

Name Arjun

Class & Section

Roll No. _____

FIRST TERM EXAMINATION—2017-18

CLASS—XI

SUBJECT—MATHEMATICS

M.M. : 100

Time : 3 Hours

General Instructions :

This question paper contains 29 questions.

Questions 1-4 in Section A carry 1 mark each.

Questions 5-12 in Section B carry 2 marks each.

Questions 13-23 in Section C carry 4 marks each.

Questions 24-29 in Section D carry 6 marks each.

All the questions are compulsory. However, internal choice has been given in few questions.

$$\tan \theta = \frac{3}{4}$$

Section-A

1. If $\sin \theta + \cos \theta = 1$, find the value of $\sin 2\theta$.

2. Find the value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$.

3. If $10^\circ + 3 \cdot 4^{n-2} + k$ is divisible by 9 for all $n \in \mathbb{N}$, then find the least positive integral value of k .

4. Find the value of $\arg(z) + \arg(\bar{z})$ ($\bar{z} \neq 0$)

Section-B

5. If $A = \{1, 3, 5, 7, \dots, 17\}$, $B = \{2, 4, 6, 8, \dots, 18\}$ and N the set of natural numbers is the universal set, then find $[A' \cup (A \cup B) \cap B']$

6. Find the range of the function $f(x) = \frac{1}{1 - 2\cos x}$

7. If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, what is the value of $\theta + \phi$?

8. Find the number of solutions of the equation :

$\tan x + \sec x = 2\cos x$ in the interval $[0, 2\pi]$

18. Find the general solution of the equation

$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x.$$

19. Using PMI, prove that

$$1.3 + 3.5 + 5.7 + \dots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}, \text{ for all } n \in \mathbb{N}.$$

OR

Using P.M.I., prove that $x^{2n} - y^{2n}$ is divisible by $x + y$ for all $n \in \mathbb{N}$.

20. Write the complex number $z = \frac{1-i}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ in polar form.

21. Solve for x : $\frac{x}{2x+1} \geq \frac{1}{4}$, $\frac{6x}{4x-1} < \frac{1}{2}$

22. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, find the value of nC_r .

23. If the letter of the word 'ZENITH' are arranged in all possible ways as listed in dictionary, then what is the rank of the word 'ZENITH'?

OR

Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the points.

Section-D

24. Find the value of the expression:

$$\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$$

OR

If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that

$$\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2.$$

10. If $\sin x + \cos x = a$, find the value of $|\sin x - \cos x|$.

11. If $\frac{(1+i)^2}{2-i} = x + iy$, find the value of $x + y$.

12. Everybody in the room shakes hands with everybody else. The total number of handshakes is 66. Find the total number of persons in the room.

13. Find the total number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line.

Section-C

14. In a town of 10000 families, it was found that 40% buy newspaper A, 20% buy newspaper B, 10% buy newspaper C, 5% buy A and B, 3% buy B and C and 4% buy A and C. If 2% buy all the three newspapers find-

- number of families which buy newspaper A only
- none of A, B and C.

$$f(x) = \frac{x}{1+x^2}$$

15. Find domain and range of the function

$$\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

OR

Find the value of $\cos^2 x + \cos^2 \left(x + \frac{\pi}{3} \right) + \cos^2 \left(x + \frac{\pi}{3} \right)$

16. Using solution of triangle, find the value of

$$(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C$$

17. If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then prove that $\tan \theta = \frac{1-m}{1+m} \cot \phi$

OR

If $a \cos 2\theta + b \sin 2\theta = c$ has α and β as its roots, then find the value of $\tan(\alpha + \beta)$ in terms of a and b .

25. If $\cos \theta = \frac{\cos \alpha \cos \beta}{1 - \sin \alpha \sin \beta}$, prove that $\tan \frac{\theta}{2} = \pm \frac{\tan \frac{\alpha}{2} - \tan \frac{\beta}{2}}{1 - \tan \frac{\alpha}{2} \tan \frac{\beta}{2}}$

26. Using PMI, prove that for all $n \in \mathbb{N}$

$$\sin \theta + \sin 2\theta + \sin 3\theta + \dots + \sin n\theta = \frac{\sin \left(\frac{n+1}{2} \theta \right) \sin \frac{n\theta}{2}}{\sin \frac{\theta}{2}}$$

OR

Using PMI, prove that $2n + 7 < (n + 3)^2$ for all $n \in \mathbb{N}$

using this, prove by induction $(n + 3)^2 \leq 2^{n+3}$ for all $n \in \mathbb{N}$

27. Solve for x :

$$2x^2 - (3 + 7i)x - (3 - 9i) = 0$$

28. Exhibit graphically the solution set of the linear inequations :

$$x + y \leq 5, 4x + y \geq 4, x + 5y \geq 5, x \leq 4, y \leq 3$$

29. How many four letter words can be formed using the letter of the word 'INEFFECTIVE' ?

Social networking sites have made personal communication less and ineffective. Comment.