First Terminal Examination 2014-2015
Mathematics
Class-XI

Time : 3 Hours   **Maximum Marks : 100**

General Instructions :

  (i)   *All questions are compulsory.*

  (ii) *The question paper consists of 26 questions divided into three Sections A, B, C.*

  (iii) *Section A comprises 6 questions of 1 mark each, Section B comprises of 13 questions of 4 marks each and Section C comprises 7 questions of 6 marks each.*

1. Write the number of non-empty subsets of set $A = \{1,2,3,4\}$.

2. Let $A = \{1,2\}$, $B = \{3,4\}$. Find number of relations from A to B.

3. Find domain and range of relation R defined by $R = \{(x, x+5) : x \in (0,1,2,3,4,5)\}$

4. Solve $x^2 + x + \dfrac{1}{\sqrt{2}} = 0$.

5. If $^nC_9 = {}^nC_8$, find $^nC_{17}$.

6. Find the general term in the expansion of $\left(x^2 + \dfrac{1}{x^2}\right)^{13}$.

7. Let $A = \{a,b,c\}, B = \{c,d\}, C = \{d,e,f\}$

    Find $A \times (B \cap C)$ and $(A \times B) \cup (A \times C)$.

8. Find domain and range of the real function $f(x) = \sqrt{16 - x^2}$.

9. Solve $2\cos^2 x + 3\sin x = 0$.

<div align="center">OR</div>

    Solve $\sin 2x - \sin 4x + \sin 6x = 0$.

10. Prove that $\cos 20° \cos 40° \cos 60° \cos 80° = \dfrac{1}{16}$

<div align="center">OR</div>

    Prove that $\sin A \sin(60 - A)\sin(60 + A) = \dfrac{1}{4}\sin 3A$.

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11. If $\sin x = \dfrac{3}{5}$, $\cos y = -\dfrac{12}{13}$, where x and y both lie in IInd quadrant, find the value of $\cos(x+y)$.

12. If $x - iy = \sqrt{\dfrac{a-ib}{c-id}}$, prove that

$$(x^2+y^2)^2 = \dfrac{a^2+b^2}{c^2+d^2}.$$

13. If α and β are different complex numbers with $|\beta| = 1$, then find the value of $\left|\dfrac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$.

14. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

15. How many words with or without meaning, each of 3 Vowels and 2 Consonants can be formed from the letters of the word "INVOLUTE".

OR

Find the number of arrangements of the letters of the word "INDEPENDENCE".

In how many of these arrangments do the vowels never occur together.

16. What is the number of ways of choosing 4 cards from a pack of 52 playing cards. In how many of these, four cards are of the same suit.

17. Using Binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

18. If the coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal, find r.

19. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4. Use principle of mathematical induction.

20. In a survey of 60 people it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspaper. Find

(i) the number of people who read atleast one of the newspaper,

(ii) the number of people who read exactly one newspaper.

Do you think reading of newspaper enhances our knowledge? Explain in brief.

21. In a $\triangle ABC$, prove that :

   (i) $\tan \dfrac{B-C}{2} = \dfrac{b-c}{b+c} \cot \dfrac{A}{2}$.

   (ii) $\dfrac{\sin(B-C)}{\sin(B+C)} = \dfrac{b^2 - c^2}{a^2}$.

22. A tree stands vertically on a hill side which makes an angle of 15° with the horizontal. From a point on the ground 35 m down the hill from the base of the tree, the angle of elevation of the top of the tree is 60°, find the height of the tree.

23. Prove by using principle of mathematical induction that :

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \frac{1}{10.13} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

24. Solve the following system of linear inequalities graphically,

$$x + 2y \le 10, \quad x + y \ge 1, \quad x - y \le 0, \quad x \ge 0, \quad y \ge 0.$$

25. The coefficient of the $(r-1)^{\text{th}}$, rth and $(r+1)^{\text{th}}$ terms in expansion of $(x+1)^n$ are in the ratio $1 : 3 : 5$. Find x and r.

26. A group consists of 4 girls and 7 boys in how many ways can a team of 5 members be selected if the team has (i) no girl, (ii) at least one boy and one girl, (iii) at least 3 girls.