APEEJAY SCHOOL PANCHSHEEL PARK

Class XII

Subject- Mathematics

MIDTERM EXAMINATION (2024-25)

Name of the student Date

Time Allowed: 3 hrs M.M. 80

General Instructions:

i. Section A has 20 questions carrying 1 mark each.
ii. Section B has 5 questions carrying 02 marks each.
iii. Section C has 6 questions carrying 03 marks each.
iv. Section D has 4 questions carrying 05 marks each.
v. Section E has 3 case-based questions (04 marks each) with subparts.
vi. All Questions are compulsory.
vii. Draw neat figures wherever required.

SECTION A (20 Marks)

1. Total number of possible matrices of 3X3 with each entry 2 or 0 is:

(A) 9 (B) 27 (C) 81 (D) 512

2. If A is square matrix of order 3X3 such that $|adj\ A|=16$, such that $(|2A|)^2=2^p$, then P is equal to

(A) 4 (B) 5 (C) 6 (D) None of these

3. If $A=\begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then A^{-1} exists if:

(A) $\lambda = 2$ (B) $\lambda \neq 2$ (C) $\lambda = -2$ (D) None

4. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying $AA^T=9I$, then ordered pair (a, b) is equal to:

A) (-2, -1) (B) (-2, 1) (C) (2, -1) (D) (2, 1)

5. If A is a square matrix of order 3, $|3A| = 27$, then $|A|$=?

(A) 9 (B) 1 (C) 3 (D) -3

6. The value of k which makes the function defined by

$f(x)=\begin{cases} x\sin\frac{1}{x} & if\ x \neq 0 \\ k, if\ x = 0 \end{cases}$ continuous at x=0 is:

(A) 8 (B) 0 (C) -1 (D) None

7. If $y = a\sin mx + b\cos mx$, then $\frac{d^2y}{dx^2}$ is equal to:

(A) $-m^2y$ (B) m^2y (C) $-my$ (D) my

8. The function $f(x) = [x]$, *where* $[x]$ *denotes the greatest integer function, is continuous at*

(A) $x = 4$ (B) $x = -2$ (C) $x = 1$ (D) $x = 1.5$

9. $\int \frac{\cos 2x}{(\cos x+\sin x)^2} dx$ is equal to:

(A) $-\frac{1}{\sin x+\cos x} + C$

(B) $\log|\sin x + cosx| + C$

(C) $\log|\sin x - cosx| + C$

(D) $\frac{1}{(\sin x+\cos x)^2} + C$

10. $\int e^x(\cos x - \sin x)\, dx$ is equal to:

(A) $e^x \cos x + C$ (B) $e^x \sin x + C$ (C) $-e^x \cos x + C$ (D) $-e^x \sin x + C$

11. The principal value branch of $\tan^{-1} x$ *is*

(A) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (B) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (C) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (D) $(0, \pi)$

12. Consider the set A having 3 elements and set B having 4 elements, the number of injective functions from set A to B is....

(A) 24 (B)12 (C)7 (D) NONE

13. *The value of* $\sin[cos^{-1}\left(\frac{1}{2}\right)]$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C)1 (D) 0

14. The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

(A) $\frac{-2\pi}{3}$ (B) $\frac{-\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$

15. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ is equal to

(A) 1 (B)-1 (C) 2 (D) -2

16. $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$ *equals to*

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

17. The sides of an equilateral triangle are increasing at the rate of 2cm/sec. The rate at which the area of equilateral triangle increases, when the side is 10cm

(A) $10cm^2/sec$ (B) $\sqrt{3}\, cm^2/sec$ (C) $10\sqrt{3}cm^2/sec$ (D) $\frac{10}{3} cm^2/sec$

18. The critical value of $[x(x-1)+1]^{1/3}, 0 \le x \le 1$

(A) $\left(\frac{1}{3}\right)^{1/3}$ (B) $\frac{1}{2}$ (C) 1 (D) 0

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion(A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

A) Both A and R are correct and R is the correct explanation of A.

B) Both A and R are correct but R is not the correct explanation of A.

C) A is true but R is false.

D) A is false but R is true.

19. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A): The value of the function $sin^{-1}(\sin {}^{2\pi}/_{3}) = {}^{2\pi}/_{3}$.

Reason (R): $\sin^{-1}(\sin\theta) = \theta$, $if\ \theta \epsilon \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

20. Assertion (A): The function $[x(x-2)]^2$ is increasing in $(0,1) \cup (2,\infty)$

Reason (R): $\frac{dy}{dx} = 0$,when x= 0,1, 2

SECTION-B (10 Marks)

Q21 Find the value of $\cos^{-1}[cos\frac{3\pi}{2}]$

Q22 Evaluate: $\int_0^1 \frac{x}{x^2+1} dx$

Q23 **If** $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases}$ is continuous at $x = 0$, then find the values of a and b.

Q24 Evaluate $\int 1/(e^x - 1)\, dx$

Q25 Using determinants, find the value of k if the points $(k, 2)$, $(1,5)$ and $(2,4)$ $are\ collinear.$

SECTION-C (18 Marks)

Q26 *Evaluate:* $\int \sqrt{10 - 4x + 4x^2}\ dx$

OR

Evaluate: $\int e^x . \sin 2x \ dx$

Q27 Find $\frac{d^2y}{dx^2}$ *if* $y = 12(1 - cost), x = 10(t - sint)$, *where* $\frac{-\pi}{2} < t < \frac{\pi}{2}$

Q28 *Evaluate* $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$

OR

Evaluate $\int_0^4 |x - 1| \ dx$

Q 29 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,find x *and* y *such that* $A^2 = xA + yI$

Q30.Express $\tan^{-1}\left(\frac{cosx}{1-sinx}\right)$, $\frac{-3\pi}{2} < x < \frac{3\pi}{2}$ in the simplest form.

Q31 Let f: **N**→ ***N*** *be defined by*

$$f(x) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$ for all nϵN

State whether the function f is one -one and onto. Justify your answer.

SECTION D (20 Marks)

Q32 *Evaluate* $\int \sqrt{tanx} \ dx$.

Q33 Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 Of the volume of sphere .

Q34 If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$

Q35 Given $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$. Find A^{-1}

Using A^{-1}, solve the system of linear equations:

$x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$

SECTION E (12 Marks)

Q36. Students of grade 10, planned to plant saplings along straight line , parallel to each other in the playground. Let us assume that they planted one of the rows of the saplings along the line

$y = x - 4$ .Let L be the set of all lines which are parallel on the ground and R be a relation on L

Based on the above information answer the following

a. Let $f: R \to R$ be defined as R=$\{(L_1, L_2) : L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$ .Check whether R Reflexive symmetric and transitive

b. Let $f: R \to R$ be defined by f(x)= x- 4 . Then find the range $f(x)$.

c. Let $f: R \to R$ be defined as R=$\{(L_1, L_2) : L_1 \perp L_2 \text{ where } L_1, L_2 \in L\}$ .Is R an Equivalence relation

Q37.The shape of a toy is given as $f(x) = 6(2x^4 - x^2)$.To make the toy beautiful 2 stick which are perpendicular to each other were placed at a point (2,3) above the toy

(i) Find the second order derivative at $x = 5$

(ii) Find all critical points.

(iii) Find the interval in which f(x) is increasing?

Q38. The sum of three numbers is 6. If we multiply the third number by 2 and add with the first number to the result we get 7. By adding second and third numbers to 3 times the first number we get 12. Using matrix representation answer the following.

(i) Represent the above situation by a matrix equation of the form $AX = B$

(ii) Find $|A|$

(iii) Determine $P = A^2 - 5A$