APEEJAY SCHOOL, SAKET
MID TERM EXAMINATION
SESSION: 2024-2025
SET A

SUBJECT: MATHEMATICS (041)　　　　　　　　　　　　　　　　　　**CLASS: XII**

TIME: 3 HOURS　　　　　　　　　　　　　　　　　　　　　　　　**M. Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.

2. Section A has 20 MCQs carrying 1 mark each

3. Section B has 5 questions carrying 02 marks each.

4. Section C has 6 questions carrying 03 marks each.

5. Section D has 4 questions carrying 05 marks each.

6. Section E has 3 case based questions carrying 04 marks each.

Section-A

1. The function $f: R \rightarrow R$ defined as $f(x) = x^3$ is

   A. one-one but not onto　　　　　　　　　B. not one-one but onto

   C. neither one-one nor onto　　　　　　　D. one-one and onto

2. Let L denote the set of all straight lines in a plane. Let a relation R be defined by l R m if and only if l is perpendicular to m $\forall$ l, m $\in$ L. Then R is

   A. reflexive　　　　B. symmetric　　　　C. transitive　　　　D. equivalence

3. The number of equivalence relations in the set $\{1,2,3\}$ containing the elements $(1,2)$ and $(2,1)$ is

   A. 0　　　　　　　B. 1　　　　　　　C. 2　　　　　　　D. 3

4. Which of the following corresponds to the principal value of branch of $\tan^{-1} x$?

   A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$　　　B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$　　　C. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$　　D. $(0, \pi)$

5. The principal value branch of $\sec^{-1} x$ is

   A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$　　B. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$　　C. $(0, \pi)$　　　D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

6. The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$

   A. $\frac{25}{24}$　　　　B. $\frac{25}{7}$　　　　C. $\frac{24}{25}$　　　　D. $\frac{7}{24}$

7. The value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$ is

   A. $\frac{3\pi}{5}$　　　　B. $\frac{-7\pi}{5}$　　　　C. $\frac{\pi}{10}$　　　　D. $-\frac{\pi}{10}$

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8. The domain of the function $\cos^{-1}(2x-1)$

A. $[0,1]$ B. $[-1,1]$ C. $(-1,1)$ D. $\dfrac{\pi}{9}$

9. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

A. square matrix   B. diagonal matrix   C. unit matrix   D. none

10. Total number of possible matrices of order 3×3 with each entry 2 or 0 is

A. 9   B. 27   C. 81   D. 512

11. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of $x + y$ is

A. $x=3, y=1$ B. $x=2, y=3$ C. $x=2, y=4$ D. $x=3, y=3$

12. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if

A. $\lambda = 2$ B. $\lambda \neq 2$ C. $\lambda \neq -2$ D. $\lambda \neq -8/5$

13. If A and B are invertible matrices, then which of the following is not correct ?

A. $\text{adj } A = |A|.A^{-1}$ B. $\det(A)^{-1} = [\det(A)]^{-1}$

C. $(AB)^{-1} = B^{-1}A^{-1}$ D. $(A+B)^{-1} = B^{-1} + A^{-1}$

14. If A is a square matrix of order 3, $|A'| = -3$, then $|AA'|$ is

A. 9   B. -9   C. 3   D. -3

15. $\int \dfrac{dx}{(\sin x \cos x)^2} =$

A. $\tan x + \cot x + C$ B. $(\tan x + \cot x)^2 + C$

C. $\tan x - \cot x + C$ D. $(\tan x - \cot x)^2 + C$

16. The function $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

A. 3   B. 2   C. 1   D. 1.5

17. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is continuous at

A. 4   B. -2 C. 1   D. 1.5

18. The number of points at which the function $f(x) = \dfrac{1}{x - [x]}$ is not continuous is

A. 1   B. 2   C. 0   D. infinite

ASSERTION-REASON BASED QUESTIONS

Each of the following contains Assertion and Reason and has the following four choices (A), (B), (C) and (D), only one of which is the correct answer.

A. Assertion and Reason are true; Reason is a correct explanation for Assertion

B. Assertion and Reason are true; Reason is not a correct explanation for Assertion

C. Assertion is True, Reason is False.

D. Assertion is False, Reason is True.

19. **Assertion:** If A and B are symmetric matrices of same order then AB-BA is also a symmetric matrix.
    Reason: Any square matrix A is said to be skew-symmetric matrix if $A = -A^T$, where A^T is the transpose of matrix A.

20. **Assertion :** $\int e^x \dfrac{x}{(x+1)^2} dx = \dfrac{e^x}{x+1} + C$

    Reason: $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

Section-B

21. Find the matrix X, if $X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

22. Verify : A (adj A) = (adj A)A = |A|I for matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$.

23. Find 'a' and 'b', if the function given by $f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 2x+1, & \text{if } x \geq 1 \end{cases}$ is differentiable at x = 1.

24. If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ then, find $\dfrac{d^2 y}{dx^2}$.

25. Volume of the cube is increasing at a rate of 9 cubic inches per second. What is the rate at which surface area is increasing when the length of the edge of the cube is 10 inches?

Section-C

26. Find the value of $\int \dfrac{\sqrt{\tan x}}{\sin x \cos x} dx$.

27. Find: $\int \dfrac{1}{x^2 - 8x + 15} dx$.

28. Find $\int \left[\sqrt{\cot x} + \sqrt{\tan x} \right] dx$

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29. Prove that

$$\cot^{-1}\left(\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right)=\frac{\pi}{2}-\frac{x}{2}, \quad 0<x<\Pi/2.$$

30. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \dfrac{x-1}{x-2}$, show that f is one one and onto. Hence, find f^{-1}.

31. (a) For what values of k and m, the system of linear equations

$2x + k y + 6z = 8, x + 2y + m z = 5, x + y + 3z = 4$ has a unique solution ?

(b) For what values of k, the system of linear equations $2x + k y + 6z = 8, x + 2y + z = 5, x + y + 3z = 4$ has infinitely many solutions?

(c) For what values of k, the system of linear equations $2x + k y + 6z = 8, x + 2y + z = 6, x + y + 3z = 4$ has no solution?

Section – D

32. If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}, Using A^{-1} solve the following system of equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=4; \quad x, y, z \neq 0.$$

OR

Use product $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve system of equations

$x + 3z = 9, -x + 2y - 2z = 4, 2x - 3y + 4z = -3$

33. Find $\dfrac{dy}{dx}$, if $y^x + x^y + x^x = d$.

34. Show that the relation R defined by $(a, b) R(c, d) \Rightarrow a + d = b + c$ on $A \times A$, where set $A = \{1, 2, 3, \dots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3, 4)]$; a, b, c, d $\in$ A.

35. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

OR

Evaluate : $\int \dfrac{\sin^{-1}x - \cos^{-1}x}{\sin^{-1}x + \cos^{-1}x} dx.$

Section E

36. A toy manufacturer wants to cut a 28 metres long wire into two pieces. One of the two pieces to be bent into the form of a square of side x and other into the form of a circle of radius y. Based on the above information answer the following questions.

28 m ──────────────────── Wire
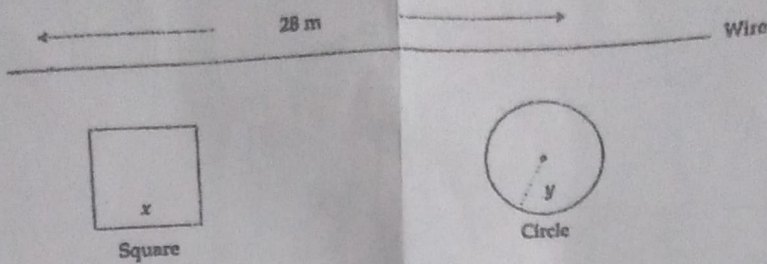
Square

Circle

(i) What is the relation between x and y ?

(ii) If A is the total area of the circle and square, then A is_____.

(iii) What is the minimum value of A ?

Or

What is maximum value of A ?

37. Three schools X, Y and Z decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs. 100 and Rs. 50 respectively. The number of articles sold are given as

School / Article	X	Y	Z
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the information given above, answer the following questions:

(i) What is the total money (in rupees) collected by school X ?

(ii) What is the total amount of money collected by all three schools X, Y and Z ?

(iii) If the number of handmade fans and plates are interchanged for all the schools, then the total money collected by all schools is_____.

OR

Total number of articles sold by three school is _____.

38. Read the following passage and answer the following questions.

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In order to set up a train water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is Rs. 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is Rs. 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.
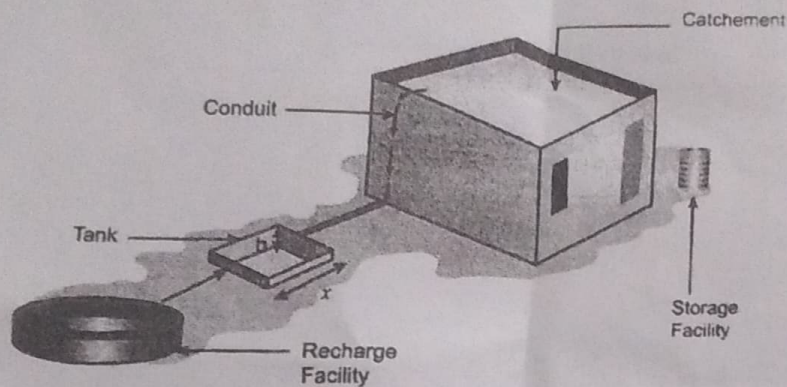
(i) Find the total cost C of digging the tank in terms of x.

(ii) Find $\dfrac{dC}{dx}$.

(iii) (a) Find the value of x for which cost C is minimum

OR

(b) Check whether the cost function C(x) expressed in term of x is increasing or not, where x > 0.