No. of Printed Pages : 10 **Roll No. 24**

FAS / Mathematics / XII / Half Yearly Examination / 2024-25

Time : 3 Hours] **[M.M. : 80 Marks**

INSTRUCTIONS:—

(i) This question paper contains 5 sections A, B, C ,D and E. Each section is compulsory. However, there are internal choices in some questions.

(ii) Section A has 18 MCQ's and 2 Assertion -Reason based questions of 1 mark each.

(iii) Section B has 5 Very Short Answer (VSA)- type questions of 2 marks each.

(iv) Section C has 6 Short Answer (SA)-type questions of 3 marks each.

(v) Section D has 4 Long Answer (LA)- type questions of 5 marks each.

(vi) Section E has 3 source based/ case based/ passage based/ integrated units of assessment (4 marks each) with parts.

[SECTION–A]

Multiple choice Questions (1 mark each)

1. Let the relation R on the set N of all natural numbers defined as $R = \{(x,y) : y = x+5 \text{ and } x < y\}$, then R is :—

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

2. The value of $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$ is:—

(a) $\pi/3$ (b) $\pi/2$

(c) 0 (d) $2\pi/3$

3. If A is a symmetric matrix, then A^3 is a _____ matrix.

(a) Skew-symmetric (b) symmetric

(c) null (d) identity

4. If a set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is :—

(a) 720 (b) 120

(c) 0 (d) None of these

5. The domain of the function $\cos^{-1}(2x-1)$ is:—

(a) $[0, 1]$ (b) $[-1, 1]$

(c) $(-1, 1)$ (d) $[0, \pi]$

6. If the area of the triangle ABD is 3 sq. units with vertices A(1, 3), B (0, 0) and D(k, 0), then k is equal to:—

(a) 3 (b) ± 3

(c) 2 (d) ± 2.

7. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, then $M_{12} + C_{22}$ is :—

(a) 4 (b) 1

(c) –4 (d) 0

8. The value of k for which $f(x) = \begin{cases} \frac{\sin 5x}{3x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x=0, is=

(a) 1/3 (b) 0

(c) 3/5 (d) 5/3

9. If $A = \begin{bmatrix} 1 & 4 \\ 3 & 15 \end{bmatrix}$, then $|A^{-1}|$ is equal to :—

(a) –1/3 (b) 1/3

(c) 2/3 (d) 4/3

10. If $\sin^{-1} x + \sin^{-1} y = \pi/2$, then value of $\cos^{-1} x + \cos^{-1} y$ is :—

(a) $\pi/2$ (b) π

(c) 0 (d) $2\pi/3$

11. If A and B are square matrices of order 3×3 such that $|A| = 5$ and $|B| = 3$, then $|3AB|$ is :—

(a) 135 (b) 45

(c) 405 (d) none of these

12. What is the rate of change of $\sqrt{x^2 + 16}$ w.r.t. x^2 at $x=3$?

(a) 1/5 (b) 1/10

(c) 1/20 (d) 1/15

13. Which of the following is true for the function $f(x) = 9x - 5$?

(a) Strictly increasing on R.

(b) Strictly decreasing on R.

(c) Both (a) & (b) are false.

(d) Both (a) & (b) are true.

14. $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$ is equal to:—

(a) π (b) 2π

(c) 3π (d) $\pi/2$

15. $\int \frac{dx}{\sqrt{x}+x}$ is equal to :—

(a) $2\log|\sqrt{x}+1|+C$ (b) $\log|x+1|+C$

(c) $\log|x-1|+C$ (d) $2\log|x+1|+C$.

16. $\int \frac{\sin^6 x}{\cos^8 x} dx$ is equal to:—

(a) $\frac{\tan^6 x}{x}+C$ (b) $\frac{\tan^7 x}{7}+C$

(c) $\frac{\tan^5 x}{5}+C$ (d) $\frac{\cot^7 x}{7}+C$.

17. Area lying in the first quadrant and bounded by the circle $x^2+y^2=4$ and the lines $x=0$ and $x=2$ is

(a) π (b) $\pi/2$

(c) $\pi/3$ (d) $\pi/4$

18. Area of the region bounded by the curve $y^2=4x$, y-axis and the line $y=3$ is :—

(a) 2 sq.units (b) 9/4 sq. units

(c) 9/2 sq. units (9) 9/3 sq. units

Assertion–Reasoning Questions

In the following statements, a statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices:—

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A) : If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ then the value of k such that $A^2=kA-2I$ is $-I$

Reason (R) : If A and B are square matrices of same order, then (A+B (A–B) is equal to $A^2+AB+BA+B^2$.

20. Assertion (A) : The absolute maximum value of the function $2x^3-24x$ in the interval [1, 3] is 89.

Reason (R) : The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

[SECTION–B]

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Show that the function f: R→R, given by $f(x) = \cos x$. $\forall\ x \in R$ is neither one-one nor onto.

22. If $x = \cos t + \sin t$ and $y = \sin t - \cos t$, then find dy/dx at $t = \pi/2$.

OR

If $y = x^{1/x}$, then find dy/dx.

23. A balloon which always remains spherical has a variable diameter $\frac{3}{2}(2x+1)$, then find the rate of change of its volume w.r.t x.

24. Evaluate $\int_{-\pi}^{\pi}\left(1-x^2\right)\sin x\cos^2 x\,dx$.

25. Evaluate: $\int\frac{1}{\sqrt{9+8x-x^2}}dx$.

OR

Evaluate: $\int x\log x\,dx$.

[SECTION–C]

This section comprises of short answer type questions (SA) of 3 marks each.

26. Show that the function f: R→R defined by $f(x) = 2x^3-5$, is a bijective function.

27. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}.

28. If the matrices $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adjA and C = 3A, then find the value of $\frac{|\text{adj } B|}{|C|}$.

29. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ prove that $\frac{dy}{dx} = -\frac{y}{x}$.

OR

Let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{if } x > 0 \end{cases}$. Determinec the value of a so that f(x) is continuos at x = 0.

30. Using properties of definite integrals, evaluate :

$$\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$$

OR

Evaluate : $\int_{-1}^{2} |x^3 - x|\, dx$.

31. Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = –1 and x = 1.

OR

Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

[SECTION–D]

This section comprises of long answer type questions (LA) of 5 marks each.

32. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$. find BA and use this to solve the system of equations : $y+2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$.

33. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$ prove that $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$ is a constant independent of a and b.

OR

If $y = e^{a\cos^{-1}x}$, $-1 < x < 1$, then show that $(1-x^2)y_2 - xy_1 - a^2y = 0$.

34. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\dfrac{2a}{\sqrt{3}}$.

OR

A figure consists of a semicircle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

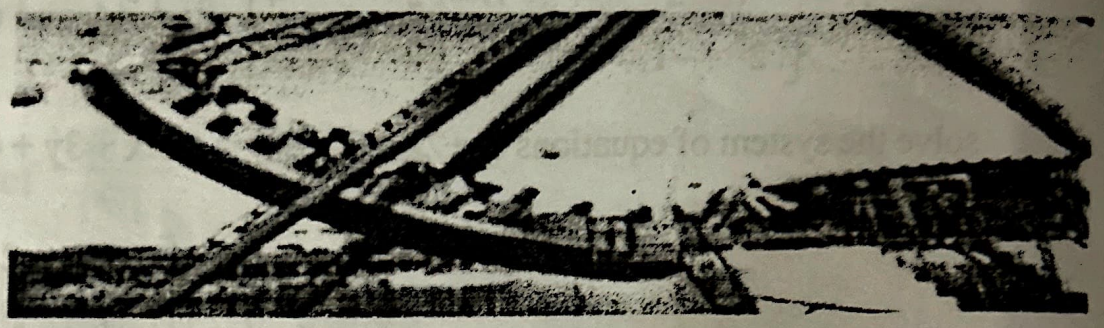
35. Evaluate : $\int_0^{\pi} \dfrac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$.

[SECTION–E]

This section comprises of 3 case study questions of 4 marks each.

36. Read the following and answer the questions given below :

Raji visited the exhibition along with her family. The exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a parabola as given by $y = x^2$.



(i) Examine surjective and injective for the function $f : R \to R$ defined by $f(x)=x^2$.

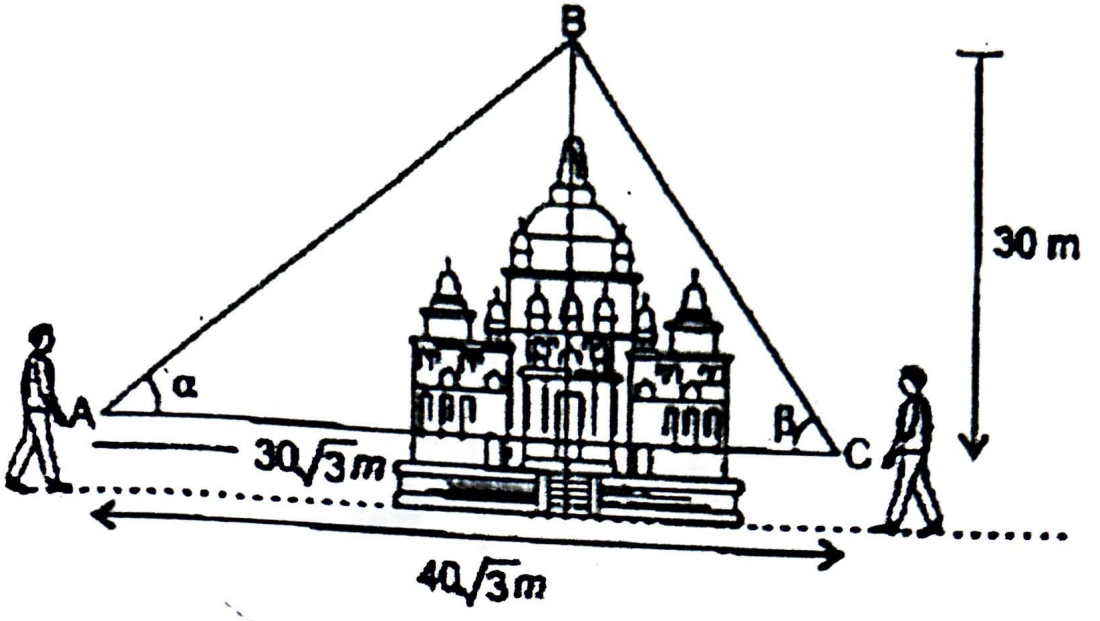
(ii) Examine surjective and injective for the function $f : N \to N$ defined by $f(x)=x^2$.

(iii) Is the function $f:\{1,2,3,.....\} \to \{1, 4, 9,.....\}$ defined by $f(x)=x^2$ is bijective ? Justify your answer.

OR

Discuss surjective and injective for the function $f : Z \to Z$ be defined by $f(x)=x^2$.

37. Albert and Mark, two friends are standing on either side of a temple of 30m height. They observe its top at the angle of elevation α and β respectively. The distance between Albert and Mark is $40\sqrt{3}$ m and the distance between Albert and the temple is $30\sqrt{3}$ m.



Based on the above information, answer the following questions :

(i) Find $\sin \alpha$.

(ii) Find $\angle BCA$.

(iii) If $\alpha = \cos^{-1}\left(\frac{k_1}{k_2}\right)$, then find $K_1 + K_2$.

OR

Find $\angle CBA$.

38. Read the following and answer the questions given below :

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$

where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to exposure to sunlight.

What is the number of days it will take for the plant to grow to the maximum height ?

(ii) What is the maximum height of the plant ?