Modern School, Vasant Vihar

Term I Examination, 2024-25

Grade XII
Mathematics
SET B

Time allowed: 3 Hours
Maximum Marks: 80

General Instructions:

1. **This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.**
2. **Section A has 18 MCQ and 02 Assertion-Reason based questions of 1 mark each.**
3. **Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.**
4. **Section C has 6 Short Answer (SA)-type questions of 3 marks each.**
5. **Section D has 4 Long Answer (LA)-type questions of 5 marks each.**
6. **Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.**

SECTION A
(Multiple Choice Questions)
(Each question carries 1 mark)

Q1. The value of k for which the function $f(x)=\begin{cases} x^2, & x\geq 0 \\ kx, & x<0 \end{cases}$ is differentiable at $x=0$ is :

(a) 1 (b) 2 (c) any real number (d) 0

Q2. If $A=\begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and $kA=\begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$, then the values of k, a and b respectively are :

(a) -6, -12, -18 (b) -6, -4, -9 (c) -6, 4, 9 (d) -6, 12, 18

Q3. If $\tan\left(\frac{x+y}{x-y}\right)=k$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\sec^2\left(\frac{y}{x}\right)$ (d) $-\sec^2\left(\frac{y}{x}\right)$

Q4. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is :

(a) 10π (b) 12π (c) 8π (d) 11π

Q5. The interval, in which the function $y=x^3+6x^2+6$ is increasing, is :

(a) $(-\infty,-4)\cup(0,\infty)$ (b) $(-\infty,4)$ (c) $(-4,0)$ (d) $(-\infty,0)\cup(4,\infty)$

Q6. If A is a square matrix such that $A^2=I$, then $(A-I)^3+(A+I)^3-7A$ is equal to :

(a) A (b) $I-A$ (c) $I+A$ (d) $3A$

Q7. $\sin(\tan^{-1}x)$, where $|x|<1$, is equal to:

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

Q8. The function $f(x)=|x|$ is :

(a) continuous and differentiable everywhere.
(b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at x = 0.
(d) continuous everywhere, but differentiable nowhere.

Q9. If A and B are matrices of same order, then $(AB' - BA')$ is a :

(a) Skew – symmetric matrix (b) Null matrix (c) Symmetric matrix (d) Unit matrix

Q10. The function $f(x)=[x]$, where $[x]$ denotes the greatest integer function, is continuous at :

(a) 4 (b) -2 (c) 1 (d) 1.5

Q11. A relation R is defined on Z as aRb if and only if $a^2-7ab+6b^2=0$, then R is :

(a) reflexive and symmetric (b) symmetric but not reflexive
(c) transitive but not reflexive (d) reflexive but not symmetric.

Q12. The function $f: N \to N$ defined by $f(n)=\begin{cases} \frac{n+1}{2}, & if \quad n \quad is \quad odd \\ \frac{n}{2} & if \quad n \quad is \quad even \end{cases}$ is :

(a) bijective (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto

Q13. Which of the following is the principal value branch of $\operatorname{cosec}^{-1}x$?

(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $(0,\pi)-\left\{\frac{\pi}{2}\right\}$ (c) $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ (d) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

Q14. The inverse of $\begin{pmatrix} -4 & 3 \\ 7 & -5 \end{pmatrix}$ is :

(a) $\begin{pmatrix} -5 & -3 \\ 7 & -4 \end{pmatrix}$ (b) $\begin{pmatrix} 5 & 3 \\ 7 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} -5 & 7 \\ 3 & -4 \end{pmatrix}$ (d) $\begin{pmatrix} -5 & -3 \\ -7 & -4 \end{pmatrix}$

Q15. If $\cos\left(\sin^{-1}\frac{3}{5}+\cos^{-1}x\right)=0$, then x is equal to :

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

Q16. The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 square units. The value of k will be :

(a) 9 (b) 3, -3 (c) -9 (d) 6

Q17. If $A=\begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, then the value of $|adj A|$ is :

(a) 64 (b) 16 (c) 0 (d) -8

Q18. The relation R in a set $A=\{1,2,3\}$ is defined as $R=\{(1,1),(1,2),(2,2),(3,3)\}$.

Which of the following ordered pairs in R shall be removed to make it an equivalence relation in A?

(a) (1,1) (b) (1,2) (c) (2,2) (d) (3,3)

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A):** $f(x) = \tan x - x$ always increases.

Reason (R): Any function $y = f(x)$ is increasing if $\frac{dy}{dx} > 0$

Q20. **Assertion (A):** $f: R \to R$ given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is bijective.

Reason (R): A function $g: A \to B$ is said to be bijective if it is one-one and onto.

SECTION B

(Very short answer type-questions (VSA) of 2 marks each)

Q21. Sand is pouring from a pipe at the rate of 12cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one – sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm ?

Q22. What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$? Given $x \neq 1$.

OR

Let $f: R \to R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}, \forall x \in R$. Find the range of f.

Q23. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find $A(adj A)$.

Q24. The matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

OR

If $A = \begin{pmatrix} -3 & 6 \\ -2 & 4 \end{pmatrix}$, then show that $A^3 = A$.

Q25. Evaluate $\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

SECTION C

(Short answer type questions (SA) of 3 marks each)

Q26. Find the intervals in which the function $f(x)=\frac{3}{2}x^4-4x^3-45x^2+51$ is

(i) strictly increasing (ii) strictly decreasing

OR

Show that $y=\log(1+x)-\frac{2x}{2+x}, x>-1$ is an increasing function of x, throughout its domain.

Q27. If $f(\alpha)=\begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}$, prove that $f(\alpha).f(\beta)=f(\alpha+\beta)$

Q28. Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a+1} is reflexive, symmetric or transitive .

OR

Consider $f:R-\{2\}\to R-\{1\}$, given by $f(x)=\frac{x-1}{x-2}$. Show that f is one-one and onto.

Q29. For what value of λ is the function defined by $f(x)=\begin{cases}\lambda(x^2-2x), & x\le 0\\ 4x+1, & x>0\end{cases}$ continuous at $x=0$? What about continuity at $x=1$?

OR

If $x=\sqrt{a^{\sin^{-1}t}}, y=\sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx}=-\frac{y}{x}$ Sq multiph

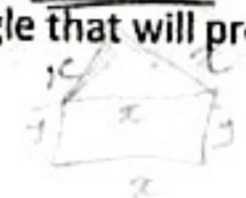
Q30. Find equation of line joining A (3, 1) and B (9, 3) using determinants and find k if D (k, 0) is a point such that area of triangle ABD is 3 square units.

Q31. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

SECTION D

(Long answer-type questions (LA) of 5 marks each)

Q32. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.



Q33. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$

OR

If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$

Q34. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Q35. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O$, then find the value of k.

OR

Express the following matrix as a sum of a symmetric and a skew- symmetric matrix

and verify your result : $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Are diagonal elements of a skew- symmetric matrix always zero ? Justify your answer.

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2marks each.

Q36. **Case-Study 1:** Read the following passage and answer the questions given below.

Let $f(x)$ be a real valued function , then its

Left Hand Derivative (LHD) = $\lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

Right Hand Derivative (RHD) = $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Also, a function $f(x)$ is said to be differentiable at $x = a$, if its LHD and RHD at $x = a$ exist and both are equal.

For the function

$$f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

(i) What is RHD of $f(x)$ at $x=1$?
(ii) What is LHD of $f(x)$ at $x=1$?
(iii) Check if the function $f(x)$ is differentiable at $x=1$.

OR

Find $f'(2)$ and $f'(-1)$

Q37. **Case-Study 2:** Read the following passage and answer the questions given below.
In a general election about 911 million people were eligible to vote and voter turnout was about 67 %, the highest ever.

ONE – NATION
ONE – ELECTION
FESTIVAL OF
DEMOCRACY
GENERAL ELECTION– 2019

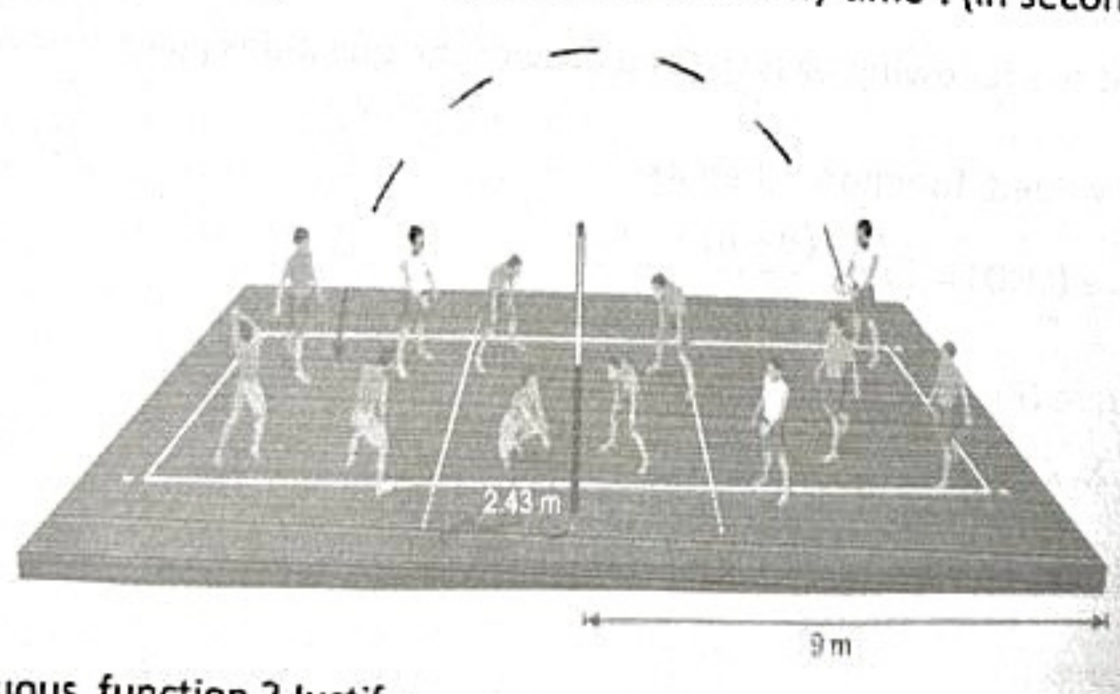
Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows :
$R=\{(V_1,V_2):V_1,V_2\in I$ and both use their voting right in GENERAL ELECTION – 2019 $\}$

(i) Two neighbours X and Y$\in I$, X exercised his voting right, while Y did not cast her vote in GENERAL ELECTION – 2019. Is X R Y ? Give reason.
(ii) Is ' R' a reflexive relation ? Explain.
(iii) Is ' R' a symmetric and transitive relation ? Explain.

OR

Is 'R' an Equivalence relation? Explain.
Find the equivalence class for Mr. Shyam, who exercised his voting right in GENERAL ELECTION – 2019.

Q38. **Case-Study 3**: Read the following passage and answer the questions given below.
A volleyball player serves the ball which takes a parabolic path given by the equation $h(t)=-\frac{7}{2}t^2+\frac{13}{2}t+1$, where $h(t)$ is the height of the ball at any time t (in seconds), $t\geq 0$



(i) Is $h(t)$ a continuous function ? Justify.
(ii) Find the time at which the height of the ball is maximum.