General Instructions:

- **This Question Paper has 5 Sections A-E.**
- **Section A has 20 MCQs carrying 1 mark each.**
- **Section B has 5 questions carrying 02 marks each.**
- **Section C has 6 questions carrying 03 marks each.**
- **Section D has 4 questions carrying 05 marks each.**
- **Section E has 3 case based integrated units of assessment (04 marks each).**
- **All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided.**
- **Draw neat figures wherever required. Take π =22/7 wherever required if not stated.**

Section A

Section A consists of 20 questions of 1 mark each.

1. If A, B are non-triangular square matrices of the same order, then$(AB^{-1})^{-1}$.
(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB

2. If A is a square matrix of order 3 and $|A| = 5$, then $|\text{adj } A| =$
(a) 5 (b) 25 (c) 125 (d) 1/5

3. If $y = \sin^{-1}x$, then $(1 - x^2)y_2$ is equal to :
(a) xy_1 (b) xy (c) xy_2 (d) x_2

4. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$, then $|\vec{a} - 2\vec{b}|$ is equal to:
(a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$

5. The angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ is:
(a) 1/3 (b) 2 / 3 (c) -1/3 (d) $\cos^{-1}(-1/3)$

6. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then $\vec{a}.\vec{b}$ is:
(a) $\sqrt{3}$ (b) 2 (c) 1/2 (d) $1/\sqrt{3}$

7. For the function $y = x^3 + 21$, the value of x, when y increases 75 times as fast as x, is:
(a) 3 (b) $5\sqrt{3}$ (c) 5 (d) none of these

8. The value of $\cos^{-1}(\cos 13\pi/6)$ is:
(a) $13\pi/6$ (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/5$

9. How many matrices are possible having 24elements?
(a) 4 (b) 6 (c) 8 (d) 2

10. The principal value of $\sec^{-1}(-2)$ is:

(a) $\pi/3$ (b) $\pi/2$ (c) $2\pi/3$ (d) not defined

11. If $\begin{vmatrix} y+2x & 5 \\ -x & 3 \end{vmatrix} = \begin{vmatrix} 7 & 5 \\ -2 & 3 \end{vmatrix}$, then the value of y is :

(a) 11 (b) 3 (c) -3 (d) 1

12. The area of the triangle with vertices (-1,2), (4,0) and (3,9) is:

(a) 43/2 sq units (b) -43/4 sq units (c) 21 sq units (d) 42 sq units

13. The minor of the element of second row and third column (a_{23}) in the determinant

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$$ is:

(a) -22 (b) 13 (c) -13 (d) 22

14. The rate of change of area of a circle with respect to its radius is :

(a) 2π (b) πr (c) $2\pi r$ (d) π

15. Given a matrix A of order 3 x 3. If $|A| = 3$, then find $|A.\text{Adj } A|$.

(a) 3 (b) 27 (c) 9 (d) 81

16. Find the order of matrix $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$

(a) 3 x 3 (b) 4 x 2 (c) 2 x 3 (d) 1 x 3

17. The scalar projection of the vector $3\hat{i} - \hat{j} - 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} - 3\hat{k}$ is :

(a) $7/\sqrt{14}$ (b) 7/14 (c) 6/13 (d) 7/2

18. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and $|\vec{a} \times \vec{b}| = \sqrt{3}$ is :

(a) $0°$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

19. **Assertion (A):** $\begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$ is a scalar matrix.

Reason (R): All the elements of the principal diagonal are equal , it is called a scalar matrix.

a. Both A and R are true and R is the correct explanation of A.
b. Both A and R are true but R is not the correct explanation of A.
c. A is true but R is false.
d. A is false but R is true.

20. **Assertion (A):** The vectors $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = 5\hat{i} + \hat{j} - 3\hat{k}$ are perpendicular to each other.

Reason (R) : $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

a. Both A and R are true and R is the correct explanation of A.
b. Both A and R are true but R is not the correct explanation of A.
c. *A is true but R is false.*
d. *A is false but R is true.*

Section B

SECTION B consists of 5 questions of 2 marks each.

21. *Find the value of* $\sin^{-1}[\sin(13\pi/7)]$.

OR

Write $\cot^{-1}(1/\sqrt{x^2 - 1}),\ x > 1$ in the simplest form.

22. A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

23. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

OR

Find the direction ratio and direction cosines of a line parallel to the line whose equations are : $6x - 12 = 3y + 9 = 2z - 2$.

24. If $y\sqrt{(1 - x^2)} + x\sqrt{(1 - y^2)} = 1$, then prove that $dy/dx = -\sqrt{(1 - y^2)/(1 - x^2)}$.

25. Find $|\vec{x}|$ if $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

Section C

SECTION C consists of 6 questions of 3 marks each.

26. Solve the following Linear Programming Problem graphically:
Maximize $Z = 400x + 300y$ subject to : $x + y \leq 200$, $x \leq 40$, $x \geq 20$, $y \geq 0$.

27. If $x = a(2\Theta - \sin 2\Theta)$ and $y = a(1 - \cos 2\Theta)$, find dy/dx when $\Theta = \pi/3$.

OR

Show that the function $f(x) = |x - 3|$, $x \in R$ is continuous but not differentiable at $x = 3$.

28. Find the intervals in which the function f given by $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly increasing or strictly decreasing.

29. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. find the value (s) of α such that $A' + A = I_2$.

OR

If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

30. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 1$.

31. Evaluate : $\tan^{-1}(\tan 9\pi/8)$.

OR

If $x^{16} y^9 = (x^2 + y)^{17}$, prove that $dy/dx = 2y/x$.

Section D

SECTION D consists of 4 questions of 5 marks each.

32. Find the shortest distance between the lines whose vector equations are:
$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$.

OR

Find the value of p, so that the lines L_1: $(1 - x)/3 = (7y + 14)/p = (z - 3)/2$ and L_2: $(7 - 7x)/3p = (y - 5)/1\ (6 - z)/5$ are perpendicular to each other. Also find the equations of a line passing through a point (3, 2,-4) and parallel to line L_1 .

33. Solve the following linear programming problem (LPP) graphically.
Maximize $Z = 20x + 40y$
Subject to constraints: $1.5x + 3y \leq 42$ and $3x + y \leq 24$, $x \geq 0$, $y \geq 0$.

34. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ verify that $BA = 6I$, how we can use the result to find the values of x, y, z from given equations. $x - y = 3$, $2x + 3y = 17$, $y + 2z = 7$.

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. How we can use A^{-1} to find x, y, z for the following system of equations:

$2x - 3y + 5z = 16$; $3x + 2y - 4z = -4$; $x + y - 2z = -3$.

35. Find the local maxima and minima, if any of the function f, given by:
$f(x) = \sin x + \cos x$, $0 < x < \pi/2$.

Section E

Case study-based questions are compulsory.

36. Three friends A, B and C are given a rectangular sheet of sides 45 cm and 24 cm. They are asked to work independently and form an open box by cutting the squares of equal length from all the four corners as shown and folding up the flaps, they want to check the volume of boxes so formed.

On the basis of above information, answer the following:

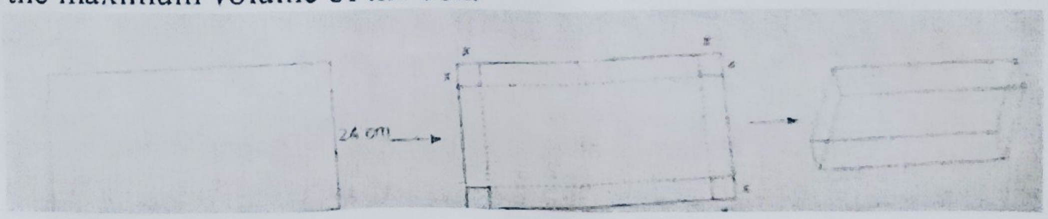
a. If a square of side x cm is cut from all corners, then find length, breadth and height o box in terms of x.

b. Find the volume of the box in term of x.

c. Find the value of x for which volume is maximum.

OR

d. Find the maximum volume of the box.



37. Amit, Biraj and Chirag were given the task of creating a square matrix of order 2. Below are the matrices created by them. A,B,C are the matrices created by Amit, Biraj and Chirag respectively.

$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

If $a = 4$ and $b = -2$.

Answer the following questions, based on the above information:

a. Find the sum of the matrices A, B and C.

b. Find $(A^T)^T$.

38. Employee in a office are following social distance and during lunch they are sitting at places marked by points A, B and C. Each one is representing position as $A(\hat{i} - 2\hat{j} + 4\hat{k})$, $B(5\hat{i} + 2\hat{k})$ and $C(3\hat{i} + 2\hat{j} + 4\hat{k})$.

On the basis of above information, answer the following questions:

a. Find the distance between B and C.

b. Find the position vector $\overrightarrow{AB}$.