Name__DAKSH THAKAR__ Class & Section_____ Roll No._____

BVN

# FIRST TERMINAL EXAMINATION—2016-17
## CLASS-XI
## SUBJECT—MATHEMATICS

Time : 3 Hours                                                            M.M. : 100

**Please check the total marks**

*General Instructions :*

1.  All questions are compulsory.

2.  The question paper consists of 29 questions divided into four sections – A, B, C and D. Section A consists 4 questions of 1 mark each, Section B consists of 8 questions of 2 marks each, Section C consists of 11 questions of 4 marks each and Section D consists of 6 questions of 6 marks each.

3.  There is no overall choice. However, internal choice has been provided.

## Section-A

1.  If $A = \{x : x \in R, x \geq 4\}$ and $B = \{x : x \in R, x < 5\}$ then find $A \cap B$.

2.  A relation R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by

    $x \, R \, y \Leftrightarrow x$ is relatively prime to y. Then find domain of R.

3.  Convert $\left(\dfrac{\pi}{8}\right)^c$ into degree measure.

4.  Find the value of $\cos\left(\dfrac{3\pi}{2} + x\right) \cos(2\pi + x) \left\{\cot\left(\dfrac{3\pi}{2} - x\right) + \cot(2\pi + x)\right\}$

## Section-B

5.  Find non-zero integral solutions of $|1 - i|^x = 2^x$

6.  If $^{10}P_r = 5040$, find the value of $r$.

7.  There are 60 students in a maths class and 40 students in a physics class. Find the number of students which are either in maths class or in the physics class when the two classes meet at the same hour.

8.  If $A + B = \dfrac{\pi}{4}$, find the value of $(1 + \tan A)(1 + \tan B)$.

9. Letters of the word 'AGAIN' are written in all possible orders. Find the 49th word.

10. If $\sin\theta = n\sin(\theta + 2\alpha)$, find the value of $\tan(\theta + \alpha)$.

11. Find the value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

12. How many liters of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

## Section-C

13. Let A and B be two non-empty sets having $n$ elements in common, then prove that $A \times B$ and $B \times A$ have $n^2$ elements in common.

**OR**

Let R be the relation on the set Z of all integers defined by $(x, y) \in R \Rightarrow x - y$ is divisible by $n$. Prove that :

(i) $(x, x) \in R$ for all $x \in Z$

(ii) $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in Z$

(iii) $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y, z \in Z$

14. Find the domain and range of the function $f(x) = \dfrac{3}{2 - x^2}$.

15. Sketch the graph of $y = \sec x$. Find its domain and range.

16. If $\dfrac{\sin A}{\sin B} = p$ and $\dfrac{\cos A}{\cos B} = q$, find $\tan A$ and $\tan B$.

**OR**

If $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = \dfrac{-3}{2}$,

prove that $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$.

17. Solve the following equations for the value of $\theta$.

(i) $\tan\theta + \tan\left(\theta + \dfrac{\pi}{3}\right) + \tan\left(\theta + \dfrac{2\pi}{3}\right) = 3$

18. Using PMI, prove that :

$1 + 2 + 3 + \ldots + n < \dfrac{(2n+1)^2}{8}$ for all $n \in N$.

19. Find real values of $x$ and $y$ for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

**OR**

If $\dfrac{(a + i)^2}{2a - i} = p + iq$, show that $p^2 + q^2 = \dfrac{(a^2 + 1)^2}{4a^2 + 1}$

20. If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4$.

21. Solve the system of inequations :

$$\frac{x}{2x + 1} \geq \frac{1}{4}, \qquad \frac{6x}{4x - 1} < \frac{1}{2}$$

22. How many even numbers are there with three digits such that if 5 is one of the digit, then 7 is the next digit ?

**OR**

In how many ways three girls and nine boys can be seated in the two vans, each having numbered seats, 3 in the front and 4 at the back ? How many seating arrangements are possible if 3 girls sit together in a back row on adjacent seats ?

23. From a class of 25 students, 10 are to be chosen for an excursion party on river side. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can they be chosen ? What precautions you should keep in the mind while being on river side ?

## Section-D

24. If $\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) = \tan^3\left(\dfrac{\pi}{4} + \dfrac{\alpha}{2}\right)$, prove that $\sin\theta = \dfrac{3\sin\alpha + \sin^3\alpha}{1 + 3\sin^2\alpha}$

25. In a $\triangle ABC$, prove that, $a = b\cos C + c\cos B$. Hence use it to prove

$\sin^3 A\cos(B - C) + \sin^3 B\cos(C - A) + \sin^3 C\cos(A - B) = 3\sin A\sin B\sin C$.

26. Prove by induction that $(2n + 7) < (n + 3)^2$ for all natural numbers $n$. Using this, prove by induction that $(n + 3)^2 \leq 2^{n+3}$ for all $n \in N$.

**OR**

Prove by induction,

$$\frac{1}{2}\tan\left(\frac{x}{2}\right) + \frac{1}{4}\tan\left(\frac{x}{4}\right) + \ldots + \frac{1}{2^n}\tan\left(\frac{x}{2^n}\right) = \frac{1}{2^n}\cot\left(\frac{x}{2^n}\right) - \cot x$$

for all $n \in N$ and $0 < x < \pi/2$.

27. Solve : $2x^2 - (3 + 7i)x - (3 - 9i) = 0$

28. Exhibit graphically the solution set of the linear inequations :

$x + y \leq 5, 4x + y \geq 4, x + 5y \geq 5, x \leq 4, y \geq 3$

29. How many four letter words can be formed using the letter of the word 'INEFFECTIVE'?

**OR**

Attempt both :

(i) In how many ways can the letters of the word PERMUTATIONS be arranged if there are always 4 letters between P and S ?

(ii) A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow chemistry part II, unless chemistry part I is also borrowed. In how many ways can he choose the three books to be borrowed ?