

## HALF YEARLY EXAMINATION (2015-16)

CLASS – XI

SUBJECT – MATHEMATICS

Max. Time: 3 Hrs.

M.M.: 100

General Instruction:

1. All Questions are compulsory
2. The question paper consists of 26 questions divided into three sections – A, B, C. Section A comprises of 6 questions of 1 mark each. Section B comprises of 13 questions of 4 marks each. Section C comprises of 7 questions of 6 marks each.
3. There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
4. Use of calculators is not permitted.

### Section – A

- ✓ 1. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both car A and car B. Is this data correct?
- ✓ 2. If  $A = \{1, 2, 3, 4, 6\}$  and  $R$  be a relation on  $A$  defined by  $\{(a, b) : a, b \in A, \text{ and } b \text{ is exactly divisible by } a\}$ , then find the range of  $R$ .
- ✓ 3. If  $P(n)$  is the statement “ $n(n+1)(n+2)$  is an integral multiple of 12”, then state whether  $P(5)$  is true or not?
- ✓ 4. Find the multiplicative inverse of  $(2 + \sqrt{3}i)^2$ .
- ✓ 5. Find the value of  $\cos(-1710^\circ)$ .
- ✓ 6. State whether the following pair of statements are negations of each other:  
 $p$ : The number  $x$  is not a rational number.  
 $q$ : The number  $x$  is not an irrational number.

### Section – B

- ✓ 7. Show that for any sets  $A$  and  $B$   
 $A = (A \cap B) \cup (A - B)$  and  $A \cup (B - A) = A \cup B$
- ✓ 8. Find the Domain and range of the function defined by  $f(x) = \frac{1}{\sqrt{x^2 - 16}}$ .
- ✓ 9. Draw the graph of the real function defined by  $f(x) = |x - 2|$ ,  $-1 < x \leq 5$ .
- ✓ 10. Prove by using Principle of Mathematical Induction :  

$$1.3 + 3.5 + 5.7 + \dots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$$
- ✓ 11. Shanta is three times of the age of her son. She has a grandson who is half of the age of her son and her granddaughter's age is seven years more than one third of the age of her son. Construct an inequation to represent the above fact considering the grandson is elder than the granddaughter. Then find the minimum age of Shanta's son.

Now, whenever Shanta goes to market she brings similar toys, clothes and eatables for both of them. **What values do you get from this information?**

✓ 12. Prove that  $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$ .

Or,

Prove that  $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$ .

✓ 13. Prove that :  $\tan 4x = \frac{4 \tan x(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$ .

✓ 14. Find real values of  $\theta$  such that  $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$  is purely real.

✓ 15. Solve :  $9x^2 - 12x + 29 = 0$ .

✓ 16. a) Identify the type of "Or" used in the following statements and check whether the statement is true or false:  $\sqrt{2}$  is a rational number or an irrational number.

b) Write the converse and contrapositive of the following statement:

"If you do not work hard then you will fail in the test"

✓ 17. Find the coordinate of the point R with x- coordinate 4, which lies on the line segment joining the points P(2,-3,4) and Q(8,0,10).

Or,

The centroid of triangle ABC is at the point (1,1,1). If the coordinate of A and B are (3,-5,7) and (-1,7,-6) respectively, find the coordinate of the point C.

✓ 18. Find the equation of the locus of points P, the sum of whose distances from the points A(4, 0,0) and B(-4,0,0) is equal to 10.

✓ 19. Solve any one of the following linear inequation and represent the solution on the number line.:

$$\frac{x-2}{x+5} > 2$$

OR

$$\left| \frac{3x-4}{2} \right| \leq \frac{5}{12}$$

### Section - C

✓ 20. a) In a university, out of 100 students 15 offered Mathematics only; 12 offered Statistics, 8 offered only physics; 40 offered Physics and Mathematics; 20 offered Physics and Statistics; 10 offered Mathematics and Statistics. 65 offered Physics. By drawing Venn-diagram, find the number of students who i) offered Mathematics, ii) offered Statistics, iii) did not offer any of the above three subjects, iv) what percentage offered exactly two subjects?

b) If  $A = [-1, 1)$  and  $B = [0, 4]$  then find i)  $A'$ , ii)  $A \cap B$ , (4+2)



21. a) Let  $R$  be a relation on  $Q$  defined by  $R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$ . Verify the following statements are true or not i)  $(a, a) \in R$  for all  $a \in Q$  ii)  $(a, b) \in R \Rightarrow (b, a) \in R$ , iii)  $(a, b) \in R$  and  $(b, c) \in R \Rightarrow (a, c) \in R$ .

b) Let  $f$  be the subset of  $Z \times Z$  defined by  $f = \{(ab, a + b) : a, b \in Z\}$ . Is  $f$  a function from  $Z \rightarrow Z$ ? Justify your answer with examples. (3+3)

22. Prove by using Principle of Mathematical Induction:  $2 \times 7^n + 3 \times 5^n - 5$  is divisible by 24, For all  $n \in N$ .

OR

Prove by using Principle of Mathematical Induction:

$$1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}, n \in N.$$

23. Solve the following system of Linear Inequations graphically :

$$3x + 2y \geq 24, \quad 3x + y \leq 15, \quad x \geq 4.$$

24. a) Find  $\sin \frac{x}{2}$  for  $\cos x = -\frac{1}{3}$ , where  $x$  lies in third quadrant.

b) Solve for general solutions :  $2 \cos^2 x + 3 \sin x = 0$   $x = n\pi + \frac{2\pi}{3}$  (3+3)

25. Two ships leave a port at the same time. One goes 24 km per hour in the direction  $N45^\circ E$  and other travels 32 km per hour in the direction  $S75^\circ E$ . Find the distance between the ships at the end of 3 hours. (6)

OR

Prove the following for a triangle ABC : (3+3)

$$a) \frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0$$

$$b) \sin \left( \frac{B - C}{2} \right) = \frac{b - c}{a} \cos \frac{A}{2}.$$

26. a) If  $\frac{(a+i)^2}{(2a-i)} = p + iq$ , then show that  $p^2 + q^2 = \frac{(a^2 + 1)^2}{4a^2 + 1}$ , where  $a, p, q$  are all real

numbers.

b) Find real values of  $x$  and  $y$  such that  $(1 + i)y^2 + (6 + i)$  and  $(2 + i)x$  are equal. (4+2)