Waves is distributed energy or distributed "disturbance (force)"

Following points regarding waves :

1. The disturbance (force) is transmitted from one point to another.

2. The energy is transmitted from one point to another.

3. The energy or distrubance passes in the form of wave without any net displacement of medium.

4. The oscillatory motion of preceding particle is imparted to the adjacent particle following it.

5. We need to keep creating disturbance in order to propagate wave (energy or disturbance) continuously.

(a) Waves classification

The waves are classified under two high level headings :

1. Mechanical waves : The motion of the particle constituting the medium follows mechanical laws i.e. Newton's laws of motion. Mechanical waves originate from a distrubance in the medium (such as a stone dropping in a pond) and the disturbance propagates through the medium. The force between the atoms in the medium are responsible for the propagation of mechanical waves. Each atom exerts a force on the atoms near it, and through this force the motion of the atom is transmitted to the others. The atoms in the medium do not experience any net displacement.

Mechanical waves is further classified in two categories such that

1. Transverse waves (waves on a string) 2. Longitudnal waves (sound waves)

2. Non Mechanical waves : These are electro magnetic waves. The electromagnetic waves do not require a medium for propagation. Its speed in vacuum is a universal constant. The motion of the electromagnetic waves in a medium depends on the electromagnetic properties of the medium.

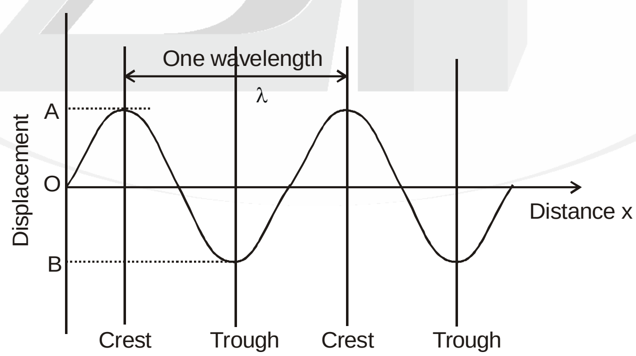
(c) Describing Waves :

Two kinds of graph may be drawn displacement - distance and displacement-time.

A displacement-distance graph for a transverse mechanical waves shows the displacement y of the vibrating particles of the transmitting medium at different distance x from the source at a certain instant i.e. it is like a photograph showing shape of the wave at that particular instant.

The maximum displacement of each particle from its undisturbed position is the amplutude of the wave.

In the figure 1, it OA or OB.



The wavelength λ of a wave is generally taken as the distance between two successive crests or two successive trough. To be more specific, it is the distance between two consecutive points on the wave which have same phase.

A displacement-time graph may also be drawn for a wave motion, showing how the displacement of one particle at a particular distance from the source varies with time. If this is simple harmonic variation then the graph is a sine curve.

- **Wave Length, Frequency, Speed**

If the source of a wave makes f vibrations per second, so they will the particles of the transmitting medium. That is, the frequency of the waves equals frequency of the source.

When the source makes one complete vibration, one wave is generated and the disturbance spreads out a distance λ from the source. If the source continues to vibrate with constant frequency f, then f waves will be produced per second and the wave advances a distance f λ in one second. If v is the wave speed then

$$v = f\lambda$$

This relationship holds for all wave motions.

☛ Frequency depends on source (not on medium), v depends on medium (not on source frequency), but wavelength depend on both medium and source.

(d) Initial Phase :

At x = 0 and t = 0, the sine function evaluates to zero and as such y-displacement is zero. However, a wave form can be such that y-displacement is not zero at x =0 and t = 0. In such case, we need to account for the displacement by introducting an angle like :

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

where "ϕ" is initial phase. At x = 0 and t = 0.

$$y(0, 0) = A\sin(\phi)$$

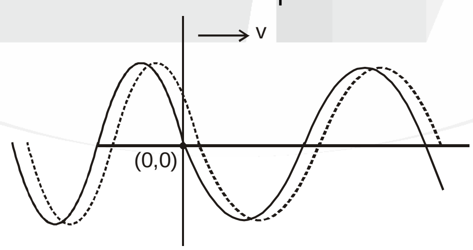
The measurement of angle determines following two aspects of wave form at x = 0, t = 0 : (i) whether the displacement is positive or negative and (ii) whether wave form has positive or negative slope.

For a harmonic wave represented by sine function, there are two values of initial phase angle for which displacement at reference origin (x = 0, t = 0) is positive and has equal magnitude. We know that the sine values of angles in first and second quadrants are positive. A pair of initial phase angles, say $\phi = \pi/3$ and $2\pi/3$, correspond to equal positive sine values are :

$\sin\theta = \sin(\pi - \theta)$

$$\sin\frac{\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{1}{2}$$

To choose the initial phase in between the two values $\pi/3$ & $\frac{2\pi}{3}$. We can look at a wave motion in yet another way. A wave form at an instant is displaced by a distance Δx in very small time interval Δt then then speed to the particle at t = 0 & x = 0 is in upward +ve direction in further time Δt



ILLUSTRATIONS

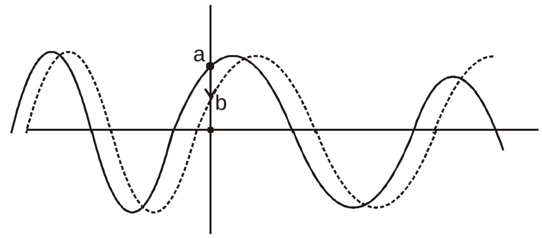
1. Find out the expression of wave equation which is moving is +ve x direction and at x = 0, t=0 $y = \frac{A}{\sqrt{2}}$

Sol. Let $y = A\sin(\omega t - kx + \phi)$

at $t = 0$ and $x = 0$

$$\frac{A}{\sqrt{2}} = A\sin\phi \quad \Rightarrow \sin\phi = \frac{1}{\sqrt{2}}$$

$$\phi = \frac{\pi}{4},\ \frac{3\pi}{4}$$

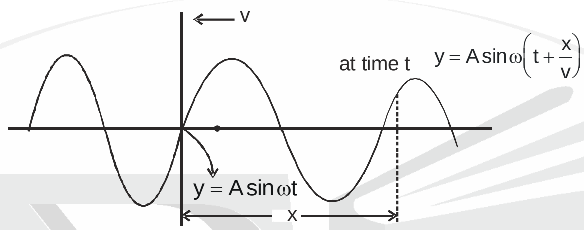


To choose the correct phase angle ϕ we displaced to wave. Slightly in +ve x direction such that
In above figure Paticle at a is move downward towards point b i.e. particle at x = 0 & $y = \frac{A}{\sqrt{2}}$ have negative velocity which gives

$$\frac{\partial y}{\partial t} = \omega A\cos(\omega - kx + \phi) \text{ at } t = 0,\ x = 0 \text{ is } \cos\phi = -\text{ ve (from figure)} \qquad ...(2)$$

from above discussion $3\pi/4$ gives $\sin\phi$ + ve and $\cos\phi$ negative i.e. $\phi = \frac{3\pi}{4}$

Note : Equation of wave which is moving –ve x direction.



$y = A\sin(\omega t + kx + \phi)$

2. If (ωt) & (kx) terms have same sign then the wave move toward –ve x direction and vice versa and with diffierent initial phase.

$y = A\sin(\omega t - kx)$ Wave move toward +ve x direction

$y = A\sin(-kx + \omega t)$

$y = A\sin(-kx - \omega t)$ Wave move toward –ve x direction.

$= A\sin(kx + \omega t + \pi)$

$y = A\sin(kx + \omega t)$

PARTICLE VELOCITY AND ACCELERATION

Particle velocity at a given position x = x is obtained by differentiating wave function with respect to time "t". We need to differentiate equation by treating "x" as constant. The partial differentiation yields particle velocity as :

$$v_p = \frac{\partial}{\partial t} y(x,t) = \frac{\partial}{\partial t} A\sin(kx - \omega t) = -\omega A\cos(kx - \omega t)$$

We can use the property of cosine function to find the maximum velocity. We obtain maximum speed when cosine function evaluates to "–1" :

$\Rightarrow v_{pmax} = \omega A$

The acceleration of the particle is obtained by differentiating expression of velocity partially with respect to time :

$$\Rightarrow\ a_p = \frac{\partial}{\partial t} v_p = \frac{\partial}{\partial t}\{-\omega A\cos(kx-\omega t)\} = -\omega^2 A \sin(kx-\omega t) = -\omega^2 y$$

Again the maximum value of the acceleration can be obtained using property of sine function :

$$\Rightarrow\ a_{pmax} = \omega^2 A$$

WAVE FUNCTION OF PROGRESSIVE WAVE / TRAVELLING WAVE

Different forms give rise to bit of confusion about the form of wave function. The forms used for describing wave are :

$$y(x, t) = A \sin(kx - \omega t)$$
$$y(x, t) = A \sin(\omega t - kx + \pi)$$

Which of the two forms is correct ? In fact, both are correct so long we are in a position to accurately interpret the equation. Starting with the first equation and using trigonometric identity :

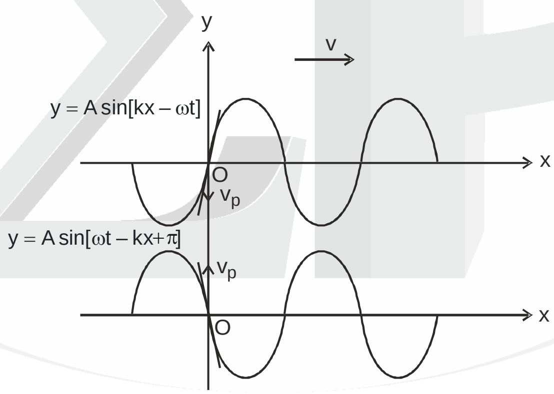
We have, $\Rightarrow$ $A\sin(kx-\omega t) = A\sin(\pi - kx + \omega t) = A\sin(\omega t - kx + \pi)$

Thus we see that two forms represent waves along at the same speed $\left(v = \frac{\omega}{k}\right)$. They differ, however, in phase. There is phase difference of "π". This has implication on the waveform and the manner particle oscillates at any given time instant and position. Let us consider two waveforms at x = 0, t = 0. The slopes of the waveforms are :

$$\frac{\partial}{\partial x} y(x,t) = kA\cos(kx-\omega t) = kA = \text{a positive number}$$

and $\frac{\partial}{\partial x} y(x,t) = -kA\cos(\omega t - kx) = -kA$ = a negative number

Forms of wave functions



Exchange of terms in the argument of sine function results in a phase difference of π.

In the first case, the slope is positive and hence particle velocity is negative. It means particle is moving from reference origin or mean position to negative extreme position. In the second case, the slope is negative and hence particle velocity is positive. It means particle is moving from positive extreme position to reference origin or mean position. Thus two forms represent waves which differ in direction in which particle is moving at a given position.

Once we select the appropriate wave form, we can write wave equation in other forms as given here:

$$y(x, t) = A\sin(kx - \omega t) = A \sin k\left(x - \frac{\omega t}{k}\right) = A\sin\frac{2\pi}{\lambda}(x - vt)$$

Further, substituting for "k" and "ω" in wave equation, we have :

$$y(x, t) = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

If we want to represent waveform moving in negative "x" direction, then we need to replace "t" by"–t".

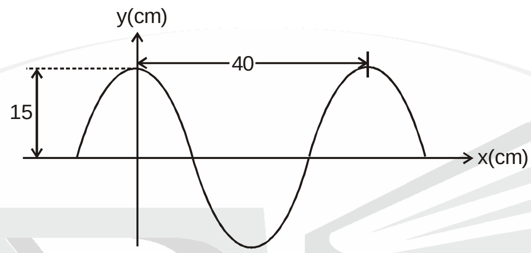
SPEED OF WAVE

Speed of wave (v) is given by, $v = \frac{\text{coefficient of } t}{\text{coefficient of } x} = \frac{b}{a}$

Thus plus (+) sign between ax and bt implies that the wave is travelling along negative x-direction and minus (–) sign shows that it is travelling along positive x-direction.

ILLUSTRATIONS

1. A sinusoidal wave travelling in the positive x direction has an amplitude of 15 cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at t =0 and x = 0 is also 15 cm, as shown



(a) Find the angular wave number, period angular frquency and speed of the wave.

(b) Determine the phase constant ϕ, and write a general expression for the wave function.

Sol. (a) $k = \frac{2\pi}{\lambda} = \frac{2\pi\text{rad}}{40\text{cm}} = \frac{\pi}{20}\text{rad/cm}$

$T = \frac{1}{f} = \frac{1}{8}s$ $\qquad\qquad$ $\omega = 2\pi f = 16\ s^{-1}$

$v = f\lambda = 320$ cm/s

(b) It is given that A = 15 cm and also y = 15 cm at x = 0 and t = 0 then using $y = A\sin(\omega t - kx + \phi)$

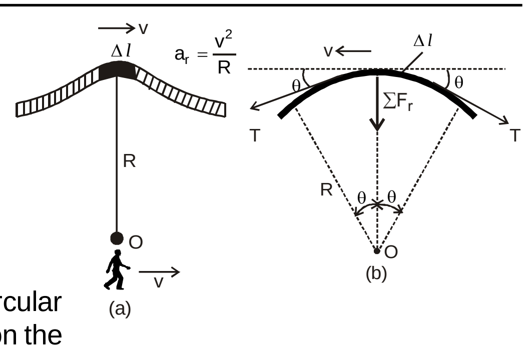
$15 = 15\sin\phi \Rightarrow \sin\phi = 1$

Therefore, the wave function is

$$y = A\sin(\omega t - kx + \frac{\pi}{2}) = (15\,\text{cm})\sin\left[(16\pi s^{-})t - \left(\frac{\pi}{20}\frac{\text{rad}}{\text{cm}}\right).x + \frac{\pi}{2}\right]$$

SPEED OF A TRANSVERSE WAVE ON A STRING

Consider a pulse travelling along a string with a speed v to the right. If the amplitude of the pulse is small compared to the length of the string, the tension T will be approximately constant along the string. In the reference frame moving with speed v to the right, the pulse in stationary and the string moves with a speed v to the left. Figure shows a small segment of the string of length Δl. This segment forms part of a circular arc of radius R.



Instantaneously the segment is moving with speed v in a circular path, so it has centripetal acceleration v^2/R. The forces acting on the

segment are the tension T at each end. The horizontal component
of these forces are equal and opposite and thus cancel.

The vertical component of these forces point radially inward towards the centre of the circular. arc. These radial forces provide centripetal acceleration. Let the angle substended by the segment at centre be 2θ. The net radial force acting on the segment is

Fig. (a) To obtain the speed v of a wave on a stretched string. It is convenient to describe the motion of a small segment of the string in a moving frame of reference.

Fig. (b) In the moving frame of reference, the small segment of length Δl moves to the left with speed v. The net force on the segment is in the radial direction because the horizontal components of the tension force cancel.

$$\sum F_r = 2T\sin\theta = 2T\theta$$

Where we have used the approximation $\sin\theta \approx \theta$ for small θ.

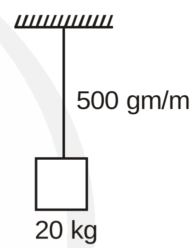
If μ is the mass per unit length of the string, the mass of the segment of length Δl is

$m = \mu\Delta l = 2\mu R\theta$ (as $\Delta l = 2R\theta$)

From Newton's second law $\sum F_r = ma = \dfrac{mv^2}{R}$ or $2T\theta = (2\mu R\theta)\left(\dfrac{v^2}{R}\right)$ $\therefore$ $v = \sqrt{\dfrac{T}{\mu}}$

ILLUSTRATIONS

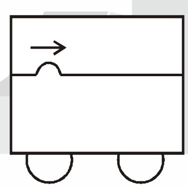
1. Find speed of the wave generated in the string as in the situation shown. Assume that the tension in not affected by the mass of the cord.



Sol. T = 20 × 10 = 200 N

$$v = \sqrt{\frac{200}{0.5}} = 20 m/s$$

2. A taut string having tension 100 N and linear mass density 0.25 kg/m is used inside a cart to generate a wave pulse starting at the left end, as shown. What should be the velocity of the cart so that pulse remains stationary w.r.t ground.



Sol. Velocity of pulse = $\sqrt{\dfrac{T}{\mu}} = 20 m/s$

Now $\vec{v}_{PG} = \vec{v}_{PC} + \vec{v}_{CG}$; $0 = 20\,\hat{i} + \vec{v}_{CG}$; $\vec{V}_{CG} = -20\hat{i}\, m/s$

3. One end of 12.0 m long rubber tube with a total mass of 0.9 kg is fastened to a fixed support. A cord attached to the other and passes over a pulley and supports an object with a mass of 5.0 kg. The tube is struck a transverse blow at one end. Find the time required for the pulse to reach the other end (g = 9.8 m/s^2)

Sol. Tension in the rubber tube AB, T = mg

T = (5.0) (9.8) = 49 N

Mass per unit length of rubber tube,

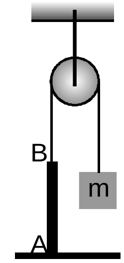
$$\mu = \frac{0.9}{12} = 0.075 \text{ kg/m}$$

$\therefore$ Speed of wave on the tube,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{0.075}} = 25.56 \text{ m/s}$$

$\therefore$ The required time is,

$$t = \frac{AB}{v} = \frac{12}{25.56} = 0.47 \text{ s}$$



ENERGY CALCULATION IN WAVES

(a) Kinetic energy per unit length

Kinetic energy per unit length, "K_L", by dividing this expression with the length of small string considered :

$$K_L = \frac{dK}{dx} = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx - \omega t)$$

Rate of transmission of kinetic energy

Average rate of transmission of kinetic energy is :

$$\left.\frac{dK}{dt}\right|_{avg} = \frac{1}{2} \times \frac{1}{2}\mu v\omega^2 A^2 = \frac{1}{4}\mu v\omega^2 A^2$$

(b) Potential energy per unit length

Potential energy density $\frac{dU}{dx} = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2$

$\frac{dy}{dx}$ = kAcos $(kx - \omega t)$ and $T = v^2 \mu$

Put above value in equation (i) then we get $\frac{dU}{dx} = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx - \omega t)$

Rate of transmission of elastic potential energy

$$\left.\frac{dU}{dt}\right|_{avg} = \frac{1}{2} \times \frac{1}{2}\mu v\omega^2 A^2 = \frac{1}{4}\mu v\omega^2 A^2$$

(c) Mechanical energy per unit length

$$E_L = \frac{dE}{dx} = 2x\frac{1}{2}\mu\omega^2 A^2 \cos^2(kx - \omega t) = \mu\omega^2 A^2 \cos^2 (kx - \omega t)$$

(d) Average power transmitted

The average power transmitted by wave is equal to time rate of transmission of mechanical energy over integral wavelengths. It is equal to :

$$P_{avg} = \left.\frac{dE}{dt}\right|_{avg} = 2 \times \frac{1}{4}\mu v\omega^2 A^2 = \frac{1}{2}\mu v\omega^2 A^2$$

If mass of the string is given in terms of mass per unit volume, "ρ", then we make appropriate change in the derivation. We exchange "μ" by "ρs" where "s" is the cross section of the string :

$$P_{avg} = \frac{1}{2}\rho s v\omega^2 A^2$$

(e) Intensity

Intensity of wave (I) is defined as power transmitted per unit cross section area of the medium:

$$I = \rho s v \omega^2 \frac{A^2}{2s} = \frac{1}{2}\rho v w^2 A^2$$

Intensity of wave (I) is a very useful concept for three dimensional waves radiating in all direction from the source. This quantity is usually referred in the context of light waves, which is transverse harmonic wave in three dimensions. Intensity is defined as the power transmitted per unit cross sectional area. Since light spreads uniformly all around, intensity is equal to power transmitted, divided by spherical surface drawn at that point with source at its center.

☛ **Phase difference between two particles in the same wave :**

The general expression for a sinusoidal wave travelling in the positive x direction is

$y(x, t) = A \sin (\omega t - kx)$

Eq^n of Particle at x_1 is given by $y_1 = A \sin (\omega t - kx_1)$

Eq^n of particle which is at x_2 from the origin

$y_2 = A\sin (\omega t - kx_2)$

Phase difference between particles is $k(x_2 - x_1) = \Delta\phi$, $K\Delta x = \Delta\phi \Rightarrow \quad \Delta x \Rightarrow \frac{\Delta\phi}{k}$ or $\Delta x \Rightarrow \left(\frac{\lambda}{2\pi}\right)\Delta\phi$

PRINCIPLE OF SUPERPOSITION :

This principle defines the displacement of a medium particle when it is oscillating under the influence of two or more than two waves. The principle of superposition is stated as :

"When two or more waves superpose on a medium particle than the resultant displacement of that medium particle is given by the vector sum of the individual displacements produced by the component waves at that medium particle independently."

Let $\vec{y}_1, \vec{y}_2, ... \vec{y}_N$ are the displacements produced by N independent waves at a medium particle in absence of others then the displacemnt of that medium, when all the waves are superposed at that point, is given as

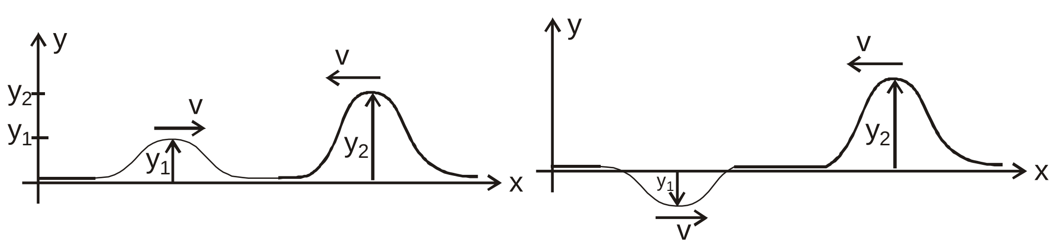
$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ... + \vec{y}_N$$

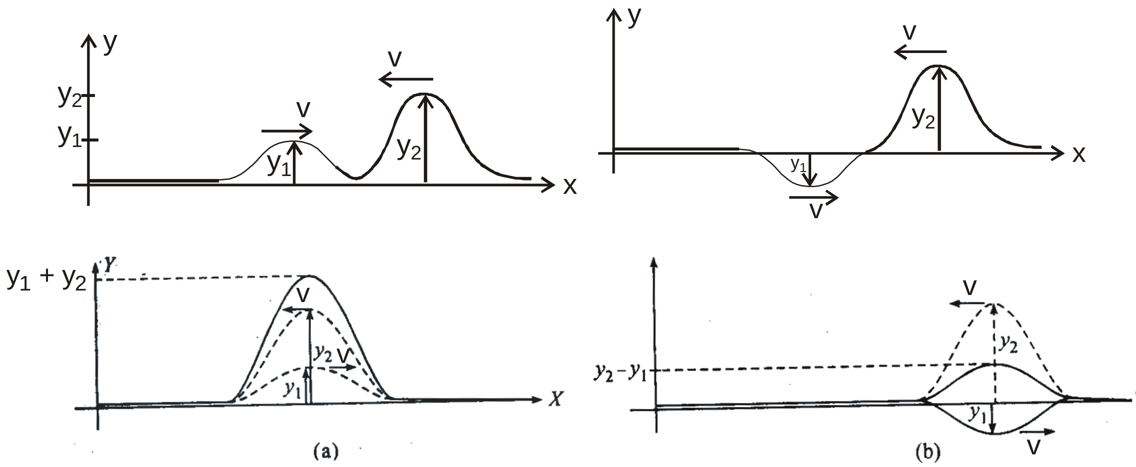
If all the waves are producing oscillations at that point are collinear then the displacement of the medium particle where superposition is taking place can be simply given by the algebric sum of the individual displacement. Thus we have

$y = y_1 + y_2 + ...+ y_N$

The above equation is valid only if all individual displacements y_1, y_2 .... y_N are along same straight line.

A simple example of superposition can be understood by figure shown. Suppose two wave pulses are travelling simultaneously in opposite directions as shown. When they overlap each other the displacement of particle on string is the algebric sum of the two displacement as the displacements of the two pulses are in same direction. Figure shown (b) also shows the similar situation when the wave pulses are in opposite side.





(a) Applications of Principle of Superposition of Waves

There are several different phenomenon which takes place during superposition of two or more wave depending on the wave characteristics which are being superposed. We'll discuss some standard phenomenons, and these are :

(1) Interference of Wave

(2) Stationary Waves

(3) Beats

(4) Lissajou's Figures (Not discussed here in detail.) Lets discuss these in detail.

REFLECTION AND TRANSMISSION IN WAVES

1. When a pulse travelling along a string reaches the end, it is reflected. If the end is fixed as shown in figure (a), the pulse returns inverted. This is bacause as the leading edge reaches the wall, the string pulls up the wall. According to Newton's third law, the wall will exert an equal and opposite force on the string as all instants. This force is therefore, directed first down and then up. It produces a pulse that is inverted but otherwise identical to the original.

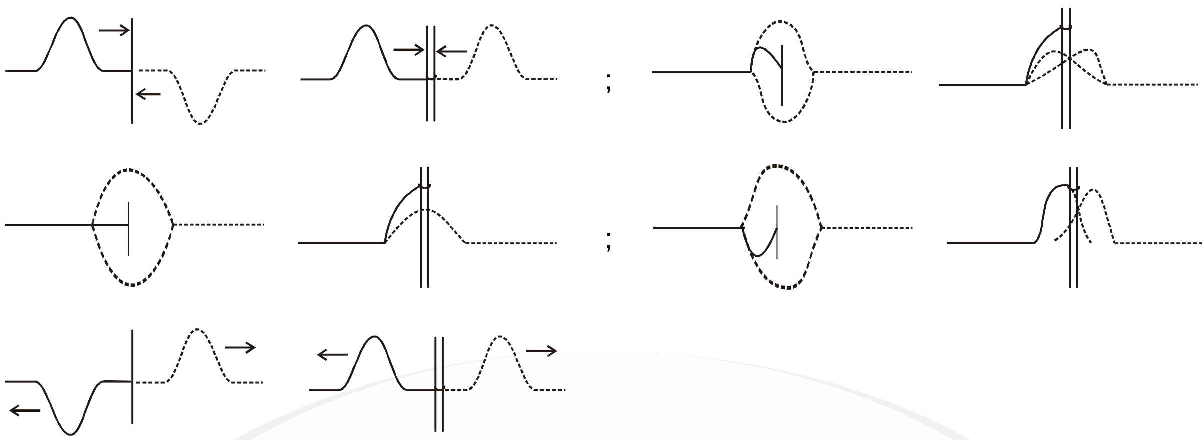
The motion of free end can be studied by letting a ring at the end of string sliding smoothly on the rod. The ring and rod maintain the tension but exert no transverse force.

(a) (b)

Reflection of wave pulse (a) at a fixed end of a string and (b) at a free end. Time increases from top to bottom in each figure.

When a wave arrives at this free end, the ring slides the rod. The ring reaches a maximum displacement. At this position the ring and string come momentarily to rest as in the fourth drawing from the top in figure (b). But the string is stretched in this position, giving increased tension, so the free end of the string is pulled back down, and again a reflected pulse is produced, but now the direction of the displacement is the same as for the initial pulse.

2. The formation of the reflected pulse is similar to the overlap of two pulses travelling in opposite directions. The net displacement at any point is given by the principle of superposition.



(a) (b)

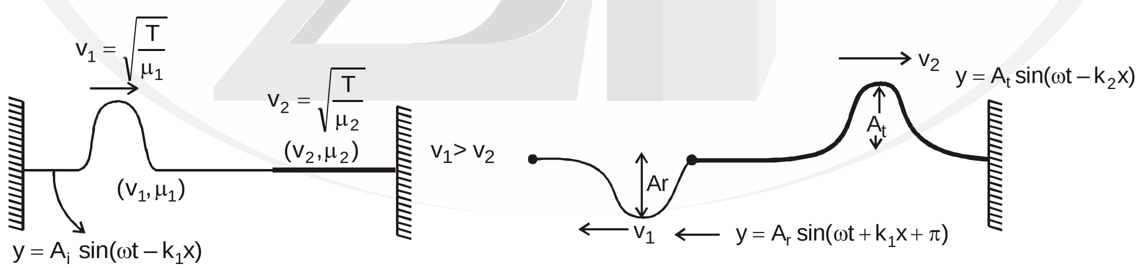
Fig (a) : shows two pulses with the same shape, one inverted with respect to the other, travelling in opposite directions. Because these two pulses have the same shape the net displacement of the point where the string is attached to the wall is zero at all times.

Fig (b) : shows two pulses with the same shape, travelling in oppoiste directions but not inverted relative to each other. Note that at one instant, the displacement of the free end is double the pulse height.

REFLECTION AND TRANSMISSION BETWEEN TWO STRING

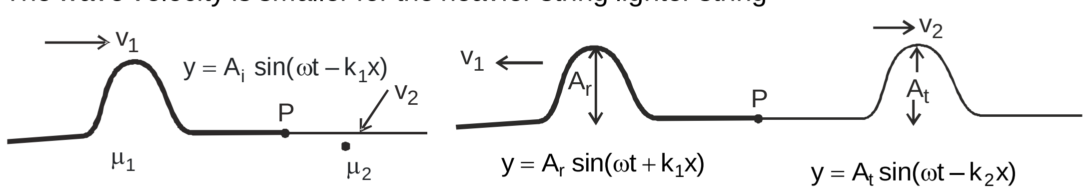
Here we are dealing with the case where the end point is neither completely fixed nor completely free to move As we consider an example where a light string is attached to a heavy string as shown is figure a.

If a wave pulse is produced on a light string moving towards the friction a part of the wave is reflected and a part is transmitted on the heavier string the reflected wave is inverted with respect to the original one.



On the other hand if the wave is produced on the heavier string which moves toward the junction a part will the reflected and a part transmitted, no inversion in waves shape will take place.

The wave velocity is smaller for the heavier string lighter string



Now to find the relation between A_i, A_r, A_t we consider the figure (b)

Incident Power = Reflected Power + Transmitted Power

$$P_i = P_r + P_t$$

$$2\pi^2 f^2 A_i^{\,2} \mu_1 v_1 = 2\pi^2 f^2 A_r^{\,2} \mu_1 v_1 + 2\pi^2 f^2 A_t^{\,2} \mu_2 v_2 \qquad \text{...(i)}$$

Put $\mu_1 = \dfrac{T}{v_1^{\,2}}$ and $\mu_2 = \dfrac{T}{v_2^{\,2}}$ in equation (i) their

$$\frac{A_i^{\,2}}{v_1} = \frac{A_r^{\,2}}{v_1} + \frac{A_t^{\,2}}{v_2}$$

$$A_i^{\,2} - A_r^{\,2} = \frac{v_1}{v_2} A_t^2 \qquad \text{.......(ii)}$$

Maximum displacement of joint particle P (as shown in figure) due to left string = $A_i + A_r$

Maximum displacement of joint particle due to right string = A_t

At the boundary (at point P) the wave must be continuous, that is there are no kinks in it. Then we must have $A_i + A_r = A_t$ ...(iii)

from equation (ii) & (iii)

$$A_i - A_r = \frac{v_1}{v_2} A_t \qquad \text{...(iv)}$$

from eq. (iii) & (iv)

$$A_t = \left[\frac{2v_2}{v_1 + v_2}\right] A_i$$

$$A_r = \left[\frac{v_2 - v_1}{v_1 + v_2}\right] A_i$$

STANDING WAVES

In previous section we've discussed that when two coherent waves superpose on a medium particle, phenomenon of interference takes place. Similarly when two coherent waves travelling in opposite direction superpose then simultaneous interference if all the medium particles takes place. These waves interfere to produce a pattern of all the medium particles what we call, a stationary wave. If the two interfering waves which travel in opposite direction carry equal energies then no net flow of energy takes place in the region of superposition. Within this region redistribution of energy takes place between medium particles. There are some medium particles where constructive interference takes place and hence energy increases and on the other hand there are some medium particles where destructive interference takes place and energy decreases. Now we'll discuss the stationary waves analytically.

Let two waves of equal amplitude are travelling in opposite direction along x-axis. The wave equation of the two waves can be given as

$y_1 = A \sin(\omega t - kx)$ [Wave travelling in +x direction] ...(1)

and $y_2 = A \sin(\omega t + kx)$ [Wave travelling in –x direction] ...(2)

When the two waves superpose on medium particles, the resultant displacement of the medium particles can be given as

$$y = y_1 + y_2$$

or $$y = A \sin(\omega t - kx) + A \sin(\omega t + kx)$$

or $\quad y = A [\sin\omega t \cos kx - \cos \omega t \sin kx + \sin \omega t \cos kx + \cos \omega t \sin kx]$

or $\quad y = 2A \cos kx \sin \omega t$ ...(3)

Equation (3) can be rewritten as

$$y = R \sin \omega t \qquad ...(4)$$

Where $\quad R = 2 A \cos kx$ ...(5)

Here equation (4) is an equation of SHM. It implies that after superposition of the two waves the medium particles executes SHM with same frequency ω and amplitude R which is given by equation (5) Here we can see that the oscillation amplitude of medium particles depends on x i.e. the position of medium particles. Thus on superposition of two coherent waves travelling in opposite direction the resulting interference pattern, we call stationary waves, the oscillation amplitude of the medium particle at different positions is different.

At some point of medium the resultant amplitude is maximum which are given as

R is maximum when $\quad \cos kx = \pm 1$

or $\quad \dfrac{2\pi}{\lambda} x = N\pi \qquad [N \in I]$

or $\quad x = \dfrac{N\lambda}{2}$

or $\quad x = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}$ .....

and the maximum value of R is given as

$$R_{max} = \pm 2A \qquad ...(6)$$

Thus in the medium at position $x = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}$, ... the waves interfere constructively and the amplitude of oscillations becomes 2A. Similarly at some points of the medium, the waves interfere destructively, the oscillation amplitude become minimum i.e. zero in this case. These are the points where R is minimum, when $\cos kx = 0$

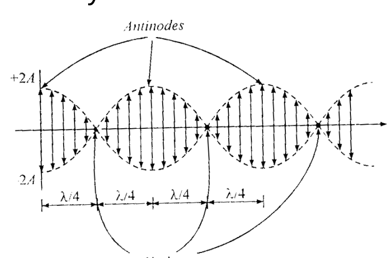
or $\quad \dfrac{2\pi x}{\lambda} = (2N+1)\dfrac{\pi}{2}$

or $\quad x = (2N + 1) \dfrac{\lambda}{4} \qquad [N \in I]$

or $\quad x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}$...........

and the minimum value of R is given as $R_{min} = 0$ [7]

Thus in the medium at position $x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}$ .... the waves interfere destructively and the amplitude of oscillation becomes zero. These points always remain at rest. Figure (a) shows the oscillation amplitude of different medium particles in a stationary waves.



(a)

In figure (a) we can see that the medium particles at which constructive interference takes place are called antinodes of stationary wave and the points of destructive interference are called nodes of stationary waves which always remain at rest.

Figure (b) explain the movement of medium particles with time in the region where stationary waves are formed. Let us assume that at an instant t = 0 all the medium particles are at their extreme positions as shown in figure - (b - 1). Here points ABCD are the nodes of stationary waves where medium particles remains at rest. All other starts moving towards their mean positions and t = $T/4$ all particles cross their mean position as shown in figure (b – 3), you can see in the figure that the particles at nodes are not moving. Now the medium crosses their mean position and starts moving on other side of mean position toward the other extreme position. At time t = T/2, all the particles reach their other extreme position as shown in figure (b - 5) and at time t = 3T/4 again all these particles cross their mean position in opposite direction as shown in figure (b - 7).

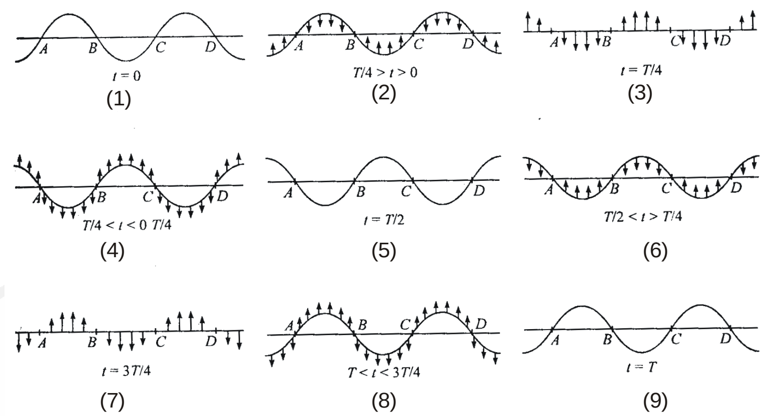


Figure (b)

Based on the above analysis of one complete oscillations of the medium particles, we can make some interference for a stationary waves. These are :

(i) In oscillations of stationary wave in a region, some points are always at rest (nodes) and some oscillates with maximum amplitudes (antinodes). All other medium particles oscillate with amplitudes less then those of antinodes.

(ii) All medium particles between two successive nodes oscillate in same phase and all medium particles on one side of a node oscillate in opposite phase with those on the other side of the same node.

(iii) In the region of a stationary wave during one complete oscillation all the medium particles come in the form of a straight line twice.

(iv) If the component wave amplitudes are equal, then in the region where stationary wave is formed, no net flow of energy takes place, only redistribution of energy takes place in the medium.

(a) Different Equation for a Stationary Wave

Consider two equal amplitude waves travelling in opposite direction as

$$y_1 = A \sin (\omega t - kx) \qquad ...(11)$$

and $$y_2 = A \sin (\omega t + kx) \qquad ...(12)$$

The result of superposition of these two waves is

$$y = 2A \cos kx \sin \omega t \qquad ...(13)$$

Which is the equation of stationary wave where 2A cos kx represents the amplitude of medium particle situated at position x and sin ωt is the time sinusoidal factor. This equation (13) can be written in several ways depending on initial phase differences in the component waves given by equation (11)) can (12). If the superposing waves are having an initial phase difference π, then the component waves can be expressed as

$$y_1 = A \sin (\omega t - kx) \qquad ...(14)$$

$y_2 = -A \sin(\omega t - kx)$ ...(15)

Superposition of the above two waves will result

$y = 2A \sin kx \cos \omega t$ ...(16)

Equation (16) is also an equation of stationary wave but here amplitude of different medium particles in the region of interference is given by

$R = 2A \sin kx$ ...(17)

Similarly the possible equations of a stationary wave can be written as

$y = A_0 \sin kx \cos(\omega t + \phi)$ ....(18)

$y = A_0 \cos kx \sin(\omega t + \phi)$ ...(19)

$y = A_0 \sin kx \sin(\omega t + \phi)$ ...(20)

$y = A_0 \cos kx \cos(\omega t + \phi)$ ...(21)

Here A_0 is the amplitude of antinodes. In a pure stationary wave it is given as

$A_0 = 2A$

Where A is the amplitude of component waves. If we care fully look at equation (18) to (21), we can see that in equation (18) and (20), the particle amplitude is given by

$R = A_0 \sin kx$ ...(22)

Here at x = 0, there is nodes as R = 0 and in equation (19) and (21) the particle amplitude is given as

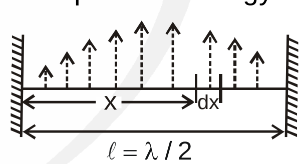
$R = A_0 \cos kx$ ...(23)

Here at x = 0, there is an antinode as $R = A_0$. Thus we can state that in a given system of co-ordinates when origin of system is at a node we use either equation (18) or (20) for analytical representation of a stationary wave and we use equation (19) or (21) for the same when an antinode is located at the origin of system.

(b) Energy of standing wave in one loop

When all the particles of one loop are at extreme position then total energy in the loop is in the form of potential energy only when the particles reaches its mean position then total potential energy converts into kinetic energy of the particles so we can say total energy of the loop remains constant

Total kinetic energy at mean position is equal to total energy of the loop because potential energy at mean position is zero.



Small kinetic energy of the particle which is in element dx is

$$d(KE) = \frac{1}{2} dm v^2$$

$$dm = \mu\, dx$$

Velocity of particle at mean position

$= 2A \sin kx\, \omega$

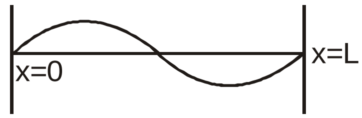
then $\quad d(KE) = \frac{1}{2}\mu dx \cdot 4A^2 \omega^2 \sin^2 kx \Rightarrow d(KE) = 2A^2\omega^2 \mu \cdot \sin^2 kx\, dx$

$$\int d(K.E) = 2A^2\omega^2\mu \int_0^{\lambda/2} \sin^2 kx\, dx$$

$$\text{Total K.E} = A^2\omega^2\mu \int_0^{\lambda/2} (1 - \cos 2kx)\, dx = A^2\omega^2\mu \left[x - \frac{\sin 2kx}{2k} \right]_0^{\lambda/2} = \frac{1}{2}\lambda A^2\omega^2\mu$$

ILLUSTRATIONS

1. Find out the equation of the standing waves for the following standing wave pattern.



(A) $A\sin\frac{2\pi}{L}x\cos\omega t$ (B) $A\sin\frac{\pi x}{L}\cos\omega t$ (C) $A\cos\frac{\pi x}{2L}\cos\omega t$ (D) $A\cos\frac{\pi x}{L}\cos\omega t$

Sol. General Equation of standing wave

$y = A'\cos\omega t$

where

$A' = A\sin(kx + \theta)$

here $\lambda = L$

$\Rightarrow$ $k = \frac{2\pi}{L}$

$A' = A\sin(kx + \theta) = A\sin\left(\frac{2\pi}{L}x + \theta\right)$

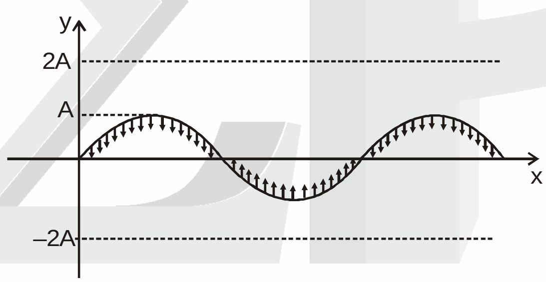
at x = 0 node

$\Rightarrow$ $A' = 0$ at $x = 0$

$\Rightarrow$ $\theta = 0$

eq. of standing wave = $A\sin\frac{2\pi}{L}x\cos\omega t$

2. Figure shows the standing waves pattern in a string at t = 0. Find out the equation of the standing wave where the amplitude of antinode is 2A.



Sol. Let we assume the equation of standing waves

is $= A'\sin(\omega t + \phi)$

where $A' = 2A\sin(kx + \theta)$

$\because$ $x = 0$ is node $\Rightarrow A' = 0$, at $x = 0$

$2A\sin\theta = 0$ $\Rightarrow$ $\theta = 0$

at t = 0 Particle at is at y = A and going towards mean position $\Rightarrow \phi = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

so eq. of standing waves is $y = 2A\sin kx\sin\left(\omega t + \frac{5\pi}{6}\right)$

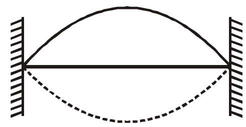
STATIONARY WAVES IN STRINGS

(a) When both end of string is fixed :

A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which the **ends are always nodes, if both ends of string are fixed.**

Fundamental Mode

(a) In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and frequency of vibration is known as the fundamental frequency or first harmonic.



Since the distance between consecutive nodes is $\frac{\lambda}{2}$

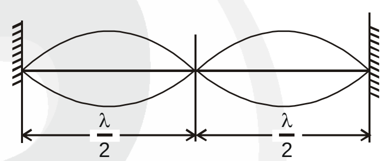
$\therefore \quad L = \frac{\lambda_1}{2} \qquad \therefore \; \lambda_1 = 2L$

If f_1 is the fundamental frequency of vibration, then the velocity of transverse waves is given as,

$$v = \lambda_1 f_1 \quad \text{or} \quad f_1 = \frac{v}{2L} \qquad \text{...(i)}$$

First Overtone

(b) The same string under the same conditions may also vibrate in two loops, such that the centre is also the node



$\therefore \quad L = \frac{2\lambda_2}{2} \qquad \therefore \; \lambda_2 = L$

If f_2 is frequency of vibrations

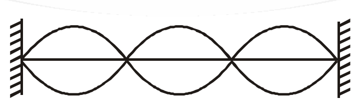
$$\therefore \quad f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$\therefore \quad f_2 = \frac{v}{L} \qquad \text{...(ii)}$$

The frequency f_2 is known as second harmonic or first overtone.

Second Overtone

(c) The same string under the same conditions may also vibrate in three segments.



$$\therefore \quad L = \frac{3\lambda_3}{2}$$

$$\therefore \quad \lambda_3 = \frac{2}{3}L$$

If f_3 is the frequency in this mode of vibration, then, $f_3 = \frac{3v}{2L}$ ...(iii)

The frequency f_3 is known as third harmonic or second overtone.

Thus a stretched string vibrates with frequencies, which are integral multiples of the fundamental frequencies. These frequencies are known as harmonics.

The velocity of transverse wave in stretched string is given as $v = \sqrt{\frac{T}{\mu}}$. Where T = tension in the string.

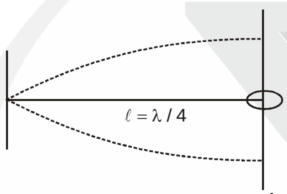
μ = linear density or mass per unit length of string. If the string fixed at two ends, vibrates in its fundamental mode, then $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ ....(17)

In general $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$; n^{th} harmonic ; $(n - 1)^{th}$ overtone

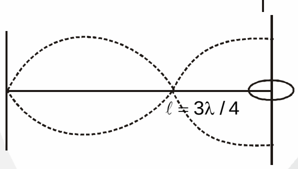
- In general, any integral multiple of the fundamental frequency is an allowed frequency. These higher frequenceis are called overtones. Thus, $v_1 = 2v_0$ is the first overtone, $v_2 = 3v_0$ is the second overtone etc. An integral multiple of a frequency is called its harmonic. Thus, for a string fixed at both the ends, all the overtones are harmonics of the fundamental frequency and all the harmonics of the fundamental frequency are overtones.

(b) When one end of the string is fixed and other is free :

free end acts as antinode

1. 

$f = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$ fundamental or I^{st} harmonic

2. 

$f = \frac{3}{4\ell}\sqrt{\frac{T}{\mu}}$ III^{rd} harmonic or I^{st} overtone

In general : $f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}$ $\Big\}$ $((2n + 1)^{th}$ harmonic, n^{th} overtone)

S.No.	Travelling waves	Stationary waves
1	These waves advance in a medium with a definite velocity	These waves remain stationary between two boundaries in the medium.
2	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3	At any instant phase of vibration varies continuosly from one particle to the other i.e., phase difference between two particles can have any value between 0 and 2 π	At any instant the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e, phase difference between any two particles can be either 0 or π
4	In these wave, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves all particles of the medium pass through their mean position simultaneously twice in each time period.
5	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

EXERCISE

1. A transverse wave is described by the equation $Y = Y_0 \sin 2\pi (ft - x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if

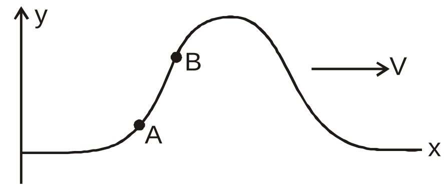
(A) $\lambda = \pi Y_0/4$ (B) $\lambda = \pi Y_0/2$ (C) $\lambda = \pi Y_0$ (D) $\lambda = 2\pi Y_0$

2. A transverse wave of amplitude 0.50 m, wavelength 1 m and frequency 2 hertz is propagating in a string in t he negative x-direction. The expression form of the wave is

(A) $y(x,t) = 0.5 \sin (2\pi x - 4\pi t)$ (B) $y(x,t) = 0.5 \cos (2\pi x + 4\pi t)$

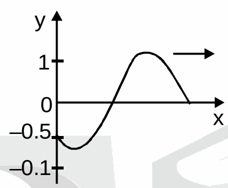
(C) $y(x,t) = 0.5 \sin (\pi x - 2\pi t)$ (D) $y(x,t) = 0.5 \cos (2\pi x - 2\pi t)$

3. A wave pulse is generated in a string that lies along x-axis. At the points A and B, as shown in figure, if R_A and R_B are ratio of wave speed to the particle speed respectively then :



(A) $R_A > R_B$ (B) $R_B > R_A$ (C) $R_A = R_B$ (D) Information is not sufficient to decide.

4. The equation of a wave travelling along the positive x-axis, as shown in figure at t=0 is given by

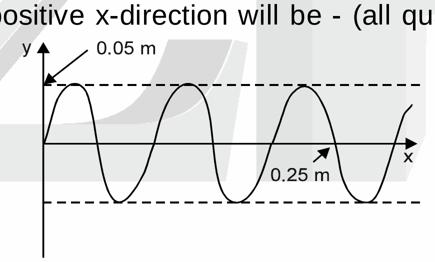


(A) $\sin\left(kx - \omega t + \frac{\pi}{6}\right)$ (B) $\sin\left(kx - \omega t - \frac{\pi}{6}\right)$ (C) $\sin\left(\omega t - kx + \frac{\pi}{6}\right)$ (D) $\sin\left(\omega t - kx - \frac{\pi}{6}\right)$

5. The velocity of a wave propagating along a stretched string is 10m/s and its frequency is 100 Hz. The phase difference between the particles situated at a distance of 2.5 cm on the string will be -

(A) $\pi/8$ (B) $\pi/4$ (C) $3\pi/8$ (D) $\pi/2$

6. If the speed of the wave shown in the figure is 330 m/s in the given medium, then the equation of the wave propagating in the positive x-direction will be - (all quantities are in MKS units)



(A) $y = 0.05 \sin 2\pi (4000\,t - 12.5\,x)$ (B) $y = 0.05 \sin 2\pi (4000\,t - 122.5\,x)$

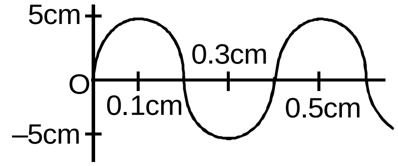
(C) $y = 0.05 \sin 2\pi (3300\,t - 10\,x)$ (D) $y = 0.05 \sin 2\pi (3300\,x - 10\,t)$

7. The displacement produced by a simple harmonic wave is : $y = \frac{10}{\pi} \sin\left(2000\pi t - \frac{\pi x}{17}\right)$ cm. The time period and maximum velocity of the particle will be respectively -

(A) 10^{-3} second and 200 m/s (B) 10^{-2} second and 2000 m/s

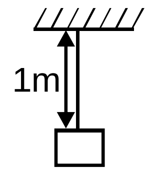
(C) 10^{-3} second and 330 m/s (D) 10^{-4} second and 20 m/s

8. Figure shown the shape of part of a long string in which transverse waves are produced by attaching one end of the string to tuning fork of frequency 250Hz. What is the velocity of the waves?



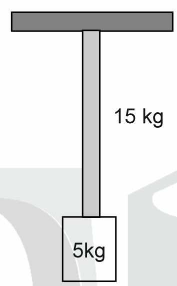
(A) 1.0 ms^{-1} (B) 1.5 ms^{-1} (C) 2.0 ms^{-1} (D) 2.5 ms^{-1}

9. A block of mass 1 kg is hanging vertically from a string of length 1 m and Mass/length =0.001 kg/m. A small pulse is generated at its lower end. The Pulse reaches the top end in approximately.



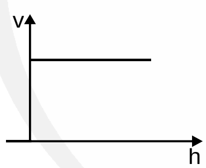
(A) 0.2 sec (B) 0.1 sec (C) 0.02 sec (D) 0.01 sec

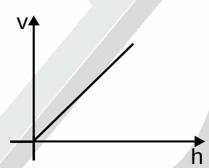
10. A uniform rope of length 10 m and mass 5 kg hangs vertically from a rigid support. A block of mass 5 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.08 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope will be-

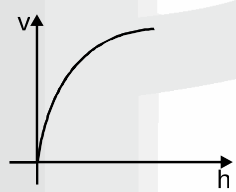


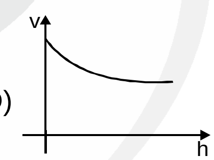
(A) 0.08 m (B) 0.04 m (C) 0.16 m (D) 0 m

11. A uniform rope having some mass hanges vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A) 

(B) 

(C) 

(D) 

12. For a wave displacement amplitude is 10^{-8} m, density of air 1.3 kg m^{-3}, velocity in air 340 ms^{-1} and frequency is 2000 Hz. The intensity of wave is -

(A) 5.3×10^{-4} Wm^{-2} (B) 5.3×10^{-6} Wm^{-2} (C) 3.5×10^{-8} Wm^{-2} (D) 3.5×10^{-6} Wm^{-2}

13. Two waves of equal amplitude A, and equal frequency travels in the same direction in a medium. The amplitude of the resultant wave is

(A) 0 (B) A (C) 2A (D) between 0 and 2A

14. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has wavelength λ and the transmitted wave λ'.

(A) $\lambda' > \lambda$ (B) $\lambda' = \lambda$ (C) $\lambda' < \lambda$

(D) nothing can be said about the relation of λ and λ'.

15. The rate of transfer of energy in a wave depends

(A) directly on the square of the wave amplitude and square of the wave frequency

(B) directly on the square of the wave amplitude and square root of the wave frequency

(C) directly on the wave frequency and square of the wave amplitude

(D) directly on the wave amplitude and square of the wave frequency.

16. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse in inverted with respect to the other.

(A) the pulses will collide with each other and vanish after collision.

(B) the pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.

(C) the pulses will pass through each other but their shapes will be modified

(D) the pulses will pass through each other without any change in their shape.

17. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is :

(A) 350 Hz (B) 5 Hz (C) 70 Hz (D) 170 Hz

18. In a stationary wave represented by $y = a \sin \omega t \cos kx$, amplitude of the component progressive wave is :

(A) $\frac{a}{2}$ (B) a (C) 2a (D) None

EXERCISE ANSWERS

1. B	**2.** B	**3.** A	**4.** D	**5.** D	**6.** C	**7.** A	**8.** A	**9.** D	**10.** C
11. C	**12.** D	**13.** D	**14.** C	**15.** A	**16.** D	**17.** C	**18.** A		

PROPAGATION OF SOUND WAVES

Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wave, sound needs a medium having properties of inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions

If the prongs vibrate in SHM, the pressure variations in the layer close to the prong also varies simple harmonically and hence increase in pressure above normal value can be written as $\delta P = \delta P_0 \sin \omega t$ where δP_0 is the maximum increase in pressure above normal value.

As this disturbance travel towards right with wave velocity v, the excess pressure at any position x at time t will be given by $\delta P = \delta P_0 \sin \omega(t - x/v)$

Using $p = \delta P$, $p_0 = \delta P_0$, the above equation of sound wave can be written as: $p = p_0 \sin \omega(t - x/v)$

FREQUENCY AND PITCH OF SOUND WAVES

FREQUENCY

Each cycle of a sound wave includes one compression and one rarefaction, and frequency is the number of cycles per second that passes by a given location. This is normally equal to the frequency of vibration of the (tuning fork) source producing sound. If the source, vibrates in SHM of a single frequency, sound produced has a single frequency and it is called a pure tone.

However a sound source may not always vibrate in SHM (this is the case with most of the common sound sources e.g. guitar string, human vocal card, surface of drum etc.) and hence the pulse generated by it may not have the shape of a sine wave. But even such a pulse may be considered to obtained by superposition of a large number of sine waves of different frequency and amplitudes. We say that the pulse contain all these frequencies.

AUDIBLE FREQUENCY RANGE FOR HUMAN

A normal person hears all frequencies between 20 & 20 KHz. This is a subjective range (obtained experimentally) which may vary slightly from person to person. The ability to hear the high frequencies decreases with age and a middle-age person can hear only upto 12 to 14 KHz.

INFRASONIC SOUND

Sound can be generated with frequency below 20 Hz called **infrasonic sound.**

ULTRASONIC SOUND

Sound can be generated with frequency above 20 Hz called **infrasonic sound.**

Even through humans cannot hear these frequencies, other animals may. For instance Rhinos communicate through infrasonic frequencies as low as 5Hz, and bats use ultrasonic frequencies as high as 100 KHz for navigating.

PITCH

Frequency as we have discussed till now is an objective property measured its units of Hz and which can be assigned a unique value. However a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called **Pitch.** A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound

ILLUSTRATIONS

1. A wave of wavelength 4 mm is produced in air and it travels at a speed of 300 m/s. Will it be audible ?

Sol. From the relation $v = \nu\lambda$, the frequency of the wave is

$$\nu = \frac{v}{\lambda} = \frac{300\ \text{m/s}}{4 \times 10^{-3}\text{m}} = 75000\ \text{Hz}.$$

This is much above the audible range. It is an ultrasonic wave and will not be audible to humans, but it will be audible to bats.

PRESSURE WAVE AND DISPLACEMENT WAVE

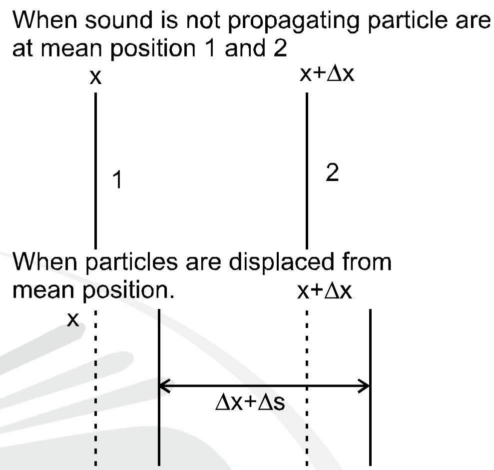
We can describe sound waves either in terms of excess pressure (equation 1.1) or in terms of the longitudinal displacement suffered by the particles of the medium w.r.t. mean position.

If $s = s_0 \sin \omega(t - x/v)$ represents a sound wave where,

s = displacement of medium particle from its mean position at x,

$s = s_0 \sin (\omega t - kx)$ ...(3.1)

Change in volume = $\Delta V = (\Delta x + \Delta s)A - \Delta xA = \Delta sA$



$$\frac{\Delta V}{V} = \frac{\Delta sA}{\Delta xA} = \frac{\Delta s}{\Delta x}$$

$$\Delta P = -\frac{B\Delta V}{V}$$

$$\Delta P = -\frac{B\Delta s}{\Delta x}$$

$$dp = -\frac{Bds}{dx}$$

$dp = -B(-k s_0) \cos (\omega t - kx)$

$dp = Bks_0 \cos (\omega t - kx)$

$dp = (dp)_{max} \cos (\omega t - kx)$

$p = p_0 \sin (\omega t - kx + \pi/2)$ .... (3.2)

where $p = dp$ = variation in pressure at position x and $p_0 = Bks_0$ = maximum pressure variation

Equation 3.2 represents that same sound wave where, P is excess pressure at position x, over and above the average atmospheric pressure and pressure amplitude p_0 is given by $P_0 = BKs_0$ .....(3.3)

(B = Bulk modulus of the medium, K = angular wave number)

Note from equation (3.1) and (3.2) that the displacement of a medium particle and excess pressure at any position are out of phase by $\frac{\pi}{2}$. Hence a displacement maxima corresponds to a pressure minima and vice-versa.

ILLUSTRATIONS

1. A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 2.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m^2.

Sol. The pressure amplitude is $p_0 = \frac{2.0 \times 10^{-3} N/m^2}{2} = 10^{-3}$ N/m^2.

The displacement amplitude s_0 is given by $p_0 = B\,k\,s_0$ or $s_0 = \frac{p_0}{Bk} = \frac{p_0 \lambda}{2\pi B}$

$$= \frac{10^{-3} N/m^2 \times (40 \times 10^{-2} m)}{2 \times \pi \times 14 \times 10^4 N/m^2} = \frac{100}{7\pi} \text{Å} = 6.6\ \text{Å}$$

SPEED OF SOUND WAVES

A. Velocity of sound waves in a linear solid medium is given by $v = \sqrt{\frac{Y}{\rho}}$ ....(4.1) where Y = young's modulus of elasticity and ρ = density.

B. Velocity of sound waves in a fluid medium (liquid or gas) is given by $v = \sqrt{\frac{B}{\rho}}$ ....(4.2)

where, ρ = density of the medium and B = Bulk modulus of the medium given by, $B = -V\frac{dP}{dV}$ ....(4.3)

Newton's formula : Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

PV = constant $\Rightarrow \frac{dP}{dV} = \frac{-P}{V}$ and hence B = P using equ. ...(4.3)

and thus he obtained for velocity of sound in a gas, $v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}}$ where M = molar mass

the density of air at 0º and pressure 76 cm of Hg column is ρ = 1.293 kg/m^3. This temperature and pressure is called standard temperature and pressure at STP. Speed of sound in air is 280 m/s. This value is some what less than messured speed of sound in air 332 m/s then laplace suggested the correction.

Laplace's correction : Later Laplace established that propagation of sound in a gas is not an isothermal but an adiabatic process and hence PV^{γ} = constant

$\Rightarrow \frac{dP}{dV} = -\gamma\frac{P}{V}$ where, $B = -V\frac{dP}{dV}$ = gP

and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \qquad .... (4.4)$$

B. Factors affecting speed of sound in atmosphere.

(a) Effect of temperature : as temperature (T) increases velocity (v) increases. $v \propto \sqrt{T}$

For small change in temperature above room temperature v increases linearly by 0.6 m/s for every 1°C rise in temp.

$$v = \sqrt{\frac{\gamma R}{M}} \times T^{1/2} \qquad \frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T} \qquad \Delta v = \left(\frac{1}{2}\frac{v}{T}\right)\Delta T \qquad \Delta v = (0.6)\Delta T$$

(b) Effect of pressure :

The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

(c) Effect of humidity : With increase in humidity density decreases. This is because the molar mass of water vapour is less than the molar mass of air.

So at constant temperature, if P changes then ρ also changes in such a way that P/ρ remains constant. Hence pressure does not have any effect on velocity of sound as long as temperature is constant.

ILLUSTRATIONS

1. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Sol. $v = \sqrt{\frac{\gamma RT}{M}}$ since temperature, T is constant,

$\therefore \ \dfrac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\dfrac{M_{O_2}}{M_{H_2}}} = \sqrt{\dfrac{32}{2}} = 4$ $\quad \Rightarrow v\,(H_2) = 4 \times 450 = 1800$ m/s **Ans.**

Aliter : The speed of sound in a gas is given by $u = \sqrt{\dfrac{\gamma P}{\rho}}$. At STP, 22.4 litres of oxygen has a mass of 32 g whereas the same volume of hydrogen has a mass of 2 g. Thus, the density of oxygen is 16 times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases,

$$\frac{f_{\text{(hydrogen)}}}{f_{\text{(oxygen)}}} = \sqrt{\frac{\rho_{\text{(oxygen)}}}{\rho_{\text{(hydrogen)}}}} \quad \text{or,} \quad f_{\text{(hydrogen)}} = 4f_{\text{(oxygen)}} = 4 \times 450 \text{ m/s} = 1800 \text{ m/s.} \qquad \textbf{Ans.}$$

INTENSITY OF SOUND WAVES

Total energy transfer = $P_{av} \times t = \dfrac{\rho A v \omega^2 s_0^2}{2} \times t$

Average intensity = average power / area the average intensity at position x is given by

$$< I > = \frac{1}{2}\,\frac{\omega^2 s_0^2 B}{v} = \frac{P_0^2 v}{2B}$$

Substituting $B = \rho v^2$, intensity can also be expressed as $I = \dfrac{P_0^2}{2\rho v}$

Note: ☞ If the source is a point source then $I \propto \dfrac{1}{r^2}$ and $s_0 \propto \dfrac{1}{r}$ and $s = \dfrac{a}{r}\sin(\omega t - kr + \theta)$

☞ If a sound source is a line source then $I \propto \dfrac{1}{r}$ and $s_0 \propto \dfrac{1}{\sqrt{r}}$ and $s = \dfrac{a}{\sqrt{r}}\sin(\omega t - kr + \theta)$

LOUDNESS

Audible intensity range for humans :

The ability of human to perceive intensity at difference frequency is different. The perception of intensity is maximum at 1000 Hz and perception of intensity decreases as the frequency decreases or increases from 1000 Hz.

☞ For a 1000 Hz tone, the smallest sound intensity that a human ear can detect is 10^{-12} watt./m^2. On the other hand, continuous exposure to intensities above 1W/m^2 can result in permanent hearing loss.

☞ The overall perception of intensity of sound to human ear is called **loudness**.

☞ Human ear do not percives loudness on a linear intensity scale rather it percives loudness on logarithmic intensity scale.

For example: If intensity is increased 10 times human ear does not perceive10 times increase in loudness. It roughly perceived that loudness is doulbed where intensity increased by 10 times. Hence it is prudent to define a logaritimic scale for intensity.

DECIBEL SCALE

The logarithmic scale which is used for comparing to sound intensity is called **decible scale**.

The intensity level β described in terms of decibels is defined as $\beta = 10 \log\left(\dfrac{I}{I_0}\right)$ (dB)

Here I_0 is the threshold intensity of hearing for human ear

i.e. $\quad I = 10^{-12}$ watt/m^2.

☞ In terms of decible threshold of human hearing is 1 dB

☞ Note that intensity level β is a dimensionless quantity and is not same as intensity expressed in W/m^2.

ILLUSTRATIONS

1. If the intensity is increased by a factor of 20, by how many decibels is the intensity level increased.

Sol. Let the initial intensity by I and the intensity level be β_1 and when the intensity is increased by 20 times, the intensity level increases to β_2.

Then $\beta_1 = 10 \log (I / I_0)$ and $\beta_2 = 10 \log (20I / I_0)$

Thus, $\beta_2 - \beta_1 = 10 \log (20I / I)$ $= 10 \log 20$ $= 13$ dB.

2. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

Sol. $b_1 = 10 \log \frac{I_1}{I_0}$ $\quad b_2 = 10 \log \frac{I_2}{I_0} \Rightarrow \quad b_2 - b_1 = 10 \log \frac{I_2}{I_1}$

or $10 = 10 \log_{10}\left(\frac{I_2}{I_1}\right) \Rightarrow \frac{I_2}{I_1} = 10^1 = 10$

for point source $I \propto \frac{1}{r^2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10}$ **Ans.**

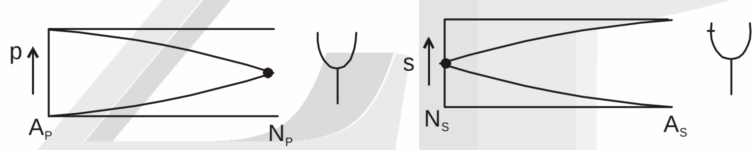
VIBRATION OF AIR COLUMNS

Standing waves can be set up in air-columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequency of air columns. In such a case the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air-column. To determine the natural frequency of the air-column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air-column.

In reality antinodes do not occurs exactly at the open end but a little distance outside. However if diameter of tube is small compared to its length, this end correction can be neglected.

A. Closed organ pipe

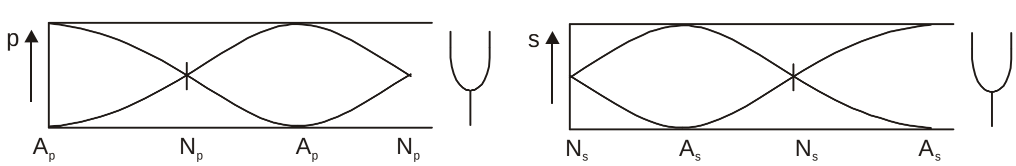
(In the diagram, A_p = Pressure antinode, A_s = displacement antinode, N_p = pressure node, N_s = displacement node.)



Fundamental mode

The smallest frequency (largest wavelength) that satisties the boundary condition for resonance (i.e. displacement node at left end and antinode at right end is $\lambda_0 = 4\ell$, where ℓ = length of closed pipe the corresponding frequency.

$n_0 = \frac{v}{\lambda} = \frac{v}{4L}$ is called the fundamental frequency. ..(9.1)



First Overtone : Here there is one node and one antinode apart from the nodes and antinodes at the ends.

$I_1 = \frac{4\ell}{3} = \frac{\lambda_0}{3}$ and corresponding frequency, $n_1 = \frac{v}{\lambda_1} = 3n_0$

This frequency is 3 times the fundamental frequency and hence is called the 3rd harmonic.

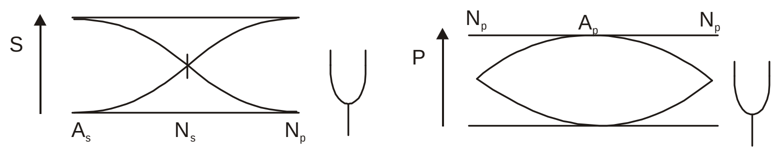
nth overtone :

In general, the nth overtone will have n nodes and n antinodes between the two nodes. The corresponding wavelength is

$$I_n = \frac{4\ell}{2n+1} = \frac{\lambda_0}{2n+1} \text{ and } \nu_n \,(2n+1)n_0 \qquad ....(9.2)$$

This corresponds to the $(2n + 1)^{th}$ harmonic. Clearly only odd harmonic are allowed in a closed pipe.

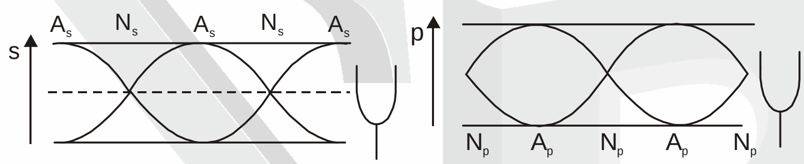
B. Open organ pipe :



Fundamental mode :

The smallest frequency (largest wave length) that satisfies the boundary condition for resonance (i.e. displacement antinodes at both ends) is, $I_0 = 2\ell$ corresponding frequency, is called the fundamental frequency

$$n_0 = \frac{v}{2\ell} \qquad ....(9.3)$$



1st Overtone : Here there is one displacement antinode between the two antinodes at the ends.

$I_1 = \frac{2\ell}{2} = I = \frac{\lambda_0}{2}$ and, corresponding frequency $n_1 = \frac{v}{\lambda_1} = 2n_0$

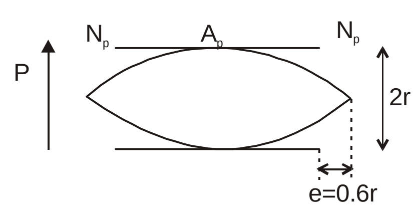
This frequency is 2 times the fundamental frequency and is called the 2nd harmonic.

nth overtone : The nth overtone has n displacement antinodes between the two antinode at the ends.

$$I_n = \frac{2\ell}{n+1} = \frac{\lambda_0}{n+1} \qquad \text{and} \qquad n_n = (n+1)\, n_0 \qquad ....(9.4)$$

This correspond to $(n + 1)^{th}$ harmonic: clearly both even and odd harmonics are allowed in an open pipe.

C. End correction : As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by e = 0.6 r



where r = radius of the organ pipe. with end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)} \quad \text{and} \quad f_0 = \frac{v}{2(\ell + 1.2r)} \qquad ....(9.5)$$

ILLUSTRATIONS

1. A tuning fork is vibrating at frequency 100 Hz. When another tuning fork is sounded simultaneously, 4 beats per second are heard. When some mass is added to the tuning fork of 100 Hz, beat frequency decreases. Find the frequency of the other tuning fork.

Sol. $|f - 100| = 4 \qquad \Rightarrow f = 95$ or 105

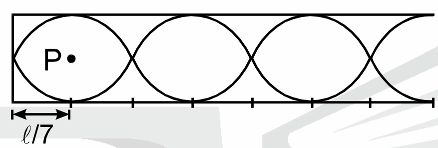
when 1st tuning fork is loaded its frequency decreases and so does beat frequency

$\Rightarrow 100 > f \qquad \Rightarrow f = 95$ Hz.

2. A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Sol. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$ or $\frac{\lambda}{4} = \frac{\ell}{7}$



Hence the amplitude at P at a distance $\frac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point

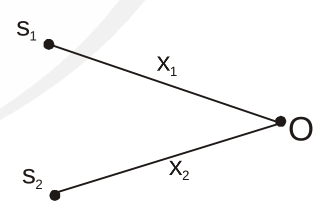
INTERFERENCE IN TIME : BEATS

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,

$p_1 = p_0 \sin(2\pi f_1 t - kx_1 + \theta_1)$

$p_2 = p_0 \sin(2\pi f_2 t - kx_2 + \theta_2)$



By principle of superposition

$p = p_1 + p_2$

$= 2p_0 \cos\{p(f_1 - f_2)t + \frac{\theta_1 + \theta_2}{2}\} \sin\{p(f_1 + f_2)t + \frac{\theta_1 + \theta_2}{2}\}$

i.e., the resultant sound at point O has frequency $\left(\frac{f_1 + f_2}{2}\right)$ while pressure amplitude p'_0 (t) variex with time as

$$p'_0(t) = 2p_0 \cos\left\{\pi(f_1 - f_2)t + \frac{\phi_1 - \phi_2}{2}\right\}$$

Hence pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1 - f_2}{2}\right)$.

Hence sound intensity will vary with a frequency $f_1 - f_2$.

This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = f_1 - f_2 = T_B = \frac{1}{f_1 - f_2} \qquad (10.1)$$

IMPORTANT POINTS :

i. The frequency $|f_1 - f_2|$ should be less than 16 Hz, for it to be audible.

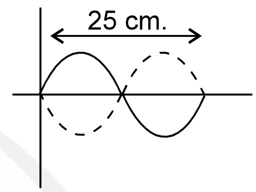
ii. Beat phenomenon can be used for determining an unknown frequency by sounding it together with a source of known frequency.

iii. If the arm of a tuning fork is waxed or loaded, then its frequency decreases.

iv. If arm of tuning fork is filed, then its frequency increases.

ILLUSTRATIONS

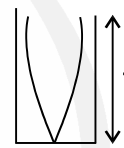
1. A string 25 cm long fixed at both ends and having a mass of 2.5 g is under tension. A pipe closed from one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s. Find tension in the string.



Sol. $\mu = \frac{2.5}{25}$ **=** 0.1 g / cm = 10^{-2} Kg/m; Ist overtone

λ_s = 25 cm = 0.25 m = $f_s = \frac{1}{\lambda_s}\sqrt{\frac{T}{\mu}}$

pipe in fundamental freq



λ_p = 160 cm = 1.6 m

$f_p = \frac{V}{\lambda_p}$

$\because$ by decreasing the tension , beat freq is decreased

$\therefore\ f_s > f_p \qquad \Rightarrow\ f_s - f_p = 8 \quad \Rightarrow\ \frac{1}{0.25}\sqrt{\frac{T}{10^{-2}}} - \frac{320}{1.6} = 8 \quad \Rightarrow \quad T = 27.04\text{ N}$

2. The wavelength of two sound waves are 49cm and 50 cm respectively. If the room temperature is 30 °C then the number of beats produced by them is approximately (velocity of sound in air at 0^0C = 332 m/s).

(A) 6 (B) 10 (C*) 14 (D) 18

Sol. $v = 332\sqrt{\frac{303}{273}}$ $\qquad \Rightarrow$ Beat frequency = $f_1 - f_2 = v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

$= 332\sqrt{\frac{303}{273}}\left(\frac{1}{49} - \frac{1}{50}\right) \times 100 \cong 14$ **Ans.**

DOPPLER'S EFFECT

When there is relative motion between the source of a sound/light wave & an observer along the line joining them, the actual frequency observed is different from the frequency of the source. This phenomenon is called Doppler's Effect. If the observer and source are moving towards each other, the observed frequency is greater than the frequency of the source. If the observer and source move away from each other, the observered frequency is less than the frequency of source.

(v = velocity of sound wrt. ground. , c = velocity of sound with respect to medium, v_m = velocity of medium, v_O = velocity of observer, v_s = velocity of source.)

a. Sound source is moving and observer is stationary :

If the source emitting a sound of frequency f is travelling with velocity v_s along the line joining the source and observer,

observed frequency, $f' = \left(\dfrac{v}{v - v_s}\right) f$ ...(11.1)

and Apparent wavelength $\lambda' = \lambda \left(\dfrac{v - v_s}{v}\right)$ ...(11.2)

In the above expression, the positive direction is taken along the velocity of sound, i.e. from source to observer. Hence v_s is positive if source is moving towards the observer, and negative if source is moving away from the observer.

b. Sound source is stationary and observer is moving with velocity v_0 along the line joining them:

The source (at rest) is emitting a sound of frequency 'f' travelling with velocity 'v' so that wavelength is $\lambda = v/f$, i.e. there is no change in wavelength. How ever since the observer is moving with a velocity v_0 along the line joining the source and observer, the observed frequency is

$f' = f\left(\dfrac{v - v_0}{v}\right)$ ...(11.3)

In the above expression, the positive direction is taken along the velocity of sound, i.e. from source to observer. Hence v_O is positive if observer is moving away from the source, and negative if observer is moving towards the source.

c. The source and observer both are moving with velocities v_s and v_0 along the line joining them:

The observed frequency, $f' = f\left(\dfrac{v - v_0}{v - v_s}\right)$ ...(11.4)

and Apparent wavelength $\lambda' = \lambda\left(\dfrac{v - v_s}{v}\right)$ ...(11.5)

In the above expression also, the positive direction is taken along the velocity of sound, i.e. from source to observer.

In all of the above expression from equation 11.1 to 11.5, v stands for velocity of sound with respect to ground.

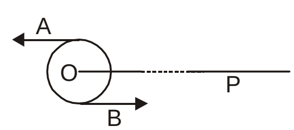
If velocity of sound with respect to medium is c and the medium is moving in the direction of sound from source to observer with speed v_m, $v = c + v_m$, and if the medium is moving opposite to the direction of sound from observer to source with speed v_m, $v = c - v_m$

ILLUSTRATIONS

1. A whistle of frequency 540 Hz is moving in a circle of radius 2 ft at a constant angular speed of 15 rad/s. What are the lowest and height frequencies heard by a listener standing at rest, a long distance away from the centre of the circle? (velocity of sound in air is 1100 ft/sec.)

Sol. The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$$v_S = \omega R$$



where, R is radius of the circle.

At points A and B, the velocity v_s of whistle is parallel to line OP; i.e., with respect to observer at P, whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP).

Observer, therefore, listens maximum frequency when source is at B moving towards observer:

$$f_{max} = f\ \frac{v}{v - v_s}$$

where, v is speed of sound in air. Similarly, observer listens minimum frequency when source is at A, moving away from observer:

$$f_{min} = \frac{f}{v + v_s}$$

For f = 540 Hz, v_s = 2 ft × 15 rad/s = 30 ft/s, and v = 1100 ft/s, we get (approx.)

f_{max} = 555 Hz and f_{min} = 525 Hz.

2. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 600 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find, (a) the frequency of the whistle as heard by an observer on the hill. (b) the distance from the hill at which the echo from the hill is heard by the driver and its frequency. (Velocity of sound in air = 1200 km/hr.)

Sol. A train is moving towards a hill with speed v_s with respect to the ground. The speed of sound in air, i.e. the speed of sound with respect to medium (air) is c, while air itself is blowing towards hill with velocity v_m (as observed from ground). For an observer standing on the ground, which is the inertial frame, the speed of sound towards hill is given by $v = c + v_m$

a. The observer on the hill is stationary while source is approaching him. Hence, frequency of whistle heard by him is

$$f' = f\frac{v}{v - v_s}$$

for f = 600 Hz, v_s = 40 km/hr, and V = (1200 + 40) km/hr, we get

$$f' = 600\ .\ \frac{1240}{1240 - 40} = 620 \text{ Hz.}$$

b. The train sounds the whistle when it is at distance x from the hill. Sound moving with velocity v with respect to ground, takes time t to reach the hill, such that,

$$t = \frac{x}{v} = \frac{x}{c + v_m} \qquad \text{....(i)}$$

After reflection from hill, sound waves move backwards, towards the train. The sound is now moving opposite to the wind direction. Hence, its velocity with respect to the ground is $v' = c - v_m$

Suppose when this reflected sound (or echo) reaches the train, it is at distance x' from hill. The time taken by echo to travel distance x' is given by

$$t' = \frac{x'}{v} = \frac{x'}{c - v_m} \qquad \text{....(ii)}$$

Thus, total time (t + t´) elapses between sounding the whistle and echo reaching back. In the same time, the train moves a distance (x – x') with constant speed v_s, as observed from ground. That is,

$x - x' = (t + t')\, v_s$.

Substituting from (i) and (ii), for t and t´, we find $x - x' = \frac{v_s}{c + v_m} x + \frac{v_s}{c + v_m} x'$

or, $\frac{c + v_m - v_s}{c + v_m} = \frac{v_s + c - v_m}{c - v_m}\ x'$

For x = 1 km, c = 1200 km/hr, v_s = 40 km/hr, and v_m = 40 km/hr, we get

$$\frac{1200+40-40}{1200-40} \times 1 = \frac{40+1200-40}{1200-40}\, x' \text{ or } x' = \frac{1160}{1240} = 0.935 \text{ km.}$$

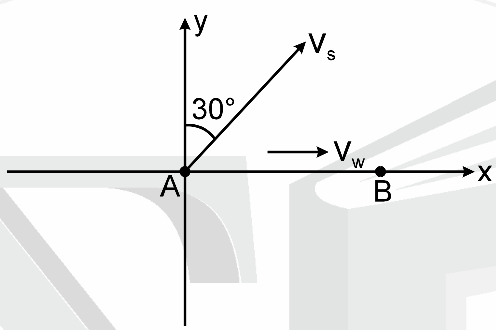
Thus, the echo is heard when train is 935 m from the hill.

c. Now, for the observer moving along with train, echo is a sound produced by a stationary source, i.e., the hill. Hence as observed from ground, source is stationary and observer is moving towards source with speed 40 km/hr. Hence v_O = – 40 km/hr. On the other hand, reflected sound travels opposite to wind velocity. That is, velocity of echo with respect to ground is v´. Further, the source (hill) is emmitting sound of frequency f´ which is the frequency observed by the hill.

Thus, frequency of echo as heard by observer on train, is given by

$$f'' = f' \frac{v'+v_O}{v'} \qquad \Rightarrow f'' = \frac{(1160-(-40))}{1160} \times 620 = \frac{18600}{29} \text{Hz}$$

3. In the figure shown a source of sound of frequency 510 Hz moves with constant velocity v_s = 20 m/s in the direction shown. The wind is blowing at a constant velocity v_w = 20 m/s towards an observer who is at rest at point B. The frequency detected by the observer corresponding to the sound emitted by the source at initial position A, will be (speed of sound relative to air = 330 m/s)



(A) 485 Hz (B) 500 Hz (C) 512 Hz (D*) 525 Hz

Sol. Apparent frequency

$$n' = n\,\frac{(u+v_w)}{(u+v_w-v_s\cos 60°)} = \frac{510\,(330+20)}{330+20-20\cos 60°} = 510 \times \frac{350}{340} = 525 \text{ Hz } \textbf{Ans.}$$

SOLVED EXAMPLES

1. Use the formula $\nu = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air:

(a) is independent of pressure. (b) increases with temperature. (c) increases with humidity.

Sol. Assume ideal gas law : $P = \frac{\rho RT}{M}$ where ρ is the density , M is the molecular mass, and T is the temperature of the gas. This gives $\nu = \sqrt{\frac{\gamma RT}{M}}$ .This shows that v is

(a) Independent of pressure.

(b) Increases as $\sqrt{T}$.

(c) The molecular mass of water (18) is less then that of N_2 (28) and O_2 (32). Therefore as humidity increases, the effective molecular mass of air decrease and hence ν increases.

2. The audible frequency for a normal human being is 20 Hz to 20 kHz. Find the corresponding wavelengths if the speed of sound in air 320 m/s

Sol. $\lambda = \frac{V}{\eta}$; $\lambda_{\text{max¼vf/kdre½}} = \frac{320}{20\,\text{Hz}} = 16$ m ; $\lambda_{\text{min¼U;wure½}} = \frac{320}{20\,\text{KHz}} = 16$ mm.

3. A sound wave of frequency 80 Hz is traveling with speed 320 m/s.

(a) Find the change in phase at a given position in 400 ms

(b) Find the phase difference between two positions separated by 20 cm at a particular instant

Sol. (a) $\Delta\phi = \frac{2\pi}{T}\Delta t = 2\pi f \Delta t = 2\pi \times 80 \times 400 \times 10^{-3} = 64\pi$

(b) $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi f}{v}\Delta x = \frac{2\pi \times 80}{320} \times \frac{20}{100} = \frac{\pi}{10}$

4. A traveling sound wave is described by the equation y = 2 sin (4t – 5x) where y is measured in centimeter, t in seconds and x in meters.

(a) Find the ratio of amplitude and wavelength of wave.

(b) Find the ratio of maximum velocity of particle to wave velocity.

Sol. (a) $\frac{A}{\lambda} = \frac{A}{2\pi/k} = \frac{Ak}{2\pi} = \frac{2 \times 10^{-2} \times 5}{2\pi} = \frac{1}{20\pi}$

(b) $\frac{\text{max. velocity particle}}{\text{wave velocity}}$ $\frac{\text{d.k dk vf/kdre os›}}{\text{rjaxosx}} = \frac{A\omega}{\omega/k} = kA = 5 \times 2 \times 10^{-2} = \frac{1}{10}$

5. Find the minimum and maximum wavelengths of sound in water that is in the audible range (20 - 20000 Hz) for an average human ear. Speed of sound in water = 1500 m/s.

Sol. $\lambda = \frac{v}{f} \Rightarrow$ $\lambda_{\text{maximum}} = \frac{v}{f_{\text{minimum}}} = \frac{1500}{20} = 75$ m $\lambda_{\text{maximum}} = \frac{v}{f_{\text{maximum}}} = \frac{1500}{20000} = 75$ mm

$\lambda = \frac{v}{f} \Rightarrow$ $\lambda_{\text{vf/kdre}} = \frac{v}{f_{\text{U;wure}}} = \frac{1500}{20} = 75$ m $\lambda_{\text{U;wure}} = \frac{v}{f_{\text{vf/kdre}}} = \frac{1500}{20000} = 75$ mm

6. A man stands before a large wall at a distance of 100.0 m and claps his hands at regular intervals In such way that echo of a clap merges with the next clap. If he claps 5 times during every 3 seconds, find the velocity of sound in air.

Sol. $T = \frac{3}{5} = \frac{2\ell}{V}$; $\frac{3}{5} = \frac{2 \times 100}{V} = V = \frac{1000}{3} = 333$ m/sec

7. (a) Find the speed of sound in a mixture of 1 mol of helium and 2 mol of oxygen at 27°C.
(b) If now temp. is raised by 1K from 300 K. Find the percentage change in the speed of sound in the gaseous mixture. [Note : This can be done after studying heat.]

Sol. (a) $\gamma_{mix} = \dfrac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \dfrac{1\times\frac{5}{2}R + 2\times\frac{7}{2}R}{1\times\frac{3}{2}R + 2\times\frac{5}{2}R} = \dfrac{19}{13}$; $m_{mix} = \dfrac{n_1 m_1 + n_2 m_2}{n_1 + n_2} = \dfrac{1\times 4 + 2\times 32}{1+2} = \dfrac{68}{3}$

$$V = \sqrt{\frac{\gamma_{mix} RT}{m_{mix}}} = \sqrt{\frac{19\times 25\times 300\times 3}{13\times 3\times 68\times 10^{-1}}} = 400.9 \text{ m/sec}$$

(b) $V = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow \ell n\, V = \dfrac{1}{2}\,\ell n\, \dfrac{\gamma R}{M} + \dfrac{1}{2}\,\ell n\, T \Rightarrow \dfrac{1}{V}\dfrac{dV}{dT} = 0 + \dfrac{1}{2T}$

$$\Rightarrow \frac{dV}{V} \times 100 = \frac{1}{2}\,\frac{dT}{T} \times 100 = \frac{1}{2\times 300} \times 100 = \frac{1}{6}\ \%$$

8. A metallic rod of length 1 m is rigidly clamped at its end points. Longitudinal stationary waves are setup in the rod in such a way that there are six antinodes of displacement wave observed along the rod. The amplitude of the antinode is 2×10^{-6} m. Write the equations of the stationary wave and the component waves at the point 0.1 m from the one end of the rod. [Young's modulus = 7.5×10^{10} N/m^2 , density = 2500 kg/m^3]

Sol. $V = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{7.5\times 10^{10}}{2500}} = \sqrt{30}\times 10^3 = \dfrac{6\lambda}{2} = 1$; $\lambda = \dfrac{1}{3}$ m

$n = \dfrac{V}{\lambda} = 3\sqrt{30}\times 10^3$; $\omega = 2\pi n = 6\pi\sqrt{30}\times 10^3$; $a = 2 \times 10^{-6}$ m

for stationary wave $y = a \sin kx \cos(\omega t + \theta)$

$y = 2 \times 10^{-6} \sin 6\pi x \cos(6\sqrt{30}\,\pi \times 10^3\, t + \theta)$

at x = 0.1; $y = 2 \times 10^{-6} \sin \dfrac{6\pi}{10} \cos(6\sqrt{30}\,\pi \times 10^3\, t + \theta)$

9. The equation of a longitudinal standing wave due to superposition of the progressive waves produced by two sources of sound is $s = -20 \sin 10\pi x \sin 100\pi t$ where s is the displacement from mean position measured in mm, x is in meters and is in seconds . The specific gravity of the medium is 10^{-3} . Density of water = 10^3 kg/m^3 . Find:
(a) Wavelength, frequency and velocity of the progressive waves .
(b) Bulk modulus of the medium and the pressure amplitude .
(c) Minimum distance between pressure antinode and a displacement antinode .
(d) Intensity at the displacement nodes.

Sol. (a) $f = \dfrac{100\pi}{2\pi} = 50$ Hz ; $k = 10\pi$; $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{10\pi} \times 100$ cm = 20 cm

$v = f\lambda = 50 \times 0.2 = 10$ m/s

(b) $B = \rho\upsilon^2 = (10^{-3} \times 1000) \times 10^2 = 100$ N/m^2

$P_m = BKS_o = 100 \times 10\pi \times 20 \times 10^{-3} = 20\pi$ N/m^2

(c) Distance between Pressure Antinode and displacement Node is $\dfrac{\lambda}{4}$ = 5 cm = .05 m

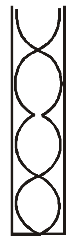
(d) $I = \dfrac{P_m^2}{2\rho V} = \dfrac{(20\pi)(20\pi)}{2\times 1\times 10} = 20\pi^2$ w/m^2

10. A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3[rd] overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Sol. Equation of stationary wave from closed end = a sink x cos ωt

Now at $x = \frac{\ell}{7}$

3rd overtone



$\frac{7\lambda}{4} = \ell$

Amplitude = a sin kx = a sin $\frac{2\pi}{\lambda}\frac{\ell}{7}$ = a sin $\frac{2\pi}{\lambda}\frac{7\lambda}{4\times 7}$ = a sin $\frac{\pi}{2}$ = a

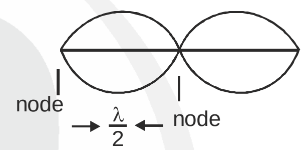
11. The speed of sound in an air column of 80 cm closed at one end is 320 m/s. Find the natural frequencies of air column between 20 Hz and 2000 Hz.

Sol. fundamental frequency of air column f = $\frac{V}{4\ell} = \frac{320}{4\times 0.8}$ = 100 other frequency which resonates are

f = 100 (2n +1)   n = 0, 1, 2, 3 .... here f $\leq$ 2 kHz

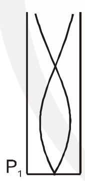
12. In an organ pipe the distance between the adjacent nodes is 4 cm. Find the frequency of source if speed of sound in air is 336 m/s

Sol. $\frac{\lambda}{2}$ = 4 × 10^{-2} m ; λ = 8 × 10^{-2} m

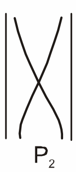


n = $\frac{V}{\lambda} = \frac{336}{8\times 10^{-2}}$ = 4200 Hz = 4.2 KHz

13. Two pipes P_1 and P_2 are closed and open respectively. P_1 has a length of 0.3 m. Find the length of P_2, if third harmonic of P_1 is same as first harmonic of P_2.

Sol. 

$\ell_1 = \frac{3\lambda}{4}$



$\ell_2 = \frac{\lambda}{2}$

First overtone fundamental $\frac{3V}{4\ell_1} = \frac{V}{2\ell_2} \simeq \ell_2 = \frac{2\ell_1}{3} = \frac{2\times 30}{3}$ = 20 cm

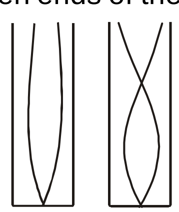
14. Find the number of possible natural oscillations of air column in a pipe whose frequencies lie below f_0 = 1250 Hz. The length of the pipe is ℓ = 85 cm. The velocity of sound is v = 340 m/s.

Consider the two cases :

(a) The pipe is closed from one end   (b) The pipe is opened from both ends.

The open ends of the pipe are assumed to be the antinodes of displacement.

Sol. $\frac{\lambda}{4} = \ell$



$\frac{3\lambda}{4} = \ell$ and for n[th] overtone $(2n+1)\frac{\lambda}{4} = \ell$

(2n + 1) $\frac{\lambda}{4} = \ell$, $\lambda = \frac{4\ell}{(2n+1)}$ = f = $\frac{V(2n+1)}{4\ell}$ < 1250 Hz = $\frac{340(2n+1)\times 100}{4\times 85}$ < 1250

$= (2n + 1) < 12.5$ n – obertone $= 2n < 11.5$; $n < \frac{11.5}{2}$

$n < 5.75$ n number of oscillation = 6. n = 0, 1, 2, 3, 4, 5

similarly for open organ pipe $f = \frac{V}{2\ell}(n + 1) < 1250 = \frac{340 \times 100}{2 \times 85}(n + 1) < 1250 = n + 1 < \frac{12.5}{2}$

$n + 1 < 6.25 = n < 5.25$

n = obertone, n = 0, 1, 2, 3, 4, 5, number of oscillation (6)

15. A source of sound with adjustable frequency produces 4 beats per second with a tuning fork when its frequency is either 474 Hz. or 482 Hz. What is the frequency of the tuning fork?

Sol. $n \pm 4 = 474$

$n \pm 4 = 482$ it is possible at n = 478.

16. Two identical piano wires have a fundamental frequency of 600 vib/sec, when kept under the same tension . What fractional increase in the tension of one wire will lead to the occurrence of six beats per second when both wires vibrate simultaneously.

Sol. $f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$; $\ell n\, f = -\ell n\, \lambda + \frac{1}{2}\ell n\, T - \frac{1}{2}\ell n\, \mu$; $\frac{1}{f}\frac{df}{dT} = 0 + \frac{1}{2}\frac{1}{T} - 0$; $\frac{df}{f} = \frac{1}{2}\frac{dT}{T}$

$$\frac{dT}{T} = \frac{2df}{f} = \frac{2 \times 6}{600} = \frac{2}{100}$$

% change $= \frac{2}{100} \times 100 = 2\%$

17. A metal wire of diameter 1 mm, is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating in its fundamental frequency and a vibrating tuning fork together produces 5 beats per sec. The tension in the wire is then reduced to 81 N. When the two are excited, beats are again at the same rate. Calculate

(a) The frequency of the fork (b) The density of the material of the wire .

Sol. $\frac{1}{2\ell}\sqrt{\frac{100}{\mu}} - n = 5$ ....(1)

$n - \frac{1}{2\ell}\sqrt{\frac{81}{\mu}} = 5$ ....(2)

add equ (1) and (2) $\frac{1}{2\ell}\left[\frac{1}{\sqrt{\mu}}\right] = 10$; $\sqrt{\mu} = \frac{1}{20\ell} = \frac{1}{20 \times .5} = \frac{1}{10}$ $\mu = 0.01$

$\rho\, \pi r^2 = 0.01$; $\rho = \frac{.01}{(0.5 \times 10^{-3})^2 \pi} = \frac{10^4}{0.5 \times 0.5\pi} = \frac{40}{\pi} \times 10^3$ kg/m³ ; put $\mu = 0.01$ in eq. (1)

$\frac{1}{2 \times 0.5}\sqrt{\frac{100}{0.01}} - n = 5$

n = 100 – 5 = 95 Hz

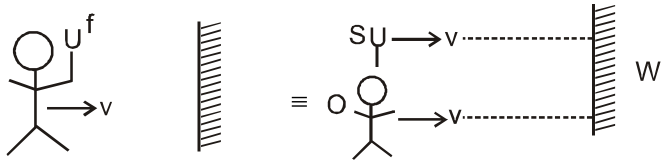
18. An observer rides with a sound source of frequency f and moving with velocity v towards a large vertical wall. Considering the velocity of sound waves as c, find:

(i) The number of waves striking the surface of wall per second

(ii) The wavelength of the reflected wave

(iii) The frequency of reflected wave as observed by observer.

(iv) Beat frequency heard by the observer.

Sol. 

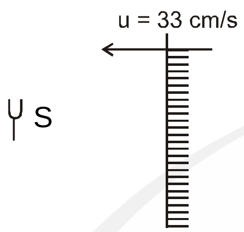
(1) $f' = f\,\dfrac{c}{c-v}$ Doppler effect between source and wall

(2) $\lambda' = \dfrac{c}{f'} = \dfrac{c}{cf}\,(c-v) = \dfrac{c-v}{f}$

(3) $f'' = f'\,\dfrac{c+v}{c} = f\,\dfrac{c+v}{c-v}$ Doppler effect between wall and observer.

(4) $f_{\text{beat foLian}} = f'' - f = f\left[\dfrac{c+v}{c-v} - 1\right] = f\,\dfrac{2v}{c-v}$

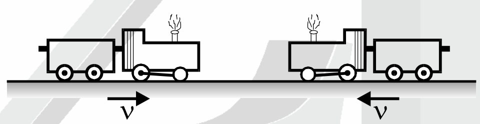
19. A stationary source emits single frequency sound. A wall approaches it with velocity u = 33 cm/s. The propagation velocity of sound in the medium is v = 330 m/s. In what way and how much, in per cent, does the wavelength of sound change on reflection from the wall ?

Sol. 

$f' = f\,\dfrac{v+u}{v}$ Doppler effect between source and wall. $f'' = f'\,\dfrac{v}{v-u}$ Doppler effect after reflection

$f'' = f\,\dfrac{v+u}{v-u}$; $\lambda'' = \dfrac{v}{f''} = \dfrac{v(v-u)}{f(v+u)} = \lambda\,\dfrac{v-u}{v+u}$; $\dfrac{\lambda'' - \lambda}{\lambda} \times 100 = \dfrac{\dfrac{\lambda(v-u)}{v+u} - \lambda}{\lambda} \times 100 = \dfrac{-2u}{v+u} \times 100 = \dfrac{-2 \times 0.33}{330 \times 0.33} \approx -0.2\,\%$

20. Two trains move towards each other with the same speed. Speed of sound is 340 ms^{-1}. If the pitch of the tone of the whistle of one when heard on the other changes to 9/8 times, then the speed of each train is :



Sol. $n' = \left(\dfrac{V+V_s}{V-V_s}\right)n$ $\qquad \dfrac{9}{8}\,n = \left(\dfrac{V+V_s}{V-V_s}\right)n$ $\qquad \dfrac{9}{8} = \dfrac{V+V_s}{V-V_s}$

$9V - 9V_s = 8V + 8V_s$ $\quad V = 17\,V_s$ $\qquad V_s = \dfrac{340}{17} = 20$ m/s

EXERCISES

1. A hospital uses an ultrasonic sound of frequency 1000 kHz tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 kms^{-1} ? The operating frequency of the scanner is 4.2 MHz.

Ans. 4.1×10^{-4} m

2. A transverse harmonic wave on a string is described by $y(x,t) = 3.0 \sin (36\, t + 0.018x + \pi/4)$ where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the starting point?

(d) What is the least distance between two successive crests in the wave?

Ans. (a) A travelling wave. It traves from light to left with a speed of 20 ms^{-1}, (b) 3.0 cm , 5.7 Hz (c) $\pi/4$ (d) 3.5 m

3. For the travelling harmonic wave $y(x,t) = 2.0 \cos 2\pi (10t - 0.0080\, x + 0.35)$. Where x and y are in cm and t in s, calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4m, (b) 0.5 m (c) $\lambda/2$, (d) $3\lambda/4$

Ans. (a) 6.4 π rad (b) 0.8 π rad (c) π rad (d) $(\pi/2)$ rad

4. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1} ? The operating frequency of the scanner of the scanner is 4.2 MHz.

Ans. 4.1×10^{-4} m

5. The transverse displacement of a string (clamped at its both ends) is given by $y(x,t) = 0.06 \sin \left(\frac{2\pi}{3} x\right) \cos (120\, \pi t)$ where x and y are in m and t in s. The length of the string 1.5 m and its mass is 3.0×10^{-2} kg. Answer the following :

(a) Does the function represent a travelling wave or a stational wave ?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength. Frequency and speed of each wave ?

(c) Determine the tension in the string.

Ans. (a) Stationary wave (b) $l = 3$ m, n = 60 Hz, and $\nu = 180$ ms^{-1} for each wave (c) 648 N

6. A meter-long tube open at one end, with movable piston at the other end. shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) When the tube length 25.5 cm or 79.3 cm. **Estimate the speed of sound in air at the temperature of the experiment**. The edge effects may be neglected.

Ans. 347 m s^{-i}