First Term Exam- 2024-2025
Class- XII
Subject- Mathematics

Time Allowed: 3 Hours | Max Marks: 80

General instructions-

1. This question paper contains 38 questions, divided into five sections- A, B, C, D and E. All questions are compulsory.
2. **Section A** comprises of **18** Multiple Choice Questions (MCQs) and **2** Assertion-Reason based questions of **1 mark** each.
3. **Section B** comprises of **5** Very Short Answer (VSA) questions of **2 marks** each.
4. **Section C** comprises of **6** Short Answer (SA) type questions of **3 marks** each.
5. **Section D** comprises of **4** Long Answer (LA) type questions of **5 marks** each.
6. **Section E** comprises of **3** Case Study Based questions of **4 marks** each.
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section- **B**, 3 questions in Section- **C**, 2 questions in Section- **D** and 2 questions in Section- **E**.
8. Please write down the serial number of the question before attempting it.
9. Use of calculator is **NOT** allowed.

SECTION-A

This section has 20 multiple choice questions of 1 mark each.

1. A function f: R→R defined as $f(x) = x^2 - 4x + 5$ is:

(a) injective but not surjective
(b) surjective but not injective
(c) both injective and surjective
(d) neither injective nor surjective

2. Let A = {3,5,7,9}. Then total possible reflexive relations on the set A will be:

(a) 2^4 (b) 2^8 (c) 2^{12} (d) 2^{16}

3. The domain of the function $f(x) = \sin^{-1}(\sqrt{x-1})$ is:

(a) [0,1] (b) [1,2] (c) [-1,1] (d) none

4. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is:

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

5. If A and B are two matrices such that AB = B and BA = A, then $A^2 - B^2$ is equal to:
(a) 0 (b) I (c) A-B (d) B-A

6. If $A = [a_{ij}]$ is a square matrix of order 2×2, whose elements are given by a_{ij} = maximum (i, j) - minimum (i, j). Then det(A^2) will be:
(a) 0 (b) 1 (c) 2 (d) 4

7. If A and B are square matrices of same order where $|B| \neq 0$, then $|B^{-1}AB|$ is:
(a) $|A|$ (b) $|B|$ (c) $|A + B|$ (d) $|A - B|$

8. If the value of the determinant of a 3×3 matrix is 10, then the value of the determinant formed by replacing each element by its cofactor will be:
(a) 10 (b) 100 (c) 1000 (d) none

9. Derivative of e^{sin^2x} with respect to cosx is:
(a) $sinx.e^{sin^2x}$ (b) $cosx.e^{sin^2x}$
(c) $-2cosx.sin^2x.e^{sin^2x}$ (d) $-2cosx.e^{sin^2x}$

10. The function $f(x) = \frac{x-1}{x(1-x^2)}$ is discontinuous at exactly:
(a) one point (b) two points (c) three points (d) four points

11. If tan(xy) = $\sqrt{2}$, then $\frac{dy}{dx}$ is equal to:
(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $-\frac{x}{y}$ (d) $-\frac{y}{x}$

12. For a curve $y = 2x - x^3$, x decreases at the rate of 2 units per second. The rate at which the slope of the curve is changing when x = 2 is:
(a) 24 units/second (b) -24 units/second
(c) 20 units/second (d) -20 units/second

13. Total number of critical points of the function $f(x) = |x| + |x - 1|$ will be:
(a) 0 (b) 1 (c) 2 (d) 3

14. The function f(x) = x + cosx is:
(a) decreasing in $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$ (b) always decreasing
(c) always increasing (d) none

15. If $\frac{d(f(x))}{dx} = logx$ and f (1) = -1, then f(x) equals:

(a) $\frac{1}{x} - 2$ (b) $-\frac{1}{x}$ (c) x (logx - 1) (d) x (logx - x)

16. The value of $\int_{-1}^{1} x\, sin^{2024}x\, dx$ is:

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{1}{2}$ (d) none

17. The objective function of linear programming problem (LPP) in a bounded feasible region attains its optimum value at:

(a) At least one of the corner points (b) at most one of the corner points

(c) at least two of the corner points (d) at all the corner points

18. The objective function Z = ax + by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the followings is true:

(a) a=9, b=1 (b) a=5, b=2 (c) a=3, b=5 (d) a=5, b=3

Assertion – Reason Based Questions

Direction: In questions numbers **19** and **20**, two statements are given, a statement of Assertion (A) is followed by a statement of Reason(R). Select the correct answer from the following options:

(a) Both A and R are true and R is correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A)**: If set A = {3,5,7} and B = {2,4,6} then total possible bijective functions from A to B are 6.

Reason (R): For two sets A and B, if n(A) = n(B) = p then total possible bijective functions from A to B are p(p-1).

20. **Assertion (A)**: The determinant of any skew symmetric matrix of odd order is always zero.

Reason (R): If a row or a column in a square matrix have all entries 'zero' then determinant of such matrix is always zero.

SECTION-B

This section has 5 very short answer type questions of 2 marks each.

21. Considering the principal values only, evaluate: $\cos^{-1}\left(-\frac{1}{2}\right) + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

22. (a) Find $\frac{dy}{dx}$ if $y = x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$.

OR

(b) If x = 4t and y = $\frac{4}{t}$ then find $\frac{dy}{dx}$ at (4, 4).

23. (a) A particle moves along the curve 6y = x^3 +2. Find the points on the curve at whic the y- coordinate is changing 8 times as fast as the x-coordinate.

OR

(b) Find the local minima of the function $f(x) = \frac{x}{2} + \frac{2}{x}$

24. Evaluate: $\int sin^3x.e^{logcosx}dx.$

25. Find the value of $\int_0^{\frac{\pi}{6}} tan^2\left(x - \frac{\pi}{6}\right)dx.$

SECTION-C

This section has 6 short answer type questions of 3 marks each.

26. (a) Let A = R - {3} and B = R - {1}. Consider the function f: A → B defined by f(x = $\left(\frac{x-2}{x-3}\right)$. Is f(x) one-one and onto? Justify your answer.

OR

(b) If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also a equivalence relation.

27. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, for $-\frac{1}{\sqrt{2}} \leq x \leq 1$.

28. (a) Let $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Show that F(x) F(y) = F (x + y).

OR

(c) How many matrices of order 3×3 is possible with each entry 0 or 1? How many these matrices will be symmetric and how many will be skew symmetric?

29. For what value of λ is the function

$$f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$$

continuous at x = 0? What about continuity at x = -1?

30. Prove that: $y=\frac{16\sin\theta}{(4+\cos\theta)}-\theta$ is strictly decreasing function in $[\frac{\pi}{2},\pi]$.

31. (a) Solve: $\int_{-1}^{2}|x^3-x|\,dx$.

OR

(b) Solve: $\int x\cos^{-1}x\,dx$.

SECTION-D

This section has 4 long answer type questions of 5 marks each.

32. Let R be a relation in set of real numbers, defined by R = {(a, b): a-1 ≤ b^2}, check if R is an equivalence relation or not?

33. (a) Find A^{-1}, where $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 0 & -3\\ 1 & 2 & 0\end{bmatrix}$. Hence solve the system of equations

$$x+2y-3z=1,\quad 2x-3z=2,\quad x+2y=3$$

OR

(b)Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations: $x-y+z=4,\quad x-2y-2z=9,\quad 2x+y+3z=1$

34. (a) Evaluate: $\int_0^{\frac{\pi}{2}}\log(\sin x)\,dx$

OR

(b)Solve: $\int\left(\sqrt{\cot x}+\sqrt{\tan x}\right)dx$

35. Solve the following L.P.P graphically:

Maximize: Z = 60x + 40y

Subject to constraints:

$$x+2y\le 12,\ 2x+y\le 12,\ 4x+5y\ge 20,\ x,\ y\ge 0.$$

SECTION-E

In this section there are 3 case study-based questions of 4 marks each.

36. A professional typist charges Rs 200 for typing 8 English and 4 Hindi pages, while he charges Rs 120 for typing 4 English and 2 Hindi pages. Based on this information, answer the following questions:

(i) If he charges Rs x for one page of English and Rs y for one page of Hindi, express the above information as a pair of linear equation and then write it in matrix form as $AX = B$ **[2Mark]**

(ii) Is it possible to find the values of x and y? Justify your answer. **[2Marks]**

37. A potter made a mud vessel, where the shape of the pot is based on the function $f(x) = |x - 2| + |x + 3|$, where $f(x)$ represents the height of the pot. Based on this information, answer the following questions-

(i) When $x > 3$, what will be the height of the pot in terms of x? **[1Mark]**

(ii) Will the slope of $f(x)$ vary with values of x? **[1Mark]**

(iii) What is $\frac{dy}{dx}$ at $x = 2$? **[2Marks]**

OR

(iii) If the potter is trying to make a pot using the function $f(x) = \{x\}$, where $\{.\}$ is fractional part function, will he get a pot of any shape or not? Justify your answer?

38. Mr. Elon started a factory of manufacturing LED bulbs. He can sell x bulbs at a price of Rs $(300 - x)$ each. The cost price of x bulbs is Rs $(2x^2 - 60x + 18)$. Based on this information, answer the following questions:

(i) Find the profit function $P(x)$ of selling x bulbs. **[1Mark]**

(ii) Find the $\frac{d(P(x))}{dx}$. **[1Mark]**

(iii) How many bulbs should he sell to earn maximum profit? **[2Marks]**

OR

How many bulbs is he selling if he is incurring a loss of Rs 18?