**BALVANTRAY MEHTA VIDYA BHAWAN (A.S.M.A.)**
**MID TERM EXAMINATION (2024 - 25)**
**CLASS: XII**
**SUBJECT: MATHEMATICS**

Time: 3 Hrs. 

Max.Marks:80

Name: _____

Roll Number: _____

**General Instructions:**
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory.However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) - type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) - type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) – type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4marks each) with sub parts.

## SECTION -A

*(handwritten: $A^T = A$)*

1. If A is a symmetric matrix, then which of the following is not Symmetric matrix,

    (a) $A + A^T$　　　　(b) $A \cdot A^T$　　　　(c) $A - A^T$　　　　(d) $A^T$

2. If $[x - 2 \quad 5 + y]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 0$, then x + y =

    (a) 0　　　　(b) -2　　　　(c) -1　　　　(d) -3

    *(handwritten: $\begin{bmatrix} 0+5+y & x-2+0 \end{bmatrix} = \begin{bmatrix} 5+y & x-2 \end{bmatrix}$. $5+y=0 \Rightarrow y=-5$; $x-2=0 \Rightarrow x=2$; $-3$)*

3. If A is a non-singular square matrix of order 3 such that |A| =3, then value of

    $| 2A^T |$ is

    (a) 3　　　　(b) 6　　　　(c) 12　　　　(d) 24

4. If A = $\begin{bmatrix} 6x & 8 \\ 3 & 2 \end{bmatrix}$ is singular matrix, then the value of x is

    (a) 3　　　　(b) -2　　　　(c) 0　　　　(d) 2

5. $A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$, then $| adjA | =$

    (a) 1/9　　　　(b) 1/81　　　　(c) -9　　　　(d) -81

6. The function given below at $x=4$ is $f(x) = \begin{cases} 2x + 3 , x \le 4 \\ x^2 - 5 , x > 4 \end{cases}$

    (a) Continuous but not differentiable

    (b) Differentiable but not continuous

    (c) Continuous as well as differentiable

    (d) Neither continuous nor differentiable

7. If $x = \sin^3 t$, $y = \cos^3 t$ then $\frac{dy}{dx}$

   (a) tan t        (b) cot t        (c) –tan t        (d) -cot t

8. Find the value of $\int \frac{dx}{\sin^2 x \cos^2 x}$

   (a) $\tan x + \cot x + C$        (b) $\tan x - \cot x + C$

   (c) $\tan x . \cot x + C$        (d) $\tan x, -\cot 2x + C$

9. $\int x^2 e^{x^3}\, dx$ is equal to

   (a) $\frac{1}{3} e^{x^3} + C$      (b) $\frac{1}{3} e^{x^4} + C$      (c) $\frac{1}{2} e^{x^3} + C$      (d) $\frac{1}{2} e^{x^2} + C$

10. If A is a non-singular matrix of order 3 and $|A| = -4$, find $|adjA|$.

    (a) 4        (b) 16        (c) 64        (d) 14

11. A is a skew-symmetric matrix and a matrix B such that B'AB is defined, then B'AB is a:

    (a) Symmetric matrix        b) skew-symmetric matrix

    (c) Diagonal matrix        (d) upper triangular symmetric

12. If $y = \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \pi/6$

    (a) 1/2        (b) -1/2        (c) 1        (d) -1

13. Which of the following is the principal value branch of $\csc^{-1} x$?

    (a) $[-\frac{\pi}{2}, \frac{\pi}{2}]$      (b) $(-\frac{\pi}{2}, \frac{\pi}{2})$      (c) $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$      (d) $(0, \pi) - \{\frac{\pi}{2}\}$

14. The area bounded by the curve $y = \cos x$, X-axis, ordinates $x = \frac{\pi}{2}$ and $x = \pi$ is

    (a) 1 sq. units      (b) 2 sq. units      (c) 3 sq. units      (d) None of these

15. The area bounded by the curve $y = \frac{x^2}{2}$, X-axis, ordinates $x = 2$ is

    (a) 1/3 sq. units      (b) 2/3 sq. units      (c) 1 sq. units      (d) 4/3 sq. units

16. The interval, in which function $y = x^3 + 6x^2 + 6$ is increasing is

    (a) $(-\infty, -4) \cup (0, \infty)$     (b) $(-\infty, 4)$     (c) $(-4, 0)$     (d) $(-\infty, 0) \cup (4, \infty)$

17. If the rate of change of volume of a sphere is equal to the rate of change of its radius, then the radius is equal to

    (a) 1 unit      (b) $\sqrt{2\pi}$ units      (c) $\frac{1}{\sqrt{2\pi}}$ units      (d) $\frac{1}{2\sqrt{\pi}}$ units

18. The function $f(x) = [x]$, $[x]$ is the greatest integer function that is less than or equal to x, is continuous at

    (a) 4      (b) -2      (c) 1.5      (d) 1

19. Assertion (A): Domain of $f(x) = \cos^{-1} 2x + \sin 2x$ is $[-1,1]$

Reason(R): Domain of a function is the set of all possible values for which function will be defined.

    (a) Both A and R are true and R is correct explanation of A.
    (b) Both A and R are true but R is NOT the correct explanation of A.

    (c) A is true but R is false.
    (d) A is false and R is true.

20. The Assertion (A): Consider a set A= {a,b,c}. A: the no of reflexive relations on the set A is $2^9$

Reason(R): The relation is said to be reflexive if $xRx$ , $x \forall A$

    (a) Both A and R are true and R is correct explanation of A.
    (b) Both A and R are true but R is NOT the correct explanation of A.

    (c) A is true but R is false.
    (d) A is false and R is true.

## SECTION – B

21. Evaluate $\sin^{-1}\left( \sin\left( \dfrac{-17\pi}{8} \right) \right)$

    OR

Find the value of the expression $\sin\left[ \cot^{-1}\left\{ \cos\left( \tan^{-1} 1 \right) \right\} \right]$

22. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & if \ x \leq 2 \\ ax + b, & if \ 2 < x < 10 \\ 21, & if \ x \geq 10 \end{cases} \ \text{is a continuous function.}$$

23. Find the least value of "a" such that function f given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1,2)

    OR

It is given that at x= 1 function $x^4 - 62 x^2 + ax + 9$ attains maximum value on the interval $[0,2]$. Find the value of a.

24. The volume of a sphere is increasing at the rate of 3 cubic centimetre per second. Find the rate of increase of its surface area, when the radius is 2cm.

25. Evaluate: $\displaystyle\int_1^2 \dfrac{dx}{x(1+\log x)}$

    OR

Evaluate: $\displaystyle\int_0^1 \dfrac{x\ dx}{1+x^2}$

26. Evaluate $\int \frac{(x^2+1)}{(x^2+2)(x^2+3)} dx$

OR

Evaluate : $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$.

27. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find $A^{-1}$.

28. Evaluate : $\int \frac{1}{9x^2 + 6x + 5} dx$.

OR

Evaluate $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

29. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}$ r.

30. Discuss the differentiability of the function $f(x) = |x-2|$ at x=2.

OR

Find the relationship between a and b so that the function defined by

$f(x) = \begin{cases} ax + 1, & if\ x \le 3 \\ bx + 3, & if\ x > 3 \end{cases}$ is continuous at x =3

31. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$.

## SECTION -D

32. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L1, L2) : L1$ is parallel to L2$\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x +4.

OR

Show that f: N → N , given by $f(x) = \begin{cases} x + 1, & if\ x\ is\ odd \\ x - 1, & if\ x\ is\ even \end{cases}$ is both one-one and onto.

33. Find the product of the matrices $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and using the result solve the following system of linear equations

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

34. Find and draw the area of region lying in the first quadrant enclosed by x-axis, the line y= x and the circle $x^2 + y^2 = 32$.

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35. If $y = \left(x + \sqrt{x^2 + a^2}\right)^n$, then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$

OR

If $\log\left(\sqrt{x^2 + y^2}\right) = \tan^{-1}\left(\frac{y}{x}\right)$, then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$

## SECTION –E

36. Read the following passage and answer the questions given below

The relation between the height of the plant (Y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{x^2}{2}$ where x is the number of days exposed to sunlight.

a. What is the rate of growth of plant?          (1mark)

b. On which day the plant attains the maximum height.  (1mark)

c. What is the maximum height of the plant?          (2mark)

OR

What is the height of the plant after two days?

37. An organization conducted bike race under 2 different categories-boys and girls. Totally there were 250 participants . Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B = {b1,b2,b3} G={g1,g2} where B represents the set of boys selected and G the set of girls who were selected for the final race .Ravi decides to explore these sets for various types of relations and functions.

1. Ravi wishes to for mall the relations possible from B to G. How many such relations are possible?          (1mark)

2. Let $R : B \rightarrow G$ be defined by R = { (b1,g1), (b2,g2),(b3,g1)}. Check R is/are injective/ surjective / bijective.
(1Mark)
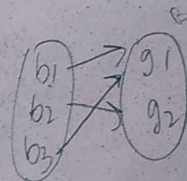
3. Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible
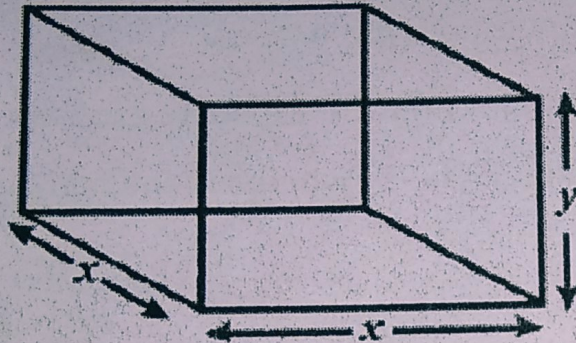( 2 Marks)

OR

Ravi wants to know among those relations, how many functions can be formed from B to G?    ( 2 Marks)

38. Read the following passage and answer the questions given below

Anuja wants to make a project for State level Science Exhibition. For this she wants to make metal box with square base and vertical sides to contain of 1024cm³ water material for top and bottom costs Rs.5 per cm² and material for slides costs Rs.2.5 per cm².



a. Find the volume of the box.                                              (1mark)

b. What is the cost of the box in terms of x?                               (1mark)

c. Find the least cost of the box.                                         (2mark)


OR

Find the dimensions of the box having minimum surface area.   (2mark)

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