No. of pages - 24

STUDENT ID

(M)

MID-TERM EXAMINATION (2024-25)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**
समय : 3 घंटे **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड-ग में प्रश्न संख्या 26 से 31 तक लघुउत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच- पाँच अंकों के प्रश्न हैं।
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में, खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, questions No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, questions No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, questions No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, questions No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, questions No. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. A function $f : R \to A$ defined as $f(x) = x^2 - 4x + 6$ is onto, if A is : 1

(a) $(1, \infty)$ (b) $[1, \infty)$

(c) $[2, \infty)$ (d) $[6, \infty)$

2. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{-1}{2}\right)\right]$ is equal to : 1

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 1

3. $\sin\left[\cos^{-1}\left(\cos\frac{7\pi}{3}\right)\right]$ is equal to : 1

(a) $\frac{-1}{2}$ (b) $\frac{-\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

4. The number of onto functions from a set P containing 4 elements to a set Q containing 2 elements is : 1

(a) 8 (b) 12

(c) 14 (d) 16

5. It is given that $A\begin{bmatrix} \cos\frac{\pi}{3} & \cos\frac{\pi}{6} \\ -\sin\frac{\pi}{3} & \sin\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A is :

(a) $\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

(b) $\frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

(c) $\frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$

(d) $\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$

6. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $(2I + 3A)(2I - 3A) = xI$, then value of x is : 1

(a) -2

(b) -5

(c) -8

(d) -1

7. If $\begin{bmatrix} p+q & s \\ r & pq \end{bmatrix} = \begin{bmatrix} 24 & 1 \\ 2 & 63 \end{bmatrix}$, then the value of $\left(\frac{21}{p} + \frac{21}{q}\right)$ is : 1

(a) 2

(b) 4

(c) 8

(d) 21

8. $\begin{vmatrix} \cos 7^\circ & -\cos 23^\circ \\ \sin 7^\circ & \sin 23^\circ \end{vmatrix}$ is equal to :. 1

(a) 0

(b) $\sin 16^\circ$

(c) $\cos 16^\circ$

(d) 0.5

9. Given a curve $y = x - x^3$ and x increases at the rate of 3 units per second. The rate at which the slope of the curve is changing, when $x = 4$ is : 1

(a) -60 units/sec

(b) -72 units/sec

(c) -48 units/sec

(d) -24 units/sec

10. If $y = \log(\sin e^x)$ then $\frac{dy}{dx}$ is :

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$

(c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

11. The derivative of e^{2025x} with respect to e^x is :

(a) 2025 (b) $2025\, e^{2025x}$

(c) $2025\, e^{2024x}$ (d) $2025\, e^x$

12. $\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} dx$ is equal to : 1

(a) $\frac{\cos 2x}{2} + c$ (b) $\frac{\sin 2x}{2} + c$

(c) $-2\sin 2x + c$ (d) $\cos 2x + c$

13. If $x = y^{1/3}$, then $\frac{d^2y}{dx^2}$ is equal to : 1

(a) $\frac{y}{6}$ (b) $\frac{x}{6}$

(c) $6y$ (d) $6x$

14. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$ is : 1

(a) 2 (b) 3

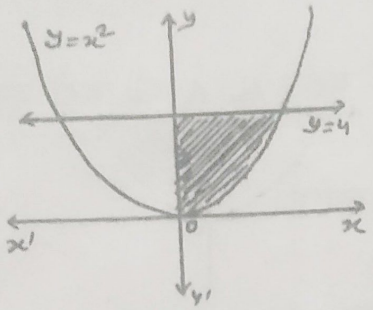
(c) 4 (d) 1

15. The integrating factor of the differential equation $\frac{dy}{dx} - \frac{1}{x}y = 1$ is :

(a) e^{-x} (b) e^{x}

(c) x (d) $\frac{1}{x}$ 1

16. In the given figure, area of the shaded region is equal to : (in sq.units)



(a) $\int_0^4 y\,dy$ (b) $\int_0^4 \sqrt{y}\,dy$

(c) $\int_0^2 x\,dx$ (d) $\int_0^2 x^2\,dx$

17. The value of $\int_{-2}^{2} |x|\,dx$ is : 1

(a) 4 (b) 3

(c) 2 (d) 0

18. The interval in which the function $f(x) = x^3 - 3x$ is decreasing is : 1

(a) $(-\infty, \infty)$ (b) $(-\infty, -1]$

(c) $[1, \infty)$ (d) $[-1, 1]$

Question number 19 and 20 each carry one mark.

In the following questions a statement of Assertion (A) is followed by statement of Reason (R) is given. Choose the correct answer out of following choices :

(a) **Both A and R are true and R is the correct explanation of A.**

(b) **Both A and Reason R are true but Reason R is not correct explanation of A.**

(c) **A is true but R is false.**

(d) **A is false but R is true.**

19. Assertion (A) : Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all diagonal elements are zero. 1

20. Assertion (A) : Domain of $y = \sec^{-1} x$ is $(-\infty, -1) \cup (1, \infty)$.

Reason (R) : Domain of $y = \cos^{-1} x$ is $[-1, 1]$. 1

SECTION-B

This section comprises of very short answer (VSA) type questions of 2 marks each.

21. Find : $I = \int \frac{dx}{x^2 - 8x + 25}$ 2

22. (a) Evaluate : $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$ 2

OR

(b) Find the domain and range of the function $f(x) = \cos^{-1}(x^2 - 4)$. 2

23. (a) If $x^y = y^x$, then find $\frac{dy}{dx}$.

OR

(b) If $\sqrt{1+x}\,.y + x\sqrt{1+y} = 0, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ 2

24. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^2 = \begin{bmatrix} 7 & 12 \\ 18 & 31 \end{bmatrix}$, then find the value of $(ad - bc)$.

(Given $ad - bc < 0$) 2

25. Prove that the function given by $f(x) = \cos x$ is neither increasing nor decreasing in $(0, 2\pi)$. 2

SECTION-C

This section comprises short answer (SA) type questions of 3 marks each.

26. Express the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. 3

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{y^2 + 2xy}{2x^2}$, given that $y = 2$ when $x = 1$. 3

OR

(b) Find the general solution of the differential equations :

$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ 3

28. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto. Justify your answer. 3

29. (a) Evaluate : 3

$$I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$$

OR

(b) Find :

$$I = \int \frac{2x}{(x+1)^2(x-1)} dx$$ 3

30. (a) Find :

$$I = \int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx$$ 3

OR

(b) Find :

$$I = \int \sin^3 x . \cos^3 x \, dx$$ 3

31. A water tank has the shape of an inverted right circular cone with its Axis Vertical and Vertex Lower most. Its semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per min. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m. 3

SECTION-D

Question number 32 to 35 are long type answer (LA) type. Each carries 5 marks.

32. (a) If $A=\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$$x-2y=10$$
$$2x-y-z=8$$
$$-2y+z=7$$

OR

(b) Solve the following system of equations using matrices : 5

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{Z}=4,$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2, \text{ where } x,y,z\neq 0$$

33. Find the area of the region bounded by the curve $9x^2+4y^2=36$ using integration. 5

34. (a) Check whether the relation S in the set of real numbers R is defined by $S=\{(a,b):$ where $a-b+\sqrt{5}$ is an irrational number$\}$ is reflexive, symmetric or transitive.

OR

(b) (i) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. 2

(ii) Draw the graph of $f(x) = \sin^{-1} x$. Also write domain and range of f(x). 3

35. Evaluate : 5

$$I = \int_{-1}^{3/2} |x \sin \pi x| \, dx$$

SECTION-E

CASE BASED QUESTIONS

Question number 36 to 38 carries 4 marks each.

36. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 400. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pay a sum of ₹ 270. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 410.

Based on the above information, answer the following questions:

(i) Convert the given above situation into a matrix equation of the form $[A]_{3\times3}[X]_{3\times1} = [B]_{3\times1}$, where $B = \begin{bmatrix} 400 \\ 270 \\ 410 \end{bmatrix}$. 1

(ii) Find $|A|$.

(iii) (a) If $\text{adj}\,A = \begin{pmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ a & b & c \end{pmatrix}$, then find the values of a, b and c. 2

OR

(b) Find the values of $|\text{adj } A|$ and $|A.\ (\text{adj } A)|$. 2

37. A fighter jet of enemy is flying along the parabolic path $2y = x^2$. A soldier is located at the point (0, 5) and is aiming to shoot down the jet when it is nearest to him.

Based on the above information, answer the following questions:

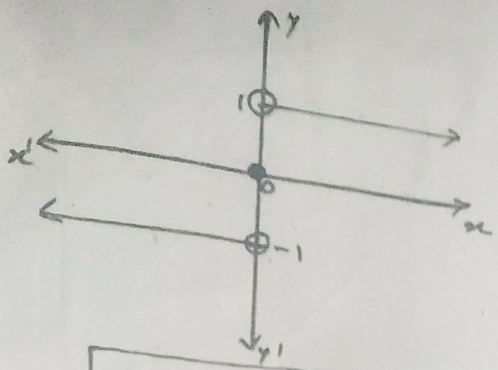
(i) Let (x, y) be the position of the jet at any instant. Express the distance between the soldier and the jet as the function f(x). 1

(ii) Taking $S = [f(x)]^2$ find $\frac{dS}{dx}$. 1

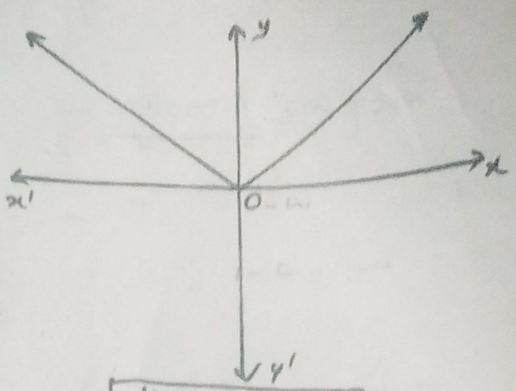
(iii) (a) What will be the position of the jet when the soldier shoots it down? 2

OR

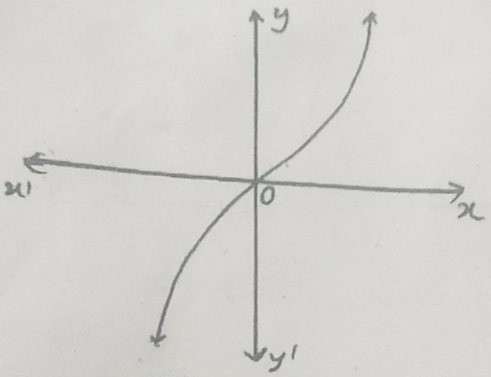
(b) What will be the distance between the soldier and the jet at any instant when he shoots it down? 2



$y = f(x) = \frac{|x|}{x}, x \neq 0$
$0, x = 0$



$y = g(x) = |x|$



$y = h(x) = x^3$

The graph of three function f(x), g(x) and h(x) are shown above.

Based on the above information, answer the following questions:

(a) Find the point of discontinuity for each of the three functions given i.e. f(x), g(x) and h(x). (If any) 2

b) Find the point of non-differentiability for each of the three functions given i.e. f(x), g(x) and h(x). (If any) 2