

First Term Exam – 2017-2018

Class – XI

Subject – MATHEMATICS

Time: 3 Hours

Max. Marks: 100

General Instructions:

- The question paper consists of 29 questions divided into four sections A, B, C and D.
- Section A comprises of 4 questions of one mark each.
- Section B comprises of 8 questions of two marks each.
- Section C comprises of 11 questions of four marks each.
- Section D comprises of 6 questions of six marks each.

Section A

1. Find the value of $\sin 22.5^\circ$.
2. If $A = \{1,3,5,\dots,17\}$ and $B = \{2,4,6,\dots,18\}$ and N the set of natural numbers is the universal set, then find $A' \cup ((A \cup B) \cap B')$.
3. If n is a positive integer, then find the value of $i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4}$.
4. Solve $4x + 3 > 6x + 7$ when x is a natural number.

Section B

5. Draw the graph of the real valued function $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{x}$, $x \in \mathbb{R} - \{0\}$ What is the domain and range of this function
6. Let $f = \{(1,1), (2,3), (0, -1), (-1, -3)\}$ be a linear function from \mathbb{Z} into \mathbb{Z} . Find $f(x)$.
7. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use $\pi = 3.14$).
8. Solve : $-12 < 4 - \frac{3x}{-5} \leq 2$.
9. The difference between any two consecutive interior angles of a polygon is 5° . If the smallest angle is 120° , find the number of the sides of the polygon?
10. Find real θ such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is purely real.
11. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1 . Find the common ratio and the terms.

12. Find the value of $\tan \frac{13\pi}{12}$.

Section C

13. By using the principle of mathematical induction prove that $(1+x)^n \geq (1+nx)$, for all natural number n , where $x > -1$.

14. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X , show that $A = B$

15. Find the domain and range of the real function $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in R \right\}$.

16. If α and β are different complex numbers with $|\beta| = 1$, then find $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$.

OR

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, then find $a^2 + b^2$

17. Find the sum of the n terms of the series: $5 + 11 + 19 + 29 + 41 + \dots$

OR

A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

18. Prove that $\frac{\sin 3x + \sin 5x + \sin 7x + \sin 9x}{\cos 3x + \cos 5x + \cos 7x + \cos 9x} = \tan 6x$.

19. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

20. Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$.

Solve the quadratic equation $2x^2 - (3 + 7i)x - (3 - 9i) = 0$, and express the roots in the standard form $a + ib$.

Prove that: $\frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\tan 8x}{\tan 2x}$.

OR

Prove that:

$$\sin \alpha + \sin \beta + \sin \gamma - \sin (\alpha + \beta + \gamma) = 4 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta + \gamma}{2} \sin \frac{\gamma + \alpha}{2}$$

23. Determine a quadratic function f defined by $f(x) = ax^2 + bx + c$ if $f(1) = 6$, $f(2) = 11$, $f(-3) = 6$.

Section D

24. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports? What are the advantages of playing sports?

OR

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find: (i) the number of people who read at least one of the newspapers, (ii) the number of people who read exactly one newspaper (iii) write the importance of reading newspaper.

25. Prove that: $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$.

OR

If $\sin x = \frac{3}{5}$, $\cos y = -\frac{12}{13}$, where x and y both lie in second quadrant, find the values of (i) $\sin(x + y)$, (ii) $\cos(x - y)$ (iii) $\tan(x + y)$.

26. By using the principle of mathematical induction prove that: $2.7^n + 3.5^n - 5$ is divisible by 24, for all $n \in \mathbb{N}$.

27. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

28. Solve graphically the following system of linear inequalities: $4x + 3y - 10 \leq 50$, $x + 2y \geq 3x + y$, $3 \leq x$, $x, y \geq 0$.

29. Find the sum of the following series up to n terms: (i) $0.6 + 0.66 + 0.666 + \dots$

OR

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m - 1)^{\text{th}}$ numbers is 5 : 9. Find the value of m .