hence, the correct option is (*a*).

**Q25.** If 6 boys and 6 girls sit in a row at random, then the probability that all the girls sit together is

(*a*) $\dfrac{1}{432}$ (*b*) $\dfrac{12}{431}$

(*c*) $\dfrac{1}{132}$ (*d*) None of these

**Sol.** If all the girls sit together, then we consider it as 1 group




$\therefore$ Total number of arrangement of 6 + 1 = 7 persons in a row = 7! and the girls also interchanged their places with 6! ways.

$$\therefore \quad \text{Required probability} = \frac{6!7!}{12!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 7!}{12 \times 11 \times 10 \times 9 \times 8 \times 7!}$$

$$= \frac{1}{132}$$

Hence, the correct option is (*c*).

**Q26.** If a single letter is selected at random from the word 'PROBABILITY', then the probability that it is a vowel is

(*a*) $\dfrac{1}{3}$ (*b*) $\dfrac{4}{11}$ (*c*) $\dfrac{2}{11}$ (*d*) $\dfrac{3}{11}$

**Sol.** Total number of alphabets in probability = 11

Number of vowels = 4

$\therefore$ Required probability = $\dfrac{4}{11}$

Hence, the correct option is (*b*).

**Q27.** If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

(*a*) $> 0.5$ (*b*) 0.5 (*c*) $\leq 0.5$ (*d*) 0

**Sol.** given that: P(A fails) = 0.2

P (B fails) = 0.3

$$\therefore \quad \text{P(either A or B fails)} \leq \text{P(A fails)} + \text{P(B fails)}$$
$$\leq 0.2 + 0.3$$
$$\leq 0.5$$

Hence, the correct option is (*c*).

**Q28.** The probability that atleast one of the events A and B occurs is 0.6. If A and B occurs simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is equal to

(*a*) 0.4 (*b*) 0.8 (*c*) 1.2 (*d*) 1.6

**Sol.** Given that: $P(A \cup B) = 0.6$

$P(A \cap B) = 0.2$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow \quad 0.6 = P(A) + P(B) - 0.2$$
$$\Rightarrow \quad P(A) + P(B) = 0.6 + 0.2 = 0.8$$
$$\text{and} \quad 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$$
$$\Rightarrow \quad P(\bar{A}) + P(\bar{B}) = 2 - 0.8 = 1.2$$

Hence, the correct option is (*c*).

**Q29.** If M and N are two events, the probability that atleast one of them occurs is

(*a*) $P(M) + P(N) - 2P(M \cap N)$

(*b*) $P(M) + P(N) - P(M \cap N)$

(*c*) $P(M) + P(N) + P(M \cap N)$

(*d*) $P(M) + P(N) + 2P(M \cap N)$

**Sol.** If M and N are any two events, then

$$P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

Hence, the correct option is (*b*).

## TRUE OR FALSE STATEMENTS

**Q30.** The probability that a person visiting a zoo will see the giraffee is 0.72, the probability that he will see the bears is 0.84 and probability that he will see both is 0.52.

**Sol.** Given that:

$$P(\text{to see giraffee}) = 0.72$$
$$P(\text{to see bears}) = 0.84$$
$$P(\text{to see both giraffee and bears}) = 0.52$$

$\therefore$ P(to see giraffee or bear)

$$= P(\text{to see giraffee}) + P((\text{to see bear}) - P(\text{to see both})$$
$$= 0.72 + 0.84 - 0.52$$
$$= 1.04 \text{ which is not possible.}$$

Hence, the given statement is **False**.

**Q31.** The probability that a student will pass his examination is 0.73, the probability of the student getting a compartment is 0.13 and the probability that a student will either pass or get compartment is 0.96.

**Sol.** Let E be the event that the student will pass and F be the event that he will get compartment

$\therefore$ $P(E) = 0.73$, $P(F) = 0.13$ and $P(E \cup F) = 0.96$

$\therefore$
$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$= 0.73 + 0.13 - 0 \qquad [\because P(E \cap F) = 0]$$
$$= 0.86$$

But $P(E \cup F) = 0.96$

Hence, the given statement is **False**.

**Q32.** The probabilities that a typist will make 0, 1, 2, 3, 4 and 5 or more mistakes in typing a report are respectively, 0.12, 0.25, 0.36, 0.14, 0.08 and 0.11.

**Sol.** Sum of all probabilities = 1

$\therefore$ $P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$
$$= 0.12 + 0.25 + 0.36 + 0.14 + 0.08 + 0.11$$
$$= 1.06 > 1$$

Hence, the given statement is **False**.

**Q33.** If A and B are two candidates seeking admission in an engineering college. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.7?

**Sol.** Given that $P(A) = 0.5$ and $P(A \cap B) \le 0.3$

Now $P(A) \times P(B) \le 0.3$

$\Rightarrow$ $0.5 \times P(B) \le 0.3$

$\Rightarrow$ $P(B) \le \dfrac{0.3}{0.5}$

$\Rightarrow$ $P(B) \le 0.6$

Hence, the given statement is **False**.

**Q34.** The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

**Sol.** Here $P(A \cap B) \leq P(A)$

Which is always true.

Hence, given statement is **True**.

**Q35.** The probability of an occurrence of event A is 0.7 and that of the occurrence of event B is 0.3 and the probability of occurrence of both is 0.4.

**Sol.** Here, $P(A) = 0.7$, $P(B) = 0.3$

$$\therefore \quad P(A \cap B) = P(A) \times P(B) = 0.7 \times 0.3 = 0.21$$

But the given probability is 0.4.

Hence, the given statement is **False**.

**Q36.** The sum of probabilities of two students getting distinction in their final examinations is 1.2.

**Sol.** Since, the two given events are not related to the same Sample space.

$\therefore$ The sum of probabilities of two students getting distinction in their final examinations may be 1.2.

Hence, the given statement is **True**.

## FILL IN THE BLANKS

**Q37.** The probability that the home team will win an upcoming football game is 0.77, the probability that it will tie the game is 0.08 and the probability that it will lose the game is ............... .

**Sol.**
$$P(\text{loosing the game}) = 1 - (0.77 + 0.08) = 1 - 0.85 = 0.15$$

Hence, the value of the filler is **0.15**.

**Q38.** If $e_1, e_2, e_3, e_4$ are the four elementary outcomes in a sample space and $P(e_1) = 0.1$, $P(e_2) = 0.5$, $P(e_3) = 0.1$, then the probability of $e_4$ is ............... .

**Sol.** We know that the sum of all probabilities = 1

$$\therefore \quad P(e_1) + P(e_2) + P(e_3) + P(e_4) = 1$$
$$\Rightarrow \quad 0.1 + 0.5 + 0.1 + P(e_4) = 1$$
$$\Rightarrow \quad 0.7 + P(e_4) = 1$$
$$\therefore \quad P(e_4) = 1 - 0.7 = 0.3$$

Hence, the value of the filler is **0.3**.

**Q39.** Let $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$, then $\overline{E}$ is ............... .

**Sol.** Given that: $S = \{1, 2, 3, 4, 5, 6\}$

$E = \{1, 3, 5\}$

$$\therefore \quad \overline{E} = S - E = \{2, 4, 6\}$$

Hence, the value of the filler is **{2, 4, 6}**.

**Q40.** If A and B are two events associated with a random experiment such that P(A) = 0.3, P(B) = 0.2 and $P(A \cap B) = 0.1$ then the value of $P(A \cap \overline{B})$ is ............... .

**Sol.** Given that: P(A) = 0.3, P(B) = 0.2

$$P(A \cap B) = 0.1$$

$$\therefore \quad P(A \cap \overline{B}) = P(A) - P(A \cap B)$$
$$= 0.3 - 0.1 = 0.2$$

Hence, the value of the filler is **0.2**.

**Q41.** The probabilities of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is ............... .

**Sol.** Given that: $P(A) = 0.5, P(B) = 0.3$

and $P(A \cap B) = 0$

[$\because$ A and B are mutually exclusive events]

$$\therefore \quad P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$$
$$= 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B)] = 1 - (0.5 + 0.3)$$
$$= 1 - 0.8 = 0.2$$

Hence, the value of the filler is **0.2**.

**Q42.** Match the proposed probability under Column $C_1$ with the appropriate written description under Column $C_2$.

| | $C_1$ **Probability** | | $C_2$ **Written description** |
|---|---|---|---|
| (*a*) | 0.95 | (*i*) | an incorrect assignment |
| (*b*) | 0.02 | (*ii*) | No chance of happening |
| (*c*) | – 0.3 | (*iii*) | As much chance of happening as not |
| (*d*) | 0.5 | (*iv*) | Very likely to happen |
| (*e*) | 0 | (*v*) | Very little chance of happening |

**Sol.** (*i*) 0.95 = Very likely to happen, so it is close to 1.

(*ii*) 0.02 = Very little chance of happening as the probability is very low.

(*iii*) – 0.3 = an incorrect assignment because probability is never negative.

(*iv*) 0.5 = as much chance of happening as not because sum of chances of happening and not happening is one.

(*v*) 0 = no chance of happening.

Hence, (*a*) $\leftrightarrow$ (*iv*), (*b*) $\leftrightarrow$ (*v*), (*c*) $\leftrightarrow$ (*i*), (*d*) $\leftrightarrow$ (*iii*), (*e*) $\leftrightarrow$ (*ii*)

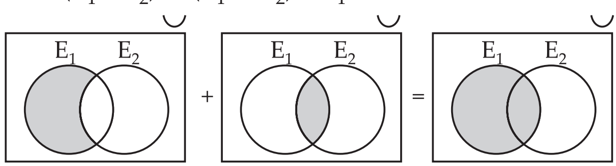
**Q43.** Match the following:

| | | | |
|---|---|---|---|
| (*a*) | If $E_1$ and $E_2$ are the two mutually exclusive events | (*i*) | $E_1 \cap E_2 = E_1$ |
| (*b*) | If $E_1$ and $E_2$ are the mutually exclusive and exhaustive events | (*ii*) | $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$ |
| (*c*) | If $E_1$ and $E_2$ have common outcomes | (*iii*) | $E_1 \cap E_2 = \phi$, $E_1 \cup E_2 = S$ |
| (*d*) | If $E_1$ and $E_2$ are two events such that $E_1 \subset E_2$ | (*iv*) | $E_1 \cap E_2 = \phi$ |

**Sol.** (*a*) If $E_1$ and $E_2$ are mutually exclusive events, then $E_1 \cap E_2 = \phi$.

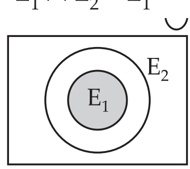
(*b*) If $E_1$ and $E_2$ are mutually exclusive and exhaustive events then $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$.

(*c*) If $E_1$ and $E_2$ have common outcomes, then
$(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$




$E_1 - E_2$ $\quad$ $E_1 \cap E_2$ $\quad$ $E_1$

(*d*) If $E_1$ and $E_2$ are two events such that
$E_1 \subset E_2 \Rightarrow E_1 \cap E_2 = E_1$




Hence, (*a*) $\leftrightarrow$ (*iv*), (*b*) $\leftrightarrow$ (*iii*), (*c*) $\leftrightarrow$ (*ii*), (*d*) $\leftrightarrow$ (*i*)