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First Terminal Examination 2015 - 2016

Class - XI  
Subject - Mathematics

Max. Marks : 100

Time : 3 Hours

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 26 questions.
3. This question paper is divided into three sections A, B and C.
4. Section A comprises of 6 questions of 1 mark each
5. Section B comprises of 13 questions of 4 marks each
6. Section C comprises of 7 questions of 6 marks each
7. There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
8. Use of calculators is not permitted.

SECTION - 'A'

(1×6=6)

1. If R is a relation from a set A having 3 elements to a set B having 2 elements, then find the number of relations from A to B.
2. Write down the power set of the set  $C = \{1, \{2\}\}$ .
3. The cost and revenue functions of a product are given by  $C(x) = 20x + 4000$  and  $R(x) = 60x + 2000$  respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?
4. If  $f(x + 1) = 3x + 7$ , then find  $f(3x)$ .
5. Express in the form  $a + ib$  :

$$i^{35} + \frac{1}{i^{35}}$$

Find the radian measure corresponding to following degree measure :

$$-47^{\circ}30'$$

SECTION - 'B'

7. Find the domain and range of the following function :

$$f(x) = \sqrt{16 - x^2}$$

8. Prove that :  $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

OR

Prove that :  $4\cos 12^\circ \cos 48^\circ \cos 72^\circ = \cos 36^\circ$

9. Using principle of mathematical induction, prove the following for all  $n \in \mathbb{N}$ .

$$1.3 + 2.3^2 + 3.3^3 + \dots + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$

10. Solve the following system of inequalities graphically.

$$2x + 3y \leq 6, \quad x \leq 2y, \quad x \geq 0, \quad y \geq 0$$

11. If A and G are A.M. and G.M. between two positive numbers, prove that the numbers are  $A \pm \sqrt{A^2 - G^2}$ .

OR

If a, b, c are in A.P.; b, c, d are in G.P. and  $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$  are in A.P., prove that a, c, e are in G.P.

12. Find the conjugate of :

$$\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$$

Find the general solution of the equation  $\sin 2x - \sin 4x + \sin 6x = 0$ .

Let A and B be two sets. If  $A \cap X = B \cap X = \phi$  and  $A \cup X = B \cup X$  for some set X, show that  $A = B$ .

OR

A and B are two sets such that  $n(A - B) = 14 + x$ ,  $n(B - A) = 3x$  and  $n(A \cap B) =$  Draw a Venn diagram to illustrate this information. If  $n(A) = n(B)$ , find :

(a) the value of x

(b)  $n(A \cup B)$

15. Convert the complex number  $z = \frac{i-1}{\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3})}$ , in the polar form.

Handwritten calculations for question 15:  
$$\begin{array}{r} 12 \\ + 17 \\ + 10 \\ \hline 20 \\ \hline 15 \\ + 15 \\ \hline 30 \\ \hline 96 \end{array}$$

OR

Find the square root of the complex number  $(-3 - 4i)$ .

16. Let R be the relation on the set Z of all integers defined by  $(x, y) \in R \Rightarrow x - y$  is divisible by n. Prove that :

- (a)  $(x, x) \in R$  for all  $x \in Z$
- (b)  $(x, y) \in R \Rightarrow (y, x) \in R$  for all  $x, y \in Z$
- (c)  $(x, y) \in R$  and  $(y, z) \in R \Rightarrow (x, z) \in R$  for all  $x, y, z \in Z$ .

17. Let  $x = 1 + a + a^2 + \dots$  and  $y = 1 + b + b^2 + \dots$ , where  $|a| < 1$  and  $|b| < 1$ . Prove that  $1 + ab + a^2b^2 + \dots = \frac{xy}{x+y-1}$ .

18. If  $a \cos 2\theta + b \sin 2\theta = c$  has  $\alpha$  and  $\beta$  as its roots, then prove that :

$$\tan \alpha + \tan \beta = \frac{2b}{a+c}$$

19. If  $A = \{x : x \in W, x < 3\}$ ,  $B = \{x : x \in N, 2 \leq x < 4\}$  and  $C = \{3, 4\}$ , then verify that,  $(A \cup B) \times C = (A \times C) \cup (B \times C)$ .

SECTION - 'C'

(6x7=42)

Handwritten scribbles and numbers: 5, 2, 1

20. A T.V. survey gives the following data for T.V. watching : 60% watch program A; 50% watch program B; 47% watch program C; 28% watch program A and B; 23% watch programs A and C; 18% watch programs B and C; 8% watch programs A, B and C. Draw a Venn diagram to illustrate this information and find :

Handwritten numbers: 50, 22, 2

- (a) What percentage watch programs A and B but not C?
- (b) What percentage watch exactly two programs?
- (c) What percentage do not watch any program?

Handwritten calculations for question 20:  
$$\begin{array}{r} 60 \\ 50 \\ 47 \\ \hline 157 \end{array}$$

(d) Do you think to some extent, parents should monitor T.V. viewing habit children? If yes, then why?

Handwritten number: 3

Handwritten calculations for question 20(d):  
$$\begin{array}{r} 28 \\ + 23 \\ + 18 \\ \hline 69 \\ \hline 188 \end{array}$$



21. In any triangle ABC, prove that :

$$\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \cos\left(\frac{A}{2}\right)$$

OR

A lamp-post is situated at the middle point M of the side AC of a triangular plot ABC with BC = 7 m, CA = 8 m and AB = 9 m. Lamp-post subtends an angle of 15° at the point B. Determine the height of the lamp-post.

22. Find the sum of the following series up to n terms :

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$$

OR

If  $a_1, a_2, a_3, \dots, a_n$  are in A.P., where  $a_i > 0$  for all i, show that :

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

23. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added ?

Using principle of mathematical induction, prove the following for all  $n \in \mathbb{N}$

$n(n + 1)(n + 5)$  is a multiple of 3.

If  $\tan x = \frac{3}{4}$ ,  $\pi < x < \frac{3\pi}{2}$ , find the value of  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$ .

The sum of n terms of two arithmetic progressions are in the ratio

$$(3n + 8) : (7n + 15).$$

Find the ratio of their 12<sup>th</sup> terms.

Handwritten calculation: 
$$\begin{array}{r} 64 \\ \times 8 \\ \hline 512 \end{array}$$

Handwritten calculation: 
$$\begin{array}{r} 4 \cancel{1} 20 \\ \times 60 \\ \hline 2400 \\ + 2400 \\ \hline 4800 \end{array}$$