

XII MATHS TEST ON DERIVATIVES AND ITS APPLICATION

M.M. : 50

TIME : 90 MIN.

- Q1.** Find dy/dx if $y = \tan^{-1}\left(\frac{1 - \cos 2x}{\sin 2x}\right)$ 1
- Q2.** find the value of a and b if the function $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{b\sqrt{x}}{\sqrt{(16 + \sqrt{x}) - 4}}, & x > 0 \end{cases}$ is continuous at $x = 0$ 4
- Q3.** Find dy/dx if a) $y = \sqrt{\tan \sqrt{x}}$ b) $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ (1+3)
- Q4.** a) if $\cos y = x \cos(a + y)$, then prove $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$
- b) Prove $\frac{d}{dx} \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\} = \sqrt{a^2 - x^2}$ (2+2)
- Q5.** If $y = a(\sin t - t \cos t)$ & $x = a(\cos t + t \sin t)$ find $\frac{d^2y}{dx^2}$ 4
- Q6.** If $y = \left\{ x + \sqrt{x^2 + 1} \right\}^m$, then prove $(x^2 + 1)y_2 + xy_1 - m^2y = 0$
- Q7.** Differentiate $\cot^{-1} \left\{ \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right\}$ w.r.t. $\cos^{-1}x^2$ 4
- Q8.** Show that the normal at any point θ to the curve $x = a \cos \theta + a \theta + \sin \theta$; $y = a \sin \theta - a \theta \cos \theta$ is at a constant distance from origin. 4
- Q9.** Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8$. 4
- Q10.** For the curve $y = 4x^3 - 2x^5$ find all the points at which the tangent passes through origin. 4
- Q11.** Find the intervals in which the function f given by $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$ on $(0, 2\pi)$ is strictly increasing or strictly decreasing. 6
- Q12.** If $(x - a)^2 + (y - b)^2 = c^2$, then prove $\frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = -c$ 6