Roll No. 12059

No. of Printed Pages : 10

MG-170+100=270

HALF YEARLY EXAMINATION 2024-25

MATHEMATICS

Time : 3 hrs.] **Class XII** **[M.M. : 80**

General Instructions—

Read the following instructions very carefully and strictly follow them—

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into five sections – A, B, C, D and E.

(iii) In section A - Q.No. 1 to 18 are Multiple Choice Questions (MCQ) type and Q.No. 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In section B - Q.No. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In section C - Q.No. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In section D - Q.No. 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.

(vii) In section E - Q.No. 36 to 38 are case study based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculators is not allowed.

SECTION-A

This section has 20 multiple choice questions of 1 mark each—

1. A function $f : R \rightarrow R$ defined by $f(x) = 2+x^2$ is :

(a) not one-one (b) not onto

(c) one-one (d) neither one-one nor onto

2. Let A = {2, 3, 5}. Then number of reflexive relations on A is :
   (a) 16
   (b) 64
   (c) 32
   (d) 128

3. A relation R in set A = {1, 2, 3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shalll be removed to make it an equivalence relations in A ?
   (a) (1, 1)
   (b) (1, 2)
   (c) (2, 2)
   (d) (3, 3)

4. The value of $\sin\left[\tan^{-1}\left(-\sqrt{3}\right)+\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$ is :
   (a) 1
   (b) −1
   (c) 0
   (d) None of these

5. The domain of the function $\sin^{-1}(3x-1)$ is :
   (a) [0, 1]
   (b) [−1, 1]
   (c) (−1, 1)
   (d) $\left[0, \frac{2}{3}\right]$

6. The number of all scalar matrices of order 3, with each entry −1, 0 or 1 is :
   (a) 1
   (b) 3
   (c) 2
   (d) 3^9

7. A matrix $A = [a_{ij}]_{3\times3}$ is defined by :

$$a_{ij} = \begin{cases} 2i+3j & ; \text{ if } i<j \\ 5 & ; \text{ if } i=j \\ 3i-2j & ; \text{ if } i>j \end{cases}$$

The number of elements in A which ~~are~~ more than 5 is :
   (a) 3
   (b) 4
   (c) 5
   (d) 6

8. If the matrix $\begin{bmatrix} 2K+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2K-3 \end{bmatrix}$ is skew-symmetric, then the value of K is :

$2K + 3 = -2K$

(a) $\frac{3}{2}$

(b) $\frac{-3}{2}$

(c) $\frac{1}{2}$

(d) None of these

9. If $|A| = |KA|$, where A is a square matrix of order 2, then sum of all possible values of K is :

(a) 1 (b) -1 (c) 2 (d) 0

10. Given that A is a square matrix of order 3 and $|A| = -2$, then $|\text{adj}(2A)|$ is equal to:

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

11. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function less than or equal x, is continuous at :

(a) $x = 1$

(b) $x = 1.5$

(c) $x = -2$

(d) $x = 4$

12. If a function f defined by $f(x) = \begin{cases} \dfrac{K\cos x}{\pi - 2x} & ; \text{if } x \neq \dfrac{\pi}{2} \\ 3 & ; \text{if } x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$,

valu $K = ?$

then :

(a) 2 (b) 3 (c) 6 (d) -6

13. If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to :

$\cos(xy)\left(x\frac{dy}{dx} + y\right) = 0$

$\cos(xy)\, x \frac{dy}{dx} = -\cos(xy)\, y$

(a) $\frac{x}{y}$

(b) $\frac{-x}{y}$

(c) $\frac{y}{x}$

(d) $\frac{-y}{x}$

14. The function $f(x) = x^3+3x$ is increasing in interval :

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) R (d) $(0, 1)$

$3x^2 + 3$

15. The maximum value of $\left(\frac{1}{x}\right)^x$ is :

$x^2+1=0$

$x^2 = \pm 1$

(a) $e^{\frac{1}{e}}$ (b) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

(c) e (d) e^o

16. If the minimum value of an objective function $z = ax+by$ occurs at two points (3, 4) and (4, 3) then :

(a) $a+b=0$ (b) $a=b$

(c) $3a=b$ (d) $a=3b$

17. If A and B are two events such that $P(A \cup B) = \frac{5}{6}$, $P(\overline{A}) = \frac{1}{4}$, $P(B) = \frac{1}{3}$, then A and B are :

(a) mutually exclusive (b) dependent

(c) independent (d) none of these

18. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is :

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$

(c) $\frac{3}{20}$ (d) $\frac{4}{5}$

Assertion-Reason based questions—

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** Domain of $y = \cos^{-1} x$ is $[-1, 1]$.

(c)

Reason (R) : The range of the principal value branch of $y = \cos^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

20. **Assertion (A) :** For non-singular matrices A and B of same order $(A^{-1}.B)^{-1} = B^{-1}.A$

(a)

Reason (R) : $(AB)^{-1} = B^{-1}.A^{-1}$ and $(A^{-1})^{-1} = A$

SECTION-B

This section comprises of very short answer questions (VSA) of 2 marks each.

21. Let $f : R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$.

Show that, in $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f, f is one-one and onto.

22. Find the value of $\sin\left[2\cos^{-1}\left(\frac{-3}{5}\right)\right]$

OR

Find the value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ $6\pi - \frac{33\pi}{5}$

23. If A is a square matrix satisfying $A^2 = I$, then find the inverse of A.

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find K so that $A^2 = 5A + KI$

24. If area of a triangle is 35 square units with vertices (2, –6), (5, 4) and (K, 4), then find the value of K.

25. If x and y are the sides of two squares such that $y = x - x^2$, then find the rate of change of the area of second square with respect to the area of first square.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Let $A = \{x \in Z : 0 \le x \le 12\}$, show that $R = \{(a, b) : a, b \in A, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

Let $A = \{1, 2, 3, \ldots\ldots, 9\}$ and R be the relation in $A \times A$ defined by (a, b) R (c, d) if $a+d = b+c$ for (a, b), (c, d) in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)].

27. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2+B^2$, then find the values of a and b.

OR

Express the matrix $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix and verify your result.

28. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}(x^2)$.

29. Find the intervals in which the functions $f(x) = \dfrac{3}{2}x^4 - 4x^3 - 45x^2 + 57$ is :

(a) strictly increasing (b) strictly decreasing

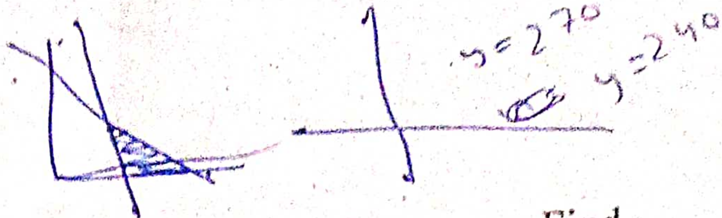
30. Solve the following linear programming problem graphically—

Minimise $z = 10x + 8y$

subject to constraints : $3x + y \geq 300$

$x + y \geq 240$

$x \geq 0, y \geq 0$



31. A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A start the game first.

OR

Assume that each born child is equally likely to be a boy or girl. If a family has two children, what is the conditional probability that both are girls given that :

(i) the youngest is a girl ? (ii) atleast one is a girl ?

SECTION-D

This section comprises of long answer type questions (LA) of 5 marks each.

32. (i) Find the values of p and q, for which 3

$$f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & ; \text{ if } x < \dfrac{\pi}{2} \\ p & ; \text{ if } x = \dfrac{\pi}{2} \\ \dfrac{q(1-\sin x)}{(\pi-2x)^2} & ; \text{ if } x > \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$.

(ii) If $x = a\cos\theta$, $y = b\sin\theta$, then find $\dfrac{d^2y}{dx^2}$. 2

33. An open box with a sqaure base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\dfrac{c^3}{6\sqrt{3}}$ cubic units.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

34. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$A^{-1} = \frac{1}{|A|}(\text{adj } M)$

$$3x+4y+7z = 14$$
$$2x-y+3z = 4$$
$$x+2y-3z = 0$$

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

and use it to solve the system of equations :

$$x-y+z = 4$$
$$x-2y-2z = 9$$
$$2x+y+3z = 1$$

35. Solve the following linear programming problem graphically :

Maximize and minimize $z = 5x+10y$

Subject to constraints : $x+2y \le 120,$

$x+y \ge 60,$

$x-2y \ge 0,$

$x \ge 0, y \ge 0$

60

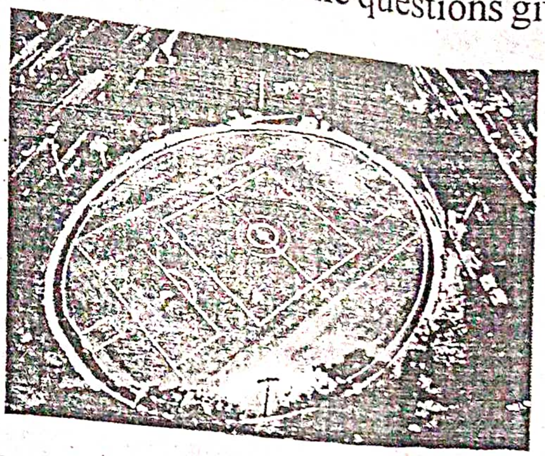
2000 46 100 200

SECTION-E

This section comprises of 3 case study based questions of 4 marks each.

36. Case-Study 1

Read the following passage and answer the questions given below: 1+1+2



In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.

(ii) Find the critical point of the function.

(iii) Use First Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

37. Case-Study 2

1+1+2

A school wants to award its students for the values of honesty, regularity and hard work with a total cash award of ₹ 6000. Three times the award money for hard work added to that given for honesty amounts to ₹ 11000. The award money given for honesty and hard work together is double the one given for regularity.

Based on the above information, answer the following questions:

(i) If ₹ x is awarded to honesty, ₹ y to regularity ₹ z is awarded to hard work, then what is the and matrix equation representing the above situation?

(ii) What is the value of | adj A | ?

(iii) What are the values of x, y, z in this case?

OR

(iii) What is the value of A(adj A) ?

38. Case-Study 3 — 1+1+2

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80. $P(A\cap B) = P(B)$

Let E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based the above information, answer the following on questions :

(i) What is the probability that all the workers are present for the job ?

(ii) What is the probability that construction will be completed on time ?

(iii) What is the probability that many workers are not present given that the construction work is completed on time?

OR

(iii) What is the probability that all workers were present given that the construction job was completed on time?