

MATHEMATICS

Time : 3 hrs.]

Class XI

[M.M. : 80

General Instructions—

- (i) This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- (ii) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- (iii) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- (iv) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- (v) Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- (vi) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A

1. The number of non-empty subsets of the set $\{1, 2, 3, 4\}$ is : 1
(a) 14 (b) 16
(c) 17 (d) 15
2. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are : 1
(a) 7, 4 (b) 6, 4
(c) 3, 3 (d) 6, 3
3. The domain of function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sqrt{x^2 - 3x + 2}$ is : 1
(a) $[2, \infty]$ (b) $(-\infty, 1] \cup [2, \infty)$
(c) $(-\infty, 1]$ (d) $[1, 2]$

P. T. O.

4. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is : 1
- (a) m^n (b) $m^n - 1$
 (c) $n^m - 1$ (d) $2^{mn} - 1$
5. The range of the function $f(x) = \frac{x}{|x|}$ is : 1
- (a) $\{-1, 1\}$ (b) $\mathbb{R} - \{0\}$
 (c) $\mathbb{R} - \{-1, 1\}$ (d) $\{-1, 2\}$
6. $\sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = ?$ 1
- (a) $2 \sin \theta$ (b) $2 \cos \theta$
 (c) $\sin 2\theta$ (d) $\cos 2\theta$
7. $\cos 18^\circ = ?$ 1
- (a) $\frac{(2\sqrt{5} + 1)}{4}$ (b) $\frac{(\sqrt{5} + 1)}{4}$
 (c) $\frac{\sqrt{10 + 2\sqrt{5}}}{4}$ (d) $\frac{\sqrt{10 - 2\sqrt{5}}}{4}$
8. $(3 \sin 40^\circ - 4 \sin^3 40^\circ) = ?$ 1
- (a) $\frac{\sqrt{3}}{2}$ (b) $3\sqrt{3}$
 (c) $2\sqrt{3}$ (d) $\frac{3}{2}$
9. $\cot 105^\circ - \tan 105^\circ = ?$ 1
- (a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$
 (c) $2\sqrt{3}$ (d) $\frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)}$

10. $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = ?$

1

(a) $2 \sin x$

(b) $2 \cos x$

(c) $\sqrt{2} \cos x$

(d) $\sqrt{2} \sin x$

11. If $\cos \theta = \frac{4}{5}$ and $\cos \phi = \frac{12}{13}$, where θ and ϕ both lie in quadrant IV, then $\sin(\theta + \phi) = ?$

1

(a) $\frac{16}{65}$

(b) $\frac{-16}{65}$

(c) $\frac{-33}{65}$

(d) $\frac{-56}{65}$

12. If $(x+iy) = \left(\frac{a+ib}{c+id}\right)$ then $(x^2+y^2) = ?$

1

(a) $\frac{(a^2 - b^2)}{(c^2 - d^2)}$

(b) $\frac{(a^2 + b^2)}{(c^2 + d^2)}$

(c) $\frac{(a^2 - b^2)}{(c^2 + d^2)}$

(d) $\frac{(a^2 + b^2)}{(c^2 - d^2)}$

13. Mark the correct answer for $(1+2i)^{-2} = ?$

1

(a) $\left(\frac{-3}{25} - \frac{4}{25}i\right)$

(b) $\left(\frac{-3}{25} + \frac{4}{25}i\right)$

(c) $\left(\frac{6}{25} - \frac{4}{25}i\right)$

(d) $\left(\frac{3}{25} - \frac{4}{25}i\right)$

14. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

1

(a) $x = 4n$

(b) $x = 2n$

(c) $x = 2n+1$

(d) $x = 4n+1$, where $n \in \mathbb{N}$

15. Mark the correct answer for $\frac{(3-5i)}{(-2+3i)} = ?$

(a) $\left(\frac{21}{13} - \frac{3}{13}i\right)$

(b) $\left(\frac{-21}{13} + \frac{1}{13}i\right)$

(c) $\left(\frac{21}{13} - \frac{1}{13}i\right)$

(d) $\left(\frac{21}{13} + \frac{1}{13}i\right)$

16. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.

(a) (3, 5), (5, 7), (7, 9)

(b) (6, 8), (8, 10), (10, 12)

(c) (8, 6), (6, 4), (10, 6)

(d) (4, 6), (6, 8), (8, 10)

17. If ${}^{n+1}C_3 = 2 \cdot {}^nC_2$, then $n =$

(a) 3

(b) 4

(c) 5

(d) 6

18. Every body in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is :

(a) 13

(b) 11

(c) 14

(d) 12

19. Find r if ${}^{10}P_r = 2 \cdot {}^9P_r$:

(a) 6

(b) 4

(c) 3

(d) 5

20. $\sum_{r=0}^n 4^r \cdot {}^nC_r$ is equal to :

(a) 6^n

(b) 5^{-n}

(c) 4^n

(d) 5^n

SECTION-B

21. Find the domain and range of the relation R given by :

$$R = \{(x, y) : y = x + \frac{6}{x}; \text{ where } x, y \in \mathbb{N} \text{ and } x < 6\}$$

OR

If $y = f(x) = \frac{1-x}{1+x}$, then show that $x = f(y)$.

22. If $\sec \theta = \frac{-4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find the values of all the other five trigonometric functions. 2

23. Express the complex number $\left(\frac{1}{3} + 3i\right)^3$ in the form of $a + ib$. 2

OR

If $f(x) = 3x^3 - 5x^2 + 10$, find $f(x-1)$

24. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements, do the vowels never occur together. 2

25. Evaluate— 2

$$\left(x^2 - \sqrt{1-x^2}\right)^4 + \left(x^2 + \sqrt{1-x^2}\right)^4$$

SECTION-C

26. Find the domain and the range of the function given below— 3

$$f(x) = |x - 1|$$

27. If $\sin \theta = \frac{3}{5}$ and $\cos \phi = \frac{-12}{13}$, where θ and ϕ both lie in the second quadrant, find the values of : 3

(i) $\sin(\theta - \phi)$

(ii) $\cos(\theta + \phi)$

(iii) $\tan(\theta - \phi)$

28. Find the square root of $-15 - 8i$. 3

OR

Determine the domain and range of the relation R defined by $R = \{(x, x+5) : x \in (0, 1, 2, 3, 4, 5)\}$

29. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second? 3

OR

Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ in the $\tan x = -\frac{4}{3}$, x in quadrant II.

30. In, how many ways can the letters of the word PERMUTATIONS be arranged if the
- words start with P and end with S 3
 - vowels are all together
 - there are always 4 letters between P and S?

OR

Prove that—

$$\sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right) = \frac{1}{\sqrt{2}} \sin x$$

31. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively. 3

SECTION-D

32. (i) Let R be the relation on the set Z of all integers defined by $R = \{(x, y) : x - y \text{ is divisible by } n\}$. Prove that— 5
- $(x, y) \in R$
 $\Rightarrow (y, x) \in R$ for all $x, y \in Z$.
 - $(x, y) \in R$ and $(y, z) \in R$
 $\Rightarrow (x, z) \in R$ for all $x, y, z \in Z$.

(ii) Find the domain and range of the function $f(x) = \frac{x^2 - 9}{x - 3}$

(iii) Find the domain of the function $f(x) = \frac{x^2 + 3x + 5}{x^2 + x - 6}$

33. Prove that : $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$ 5

OR

Prove that—

$$\cos 2x \cdot \cos \frac{x}{2} - \cos 3x \cdot \cos \frac{9x}{2} = \sin 5x \cdot \sin \frac{5x}{2}$$

34. Find the modulus and argument of $z = \frac{(1+i)^{13}}{(1-i)^7}$ 5

35. A group consist of 4 girls and 7 boys. In how many ways, a team of 5 members be selected, if the team has :

- (i) no girl
- (ii) at least one boy and one girl
- (iii) at least 3 girl

OR

In an examination, a question paper consists of 12 questions divided into two parts i.e., part I and part II containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

SECTION-E

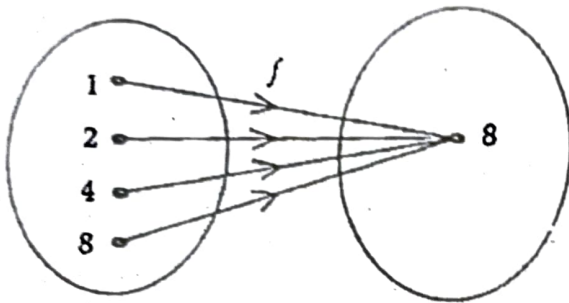
36. Read the text carefully and answer the questions— 4

A Relation R from A to B can be depicted pictorially using arrow diagram. In arrow diagram, we write down the elements of two sets A and B in two disjoint circles. Then we draw arrow from set A to set B whenever $(A, B) \in R$. An example of information depicted through an arrow diagram is shown below. For example: A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provides ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 to the people who work in the categories A, C,

Which among the above figures shows a Relation between the two non-empty sets?

- (a) A, B, C (b) B, C
 (c) A, C (d) A, B

(4)



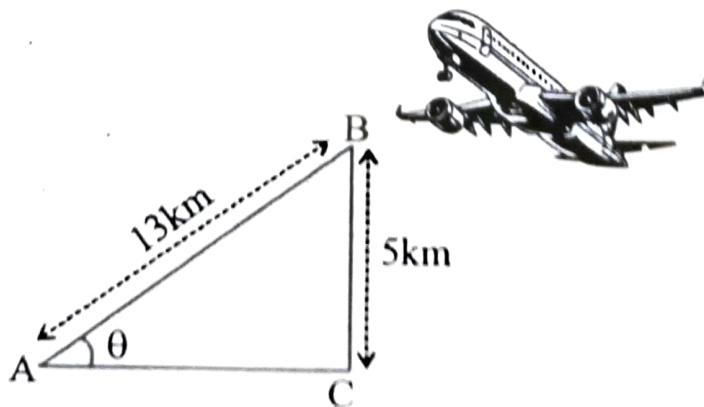
The relation defined in the above arrow diagram from set A to set B is:

- (a) x is a factor of y (b) $x = 2y$
 (c) x is a square of y (d) x is a square root of y

37. Read the text carefully and answer the questions—

4

An airplane is observed to be approaching a point that is at a distance of 13 km from the point of observation and makes an angle of elevation of θ and the height of the airplane above the ground is 5 km. Based on the above information answer the following questions.



- (a) Find the value of $\sin 2\theta$.
 (b) Find the value of $\cos 2\theta$.
 (c) Find the value of $\sin\left(\frac{\theta}{2}\right)$

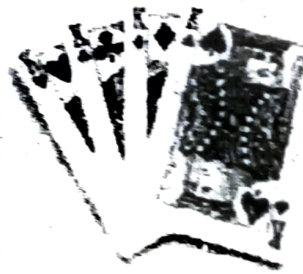
(d) Find the value of $\cos\left(\frac{\theta}{2}\right)$

38. Read the text carefully and answer the questions—

4

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards in valid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on.



Four cards are drawn from a pack of 52 playing cards, then:

- In how many ways can four cards be drawn from a pack of 52 playing cards?
- In how many ways can four cards be drawn from a pack of 52 playing cards such that all 4 cards are from the same suit?
- In how many ways can four cards be drawn from a pack of 52 playing cards such that 2 cards are Aces?
- In how many ways can four cards be drawn from a pack of 52 playing cards such that all are club cards?