

ST. GEORGE'S SCHOOL,
MID-TERM EXAMINATION 2015-16
SUBJECT: MATHEMATICS
CLASS XI

MAX. MARKS: 100
NO. OF PAGES: 2

TIME : 3 hrs
DATE: 23.7.15
Q.1 to Q6 \Rightarrow 1 marks
Q.7 to Q19 \Rightarrow 4 marks
Q.20 to Q26 \Rightarrow 6 marks

SECTION A

1. Write power set of $\{0, \phi, \{\phi\}\}$?
2. Convert $-47^{\circ}30'$ in radians?
3. Find the domain of $f(x) = \sqrt{a^2 - x^2}$ ($a > 0$)
4. If $|1 + i|^x = 2$ then find the value of 'x'.
5. How many chords can be drawn through 15 points on a circle?
6. Solve $-(x-3) + 4 < 5 - 2x$ and represent it on the number line.

SECTION B

7. Letters of the word "MOTHER" are arranged (with or without meaning) to form a dictionary. Find the rank of the word "MOTHER"

8. If $a + ib = \frac{c+i}{c-i}$ where c is a real number prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2 - 1}$

9. Prove that $\frac{\cos 8A \cos 5A - \cos 12A \cos 9A}{\sin 8A \cos 5A + \cos 12A \sin 9A} = \tan 4A$

10. Using Principle of Mathematical Induction prove that for all $n \in N$

$$\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{2n+1}{n^2}\right) = (n+1)^2$$

11. If f is real function defined by $f(x) = \frac{x-1}{x+1}$, then prove that $f(2x) = \frac{3f(x)+1}{f(x)+3}$

12. There are 200 individuals with a skin disorder, 120 has been exposed to chemical C1, 50 to chemical C2 and 30 to both chemicals C1 and C2. Find the number of individuals exposed to
(i) chemical C1 or chemical C2 (ii) chemical C1 but not chemical C2 (iii) chemical C2 but not chemical C1.

13. Find the real numbers 'x' and 'y' if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$

14. If the fourth term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $\frac{20}{27}$, then find the value of 'a' and 'n'
15. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$ respectively then find the value of 'n'.
16. Prove that $\cos 5A = 16 \cos^5 A - 20 \cos^3 A + 5 \cos A$
17. Convert the complex number $-2 - 2\sqrt{3}i$ in polar form.
18. Let R be the relation on the set Z of integers defined by $R = \{(a, b) : a^2 + b^2 = 100, a \in Z, b \in Z\}$. Write the domain and range.
19. Solve the inequality $\frac{x+3}{x+7} > 3$

SECTION C

20. (i) Prove that $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}$
- (ii) Prove that $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos \theta$
21. Prove that $\tan A + \tan(60^\circ + A) + \tan(120^\circ + A) = 3 \tan 3A$
22. Solve graphically the system of linear inequations: $4x + 3y \leq 60, y \geq 2x, x \geq 3, y \geq 0$
23. Find $(x+1)^6 + (x-1)^6$. Hence evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$
24. Using Principle of Mathematical Induction prove that for all $n \in \mathbb{N}$ $n(n+1)(n+2)$ is divisible by 6.
25. A committee of 7 has to be formed from 9 Boys and 4 Girls. In how many ways can this be done when the committee consists of:
 (i) exactly 3 Girls? (ii) atleast 3 Girls? (iii) atmost 3 Girls?
26. (a) Given $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$ and $C = \{1, 3, 5\}$. Verify that
 (i) $A - (B \cup C) = (A - B) \cup (A - C)$
 (ii) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$
 (iii) If $a \in \mathbb{N}$ such that $a\mathbb{N} = \{ax : x \in \mathbb{N}\}$. Describe the set $3\mathbb{N} \cap 7\mathbb{N}$