HALF-YEARLY EXAMINATION

(SESSION : 2019-2020)

CLASS : XII

SUBJECT : MATHS

SUBJECT CODE : 041

TIME: 3 HRS. **MAXIMUM MARKS : 80**

GENERAL INSTRUCTIONS

(i) The question paper is divided into four sections. Section A, section B, section C and section D.

(ii) This question paper contains **36** questions. All questions are compulsory

(iii) Question nos. **1** to **20** in **SECTION-A** are **very short answer questions** carrying **1 mark** each.

(iv) Question nos. **21-26** in **SECTION-B** are **short answer questions** carrying **2 marks** each.

(v) Question nos. **27-32** in **SECTION-C** are **long answer-I type questions** carrying **4 marks** each.

(vi) Question nos. **33-36** in **SECTION-D** are **long answer-II type questions** carrying **6 marks** each

(vii) Use of calculator is not permitted. You may ask for logarithmic tables if required.

SECTION—A

1. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

a. A **b.** I – A **c.** I + A **d.** 3A

2. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is:

a. 9 **b.** 27 **c.** 81 **d.** 512

3. The point on the curve $y^2 = x$, where the tangent makes an angle of $\pi/4$ with x–axis is

a. $\left(\frac{1}{2}, \frac{1}{4}\right)$ **b.** $\left(\frac{1}{4}, \frac{1}{2}\right)$ **c.** (4, 2) **d.** (1, 1)

4. If the value of a third order determinant is 12, then, the value of the determinant formed by replacing each element by its co-factor is ________ .

5. If $f(x) = x^{2/3}$ on [–1, 1] then is Rolle's theorem applicable to f(x) in [–1, 1].

6. If *A* is an invertible matrix of order 3 and | A | = 5, then find | adj. A |.

7. Find rate of change of gradient of $x^3 - 3x^2 + 1$ at x = 1.

8. If $y = \log_7 (\log x)$ then find the value of dy/dx.

9. Evaluate : $\int \frac{\sin 2x}{a^2 + b^2 \sin^2 x}\,dx$

10. Evaluate : $\int \frac{dx}{\sin^2 x \cos^2 x}$

11. Evaluate : $\int \frac{dx}{x^2 (x^4 + 1)^{3/4}}$

12. Evaluate : $\int \frac{dx}{(x-2)(x-3)}$

13. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$

a. 1 **b.** 1/2 **c.** 0 **d.** not possible to find

14. The value of $\int_1^2 \frac{\sqrt{x}\, dx}{\sqrt{x} + \sqrt{3-x}}$

a. 1 **b.** 1/2 **c.** 0 **d.** not possible to find

15. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6.

The probability that B is selected is :

a. 0.25 **b.** 1/2 **c.** 0.75 **d.** none of these

16. Let A and B be two events such that P (A) = 0.6, P (B) = 0.2, and P (A | B) = 0.5. Then P (A′ | B′) equals

a. 1/10 **b.** 3/10 **c.** 3/8 **d.** 6/7

17. $e^{\int \frac{-x\,dx}{1-x^2}}$ is equal to

a. $-x$ **b.** $-\sqrt{1-x^2}$ **c.** $\sqrt{1-x^2}$ **d.** $\frac{1}{2}\log\left(1-x^2\right)$

18. Write anti derivative of $\frac{1}{x\sqrt{x^3-1}}$.

19. The minimum value of the objective function Z = $ax + by$ in a linear programming problem always occurs at only one corner point of the feasible region. State whether statement is **True or False.**

20. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$.

Conditions on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

a. $p = q$ **b.** $p = 2p$ **c.** $q = 2p$ **d.** $q = 3p$

SECTION—B

21. Evaluate : $\int \frac{x^5 - 1\, dx}{x^2 + 1}$ **OR** $\int \frac{dx}{(2\sin x + 3\cos x)^2}$

22. Evaluate : $\int \frac{\cos 2\mathrm{x} - \cos 2\alpha}{\cos \mathrm{x} - \cos\alpha} \mathrm{dx}$ **OR** $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

23. Evaluate : $\int_0^{\pi/4} \log(1+\tan x)\, dx$ **OR** $\int_{-a}^{a} \cos x \log\left(\frac{1+x}{1-x}\right) dx$

24. Evaluate : $\int \frac{dx}{(x+2)(x-3)}$

25. If $\cos y = x\cos(a+y)$, then prove $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

26. If $\mathrm{y} = \tan^{-1}\left(\frac{\cos \mathrm{x}}{1 - \sin \mathrm{x}}\right)$ find dy/dx.

SECTION—C

27. Determine the values of a, b, c for which the function *f* defined by

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{when } x < 0 \\ c, & \text{when } x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{when } x > 0 \end{cases}$$ is continuous at x = 0.

28. Find intervals for which following function f(x) = sinx –cosx is strictly increasing and decreasing. $(0 \le x \le 2\pi)$

OR

Find the equation of the normal line to $y = x^3 + 2x + 6$ which is parallel to the line $14y + x + 4 = 0$.

29. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$

30. Solve using properties of determinants $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

31. If a young man rides his motor cycle at 25 kmph, he has to spend Rs.2 per km on petrol, if he rides at a faster speed of 40 kmph, the petrol cost increases to Rs.5 per km. He has Rs.100 to spend on petrol and wishes to cover maximum distance. Find what is the maximum distance, he can travel within one hour, Express as L.P.P. and solve it.

32. In a bag there are five balls. Two balls are drawn and found to be white. Find the probability that all balls are white.

OR

In a competitive examination, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that the answer is correct, given that the copies it is 1/8. Find the probability that he knows the answer to the question given that he correctly answered it.

SECTION—D

33. Show that the semi vertical angle of cone of maximum volume and given surface is $\sin^{-1}\frac{1}{3}$.

34. If $x = \sin t, y = \sin pt$, prove that $(1-x^2)y_2 - xy_1 + p^2y = 0$.

35. Evaluate : $\displaystyle\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$

OR

Evaluate : $\displaystyle\int_1^4 f(x)\,dx$ where $f(x) = |x-1| + |x-2| + |x-3|$

36. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations

$x + 2y + z = 4, -x + y + z = 0, x - 3y + z = 2.$

OR

Using Properties of determinants prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.