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XII-K

SS4

HALF YEARLY EXAM-SEPTEMBER 2024

MATHEMATICS(SET II)

TIME DURATION : 3 HOURS MM: 80

General Instructions:

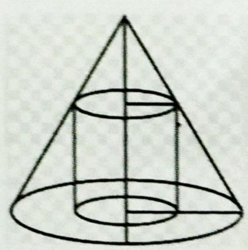
1. This Question paper contains five sections A, B, C, D, and E. Each section is compulsory . However, there are internal choices in some questions.
2. Section A has 20 questions of 1 mark each.
3. Section B has 5 questions of 2 marks each.
4. Section C has 6 questions of 3 marks each.
5. Section D has 4 questions of 5 marks each.
6. Section E has 3 case based questions of 4 marks each.

SECTION A (1 MARK EACH)	
1	Evaluate $\int \frac{sec^2 x}{cosec^2 x} dx$
2	If $y(2-x^2) = x$, $find\ \frac{dy}{dx}\ at\ (1,1)$
3	If function $f(x)$ is continuous at $x = 0$ then Find the value of a $f(x) = \begin{cases} \frac{sin^2 3x}{3x^2}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$
4	If A is a square matrix of order 3 such that $\|A\| = 16$, then find the value of $\|adjA\|$.
5	Find the domain of function $f(x) = sin^{-1}(3x-1)$
6	Find the number of one - one functions that can be defined from the set $\{a, b\}\ to\ \{3, 4, 5\}$
7	Find $\frac{dy}{dx}$ if $y = (3^{-x}.5^{x})$
8	Corner points of the feasible region for an Linear programming problem are $(0,0), (5,0), (0,5)\ and\ (3,4)$. Let $Z = px + qy$ be the objective function. The maximum value of Z occurs at both the points (0, 5) and (3, 4) then find the relation between $p\ and\ q$
9	Find an angle θ, $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine.
10	Find the value of $Sin\left(cot^{-1}\left(\frac{-1}{5}\right)\right)$

11	If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then find A^2
12	Evaluate $\int \frac{dx}{\sqrt{4x^2+9}}$
13	If $y = a(1 + \sin\theta),\ and\ x = a\cos\theta$ then find $\frac{dy}{dx}$.
14	Evaluate $\int \sin 3x.\cos 2x\, dx$
15	Write an equivalence relation R on the set $A = \{1, 2, 3\}$ containing both $(1, 2)\ and\ (2, 1)$.
16	Find the derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to x.
17	If A is the matrix of order 2×3 and B is the matrix of order 3×5 then what will be the order of matrix $(AB)^T$
18	Evaluate within the principal value $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$
	DIRECTION: In question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). **Choose the correct option** **(a) Both A and R are true and R is the correct explanation of A.** **(b) Both A and R are true but R is not the correct explanation of A.** **(c) A is true but R is false.** **(d) A is false but R is true.**
19	Statement A (Assertion): $\int \frac{dx}{1+(\sqrt{x})^2} = \tan^{-1}(\sqrt{x}) + c$ Statement R(Reason) : $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
20	Statement A (Assertion): $f(x) = \frac{1}{x-2}$ is a decreasing function for all $x \in R - \{2\}$ Statement R(Reason) : Any function $y = f(x)$ is decreasing if $\frac{dy}{dx} \leq 0$.
	SECTION B (2 MARKS EACH)
21	Show that the relation R defined on the set of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither symmetric nor transitive by giving example of each. **OR**

	If $f(x): [0, \infty) \to R$ be a function defined as $f(x) = 9x^2 + 6x - 5$,then show that function $f(x)$ is not onto . where R is the set of real numbers
22	Show that f(x) = $Cos\left(2x + \frac{\pi}{4}\right)$ is an increasing function on $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$
23	Show that $f(x) = \lvert x \rvert$, where $\lvert x \rvert$ represents modulus function, is not differentiable at $x = 1$. OR If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, Find $\frac{dy}{dx}$
24	Using determinants find all possible values of k for which the points $(k + 5, k - 4), (k - 2, k + 3)$ and (k, k) lie on the straight line.
25	Evaluate $\int \frac{dx}{\sin x \cos^3 x}$
	SECTION C (3 MARKS EACH)
26	If $f(x)$ is continuous at $x = \frac{\pi}{2}$, find a and b $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} &, if\ x < \frac{\pi}{2} \\ a &, if\ x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} &, if\ x > \frac{\pi}{2} \end{cases}$ OR If $y = \tan^{-1} x$, $then\ show\ that$ $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$
27	Show that the function $f: R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ $given\ by$ $f(x) = \frac{4x+3}{3x+4}$ is bijective.
28	Two equal sides of an isosceles triangle with fixed base a are decreasing at the rate of 9 cm/sec. How fast is the area of the triangle decreasing when equal sides are equal to the base. OR A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost.Its semi vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute.Find the rate at which the level of the water is rising at the instant when depth of the water in the tank is 10 m.

29	Evaluate $\int \frac{2x\,dx}{(x^2+1)(x^2+2)}$
30	If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$
31	Prove that $4\left(\cos^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{10}}\right) = \pi$ **OR** Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$
	SECTION D (5 MARKS EACH)
32	If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ and I is the unit matrix of order 2×2 then show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$
33	Evaluate $\int \frac{(\sin 2x + \cos x)dx}{5 - \cos^2 x + 2\sin x}$ **OR** Evaluate $\int \frac{(\sin x + \cos x)dx}{\sqrt{9 + 16\sin 2x}}$
34	Find the intervals in which the function $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$ is strictly increasing or strictly decreasing.
35	Find the minimum value of $Z = 3x + 9y$ Subject to constraints $x + 3y \le 60$, $x \le y$, $x + y \ge 10$, $x \ge 0$, $y \ge 0$ **OR** Maximize $Z = x + 2y$ Subject to constraints $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$, $x \ge 0$, $y \ge 0$

SECTION E (CASE BASED QUESTIONS 4 MARKS EACH)	
36	Let A be the set $\{0, 1, 2, 3, \ldots\ldots 15\}$ If R is a relation defined on set A defined as $R = \{(x, y): \|x - y\| \text{ is divisible by } 4\}$. (i) show that R is an equivalence relation. (ii) write the set of all numbers belonging to set A and related to 2.
37	Two shopkeepers A and B are using handmade bags and paper bags. A is using 8 Handmade bags and 7 paper bags per day and B is using 3 handmade bags and 5 paper bags per day .The shopkeepers A and B spend ₹160 and ₹79 respectively on these bags per day. Based on the above information answer the following questions: (i) Represent the given information in the form of a matrix. (ii) What is the cost of one handmade bag? (iii) What is the cost of one paper bag? 
38	Show that the height of cylinder of greatest volume,which can be inscribed in a right circular cone of height H and semi vertical angle α, is one third that of the cone. Also prove that the greatest volume of cylinder is $\frac{4}{27}\pi H^3 \tan^2 \alpha$ 

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